MSDS 460: Final Project Report

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Case Report

Product Profit Optimization Using a Linear Programming Model: New Chemicals Company in Tire Manufacturing Industry

Hassan Ali, Ethan Norton, Jamia Russell

Abstract

The scale-up of chemical processes from lab scale to pilot scale and finally to commercial scale has proven a difficult problem to solve for companies from a wide variety of industries. This paper examines one use-case in this field of a new company trying to model their approach to scale production for their new R&D product formulations to prospective customers using linear programming to optimize for the most profitable product mix. To develop the model, data was gathered from trial runs at their existing sites to obtain the process inputs and raw material formulations for the existing product portfolio. Knowing this, along with other model assumptions, the objective function was designed to maximize profit within the constraints set by each site's available resources including raw material availability and labor.

The model itself was coded and solved utilizing GLPK Solver in Python. Key findings uncovered the importance of aligning production with site-tailored constraints to optimize resource allocation. Sensitivity analysis indicated opportunities for increase in business performance and total profit through varied changes in raw material cost, availability, and labor hour variation across sites. Results indicate that while the model and production system are efficient, there is room for optimization with adjusted constraints reflecting the positive marginal values for enhanced total profit. Flexibility in constraints can result in significant fluctuations in profit margins.

This research concludes with the hypothetical idea of cooperative trading, which would allow optimal materials to be sourced dynamically in the market, leading to even higher profit levels.

Keywords: Profit Optimization, Linear Programming, Sensitivity Analysis, Product Mix, Manufacturing

1. Introduction

Background and Goals

CB Corporation wants to begin pilot scale production for their new R&D product formulations that they seek to sell to three prospective tire industry customers: Wheel Works, Tirevana, and Grip Gurus. Each customer has selected three products they are interested in buying based on their unique combination of rubber, fillers, and chemicals which will lead to differing performance in their tires. CB Corporation examined the raw material, labor, and transportation costs and negotiated a sale price for the selected products with each customer. Considering these factors, CB Corporation now needs to decide on what products are worth making and how much of the product to sell. They have a production trial week planned for the given sale period and two available sites ready for production.

CB Corporation desires to reach the following goals:

- Increase customer engagement for future sales by delivering on price and performance targets
- Maximize efficiency of labor and resources to products chosen for sale and keep in line with internal sustainability targets
- Select products that will maximize short-term profit for business

Problem Statement

The pilot plant works in a batch process that starts with raw material collection, weighing, mixing, QC testing, and packaging. One batch of product is equivalent to 15 kg. The pilot plant can produce and ship from two sites.The business manager for the pilot plant wants to know what product portfolio should be produced based on the current product sale price and the associated costs for the inputs. In other words, what products should be made, and how much of each should be made, at which site to make the most profit after one trial period (1 Week or 168 Total Hours per site).

The objective function is to optimize profit, which will stem from formatting a linear program, and that will give us a better understanding of the problem, and to build the best model, given the various constraints.

See **Raw Material**, **Product**, and **Site** details below.

Product	Product Price ($)	Site 1 Labor (Hours)	Site 2 Labor (Hours)
Product 1	90	0.25	0.35
Product 2	70	0.25	0.35
Product 8B*	100	0.35	0.50
Product 3	75	0.25	0.35
Product 4	80	0.25	0.35
Product 4*	100	0.35	0.50
Product 10	150	0.25	0.35
Product 6	90	0.25	0.35
Product 9	120	0.2	0.35

Table 1: Product Sale Price per Batch Unit and Labor Hours per Batch Unit per Site

Raw Material	Raw Material Cost ($/kg)	Site 1 Inventory (kg)	Site 2 Inventory (kg)
RSXX	.40	3000	5000
STYY	.60	4000	2500
CBAA	1.1	1000	1750
CBZZ	1.1	500	250
SBRXL	2.0	200	400
SLSM	2.0	900	1200
CARBON 10	8.5	1500	1250
CARBON 20	5.0	2000	1900
CARBON 30	6.0	800	800
CARBON 40	3.5	800	800
CARBON 60	6.0	500	250

Table 2: Raw Material List with Cost and Inventory per Site

P	RSXX	STYY	CBAA	CBZZ	SBRXL	SLSM	C10	C20	C30	C40	C60
1	50%		15%				35%				
2	30%		12%		20%						38%
8B*		50%	13%				37%				
3	33%		13%			20%		34%			
4	25%	25%		13%			37%				
4*	53%			13%			34%				
10	53%			13%						34%	
6	25%	25%	15%						35%		
9	66%									34%	

Table 3: Product Code with Product Formulation (Raw Material Input wt.%)

Research Questions

1. Will labor hours be a bigger constraint on **site choice** than inventory availability?
2. How many products will prove **unprofitable** and will not be sold due to low sale price and high raw material costs?
3. Can we sell or trade **unused raw materials** to gain more resources for profitable products?
4. What **model solving method** will be best for the decision problem described above?
5. How should we allocate or **forecast resources** in the future to maximize profit?
6. If we migrate this to a circular economy, how would that meet **sustainability goals** and further drive profit?

2. Literature Review

Overview of Existing Research Related to Business Problem

Product mix optimization is a specific niche area within operations research, focusing on determining the most profitable combination of products to manufacture under specific constraints.

