BOSTON UNIVERSITY
College of Engineering

EK 301	Engineering Mechanics I	Fall 2023

Truss Design Project
SECTIONS: **INTRODUCTION & BUCKLING LAB**
draft dated September 27, 2023

Schedule:

Part 1: Buckling Lab Report	20%	Oct 27 2023; 11:00pm	*to Gradescope*
Part 2: Prelim Design Report	35%	Nov 10 2023; 11:00pm	*to Gradescope*
Part 3: Final Design Report	30%	Dec 8 2023; 5:00pm	*to Gradescope*
Part 4: Truss Testing	15%	Dec 9 2023; or by appt	*Room 113a, 110 Cummington Mall*

Project materials: You will obtain all truss construction materials via distribution from ME Department. For Buckling Lab, you will obtain your bars at your signup time of testing. All other materials below will be at the testing room for your use.

- Acrylic bars (4 ft length; quantity 4)
- Wooden yardstick
- Binding tape
- Scoring knife
- Kitchen scale

Memo from Washington Roebling, Chief Engineer of design and construction of the Brooklyn Bridge, to his assistant:

> There are so many points to be considered, so many conflicting interests to be reconciled on the parts of the truss that it is perfectly bewildering to pick out the best thing. For example, I want to reduce the aggregate weight so as to keep down the pressure on the masonry. I want to simplify the superstructure so as to make work in the shop easy and erection easy and safe, and I also want to keep down the wind surface as much as possible. On the other hand I want the truss sufficiently strong to resist a reasonable amount of bending, and this goes against the other points. But the only possible way in which I can reduce pressure on masonry and wind surface is by reducing the height and weight of the trusses and increasing the strain per square inch on the iron. I do not see that any reduction of weight is possible in any other parts of the structure. By making the truss rods of steel we make some reduction in weight but it is only in the low truss that the rod section is great enough to enable us to attain any appreciable advantage by the substitution of steel for iron. In the high truss with rods through two panels the section is hardly sufficient to make it worthwhile to change. This, therefore, would be one argument in favor again of reducing the weight of the intermediate truss and leaving the rods in all the trusses within one panel. This includes the two central trusses even if they are arranged with a square bar in the middle of two flat ones outside.

Source: McCullough, David G. The Great Bridge: The Epic Story of the Building of the Brooklyn Bridge. New York: Simon and Schuster, 1972. Print.

Contents

1 Overview

1.1 Goal and approach

The overall goal is to design a truss capable of supporting a given load from given materials. The design options are to be guided through engineering analysis. The key assumptions used in our analysis are:

1. The structure is well modeled as a pin-jointed two dimensional truss.
2. The strength of the truss members in tension is practically infinite.[1]
3. The strength of the joints is practically infinite[2]
4. The dominant failure mechanism is buckling of the individual members.

Based on these assumptions, you will create a computer program to predict the performance of candidate truss designs. The program will allow you to quantitatively evaluate the performance of a number of designs without going through the effort (in engineering terms, the expense) of building all of them.

The process leading to a successful design based on computational engineering analysis requires several steps:

- Step 1: *Characterize the materials.* Materials can vary from batch-to-batch. To quantify the buckling behavior and range of uncertainty in our chosen materials, a large selection needs to be measured. This is the purpose of the buckling lab.
- Step 2: *Model development & verification.* Create a computer program that uses assumptions 1-4 above to predict the performance of candidate truss designs and verify that it can correctly analyze a truss.
- Step 3: *Iterative Design.* Brainstorming multiple designs that adhere to a common set of specifications allows you to creatively tackle a common problem and evaluate the pros and cons of various approaches. Your computer program will allow you to quickly evaluate the relative performance of these designs and give us a chance to provide feedback.
- Step 4: *Final design.* Based off feedback on your design, verification of your analysis program, and general knowledge of the performance of your peers' designs, you will have the ability to iterate and optimize a final design. Try a bunch of variations!
- Step 5: *Truss testing.* The performance of your design (and those of all other groups) will be evaluated in a truss testing setting. Since each member of your team will build and test the same design you'll have the unique ability to explore variability in performance when real-world materials are used.

Each of these steps corresponds to a unique and specific part of the overall project design. This is organized to mimic the engineering design process that one might find in practice, e.g. in designing a new plane.

[1] The terminology "practically infinite" is used here in an engineering sense and not a mathematical sense. Nobody believes that the truss members can really sustain an infinite tensile force. Really what we mean is that other failure mechanisms become important long before fracture of the tension members.

[2] idem.

