EEE4118F - Controls Lab Cascade Control Design Report

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Contents

Abstract

This report features the design of a robust digital cascade controller system for the output potentiometer of a laboratory servo system. The design process involved three crucial stages: system identification, cascade controller design, and controller implementation. The system identification involved a series of tests to determine the following system's characteristics, such as the open loop gain ranged between 6.73 and 37.6, the time constant ranged from 0.08 to 0.46, and a constant gear reduction ratio of 9.29. The nominal transfer function of the was derived as $TF = \frac{9.8}{0.1s+1}$. The cascade controller design involved plotting the sensitivity bode plot, obtaining low and high frequency bounds specifications, and obtaining the speed and position controllers using the QFT design tool. The system was tested for all required specifications under all possible loading cases through simulations. The controllers were then implemented, which showed some steady-state error, which was corrected by increasing the gain of the position controller. The system was shown to be robust and meeting all specifications besides the set point tracking which remained slightly off. The speed and position controllers were defined as $G_s(z) = 0.1$ and $G_p(z) = 0.33$ respectively.

I. Introduction and Model Summary

This technical report details the development of a robust digital cascade controller system, engineered to analyze and rectify the output of a servo motor system in order to track the setpoint. The design process features three critical stages, namely system identification, cascade controller design, and controller implementation. Additionally, a comprehensive requirement analysis is featured to derive the design specifications from the client's requirements. The system identification aims in determining the key characteristics of the system, including the open loop gain "A", the time constant "τ", and the gear reduction ratio "k". Subsequently, the cascade controller design phase aims in utilizing these characteristics, alongside the requirement analysis, to develop both a speed and position controllers and test them through simulations. Finally, the implementation phase aims for the physical implementation of the controller system on the servo motor and the final testing of its performance.

The overall system model, illustrated in Figure 1 below, comprises the plant, as well as the position and speed controllers that have been implemented. The input of the system is a unit step, expressed in voltage [V]. The system features two negative feedback loops that are cascaded. The first feedback loop consists of the position controller, a second feedback loop, an amplifier for the gear reduction ratio, and an integrator block.

The position controller is implemented digitally and discretely in the z-domain, and its role is to regulate and manipulate the position of the system to achieve the desired output. The position controller receives information on the current state of the system through a feedback loop, and then adjusts the position of the overall system using that data.

The second feedback loop comprises of a DAQ and a motor, with a velocity controller in the feedback branch. The DAQ allows for the conversion from digital to analog signals and features a zero-order hold which converts continuous-time signals into a discrete-time signals. The DAQ block also features a saturation block, which limits the input voltage in the system to be between -10V and 10V. The DAQ is directly connected to the servo motor system which is further discussed in the system identification. It is represented as a transfer function, which contains all the essential characteristics of the plant. The speed controller in the feedback branch is also implemented digitally and discretely in the z-domain, and its purpose is to adjust the current velocity state of the motor to the desired velocity. This second feedback loop is then connected to a gain amplifier block, which holds the constant value of the gear reduction ratio. This ratio is crucial to the system, as it ensures that the system responds to controller inputs appropriately.

The output of the second feedback loop is multiplied by the amplifier block and passed onto an integrator block. The integrator block reduces set point tracking errors in order to achieve zero steady-state error. Finally, a saturation block is featured at the end of the system to limit velocity and position outputs to be between -10V and 10V. The final output of the overall system is given in voltage but it represents the angle of rotation of output potentiometer.