Models in this field can range from simplified to complex depending on the production process steps and inputs. In a paper on food production, a simple model used basic ingredient formulations and availability to decide on the most profitable bread selection for sale (Oluwaseyi, Elizabeth, and Olaoluwa 2020). The advantage of simple input and output models is they allow for companies to easily see the impact of adjustments to constraints on raw materials. The drawback of this approach is that it is often much too simplified, removing complexity from other considerations such as location and labor availability.

More state-of-the-art investigations look at a wider range of constraints. Another paper examined profit optimization of a chemical company through a linear programming model. The objective of this study was to formulate a linear programming model that would suggest a viable product-mix of aluminum sulfate and sulfuric acid to ensure optimum profit (Maurya et al. 2015). Constraints included machine hours on reaction, filtration, and evaporation process steps as well as demand. From this approach, they were able to move away from ambiguous purchasing decisions towards knowing which product-mix maximized demand and minimized material waste.

Further studies show even more complexity, breaking down the batch production process into a series of steps flowing from one stage to the next. Mass balances and energy balances can then be formulated on each stage creating new constraints on time and resource availability. This "flowchart" process is useful for taking into account equipment and labor availability which often cannot be shared, skipped, or reordered (Kimutai, Maina, and Makokha 2019).

The investigation in this paper follows a similar approach to the field with the exception that the objective function is not simplified to a simple profit contribution term but rather a formula that takes in per-product sales revenue and subtracts per-product cost. Additionally, the process itself is not broken down into stages. Rather, it is assumed that the constraints on the preparation stages, product testing stages, and packaging and shipment stages are inconsequential to the specific product-making decisions being made.

Key Characteristics and Approaches to Literature

The literature presents a range of methodologies, from deterministic models, which use fixed input data, to stochastic models that incorporate variability and uncertainty. Deterministic approaches are typically praised for their simplicity and readability by key stakeholders. However, these models could fail to account for real-world complexities such as demand fluctuations and supply chain disruptions, but this may also be offset in the event of purchasing agreements, or more stable demand environments, such as the one in our development.

In contrast, stochastic models, including Monte Carlo simulations and simulated annealing, offer a more dynamic approach to optimization. One study explores and validates the use of these techniques to introduce flexibility, variability, and probability into the optimization process, incorporating a multi-objective approach and attempting to simulate these into a manufacturing with probabilistic outcomes (Janekova et al. 2018). However, these models may only be applicable to scenarios with large enough scale, and not suitable for medium to small enterprises.

Advantages and Disadvantages of Literature Approaches

Deterministic models provide a more defined and interpretable optimization path, making them ideal for environments with stable, predictable input data. However, their deterministic nature limits effectiveness in dynamic or uncertain markets, and could be risky if competition attempts price gouging. This downside was not relevant to our problem, as our inputs are static for a given time period, and can update when markets change or upon contract renegotiations. As CB Corporations has minimal products, this is manageable to update on a quarterly basis. On the other hand, stochastic models, such as those incorporating Monte Carlo simulations, are better equipped to handle uncertainty, providing a broader range of possible outcomes (Janekova et al. 2018). Despite this advantage, the complexity and resource demands of stochastic models could be considered a drawback, and would likely not be relevant in the context of this problem/our proposed methodology.

Model Comparison and Result Comparison

The linear programming models discussed in the literature primarily aim to maximize profit through product mix and process optimization, often assuming a static environment and constraints utilizing the simplex method, while also including a section on sensitivity analysis (Kimutai, Maina, and Makokha 2019). This approach not only enhances current profitability but also contributes to long-term sustainability and resource efficiency. Moreover, our model also includes a sensitivity analysis, providing business relevant insights into how changes in input variables affect the optimal solution—an area often underexplored in existing studies. This addition allows our model to adapt more effectively to real-time data and evolving market conditions, offering a more comprehensive and sequential tool for decision-making in competitive industries. Therefore, while our model is deterministic, it is still comprehensive and likely the best approach to the problem at hand. Furthermore, our methodology and adaptation to insights drawn from sensitivity analysis facilitated business insights that could not have been derived from a Monte Carlo analysis or simulation.

The conventional linear program models found in most literature fall short in considering the relationships between products that share resources in product mixing in differing production environments. The dependency of these resources on production output can affect production structure and total profit (Mohammed and Kassem 2020). Model limitations are enhanced when resources are limited and at high cost, negatively affecting business performance and satisfaction of customer needs (Xu et al. 2009). Incorporating an approach that designates the optimal location of resource use improves the conventional linear programming model to reflect the relationship between resource locations, availability, and labor costs to the optimization model (Ji, Gao, and Wang 2022). This application provides the ability to make changes in production that are concurrent with market fluctuations and customer needs. Our model's site designation of production lines, material and labor hour use reach an optimal production strategy not seen in other models.

Key Takeaways and Room for Further Growth

The majority of studies indicate linear programming models are effective in optimizing resource allocation across various industries including, finance, transportation, and supply-chain manufacturing to minimize costs. They perform best with problems obtaining linear constraints and objective functions. The development of the simplex and interior-point methods has increased the ability for linear programming models to efficiently and accurately solve large scale real-world linear programming problems. Oluwaseyi, Elizabeth, and Olaoluwa (2020) illustrate a simplified model focusing on basic ingredient formulations and availability, but have overlooked the potential constraints of location and labor cost. Advanced models provide a wider range for constraints and increase the applicability of the model to real-world problems. Other researchers like Maurya et al. (2015) present a model incorporating machine hours, process steps, and demand to optimize product mix and profitability impacting decision making. Model advancement and detail increases as flowcharts are used to account for labor and equipment availability. There is room for improvement as simplification of models, limited constraint considerations, and the lack of account for the interdependencies of production stages could be addressed.

