1. Let S_1 and S_2 be subspaces of R^n. Define the *union* S_1 U S_2, the *intersection* S1 ∩ S2, and the *direct sum* S_1 and S_2, denoted $S_1 \oplus S_2$. Of these new sets, which are and which are not subspaces of R^n? Give an argument or counterexample to justify your answer.

The *union* of two subspaces is a subspace if and only if one of the subspaces is contained by the other.

- These two subspaces would only remain subspace iff they were null vectors because then it would be closed under both scalar multiplication/addition.

The *intersection* of S1 and S2 is when they both are at the null vector (assuming they are linearly independent vectors), “same page”, or wavelength in physics? This is pictured as the meeting point in the bottom, so the intersection is the subspace, yes.

The *direct sum* S1 and S2 are subspaces iff both are null vectors.

- This is because it is closed under both scalar multiplication/addition.

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2. Is the set containing just the zero vector, S = {**0**} is a subspace of any R^n? If so, find a basis and clearly argue why that basis is linearly independent. What is its dimension?

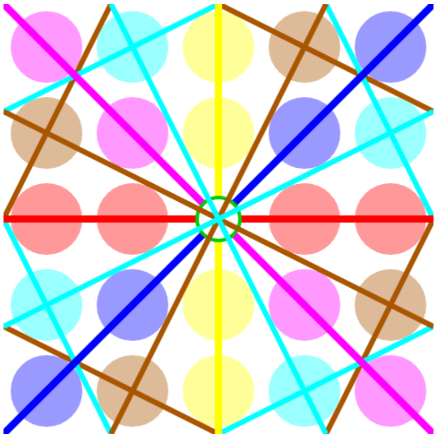
-Yes, the set containing the zero vector is a subspace of any R^n.

1) Any span of S consists of the zero vector 0.

2) However, the only item in S = {0} (null vector) should have 0 entries.
    a) Building from other discussion posts,there should be no other variables within the vector either. For example, all items should be 0.
        i) So, I believe that the zero vector to any R^n should be one-to-one.
            (1) And any vector relies on the zero vector subspace (kind of like a preimage).

-Dimension = # of vectors in a given basis.
-Basis of s (given bases)
- must be linearly independent (otherwise it cannot span).
    - this is false as there is a non-trivial linear combination of the zero vector.
        - Therefore, there is a basis and the dimension is 0 for only the subspace.
        - the rest of the vector would not be able to span, scale, or extrapolate without the basis of the null vector.

However, the span of the rest of the space is linearly independent, this means that the rest of the span would have infinitely many bases in any given R^n, as there are infinitely many scalars



Here is a cool picture. The null vector for everyone, in every dimension!

i would have liked more time to think through this one for sure, open to comments and critiques. Thanks for reading!