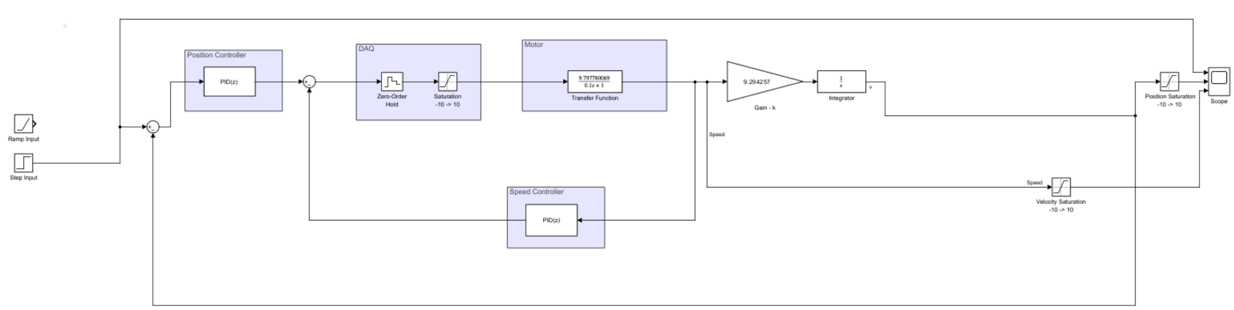

Fig. 1: Block diagram showing the overall system

II. Requirement Analysis

A. Requirements from the client

The client's requirements were highly specific, including crucial aspects such as set point tracking, over/undershoot, settling time, output disturbance rejection and stability check during plant alterations. Additionally, penalties would be awarded for the type of controller used and the controller output u(t).

Set point tracking refers to the precision with which the control system can accurately follow the desired setpoint, where the setpoint corresponds to the desired output of the system. The requirement for this is that there must be a zero steady state error.

The overshoot of the system, expressed as a percentage, signifies the amount by which the output exceeds the setpoint before ultimately settling down and achieving steady state. The requirement from the client stipulates that the overshoot should not exceed 20% of the final value.

The settling time of a system refers to the time required for the system to reach steady state, which represents how fast the system reacts to a change in input. The requirement for the settling time is that is must remain below 1 second.

Output disturbance rejection refers to how long the controller takes to recover from external disturbances and return to the previous final value. This aspect is an essential part of the system's performance and the recovery time be within 1 second.

Regarding the potential penalties, the use of an On/Off Bang-Bang Controller as a controller type is not acceptable. Furthermore, the controller output u(t) must have minimal high amplitude oscillations, not exceeding 0.4V.

B. Specifications derived from the requirements

The set point tracking specification was that zero steady state error must be achieved. In control design, this specification is achieved through the implementation of an integrator, $\frac{1}{s}$.

Notably, the overshoot specification of being less than 20% applies for the entire system. Hence, both position and velocity overshoot can be calculated utilizing the same equation:

Over/Undershoot - Speed

WITHIN 20%

$$M_{pv} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$
$$M_{pv} = 0.2$$
$$\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = -\ln(0.2)$$
$$\zeta^2\pi^2 = \ln(0.2)^2\,(1-\zeta^2)$$
$$\zeta^2 = \frac{\ln(0.2)^2}{\pi^2+\ln(0.2)^2}$$
$$\zeta = \sqrt{\frac{\ln(0.2)^2}{\pi^2+\ln(0.2)^2}} = 0.456$$

Over/Undershoot – Position

WITHIN 20%

$$M_{pp} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$
$$M_{pp} = 0.2$$
$$\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = -\ln(0.2)$$
$$\zeta^2\pi^2 = \ln(0.2)^2\,(1-\zeta^2)$$
$$\zeta^2 = \frac{\ln(0.2)^2}{\pi^2+\ln(0.2)^2}$$
$$\zeta = \sqrt{\frac{\ln(0.2)^2}{\pi^2+\ln(0.2)^2}} = 0.456$$

The damping factor for both the velocity and position controllers has therefore been established at ζ= 0.456 in order to obtain less than 20% overshoot.

The settling time for the system must be within 1 second. This requirement is reliant on the natural frequency, ω_n, which was determined using the specification:

Settling Time

WITHIN 1S – NO OSCILLATIONS

POSITION

$$t_{2\%} = \frac{\ln\left(\frac{2}{100}\sqrt{1-\zeta^2}\right)}{-\zeta\omega_n}$$
$$\therefore \omega_n = \frac{\ln\left(\frac{2}{100}\sqrt{1-\zeta^2}\right)}{-\zeta t_{2\%}} = 8.836\,rad$$

An additional specification was further incorporated to ensure the adequacy and precision of the QFT design. The margin for digital design was set to be ϕ_d = 15°and the unsaturated step size was specified to be y_u = 10°.

From these the final specifications were obtained for the frequency response:

i. Zero Steady State error

ii. $|\frac{1}{1+L}| \leq -20dB$ when $\omega \leq 0.95$ rad

iii. $|\frac{1}{1+L}| \leq 3.7dB \; \forall \; \omega$

Specification *ii* and *iii* were determined during the cascade controller design using the sensitivity bode.