3. Methodology

Problem Overview

This project solves a linear program model with finding the optimal product and material selection based on defined constraints and profitability per product. The objective function defined below is to maximize total profit while following constraints on labor hours and raw material inventory at each site.

Linear Programming Model Formulation

Objective Function:

The single goal is to maximize profit, which was further broken down by calculating the profit for each product per site. The profit for each product at both sites is calculated and summed to form the **objective function**:

Maximize P where

$$P = \sum_{j=1}^{2} \sum_{i=1}^{9} (R_i x_{i,j} - C_i x_{i,j})$$

where

R_i = Revenue (Product Sales Price) per Batch Unit of Product *i* (equiv. 15 kg)

C_i = Raw Material Cost per Batch Unit of Product *i* (equiv. 15 kg)

$$= 15 * \sum_{i=1}^{9} (V_m * M_{m,i})$$

where

V_m = Vector [1x11] of Cost (per kg) of Raw Material *m*

$M_{m,i}$ = Matrix [11x9] of Fraction of Raw Material *m* per Product *i*

$x_{i,j}$ = Quantity of Batch Units of Product *i* produced in Site *j*

where

i = {P1, P2, P8*, P3, P4, P4*, P10, P6, P9}

j = {Site 1, Site 2}

m = {RSXX, STYY, CBAA, CBZZ, SBRXL, SLSM, C10, C20, C30, C40, C60}

Decision Variables:

The decision variables represent the quantity of each product to be produced at each site which is not limited by any demand. These variables are continuous and non-negative:

$x_{i,j}$ = Quantity of Batch Units of Product *i* produced in Site *j*

$x_{i,j} = \geq 0$

Constraints:

- **Labor Hours Constraint:** Each site has a maximum number of labor hours available and the total labor required for producing the products at each site must not exceed these limits:

$\sum L_{i,j} \leq H_j$

where

$L_{i,j}$ = Labor Hours to make Product *i* at Site *j*

H = Total Available Labor Hours per Site *j* (168 Hours)

- **Raw Material Inventory Constraint:** Each site has a limited inventory of raw materials and the amount used in production must not exceed the available inventory:

$\sum Q_{m,j} \leq I_{m,j}$

where

$Q_{m,j}$ = Quantity (kg) of Raw Material *m* used in Site *j* for production

$I_{m,j}$ = Available Inventory (kg) of Raw Material *m* at Site *j*

Model Implementation

The model was implemented utilizing Python and the PuLP library, which provides a straightforward interface for defining linear programming problems, with the ability to perform cross product, which was relevant to this problem. The GLPK solver was used to solve the problem and generate sensitivity analysis results.

- **Data Preparation:** Raw materials, product prices, labor hours, and inventories were defined as arrays. The raw material usage for each product was represented in a matrix, where rows correspond to raw materials and columns to products.This was defined by operations, and transformed into this to easily formulate the problem.
  - **Data Modification:** This was derived from Hassan Ali's work, but the values were modified and skewed to protect company secrets and internal strategy. This includes labor hours per site, and available materials.
- **Model Formulation:** The decision variables were defined for each product at both pilot sites. The profit function was formulated by subtracting the raw material costs from the product prices, after standardizing the unit size of both. Constraints for labor hours and raw material inventory were added to the model, where this additional layer of choosing the best profit based on that constraint.
- **Optimization:** While we found PuLP solver generated an optimal solution, we were able to get a more refined and constraint optimized solution by utilizing GLPK (version GLPSOL--GLPK LP/MIP Solver 5.0). GLPK (GNU Linear Programming Kit) is an

open-source linear programming solver that is well known for applications to complex optimization problems. The difference between solvers has been studied in cases (Dedović, Gušavac 2023), and the benefit of GLPK is that it is implemented through C and able to be utilized in python, which allows the user-friendly interface of Python, while also being able to utilize C's accuracy and computational efficiency, by tapping into the Conda environment. In this specific example, GLPK yielded a higher optimal solution by optimizing the labor constraint max, so we decided to build off that base model and validate in the following steps.

- **Data and Model Validation:** We performed additional calculations to cross check the solver, as well as performed a robustness check where we evaluated how changes would impact the objective function, thus thoroughly cross referencing the solver's outputs.
    - Robustness Check Test 1: Remove site Labor Hours constraint (Capped at 10,000 hours for Site 1 and Site 2)
        - Expectation: Non-unique raw material inventory used up at each site
        - Result: All inventory at each site used up except RSXX and STYY, which there was excess of, and SBRXL and Carbon 60, which are unique materials tied to other common materials that were used up in other more profitable formulations
    - Robustness Check Test 2: Remove site Raw Material Inventory constraint (Capped at 20,000 kgs per material at Site 1 and Site 2)
        - Expectation: Only make most profitable product(s) until available labors are all used up
        - Result: Only the most profitable product, Product 10, was made from RSXX and Carbon 40 until labor hour limit was reached (168 hours)
- **Sensitivity Analysis:** The GLPK solver was configured to generate a sensitivity analysis report, which provides insights into how changes in constraints or objective coefficients affect the optimal solution.