1.2 Specifications

The truss you design must conform to the following specifications.

1. **The truss must be a single, planar, simple truss. No truss member may be designed to extend below the line connecting the two end joints.**

   A simple truss starts with three members and three joints configured in a triangle. New triangles may be added by adding two new members with a single joint between them. Members may not cross each other or be doubled-up, and can only be joined to other members at their ends.

   Make sure your truss is a simple truss by checking first that it is comprised only of triangles. If any quadrilaterals are formed, your truss is incorrect! Second, the number of joints J and members M must be related[3] by

   $$M = 2J - 3.$$

2. **The truss must be designed to support a minimum live[4] load of 32** $oz.$**= 2** $lb.$ **The load must be placed on a joint located at a horizontal distance of 9-12** $in.$ **away from the pin support; note this is a range for this distance.**

   This implies that there must be a joint at that location along the bottom line. You are free to have any other number of joints along that line but you must have at least that one. Feel free to deviate from the classic 'train bridge' design where all the bottom-most joints are horizontal and in-line with each other (e.g. it doesn't have to *look* like the truss shown in Fig. 1, only be supported and loaded as shown).

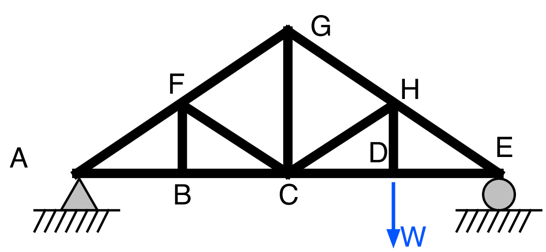

Figure 1: Simply supported two dimensional truss.

3. **The total (virtual) cost of the truss must be less than \$305** where the cost of the truss is defined as

$$\text{Cost} = C_1 J + C_2 L \quad (1)$$
$$\text{where:} \quad C_1 = \$10/\text{joint} \quad (2)$$
$$C_2 = \$1/\,in., \quad (3)$$

   with J = number of joints and L = total length of all members summed together.

Summary of distance and cost specifications

Joint-to-joint span L_{jj}	$8\ in. \leq L_{jj} \leq 16\ in.$
Truss span	33 $in.$
Load to pin support span	9-12 $in.$
Total virtual cost	< \$305

[3] Fun puzzle! Derive this!

[4] The "live load" is the load that is added to the structure. The structure's own weight is called the "dead load".

1.3 Group work

Working in groups is a vital component of most engineering projects. You will encounter group projects that involve scenarios ranging from working with close friends to complete strangers, trustworthy and seasoned engineers who speak your language & have a similar work approach to non-engineers who have no knowledge of the technical jargon and analysis (I know, it's sad but true). Regardless of the setting, your engineering profession demands a responsibility to the challenge presented to you, and to conduct yourself in a professional manner. This responsibility includes integrating yourself with your team.

The EK301 Truss Design Project is no different. You may find yourself working with peers who you've never met prior to EK301, both within and outside of your major. Future engineering classes are likely to have significant group components as well. To assist with achieving optimal group dynamics, your **buckling lab report** should include the two deliverables described below regarding your group effort:

1. Effort-based contract: You should read through the entirety of this project manual to better understand what is required in completing all aspects of the project. Based on this project, your group should write a contract wherein you will briefly describe the following:
    - Strengths/weaknesses: Your particular strengths and weaknesses that pertain to accomplishing a group-based engineering design project. For example, if you feel especially confident in your Matlab/coding skills but less confident in technical writing, write these thoughts down and discuss them with your group.
    - Scheduling issues: You should discuss hard deadlines or extenuating non-academic circumstances that your group should be aware of, for scheduling purposes.
    - Group contract: As a group, you should collectively write a short document that describes what measures you feel comfortable with if someone misses a deadline, and the parameters involved with keeping everyone updated on individual deliverables that you're each responsible for. Note this is something **YOU** write; we do not distribute a downloadable form for this exercise.
2. Case study: You will need to individually read through a short case study on group dynamics ('It Takes Two to Tango'[1]), and discuss the case study with your group setting. You should take meeting notes ("minutes") and especially focus your discussion on the group dynamics discussed in the case study. The case study is posted in the project folder on the course website.
    - The participants, date, time, and place of the meeting should be recorded. The chair of the meeting and the recorder (minute taker) should be identified.
    - The planned agenda should be attached.
    - The important points should be summarized, along with who suggested each of them.
    - The conclusions should be recorded and any action items listed, with the responsible person labeled.