III. System Identification

The system identification was a pivotal aspect of the design that enabled the identification of the necessary initial conditions and parameters. The accuracy and adequacy of the desired controllers were entirely due to a precise system identification. The characteristics of the plant's transfer function were determined via a set of tests conducted on the laboratory's servo system while it was operating under different conditions, such as an increase in load or output disturbances. The critical parameters that need to be identified were denoted by "A", the open loop gain of the plant, "τ" the time constant, and "k" the gear reduction ratio.

A. Experimental Procedure

The servo system was made up of several components connected together. The system featured an Operation-Amplifier circuit which was connected to a power amplifier and a D.C motor. Additionally, a Tachometer was used to show the speed of the system and the output was determined using a potentiometer. A magnet be could used to load the D.C motor, and allowed for the determination of the plant's characteristics under different conditions. There were three possible cases which included when there was no load (the magnet was not affecting the motor), partial load (the magnet partially affected the D.C motor) and maximum load (the magnet exerted a full load on the D.C motor). Finally, the DAQ, was located just before the servo system which facilitated the conversion from digital to analog signals. Situated on left side of the servo system was a computer which acted as the system's controller and was linked via the DAQ.

A Csharp simulation software was utilized for the recording of the data and the implementation of the controller. The software also enabled the testing of various voltage [V] inputs in the form of a unit step or a ramp input. The system output was the angle of the motor and corresponded to a voltage value derived from the potentiometer. Multiple tests were conducted using the simulation software which enabled the determination of the required parameters.

A first test was performed using a ramp input and no load onto the D.C motor. This test allowed the determination of the system's saturation point and the range of input voltage values leading to it. This test was crucial as it allowed for the identification of a suitable unit step input that does not result in the saturation of the system but still induced the motor to move. As a result, this test was repeated numerous times until an adequate unit step input voltage was obtained.

The second set of test involved the input of a unit step. This test recorded the speed of the motor as well as its position in reaction to the input. The setpoint was also being recorded, and the test was repeated for the three possible cases of the motor mentioned above, i.e., no load, partial load, and maximum load. This data was logged and put into a ".csv" file. The collected data was then analyzed using control theory allowing for the determination of the required parameters at different loads. This allowed for a range of values for "A" the open loop gain and "τ" the time constant to be derived. The gear reduction ratio "k", could be determined using the slope of the system output (dV/dt) and dividing it by the motor velocity.

Finally, the servo system's behavior was simulated on MATLAB Simulink to confirm the accuracy and adequacy of the parameters gathered in the system identification tests. The structure used for the simulations is depicted in Figure 2 below.

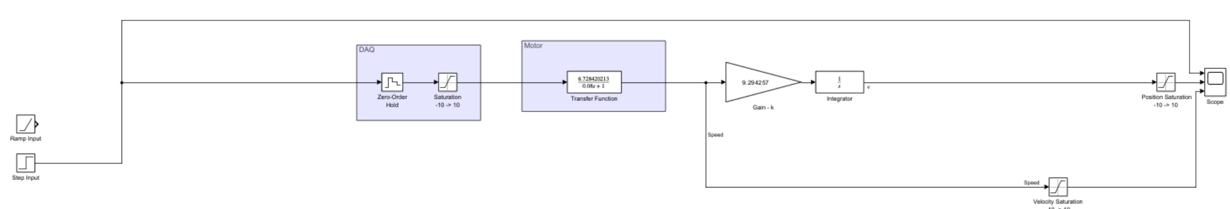

Fig. 2: Block diagram of the Simulink structure used for the simulations

B. Data and Results

From the ramp test, the unit step input was gathered. This was a step of 0.25V, from 0.3 to 0.55V.