Modeling Assumptions

Our product mix optimization model operates under several key assumptions. It employs a deterministic approach, focusing on a fixed set of variables and conditions, which contrasts with stochastic methods like Monte Carlo simulations. While Monte Carlo simulations are valuable for handling uncertainty and variability in market dynamics, our model simplifies by not considering all potential combinations. This decision makes it better suited for scenarios with fewer variables or where production is governed by contracts, which is a large assumption of this model.

Another key assumption is that there are quarterly or periodic updates to prices for products, resources, and possibly even labor hours per product to be updated, thus updating the model. It is also assumed that the constraints on the preparation stages, product testing stages, and packaging and shipment stages are inconsequential to the specific product-making decisions being made.

The following standard linear programming assumptions apply:

- Proportionality Assumption: Contributions from decision variables lead to proportional changes in the objective function value
- Additive Assumption: Every function is the sum of contributions from each individual variable
- Divisibility Assumption: Decision variables can take on any Real Number value (Integer, Fraction, and Irrational)
- Certainty Assumption: Values assigned to parameters (non-variable terms) are known constants

4. Computational Experiment and Results

Total Maximized Profit Across Site 1 and Site 2: $75, 203.05

Optimal Product Value at Site 1			**Optimal Product Value at Site 2**	
Product	**Value (Batch)**	**Profit ($)**	**Value (Batch)**	**Profit ($)**
Product 1	0.000	0.00	170.756	6,813.16
Product 2	0.000	0.00	0.000	0.00
Product 8B*	88.610	4,092.43	0.000	0.00
Product 3	238.689	9,398.37	0.000	0.00
Product 4	0.000	0.00	0.000	0.00
Product 4*	0.000	0.00	0.000	0.00
Product 10	156.863	19,894.14	128.205	16,259.60
Product 6	152.381	7,965.71	152.381	7,965.71
Product 9	0.000	0.00	28.657	2,813.88
	Total	41,350.68	**Total**	33,852.37
Optimal Material Use at Site 1			**Optimal Material Use at Site 2**	
Material	**Value (kg)**	**Profit ($)**	**Value (kg)**	**Profit ($)**
RSXX	3000.001	20,606.73	3155.030	22,312.55
STYY	1236.070	7,117.88	571.428	3.085.71
CBAA	981.100	4,457.19	727.050	3,562.58

CBZZ	305.880	2,722.35	250.000	2,225
SBRXL	0.000	0.00	0.000	0.00
SLSM	716.067	2,148.20	0.000	0.00
CARBON 10	491.830	-901.70	896.460	-2,241.17
CARBON 20	1217.310	0.00	0.000	0.00
CARBON 30	800.000	9.094	800.000	9.094
CARBON 40	800.000	5,200	799.990	4,907.68
CARBON 60	0.000	0.00	0.000	0.00
	Total	41,350.68	**Total**	33,852.37

Results Interpretation

The output includes the optimal production quantities for each product at both sites, the total profit, and the amount of raw material used. The sensitivity analysis report provides additional information on the robustness of the solution to changes in the model parameters. The methodology ensures that production is optimized in terms of profit while respecting the constraints on labor and raw materials, providing a practical solution to resource allocation in manufacturing. The results of the optimization show that the production scheme across the two sites maximizes total profit as $75,203.05 utilizing the full 168 labor hours available at each site. Site 1's optimal production amount designates large quantities of Products 3, Product 10 and Product 6, similar to Site 2 where Products 10 and Product 6 are in larger quantities. The model finds the optimal value of products to reach maximized profit as 88.61 batches for Product 8B*, 238.69 batches for Product 3, and 156.86 batches for Product 10 and 152.381 batches for Product 6 at Site 1. At Site 2, the optimal production portfolio is 170.75, 128.205, 152.381, 28.65 batches respectively for Products, 1, 10, 6, and 9 respectively. Site 1 utilized a significant amount of RSXX, STYY, CBAA, CARBON20, CARBON30, and CARBON40. SBRXL and CARBON 60 were not utilized in the production strategy. Site 2's material utilization included RSXX, STYY, CBAA CBZZ and CARBON 10,20, and 40. The results show that Site 1 generates the most profit compared to Site 2 during the production period.

Sensitivity analysis of the model's constraints reveal opportunities for an increase in business performance and total profit. For instance, CARBON40 and CARBON30's shadow prices (16.387 and 2.752, respectively) and their binding constraint status indicate an increase in the availability of material should increase profit. Specifically, CARBON40's marginal value predicts a substantial increase to profit compared to CARBON30. Conversely, negative incremental changes in material availability show the reducing materials led to diverse impacts.

Materials RSXX, CBZZ, and CARBON30 showed limited decrease in profit from ranges of $100-200 compared to CARBON40 reflecting a loss that varied from $1100-$3400.

