1. My favorite theorem so far (i think it's beautiful): T is one-to-one if and only if T(x) = 0 has only trivial solution.✅
    a. Proof.
        i. T(u) = T(v) = y. Then T(u) - T(v) = 0
            1. This will lead to u = v.
    b. Also, the preimage should be zero. and have at most one element(?) (That is all you have to check)

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2. T/F example (with full solution and explanation)✅

- 14. There is a linear transformation T : R 2 → R 1 that is one-to-one but not onto.
    - True.
        - **Proving** *not* Onto: This would have to range to domain. (Similar to Book page 76, invert the arrows on the left picture, but it would be a vector transferring to a single point)
            - Application: Perhaps think of the instance when you first meet someone, that would be r1 in your life, and then r2 would be the physical memories that are built with one another (r2+ would be projections?). So, r2 to r1 would be an individual looking back at a memory. This would be not onto as other vectors have not seen this domain, and this domain can be referred to to prevent *onto.*
        - **Proving** One-to-one: Page 77 in the book, just invert the arrows on the right picture, one solution.
            - Application: two vectors having access to a domain that is inaccessible to other vectors.
    - I think of this as two vectors perhaps wanting to maximize the growth together, and "sticking" together (u = v) while optimizing the outcome of a certain domain, while acknowledging fluidity.

My interpretation of domain:

- vector space in which a transformation acts
    - I think of this as an image or short film flashing across my consciousness

(http://www.math.umbc.edu/~campbell/Math221/Glossary/#:~:text=1%2Ci%7D).-,Domain,%E2%86%92R5%20is%20R)

My interpretation of vector:

- An algorithm or entity, vehicle capable of transferring a domain.

Google dictionary

My interpretation of range:

- All possible projections, visualizations, etc that a vector is capable of. The sky's the limit!

My interpretation from https://en.wikipedia.org/wiki/Row_and_column_spaces#:~:text=In%20linear%20algebra%2C%20the%20column,combinations)%20of%20its%20column%20vectors.&text=The%20row%20space%20is%20defined%20similarly.

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3. Practice example (with full solution and explanation) ✅
   - ☐ 10. Find all values (if any) of s and t so that, for the matrix

$$A = \begin{matrix} 2 & 0 & 4 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & s & t \end{matrix}$$

   (a) the matrix transformation determined by A is onto.
   - Onto if and only iff every row in A contains a pivot in each *row.* This is clear as long as s DNE 3 or S = 3 and t DNE 0
     - Both outcomes as onto implies consistent(?)

   (b) the matrix transformation determined by A is one-to-one.
   - One-to-one if and only iff every row in A contains a pivot in each *column.* Not possible in given matrix A. (columns > pivots).
   - Free variable = one variable is onto.

   (c) there is exactly one free variable in the matrix equation A~x = ~0.
   - Same as A. either s or t will be the free variable, but not both. So, there must be three basic variables.

   https://textbooks.math.gatech.edu/ila/one-to-one-onto.html
   http://www.sci.wsu.edu/math/faculty/hudelson/free.html#:~:text=Free%20and%20Basic%20Variables,augmented%20matrix%20to%20echelon%20form.

Open to comments, thanks for reading!