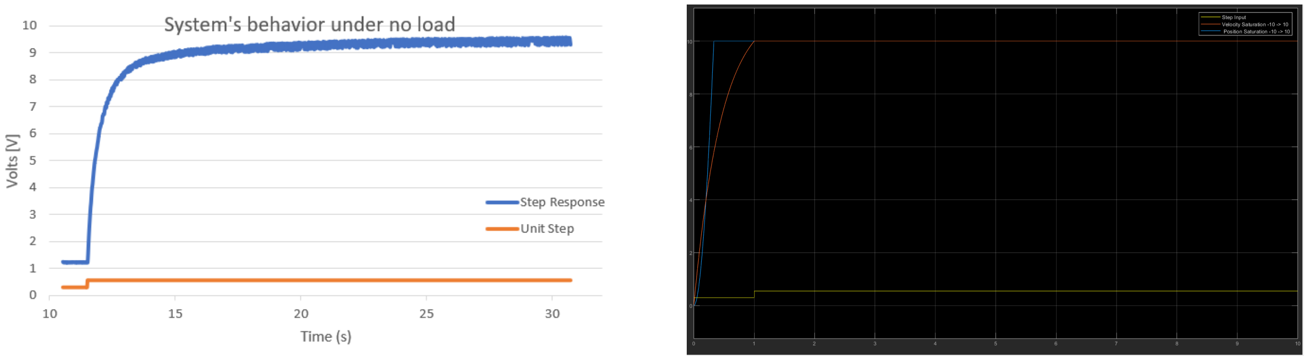

Fig. 3: Graphs showing the data gathered and the simulations of the relationship between the motor velocity and time when no load is applied

From the data, the gain A and the time constant τ is gathered:

$A = 37.61322156$

$\tau = 0.46s$

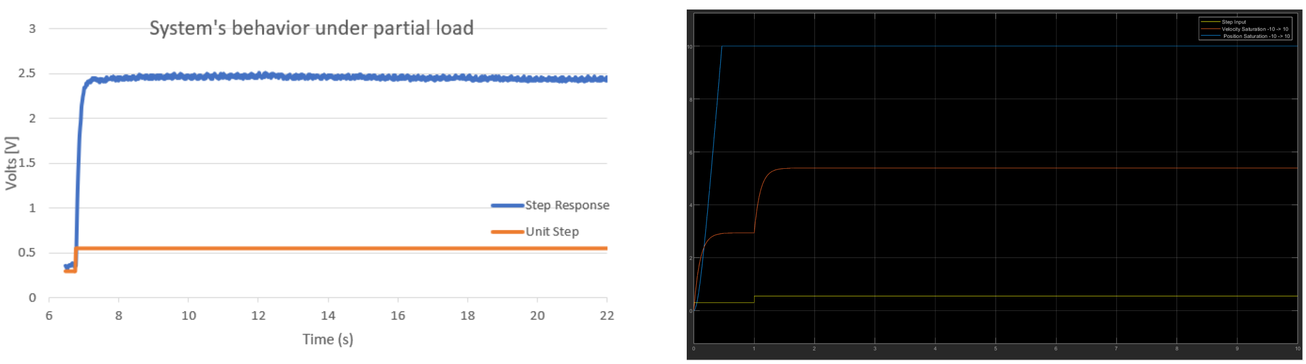

Fig. 4: Graphs showing the data gathered and the simulations of the relationship between the motor velocity and time when partial load is applied

From the data, the gain A and the time constant τ is gathered:

$A = 9.797780069$

$\tau = 0.1s$

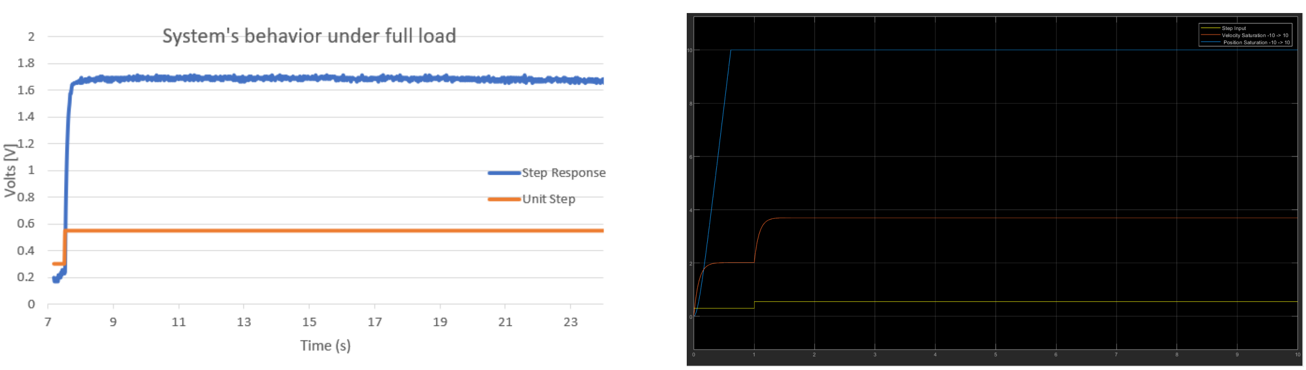

Fig. 5: Graphs showing the data gathered and the simulations of the relationship between the motor velocity and time when full load is applied