It is evident the model accomplished its goal to maximize the use of available materials. The absence of production of materials with limited use in the product mix, shows the model's efficacy in determining the notable changes in total profit. Sensitivity analysis of materials not utilized in production across both sites - SBRXL and CARBON 60 reveals that increasing the utilization and availability for these materials in ranges within and outside of the models existing constraints does not change the optimal solution. In addition, material cost provides an opportunity to assess production as market prices change. Both Site 1 and Site 2 utilize CARBON10 in their product portfolio which sensitivity analysis shows that a potential decrease in cost can significantly increase profit compared to SLSM and CARBON40.

Labor hour adjustments impact the production plan and total profit for the CB pilot company as increases in labor hours at Site 1 depict a consistent increase in profit despite the marginal benefit decreasing slightly. Sensitivity analysis shows that additional labor hours improve the output of Product 30 by 7.52 units per hour. On the other hand, Product 4's batch output shows sensitivity to increased labor hours indicating that as labor hours increase priority may be given to other products that have a higher profit margin and utilization rate.

In terms of site choice between producing at Site 1 or Site 2, the biggest constraint appears to be inventory availability. This is because the biggest constraint on product-mix choice is actually sales price and the formulation of the given product. This can be seen when modifying the model to be unconstrained in either labor hours or inventory availability, which pushes production to only high-price products. Even when increasing per-product cost or per-product labor requirements, the highest price product is always chosen. If the given site cannot produce the best available priced product, it is because the product shares a similar formulation with a higher priced product and therefore the only way to offset the price difference is to increase that material's inventory. That is why some products are produced at Site 1 but not Site 2 and vice-versa.

5. Discussion and Conclusions

The results demonstrate how formulating and modifying a linear program on product mix optimization could lead to operational improvements. By optimizing the mix of products produced at each site, the model builds upon an optimal strategy that maximizes profit while accounting for constraints such as labor hours, raw material availability, and additionally differentiates per site. This approach not only demonstrates the effectiveness of linear programming in handling complex manufacturing decisions but also showcases how sensitivity analysis can be utilized dynamically to encourage pivots that yield new optimal solutions.

Implications for the Decision Problem

By exploring how reintroducing unused materials or products into the production cycle could further optimize profit and sustainability, future studies could extend the model's applicability across different industries. For example, one researcher quantitatively evaluated the differences between negotiation and trade between a manufacturer and retailer, and found that overall profit was higher with cooperative trade, even accounting for variable costs and products (Matsuno and Weng 2022). Additionally, Ezimadu (2023) evaluated the impact of trade credits on pay-offs for a manufacturer-retailer pair, detailing significant increase in profit pay-offs after the initial period. This finding highlights the possible benefits of cooperative game theory approaches, which have been shown to be mutually beneficial in other contexts, making them a compelling application for further investigation.

In our specific model, the findings suggest that a specialization approach to product manufacturing, where each site focuses on producing the most profitable products within its capacity, can lead to optimal profitability. If we remove the constraints on the material (increase the amount of inventory available at each site), it is noticed that the model will produce Product 10 solely at both sites. The main recommendation to key stakeholders in this case is to optimize availability of materials that will go into producing Product 10, which should be achievable by reducing other materials on stock, while also changing inputs based on business requirements for further production decisions. Specifically, the integration of a circular economy concept, where unused products can be sold off and reinvested into more profitable lines, presents a creative way to increase overall efficiency and meet sustainability goals. This approach is relevant in industries that do not face infrastructure risks, and is feasible within the business assumptions mentioned previously. Overall, this model is comprehensive for the business problem, and shows robustness to changing parameters.

Model Limitations

Despite the results mentioned above, the model does have limitations to consider. One major limitation is the assumption of linearity in cost and revenue relationships. We assumed that resource input - labor hours and raw materials, and production outputs - quantity of production, would provide constant returns to scale. This limitation simplifies the model to provide interpretable results, however does not fully reflect real-world production environments where complex production processes often have proportional increases and decreases that are not do not reflect resource input. Failing to account for diseconomies of scale could cause the model to suggest production outputs that are infeasible resulting in an overestimation of profit, thus causing suboptimal resource allocation, and accuracy of sensitivity analysis. Additionally, the model's exclusion of materials in production due to the suboptimality within existing constraints is a missed opportunity for flexibility in response to future demand or supply changes. The narrow scope of analysis focused on short term profit maximization does not consider strategic planning with long term benefits. There is a risk of neglecting the value in future productions that present differing constraints. Lastly, the models single focus on profit optimization overlooking other potential objectives such as product diversification as means to increase utilization of available resources and materials.

The business assumptions also do not consider the inputs and constraints from process stages outside of batch mixing including, but not limited to, weighing, staging, packaging, transportation, and storage. These stages were removed from the model because it was assumed these factors would not be product or site dependent, however an outside analysis would have to be done to confirm this theory.

Areas for Future Work

Future research could build on this model by incorporating non-linear cost functions and dynamic pricing mechanisms to better reflect real-world complexities. Additionally, integrating stochastic elements, such as demand uncertainty or supply chain disruptions, could enhance the robustness of the model. Exploring the use of Monte Carlo simulations in combination with the linear programming approach could provide a more thorough understanding of potential outcomes under varying market conditions. For example, one could take sensitivity analysis outputs and explore the interaction of simulating how different improvements would lead to profitable gains, while also factoring in various risks. Another area could be to further analyze various solvers outputs and compare and contrast results, as done in other studies (Machado 2023). Overall, while this Model was deemed the best fit for the problem statement after a thorough literature review, there is still room for further validation, both from a methodology and optimization approach.