The goal of this effort is have clear communication and a healthy working relationship with each other. If problems arise throughout the course of the project timeline, try to resolve them amongst yourselves and by referencing your group contract. You will individually fill out a peer evaluation for yourself and your group members at the end of the project to let us know things went, and negative feedback may result in a grade adjustment. Don't hesitate to approach your section instructor if problems crop up before then or if you need more guidance.

[1] Oakley B., J. Student Centered Learning, Vol 1 (1), 19-27 (2002)

2 Buckling Lab

This is a "lab" assignment. That means that you will work with your group to do an experiment (described below), and submit a report that describes your results. Sections 2.1, 2.2, 2.3 describe the lab procedure itself. Section 2.5 describes what you need to submit for evaluation.

Lab assignment due date: Oct 27 2023; 11:00pm

2.1 Background

We desire to design, build, and test an acrylic truss structure to meet specified load requirements. The strength of the truss is expected to be limited by the buckling strength of the truss members. The buckling load is expected to depend on the length of the member. The purpose of this lab, therefore, is to measure the buckling strength of acrylic strips as a function of their length.

Definition: Critical Buckling Load We wish to determine P_{crit}, which represents the critical buckling load for an acrylic strip, which is loaded in pure axial compression, and simply supported at its ends. P_{crit} depends on the material properties of the strip, its cross sectional dimensions, and its length. We will attempt to keep all those parameters the same from experiment to experiment except the length.

"Buckling" is a geometric instability which is characterized by the possibility that the given structure can support the applied load in more than one equilibrium configuration. When this happens, the desired configuration (the desired shape of the structure - in this case a straight strip) is usually unstable, while the undesired configuration (or shape - in this case a bent or wavy strip) is nominally stable. For pure axial loads above P_{crit}, the straight configuration becomes unstable and the structure can suddenly (and catastrophically) change to the collapsed or buckled (wavy) configuration. For pure axial loads below P_{crit}, the straight configuration is the only equilibrium configuration and it is stable.

2.2 Methods

In brief, each group member will cut 3 acrylic strips, each to a different length. You will measure the buckling load of each sample using a scale and recording the reading when the strip buckles. Results are combined with those of other groups and fit to determine the buckling load as a function of length. The specific instructions below should be followed in your lab session by your group.

2.2.1 Sample preparation

As a group, choose 3 lengths L_1, L_2, and L_3 within the bounds of:

$$8\,in. \leq L_j \leq 16\,in. \tag{4}$$

Prepare one sample per length. You should use *part* of one of the **acrylic bars**, **the scoring knife**, and the **scale** for this step. Return the remaining stock to the GSTs.

To cut the sample, put the strip flat on the lab bench. If possible, put something (e.g. a piece of cardboard or a cutting board) between the strip and the table, though, because the knife is likely to slip and would otherwise mar the desk surface. Carefully hold the strip still and away from the cut. Draw your scoring knife across the strip where you would like the cut to be, to make a deep scratch on the surface. This scratch is the start of a crack. If it's deep enough, then when you try to break the acrylic at that place, it will break by extending this initial crack through the piece.

Once each sample is cut to the desired length, precondition the sample by gently compressing it until it bends slightly a few times. The goal is to "break in" the sample so that it is more repeatable for its buckling load measurements.

2.2.2 Buckling load measurement

Test each sample nine times, giving a total of 27 measurements for your group. Note that the video shows measurements in grams, but you should work with units of ounces (oz.) throughout the project. **Be sure to set your scale to units of oz.**

Because the measurement is dynamic and the scale reading will fluctuate, we recommend that you use a phone to make a video recording of the scale reading as you load the beam. Making this recording isn't a requirement but you might find it helpful for getting an accurate reading of the maximum value when the sample buckles.

- Place the scale (note the model of your scales) on a flat work surface.
- Zero the scale by pressing the "tare" button.
- Place the strip on the scale and record its weight.
- Zero the scale again with the strip on the scale.
- Remove the strip from the scale and reposition it vertically, with the top gently supported between two fingers to prevent the strip from falling over. Verify that the scale reading is zero in this configuration!
- Touching only its top edge, compress the strip vertically downward against the scale. The scale should read the total force you've applied to the top of the strip.
- Slowly increase the applied force until the strip starts to bow.
- You should find that the scale readings plateau (max) at or near a single value when the sample first bows (buckles). This is one criterion for buckling. Record this plateau reading as the buckling load for the tested sample.

In cases where a single value of the plateau is not clear, you should record the narrowest range within which the buckling load lies.