From the data, the gain A and the time constant τ is gathered:

$A = 6.728420213$

$\tau = 0.08s$

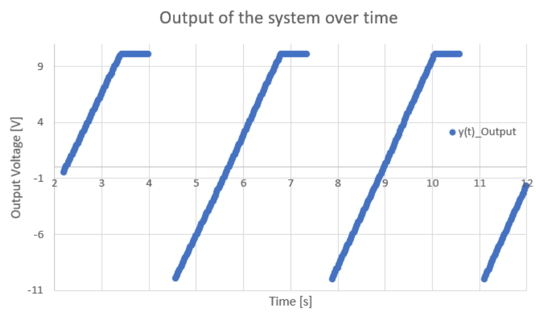

Fig. 6: Graph showing the output of the system over time

From the data, the gear reduction ratio k was determined using the slope and dividing it by the motor velocity: $k = 9.294257$

C. Discussion and Analysis of the results

System Identification was conducted on the servo motor system to construct a robust digital position controller. This involved determining the open-loop gain "A" and time constant "τ" to construct the transfer function of the plant. In addition, the gear reduction ratio "k" was calculated for the system using the output slope. The values for "A" and "τ" were obtained through three unit step tests, each with varying load amounts.

The range of values for "A" was measured and calculated to be between 6.728420213 and 37.61322156, while the values for "τ" ranged from 0.08 to 0.46. The gear reduction ratio, "k", was determined to be a constant value of 9.294257 which is independent of the load variations.

Additionally, the accuracy and adequacy of these values was verified by incorporating them into the MATLAB Simulink system, as illustrated in figures 3, 4 and 5. The step response from the gathered data and simulations was consistent for all load scenarios. These values were then utilized to determine the nominal plant, allowing for the implementation of the cascade controller design. The final transfer function of the plant from the system identification is considered as the "nominal plant" and is presented below:

$TF = \frac{9.797780069}{0.1s+1}$

IV. Controller Design using Simulations

The cascade controller design was the major part of the entire design. The aim of this part of the design is to determine transfer functions of both controllers that meet all the specifications. It was crucial that the simulations are done accurately and that the transfer functions of each controller are determined precisely. This part is critical as it will determine the success of the implementation. All the required specifications must be met in the simulations for the controller to be implemented correctly.

The cascade controller design represented a fundamental aspect of the overall system. Its primary task was to determine transfer functions for both controllers that meet all the required specifications. The accuracy of simulations and precision in the determination of each controller's transfer function were crucial, as it ultimately dictated the success of the controller implementation. Meeting all the specifications within simulations was a critical part, as it ensured that the controllers were correct theoretically (in an ideal scenario).

A. Experimental procedure

Firstly, setting the bounds of the system was a critical step which requires the careful consideration of the system's specifications. In order to achieve this, the optimal plant transfer function was determined by analyzing the specifications derived in the requirement analysis. Various parameters such as "ζ" the damping factor, "A" the open loop gain and "ω_n the natural frequency were used to generate a sensitivity bode plot. This plot allowed for the determination of the low and high-frequency bounds specifications. The low frequency bound was defined as the frequency associated to a close-loop magnitude value of -20dB. The robust stability margin was defined by analyzing the peak of the bode plot (highest magnitude value in dB). The low and high frequency bounds were then implemented into a QFT design tool in MATLAB, which enabled the design of the position and velocity controllers using an Inverse Nichols Chart (INC).

When implementing QFT designs for controllers, the selection of a nominal plant was a critical step. This plant is obtained from system identification and serves as a reference model for the system's behavior without the controllers. In this case, the nominal plant transfer function was defined by the equation $TF = \frac{9.797780069}{0.1s+1}$. The time constant of the system, denoted by "τ", was already sufficiently low, modifications for system's speed were generally unnecessary, as the system was already inherently fast. The speed controller was set to be a proportional controller in with a gain value of less than 1 allowing the increase of the system's speed.