CB Corporation is missing profit opportunities by not offsetting its raw material cost at this time. According to the model, the revenue is being maximized within the set available trial time labor hours. However, cost is also high due to unused raw materials. In future trials, historical data should be used to forecast resource acquisition and allocation through quantitative models such as linear regression. The model inputs would include expected product-mix based on historical trends and expected raw material costs which may fluctuate over time. Without these models, profitability cannot be entirely maximized.

One of the goals for CB Corporation is to maximize efficiency of labor and resources to products chosen for sale and keep in line with internal sustainability targets. Part of this sustainability effort could be in recycling raw materials into the economy to offset internal costs and reduce waste. This can be done by selling unused raw materials to other companies that need it and potentially buying more needed raw materials if the model reports CB Corporation is constrained by its available inventory.

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Appendix

Base Model Code [GLPK]

```
import os

def find_glpsol():
    # Common directories to search
    possible_directories = [
        os.path.expanduser("~/AppData/Local/Anaconda3/Library/bin/"),  # 
Conda default location
        os.path.expanduser("~/AppData/Local/Anaconda3/envs/"),         # 
Conda environments
        "C:/Program Files/GLPK/bin/",                     # Typical Program 
Files location
        "C:/Program Files (x86)/GLPK/bin/",               # Another 
potential Program Files location
    ]

    # Search through Conda environments
    conda_env_path = os.path.expanduser("~/AppData/Local/Anaconda3/envs/")
    if os.path.exists(conda_env_path):
        for env in os.listdir(conda_env_path):
            possible_directories.append(os.path.join(conda_env_path, env, 
"Library/bin/"))

    # Check if glpsol.exe exists in any of the possible directories
    for directory in possible_directories:
        glpsol_path = os.path.join(directory, 
"glpsol.exe").replace("\\","/")
        if os.path.isfile(glpsol_path):
            return glpsol_path

    # Search system PATH as a last resort
    for path in os.environ["PATH"].split(os.pathsep):
        glpsol_path = os.path.join(path, "glpsol.exe")
        if os.path.isfile(glpsol_path):
            return glpsol_path

    return None
```

```
# Run the script
glpsol_path = find_glpsol()

if glpsol_path:
    print(f"Found glpsol.exe at: {glpsol_path}")
else:
    print("glpsol.exe not found on the system.")

#Original Model w/ Sensitivity Analysis Report
import pulp
from pulp import GLPK_CMD, LpProblem, LpMaximize, LpVariable, LpStatus

# Define raw materials and products
raw_materials = ['RSXX', 'STYY', 'CBAA', 'CBZZ', 'SBRXL', 'SLSM', 'CARBON
10', 'CARBON 20', 'CARBON 30', 'CARBON 40', 'CARBON 60']
products = ['Product 1', 'Product 2', 'Product 8B*', 'Product 3', 'Product
4', 'Product 4*', 'Product 10', 'Product 6', 'Product 9']

# Define data as arrays
raw_material_costs = [0.40, 0.60, 1.1, 1.1, 2.0, 2.0, 8.5, 5.0, 6.0, 3.5,
6.0]
product_prices = [90, 70, 100, 75, 80, 100, 150, 90, 120]
site1_inventory = [3000, 4000, 1000, 500, 200, 900, 1500, 2000, 800, 800,
500]
site2_inventory = [5000, 2500, 1750, 250, 400, 1200, 1250, 1900, 800, 800,
250]

# Define raw material usage per product (rows: raw materials, columns:
products)
raw_material_usage = [
    [0.50, 0.30, 0.00, 0.33, 0.25, 0.53, 0.53, 0.25, 0.66],  # RSXX
    [0.00, 0.00, 0.50, 0.00, 0.25, 0.00, 0.00, 0.25, 0.00],  # STYY
    [0.15, 0.12, 0.13, 0.13, 0.00, 0.00, 0.00, 0.15, 0.00],  # CBAA
    [0.00, 0.00, 0.00, 0.00, 0.13, 0.13, 0.13, 0.00, 0.00],  # CBZZ
    [0.00, 0.20, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00],  # SBRXL
    [0.00, 0.00, 0.00, 0.20, 0.00, 0.00, 0.00, 0.00, 0.00],  # SLSM
    [0.35, 0.00, 0.37, 0.00, 0.37, 0.34, 0.00, 0.00, 0.00],  # CARBON 10
    [0.00, 0.00, 0.00, 0.34, 0.00, 0.00, 0.00, 0.00, 0.00],  # CARBON 20
    [0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.35, 0.00],  # CARBON 30
```

```
    [0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.34, 0.00, 0.34],  # CARBON 40
    [0.00, 0.38, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00],  # CARBON 60
]

# Define labor hours per product at each site
site1_labor_hours = [0.25, 0.25, 0.35, 0.25, 0.25, 0.35, 0.25, 0.25, 0.20]
site2_labor_hours = [0.35, 0.35, 0.50, 0.35, 0.35, 0.50, 0.35, 0.35, 0.35]

# Define the problem
prob = pulp.LpProblem("Maximize_Profit", pulp.LpMaximize)

# Define decision variables for the quantity of each product to produce at
each site
product_vars_site1 = [pulp.LpVariable(f"Product{i+1}_Site1", lowBound=0,
cat='Continuous') for i in range(len(products))]
product_vars_site2 = [pulp.LpVariable(f"Product{i+1}_Site2", lowBound=0,
cat='Continuous') for i in range(len(products))]

# Calculate profit per product per site
profit_values_site1 = []
profit_values_site2 = []

for i in range(len(products)):
    cost = 15 * sum(raw_material_costs[j] * raw_material_usage[j][i] for j
in range(len(raw_materials)))
    profit_values_site1.append(product_prices[i] - cost)
    profit_values_site2.append(product_prices[i] - cost)

profits_site1 = [product_vars_site1[i] * profit_values_site1[i] for i in
range(len(products))]
profits_site2 = [product_vars_site2[i] * profit_values_site2[i] for i in
range(len(products))]

# Objective function
prob += pulp.lpSum(profits_site1 + profits_site2), "Total_Profit"

# Calculate total labor hours per product per site
labor_hours_site1 = [product_vars_site1[i] * site1_labor_hours[i] for i in
range(len(products))]
```

```
labor_hours_site2 = [product_vars_site2[i] * site2_labor_hours[i] for i in
range(len(products))]

# Sum the total labor hours for each site
sum_labor_site1 = pulp.lpSum(labor_hours_site1)
sum_labor_site2 = pulp.lpSum(labor_hours_site2)

# Add labor hours constraints
prob += sum_labor_site1 <= 168, "Site1_Labor_Constraint"
prob += sum_labor_site2 <= 168, "Site2_Labor_Constraint"

# Calculate inventory constraints per raw material per site
for i in range(len(raw_materials)):
    # Constraint for Site 1
    prob += pulp.lpSum(product_vars_site1[j] * raw_material_usage[i][j] *
15 for j in range(len(products))) <= site1_inventory[i],
f"Site1_{raw_materials[i]}_Constraint"

    # Constraint for Site 2
    prob += pulp.lpSum(product_vars_site2[j] * raw_material_usage[i][j] *
15 for j in range(len(products))) <= site2_inventory[i],
f"Site2_{raw_materials[i]}_Constraint"

# Solve the problem using GLPK
glpk_path =
"C:/Users/hali/AppData/Local/Anaconda3/envs/myenv/Library/bin/glpsol.exe"
status = prob.solve(GLPK_CMD(path=glpk_path, options=['--ranges',
'Product_Optimization.sen']))

# Output results
optimal_values_site1 = [product_vars_site1[i].varValue for i in
range(len(products))]
optimal_values_site2 = [product_vars_site2[i].varValue for i in
range(len(products))]
objective_value = pulp.value(prob.objective)

# Calculate inventory used per raw material
inventory_used_site1 = {raw_materials[i]: sum(optimal_values_site1[j] *
raw_material_usage[i][j] * 15 for j in range(len(products))) for i in
range(len(raw_materials))}
```

```
inventory_used_site2 = {raw_materials[i]: sum(optimal_values_site2[j] *
raw_material_usage[i][j] * 15 for j in range(len(products))) for i in
range(len(raw_materials))}

# Calculate total labor used
total_labor_used_site1 = sum([optimal_values_site1[i] *
site1_labor_hours[i] for i in range(len(products))])
total_labor_used_site2 = sum([optimal_values_site2[i] *
site2_labor_hours[i] for i in range(len(products))])

# Print results
print("Optimal production values for Site 1:", optimal_values_site1)
print("Optimal production values for Site 2:", optimal_values_site2)
print("Total Profit:", objective_value)

print("Inventory used per raw material at Site 1:")
for material, usage in inventory_used_site1.items():
    print(f"{material}: {usage}")

print("Inventory used per raw material at Site 2:")
for material, usage in inventory_used_site2.items():
    print(f"{material}: {usage}")

print("Total labor used at Site 1:", total_labor_used_site1)
print("Total labor used at Site 2:", total_labor_used_site2)

# To extract sensitivity analysis, read the generated .sen file
try:
    with open("Product_Optimization.sen", "r") as f:
        sensitivity_analysis = f.read()
        print("\nSensitivity Analysis:\n")
        print(sensitivity_analysis)
except FileNotFoundError:
    print("Sensitivity analysis file not found. Ensure GLPK has generated
the sensitivity report.")
```