2.3 Results

2.3.1 Data

Record your results in a table; we recommend using the format below.

	Length 1 (in.)		Length 2 (in.)		Length 3 (in.)	
nominal length:						
trial #	Wgt (oz)	P_{meas} (oz)	Wgt (oz)	P_{meas} (oz)	Wgt (oz)	P_{meas} (oz)
trial 1						
trial 2						
trial 3						
trial 4						
trial 5						
trial 6						
trial 7						
trial 8						
trial 9						
Avg						
range						

The range represents your best estimate of the *smallest* range within which you are pretty confident[5] that the true value of P_{crit} lies for your samples.

2.3.2 Plot and functional fit

Functional fitting: background. The critical buckling load P_{crit} varies with the length of the strip. In order to estimate P_{crit} for strip lengths that were not measured, it is useful to have an approximation for $P_{crit}(L)$. Here we will base $P_{crit}(L)$ by fitting the measured values P_{meas} across the several lengths that were obtained.

We either derive or guess a functional form for $P_{crit}(L)$ with some unspecified coefficients in it, and then we evaluate the coefficients so that our assumed form is consistent with our measurements. The most popular form

$$P_{crit}(L) = A\,L + B. \tag{5}$$

assumes that P_{crit} is a straight line with slope A and $y-$intercept B. Another popular form

$$P_{crit}(L) = C\,L^p. \tag{6}$$

assumes that P_{crit} is proportional to a power of L. Note that (6) is the same as assuming (5) for $\log(P_{crit})$. Both methods require identifying two parameters (i.e. A and B, or C and p) so that the curves approximately represent the measured data.

[5] Here we use "pretty confident" to avoid more precise statistical computations. An acceptable level of confidence in reported measurements will vary from field to field. In some fields, it is as low as 95%, while in others it can be 99% or even higher.

Functional fitting: Steps.

1. Plot P_{crit} versus L:[6] Plot the average value of P_{crit} for each strip length. Some plot tips:
   - Choose the axes so that the data points use the most of the available graph space. For example, if your measured lengths are 12 *in.*, 15 *in.*, and 16 *in.*, then a range for L of 11 *in.* $< L <$ 17 *in.* is a better choice than 0 *in.* $< L <$ 40 *in.*.
   - Axis labels must include units!
   - In your report, be sure to include appropriate axis labels, a legend, a figure number, and a descriptive caption.
   - Fig. 2 shows an example of an acceptable plot, albeit from a completely different application.

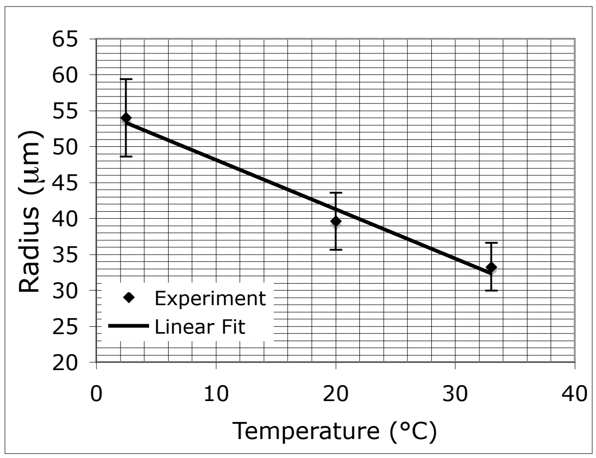

Figure 2: Example of an acceptable plot: Maximum expansion radius vs. temperature for a single bubble sonoluminescence experiment at 23.1 kHz. (Courtesy of Prof. R.G. Holt.)

2. Add error bars: Above and below the average values plotted above, add vertical error bars that indicate your confidence range in P_{crit}. You can see such error bars on the example plot in Fig. 2.

3. Fit a line to the data by drawing the line that visually minimizes the sum of the vertical distances between the points and the line. Determine the equation of this line by finding the slope and intercept. Report these values. Note that this line represents the "best estimate" of the buckling strength of any length of strip, given the data that you collected *and* the assumed form of P_{crit} given in (5).

4. Estimate the error of the fit by determining the average vertical distance from your data points to the line. This error will have units of ounces. In your report, discuss whether the error of the fit is small or large relative to the error bars. Discuss in your report whether it is more appropriate to use the error of the fit or the range of any single measurement when determining the failure load of your truss.

[6] The conventional way to plot a versus b, is with a on the $y-$axis, and b on the $x-$axis. That is, plots are typically y versus x.