The next step involved simplifying the second feedback loop into a single transfer function. This new block was then multiplied by the gear reduction ratio block and the integrator block, resulting in the entire system's simplification, except for the position controller. The remaining elements were just the position controller and the new simplified transfer function. The QFT design tool was utilized to determine the transfer function for the position controller while still ensuring that it satisfied the bounds specifications.

Finally, the entire system with the controllers was tested by simulating it on MATLAB Simulink. Both controllers were implemented into the Simulink model and the step responses were analyzed to determine if they were accurate and adequate enough and that they met all the specifications. All three possible cases were tested through their different plant characteristics defined by their transfer function. The steady state tracking, the overshoot as well as the settling time specifications were tested.

B. Data and Results

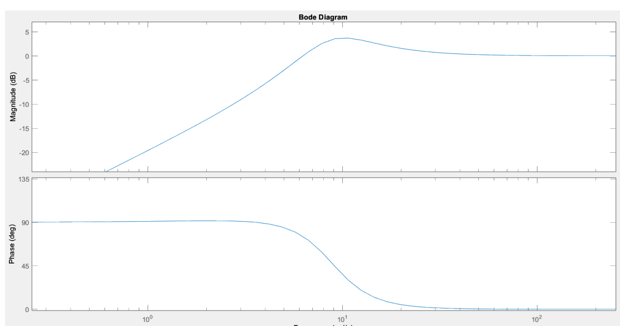

Fig. 7: Sensitivity Bode plot of the system, allowing for the determination of the low and high frequency bounds.

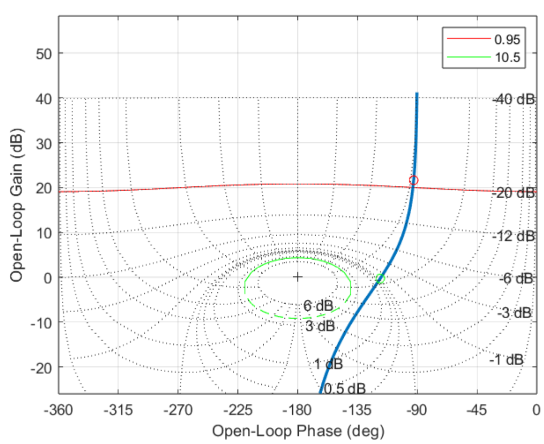

Fig. 8: QFT design tool showing the implementation of the position controller and the effect it has on the system relative to the bounds.