Sensitivity Analysis Report

```
GLPK 5.0  - SENSITIVITY ANALYSIS REPORT                                                         Page   1

Problem:
Objective:  Total_Profit = 75203.0601 (MAXimum)

   No. Row name     St      Activity         Slack   Lower bound       Activity      Obj coef  Obj value at Limiting
                                          Marginal   Upper bound          range         range   break point variable
------ ------------ -- ------------- ------------- -------------  ------------- ------------- ------------- ------------
     1 Site1_CARBON_10_Constraint
                    BS     491.83659    1008.16341          -Inf            .        -8.32072   71110.62520 Site1_Labor_Constraint
                                              .       1500.00000     1640.61465        .37937   75389.64850 Product1_Site1

     2 Site1_CARBON_20_Constraint
                    BS    1217.31602     782.68398          -Inf      149.00394       -.35801   74767.24732 Product1_Site1
                                              .       2000.00000     1530.00000       3.73972   79755.48102 Site1_CARBON_30_Constraint

     3 Site1_CARBON_30_Constraint
                    NU     800.00000          .             -Inf      375.11765      -2.75217   74033.70959 Site1_SLSM_Constraint
                                           2.75217     800.00000     1027.96137          +Inf   75830.44964 Site1_CBAA_Constraint

     4 Site1_CARBON_40_Constraint
                    NU     800.00000          .             -Inf      757.88524     -16.38779   74512.89251 Site1_CBAA_Constraint
                                          16.38779     800.00000     1307.69231          +Inf   83523.01269 Site1_CBZZ_Constraint

     5 Site1_CARBON_60_Constraint
                    BS          .        500.00000          -Inf            .            -Inf   75203.06010
                                              .        500.00000      380.00000       2.24108   75203.06010 Product2_Site1

     6 Site1_CBAA_Constraint
                    BS     981.10895      18.89105          -Inf      697.91568      -1.79857   73438.46738 Product6_Site1
                                              .       1000.00000     1068.43935      36.53411  111047.00170 Site1_CARBON_40_Constraint

     7 Site1_CBZZ_Constraint
                    BS     305.88235     194.11765          -Inf            .       -12.59223   71351.31916 Product9_Site1
                                              .        500.00000      595.68733       2.62389   76005.66090 Product6_Site1

     8 Site1_Labor_Constraint
                    NU     168.00000          .             -Inf      136.98328    -131.94286   71110.62520 Product3_Site1
                                         131.94286     168.00000      171.39070          +Inf   75650.43885 Site1_CBAA_Constraint

     9 Site1_RSXX_Constraint
                    NU    3000.00000          .             -Inf     1818.48739      -1.29076   73678.00523 Product4_Site1
                                           1.29076    3000.00000     3167.83969          +Inf   75419.70166 Site1_CBAA_Constraint

    10 Site1_SBRXL_Constraint
                    BS          .        200.00000          -Inf            .            -Inf   75203.06010
                                              .        200.00000      263.15789       4.25805   75203.06010 Product2_Site1
```

