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3. Describe the *Power Method* for computing eigenvalues and provide an example. If possible, provide code to run this algorithm.

The power method finds the absolute value of a dominant eigenvalue. An application of this could be utilized in order to calculate the density of given particles within a matrix.

The *power method* could be calculated to find the absolute value of a dominant eigenvalue. I would imagine that the most powerful eigenvector pierces the highest levels of eigenvalues. This eigenvector would likely be able to detect power states within other vectors, and it relies heavily on the null vector state.

It would be interesting to find a generalized algorithm for the construction of a global Lyapunov function which can be used to prove the stability of a wide class of stochastic differential equations. This could probably be used to have a stable level of power, distributed across multiple states. I imagine that the power level could be extrapolated to various “well-being” variables, accounting for seasonality.

The example is listed below, showcased in the linked articles.

Code to run algorithm (this merges both examples):

Code with further examples found here:
-https://pythonnumericalmethods.berkeley.edu/notebooks/chapter15.02-The-Power-Method.html
-https://www.sciencedirect.com/topics/mathematics/power-method#:~:text=The%20Power%20Method%20is%20used,exists%2C%20and%20a%20corresponding%20eigenvector.&text=To%20apply%20the%20Power%20Method,eigenvector%20of%20the%20dominant%20eigenvalue.

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4. Provide a brief summary and small application of eigenvectors and eigenvalues. Interesting places to look are Principal Component Analysis in data science, graph theory, Schrödinger equation in quantum physics, or image processing.

Schrödinger equation in quantum physics:
“Elvet” is a tool that is calculated to find an energy eigenstate, within the google colab environment (highly recommended for coders due to the interconnectedness).

Application:
The idea utilized in the article from the second linked source mentions the energy state in figure B. The importance of eigenvectors/eigenvalues are important in the sense that these eigenvalues are step like (**discrete values).** Essentially, it seems like energy can be distributed within this null vector, that eigenvectors have accessibility to. Also, it seems that the increased number of excited states (eigenvalues), the more consistent the frequency.

Sources:

1. (Great Programming Source) https://colab.research.google.com/github/jccriado/elvet_examples/blob/main/differential_equations/2_quantum_harmonic_oscillator.ipynb
2. (On Eigenvectors, Eigenvalues, and Harmonic Oscillation) Love this one http://ne.phys.kyushu-u.ac.jp/seminar/MicroWorld2_E/2Part3_E/2P31_E/energy_eigenvalue_E.htm#:~:text=Thus%20the%20allowed%20energies%20are,that%20the%20energy%20is%20quantized.
3. (A Generalized Lyapunov Construction for Proving Stabilization by Noise) I find all areas of Kolba's research interesting. https://dukespace.lib.duke.edu/dspace/bitstream/handle/10161/5444/Kolba_duke_0066D_11285.pdf;sequence=1

No need for LaTeX in this problem, colab saves resources!

Further Research:
-How to optimize the levels within the harmonic oscillator… turn up the frequency!
-Breaking the Barrier, it would be ideal to release the energy from outside of the potential barrier. Once this is feasible, it would be great to imagine the possibilities.

Some outlandish ideas:
- Applying the "breaking" of the barrier to *push* carbon dioxide out of the planet. I don't see this idea in modern research. Perhaps this could be done to stabilize mars, for less work, and could be focused on areas most impacted by climate change.
- Utilizing the A Generalized Lyapunov Construction for Proving Stabilization by Noise to colonize future solar systems, while acknowledging that it is plausible for different solar systems data structures.

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