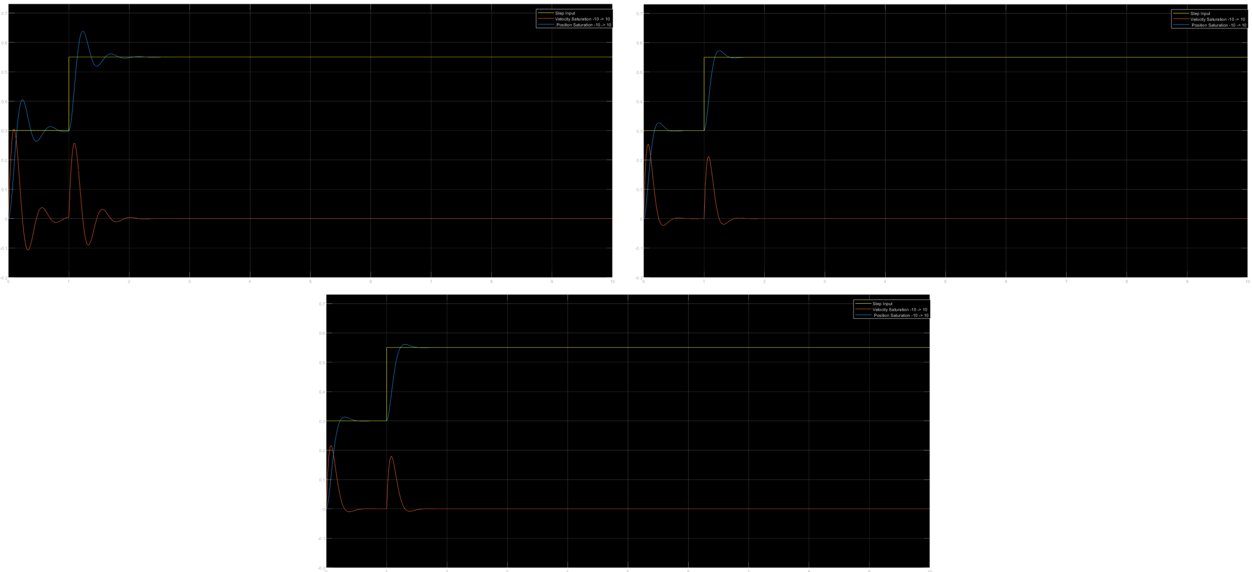

Fig. 9: Graphs showing the simulations of the relationship between the position, motor velocity and time for all three load cases.

C. Discussion and Analysis of the results

The sensitivity bode plot was utilized to identify the sensitivity bounds specifications. Specifically, the frequency associated with a magnitude value of -20dB was found to be 0.95 rad/s, revealing the low frequency bounds of the system. Additionally, the peak of the magnitude bode plot was at 3.7dB, indicating a robust stability margin of -3.7dB. The velocity proportional controller transfer function was determined to be $G_s(z) = 0.1$. The low and high frequency specifications were then implemented into the QFT design tool, resulting in the determination of the position controller transfer function, $G_p(z) = 0.25$. It is worth noting that, the cascade controller design produced two proportional controllers only featuring a gain value. Subsequently, the controllers were integrated back into the Simulink model, as depicted in Figure 1, and the overall system was simulated and tested under all possible loading scenarios. As illustrated in Figure 9, the step responses of each cases met all the required specifications.

V. Controller Implementation

From the cascade controller design, the essential tasks of determining the speed and position controllers were achieved. However, the efficiency and performance of these controllers can only be accurately evaluated via testing using the physical servo system. This required the physical implementation of the controllers, which was performed through the conversion of both controller transfer functions into differential equations before they could be integrated into the CSharp software. Once implemented, the behavior of the entire system was evaluated and various tests were performed to confirm the performance and adequacy of the controllers. These tests assessed each specification, and the controllers were adjusted in order to achieve maximum performance.

A. Experimental Procedure

The initial step in the implementation process involved converting the transfer function into the controller algorithm. However, this task was relatively trivial given that the controllers acquired were gain controllers. As a result, the gain values could easily be integrated into the software, and subsequently, the acceptance tests could be conducted to verify if the system meets the required specifications. The controller gains were adapted based on the behavior of the system to satisfy specific specifications. Finally, a comparison between of the final step response of the system and the step response of the simulated model was performed allowing for the final evaluation of the controllers.

B. Data and results

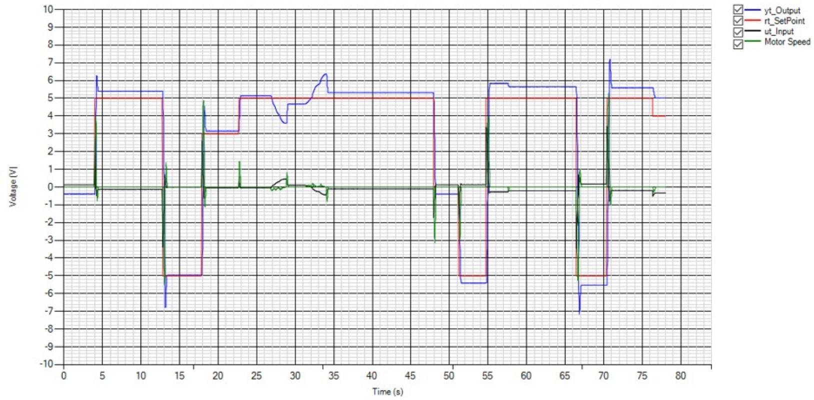

Fig. 10: Graph showing the final system with the controllers implemented and each specification tests ran

C. Discussion and Analysis of the results

The plot depicted in Figure 10 demonstrates the system's performance across a series of tests. Each specification was tested at different intervals, by analyzing the system output and comparing it to the desired setpoint, represented by the red line. The blue line represented the output of the system with the controllers implemented.