2.4 FYI: What the Graduate Student Teachers (GSTs) (not you!) will do

2.4.1 Data analysis

For your reference, a detailed discussion on a curve fit analysis for the data and buckling strength is included in Appendix A. The GSTs will conduct an analysis of the full data set that was generated by the entire class based off this comprehensive curve fit analysis. They will use this large data set to fit the data to both the theoretical model (allowing the Young's modulus to be varied to find the best fit) and a semi-empirical model (allowing both the constant A and the exponent α to be varied to find the best fit). This information will be posted to Blackboard, with an indication of the best fit and its values.

Note that the GSTs' result will be more accurate than yours since it is based on a larger data set and a better model. Therefore, despite all your hard work in doing your own analysis and generating a linear model, for your truss design and failure analysis, **you should use the curve fit produced by the GSTs in all the analyses of your truss designs**.

2.4.2 Uncertainty analysis

Experimental expectation and historical data both agree that the curve fit will not perfectly describe the data. The GSTs will apply an uncertainty analysis that will account for the scatter in the data, resulting in a length-dependent uncertainty in the critical buckling load. This uncertainty is important to recognize and incorporate in your design analysis, since it will allow you to determine a range of loads that your truss can reasonably be expected to hold before collapse. Pay close attention to the uncertainty results that the GSTs report after they analyze the data. You'll need these numbers when you design your truss.

2.5 Assignment: The lab report sections

This section describes what you need to turn in for the lab assignment, i.e. the "deliverables." Submit one report per group. Lab reports follow a standard format which we've adapted *this* project into the section headings below.

Lab assignment due date: Oct 27 2023; 11:00pm

Submit lab report location: *to Gradescope*

- **Title page:** Include all student names, section number, semester, and lab title.
- **Background:** For this section, you may copy the first paragraph from section 2.1.
- **Methods & Data:** The methods section of a lab report describes both the idea behind a measurement and also how you made the measurement. Describe the procedures you used in your own words in a way that a new classmate without access to this handout would be able to duplicate your steps.
- **Analysis:** Include a *Free Body Diagram* of the loaded acrylic strip. The FBD should indicate:
  - *The applied load* (i.e. the force applied by your hand to the top edge).
  - *The gravity load.*
  - *The reaction force* from the scale acting on the strip.

  The reading on the scale is a force, F_{scale}. Give an equation for F_{scale} in terms of the forces appearing in your *Free Body Diagram*. Explain the equation.
- **Results:** Show the data table and plot described in section 2.3.
- **Discussion:** Write *one short* paragraph addressing each of the following questions:
  - How did you determine the range estimate for your measurements?
  - Compare the measurements made by everyone in your group; remark on consistencies and inconsistencies. Why (in retrospect) should we expect those? (For example, were the lengths all exactly the same? Given what you know about the weight per unit length of the strips, how big a length difference would show up as a weight difference? Can you infer anything about consistency of width or thickness in the samples?)
  - In your fit of P_{crit}, compare the mismatch between the measurements and the curve to the estimated range of P_{crit} at a single length. Discuss which, if either, is the more appropriate uncertainty to consider in the design of your truss.
- **Conclusion:** Write *one very short* paragraph that summarizes your conclusions about your estimate of the buckling load of the strips and how it depends on length, and your quantitative estimate of the accuracy of your buckling load measurements.
- **Appendix:** The appendix should include:
  - The minutes of your case study discussion.
  - A copy of your group contract.
  - Other additional information you feel is pertinent (e.g. calculations, etc).

A About fitting a curve to data

You and your classmates have collected a large set of data that determines experimentally the dependence of beam buckling strength on beam length. There are several different techniques for fitting a curve to a data set, depending on how much information and knowledge you have about the system you were testing. Below we discuss three approaches. The first, an empirical fit, is the one **you** will use in your analysis. The second and third fit types will be investigated by GSTs, where the second is a fit to a functional form that is based on a theoretical model of buckling beam. The final is a semi-empirical fit, that relaxes the theoretical relationship to allow for the possibility of a different fit.

A.1 Empirical fitting

This method is perhaps the most straightforward to apply. It is not, however, based on a physical theory that relates buckling strength and beam length. This method relies on the belief that a physical relationship does exist and that the data will reveal this fit. One method to display this relationship is to use a polynomial function given by

$$W(l) = a_0 + a_1 l + a_2 l^2 + a_3 l^3 + \cdots + a_m l^m$$

where m must be smaller than the number of data points in your data set.

