Answer any **two** of the following:

1. Does diagonalizability imply invertibility? Clearly state the definition of a diagonalizable matrix and an invertible matrix. Give an example of a 2x2 matrix that is diagonalizable but not invertible. Then give an example of a 2x2 matrix that is invertible but not diagonalizable. Show all work to justify your examples.

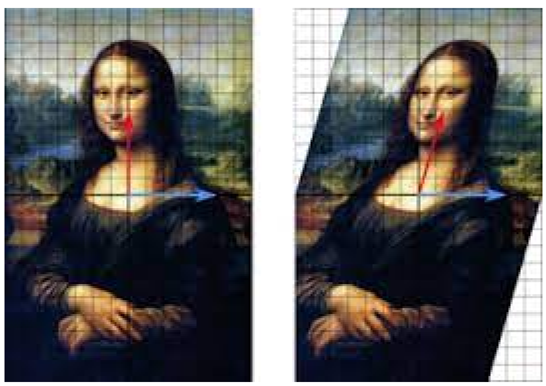
2. Properties of Similar Matrices: State the definition when two matrices A & B are similar. Then give reasoning to justify or disprove the following:

- the notion of "similar" is an equivalence relation
- if A is diagonalizable and B is similar to A, then B is also diagonalizable
- if A is invertible and B is similar to A, then B is also invertible.

3. Complex Eigenvectors: Theorem 9 on pg 301 oddly uses the "minus" complex eigenvalue $\lambda = a - bi$. Why is the "minus" is used here and not $\lambda = a+bi$? If the "+" eigenvalue were used, what changes would have to be made to this theorem if any? Verify your answers with examples.

4. Give applications of at least two of the concepts of this module: diagonalization, eigenbasis, and complex eigenvalues.

3. Complex Eigenvectors:

- The minus(-) sign is used here and not the +bi so that the eigenvalue and complex eigenvectors are able to span in any direction, and not be limited to the constraints of purely the (+) function. This allows the orchestration of “b” through i to allow for both rotation and scaling within a given matrix (a), without requiring a to change its properties.
- Changes to be made: A = D PCP^-1 would need to be tweaked, which would require more work, instead of just tweaking b with i.
- Verification with examples:
    - 
    - In this example, the complex eigenvector is able to alter the image to the viewer, so it appears different from the typical image.
    - Perhaps this is being done through youtube and videos, where various creators are manufacturing false images/videos and catching people by surprise, which in my opinion is deceitful. I believe in transparency.

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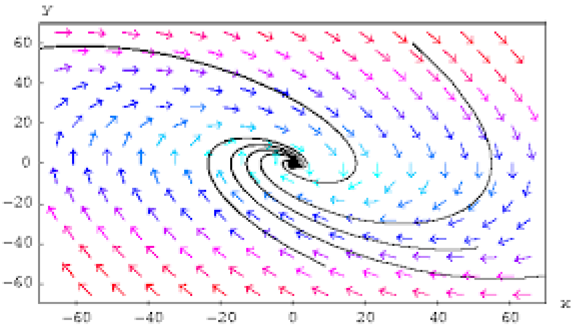
4. Applications of at least two of the concepts of the model:

An application of Complex Eigenvalues is:

- An application of a complex eigenvalue will demonstrate that it comes in “conjugate pairs”. These complex eigenvalues deviate in ways that are eccentric to eigenvalues. These can be considered more fluid and abstract, rather than linear and concrete. An application of a complex eigenvalue could be applicable to flying autonomous vehicles. As airspace does not have physical visible lanes(?). Complex Eigenvalues could likely be employed to predict paths, at a much more dynamic level, with flows in every direction.

- 

An application of Eigenbasis is:

- An eigenbasis is a space that supports eigenvectors. An application of this could be a reservoir of energy within an area that eigenvectors can tap into. For example, in machine learning applications this could potentially be natural/shared energy within a given space that is allocated to empower various artificial intelligence devices.
- Source with an image: https://www.researchgate.net/figure/a-Overlap-of-Z2-with-energy-eigenbasis-of-H-The-overlap-is-seen-to-be-dominated-by-N_fig2_329641673