During the interval of 4s to 13s, the system's steady state tracking, settling time, and overshoot were tested using a positive unit step input in volts [V]. Initially, the steady state tracking had errors and adequate tracking of the setpoint was not achieved. To correct this, the position controller's gain was meticulously increased to achieve zero steady state error. However, too much gain led to oscillations, so the correct gain value had to be identified. Ultimately, the position controller transfer function was adjusted to a value of $G_p(z) = 0.33$, resulting in improved steady state tracking. While zero steady state tracking was not achieved, the tracking error was within a margin of 0.4V relative to the setpoint. Using the plot, the settling time was estimated to be approximately 0.3 seconds, indicating a very fast system that met the settling time specification. The overshoot specification was also met, with the system's overshoot measured at 16%.

Between 13s and 18s, the system's response to a negative unit step input in volts [v] was tested, which it passed.

From 18s to 23s, a lower unit step voltage was applied to ensure that the system continued to perform adequately and met the required specifications, which it did successfully.

From 23s to 48s, the system was tested for its ability to handle output disturbances caused by rotating the motor axle. The time taken for the system to recover and return to tracking after the disruption was found to be 0.1-0.2 seconds which met the specification that it must be within 1 second.

The system's response to changes in plant load was tested between 43s and 67s. A magnet was placed on the motor, introducing an additional load and affecting the system's behavior. Despite this, the controller still met all the specifications, indicating that it was robust and capable of adapting to a change in loading.

VI. Conclusion and recommendations

A robust digital cascade controller system was created to control the output potentiometer of the laboratory servo system. The design process involved three key stages: system identification, cascade controller design, and controller implementation. A requirement analysis was also performed to obtain design specifications from the client's requirements.

The system identification was done through a series of tests, including a ramp test and multiple unit step tests for varying loading cases, to determine the characteristics of the plant. These tests enabled the gathering of the necessary parameters of the system's behavior such as the open loop gain, time constant, and gear reduction ratio. The range for the open loop gain "A" was calculated to be between 6.728420213 and 37.61322156, while the time constant "τ" ranged from 0.08 to 0.46. The gear reduction ratio "k" was determined to be a constant value of 9.294257. The accuracy and adequacy of these values were then verified through simulations using MATLAB Simulink for each possible loading cases. The nominal transfer function of the plant was derived as $TF = \frac{9.797780069}{0.1s+1}$.

The cascade controller design featured the plotting of the sensitivity bode plot which was obtained using the nominal plant transfer function and the requirement analysis. The low and high frequency bounds specifications were obtained from this plot, and the speed controller was implemented as a proportional controller with transfer function $G_s(z) = 0.1$. The system was simplified and the bounds were implemented on the MATLAB QFT design tool which ultimately led for the determination of the position controller transfer function, $G_p(z) = 0.25$, meaning both controllers only featured a gain value These controllers were tested for performance under all possible loading cases, and the system met all required specifications, including set point tracking, settling time, and overshoot.

The controller was then implemented into the physical system to test its performance. The setpoint tracking for the physical controller did not achieve zero steady state error. To correct this, the gain of the position controller was increased, resulting in a final transfer function of $G_p(z) = 0.33$ resulting of a set point tracking within 0.4V relative to the desire value. The overall system was shown to be robust and capable of adapting to different loading cases. All the other specifications were met with a settling time of 0.3 seconds, an overshoot of 16%, and an output disturbance rejection time of 0.1-0.2 seconds. However, some uncertainties and flaws in the design could be addressed to further improve the system.

The overall design can be enhanced by addressing uncertainties in the system. Due to the nature of the physical servo motor system, certain components are subject to variations, leading to measurement uncertainties. These component uncertainties are usually caused by noise which come from a range of sources, including changes in temperature, faulty wiring, or vibrations in the motor. Furthermore, the data gathering was done with approximations and rounding of certain figures. These measurement errors inevitably affected the system identification, which had an impact on the cascade controller design and implementation. However, these uncertainties can be reduced by increasing the number of tests and data samples collected, and by taking an average of the test, resulting in more reliable parameters. This would improve the accuracy and performance of the design.

In addition to these uncertainties, flaws in the system were identified which prevented the setpoint tracking from achieving zero steady state error. To address this, an additional integrator can be implemented to the controllers, which would allow for zero steady state error. However, this would require reevaluating the QFT design of the controllers.

Addressing these uncertainties and flaws in the system would result in a significant improvement in the accuracy and performance of the digital cascade controller system.