Since you have only three data points to work with, it is most appropriate to select a *linear* fit, so ignore the nonlinear terms and fit the data with the linear fit equation.

A.2 Fit to a theoretical curve

Since the GSTs will have the entire set of class data to analyze, they have a basis for investigating a fit that better matches buckling theory. A treatment of the beam as a long, thin, and massless rod yields the relationship for the buckling strength W as a function of the beam length l to be

$$W(l) = \frac{\pi^2 EI}{l^2}.$$

Here E is a material property known as Young's modulus, I is the moment of inertia, and l is the length. The moment of inertia and the length are easily determined from the beam geometry but E is not easily found. It can therefore serve as an adjustable parameter to find the best fit.

A.3 Semi-empirical fit

The equation relating the buckling strength to the beam length given above is theoretically justified. However, the beams may not be kind enough to obey our theoretical derivation (perhaps due to material imperfections, experimental error, invalid modeling assumptions, etc). A better fit may be achieved by selecting a function motivated by the theory but which can be adjusted based on the measured data. For example, recognizing that the theory predicts a buckling strength inversely proportional to some power of the length, we could define

$$W(L) = \frac{A}{L^\alpha}$$

where both A and α are to be chosen to get the best fit to the data.

B Guidelines on writing reports

For all reports, proper format and proper English usage are required. All reports must be clear and user friendly. Each deliverable should be interpreted as one per design group. Be sure to include your section number on the title page, along with your team and member names. All of your reports for this course must be at least two pages long, though you should find that they will be longer than that.

General guidelines to follow include the following.

1. Include all experimental data you collect.
2. Show all equations used and define all variables.
3. Typeset your document for maximum readability. The only exceptions are handwritten meeting notes or hand-drawn schematics.
4. Be consistent with your variable names.
5. Number your plots, figures, and equations throughout the text.
6. Cite all references
7. Proofread for typos, spelling, grammar, formatting, and general writing flow. This is especially important when multiple authors are involved.

A comprehensive report should include the following:

1. Cover page

   This should contain the title of the report, the name of the group, the names of the members of the group, the section, and the name of the professor of the section.

2. Introduction

   This explains the purpose of the report/lab/testing that was done. Discuss what you intended to learn from the experiment/test/analysis/design and how you plan on using what you have learned.

3. Procedure (if applicable)

   This describes what you did to gather your data. You should describe any experimental apparatus in detail.

4. Data

   Present the data in tabular form. Do not forget to include units! Be sure to include a brief description that orients the reader to the source of the data.

5. Analysis

   Describe the analysis that you performed on the data. Show all equations with all symbols defined. Show an example for each unique calculation (e.g. sum of moments, standard deviation, mean buckling load, etc.). Refer to any figures (such as FBDs), data tables, or tabular presentations of your results as needed.

6. Results

   Present the results of your analyses in tabular and/or graphical form in a way that is easy for the reader to follow and interpret. Label and reference each table and graph.

7. Discussion & Conclusion

   Answer any questions that you have been asked to answer in the lab/project manual. Discuss your results. Do they make physical sense? Do they help you accomplish the goals you set out in your introduction? Discuss the uncertainty in your data and the sources of error. What did you learn? How will the results be used? What might you suggest doing differently next time?

8. Appendices

   Attach any hand calculations, computational code, extra data tables, alternative designs you considered but didn't use, and other relevant information that are important but did not make their way into the main report. Be sure to reference your appendices at some relevant point in your report. (Notice, for example, that this appendix was referenced earlier in this document.)

C Rubrics

C.1 Buckling Lab Rubric

Buckling Report Rubric	
Report Section	**Points**
Title Page	2
Background	2
Methods	5
Analysis	15
Results: Data table	15
Results: Plot	15
Discussion	20
Conclusion	5
Appendix	10
Quality of Writing and Presentation	11
Total	100

C.2 Preliminary Design Report Rubric

Preliminary Design Report Rubric	
Report Section	**Points**
Title Page	2
Introduction	5
Theory	5
Data and Design	
Designs: Diagrams	6
Designs: Cost and specifications	12
Analysis	
Input file: submitted & correct	5
Model verification	11
Designs: Max load calculation	18
Results	7
Discussion	20
Quality of Writing and Presentation	9
Total	100

C.3 Final Design Report Rubric

Final Design Report Rubric	
Report Section	**Points**
Title Page	2
Introduction	5
Procedure	5
Analysis	5
Results	
Final design diagram	7
Final design table	20
Summary	10
Discussion	20
Appendix	10
Quality of Writing and Presentation	16
Total	100