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GLPK 5.0  - SENSITIVITY ANALYSIS REPORT                                                         Page   2

Problem:
Objective:  Total_Profit = 75203.0601 (MAXimum)

   No. Row name     St      Activity         Slack   Lower bound       Activity      Obj coef  Obj value at Limiting
                                          Marginal   Upper bound          range         range   break point variable
------ ------------ -- ------------- ------------- -------------  ------------- ------------- ------------- ------------
    11 Site1_SLSM_Constraint
                    BS     716.06825     183.93175          -Inf       87.64938       -.60862   74767.24732 Product1_Site1
                                             .         900.00000     1062.38859       6.35752   79755.48102 Site1_CARBON_30_Constraint

    12 Site1_STYY_Constraint
                    BS    1236.07261    2763.92739          -Inf     1037.68908      -5.89439   67917.16243 Site1_CARBON_30_Constraint
                                             .        4000.00000     1617.61264       1.00243   76442.13806 Product1_Site1

    13 Site2_CARBON_10_Constraint
                    BS     896.47059     353.52941          -Inf           .          -.10000   75113.41304 Product4_Site2
                                             .        1250.00000     1696.47059       2.35714   77316.16934 Site2_CARBON_30_Constraint

    14 Site2_CARBON_20_Constraint
                    BS         .        1900.00000          -Inf           .             -Inf   75203.06010
                                             .        1900.00000      870.85714        .10294   75203.06010 Product4_Site2

    15 Site2_CARBON_30_Constraint
                    NU     800.00000          .             -Inf      446.47059      -2.35714   74369.74077 Site2_CARBON_10_Constraint
                                           2.35714     800.00000     1696.47059          +Inf   77316.16934 Product1_Site2

    16 Site2_CARBON_40_Constraint
                    NU     800.00000          .             -Inf      653.84615     -11.42941   73532.60761 Product9_Site2
                                          11.42941     800.00000     1670.85714          +Inf   85156.44497 Product1_Site2

    17 Site2_CARBON_60_Constraint
                    BS         .         250.00000          -Inf           .             -Inf   75203.06010
                                             .         250.00000      760.00000       2.43509   75203.06010 Product2_Site2

    18 Site2_CBAA_Constraint
                    BS     727.05882    1022.94118          -Inf      675.83193      -1.75000   73930.70716 Product4_Site2
                                             .        1750.00000      791.53846      25.90667   94038.73069 Site2_CARBON_40_Constraint

    19 Site2_CBZZ_Constraint
                    NU     250.00000          .             -Inf           .        -14.68462   71531.90625 Product7_Site2
                                          14.68462     250.00000      305.88235          +Inf   76023.67096 Product9_Site2

    20 Site2_Labor_Constraint
                    NU     168.00000          .             -Inf      108.23529    -114.00000   68389.88363 Product1_Site2
                                         114.00000     168.00000      191.56863          +Inf   77889.88363 Site2_CARBON_10_Constraint
```

```
GLPK 5.0  - SENSITIVITY ANALYSIS REPORT                                                         Page   3

Problem:
Objective:  Total_Profit = 75203.0601 (MAXimum)

   No. Row name     St      Activity         Slack   Lower bound       Activity      Obj coef  Obj value at Limiting
                                          Marginal   Upper bound          range         range   break point variable
------ ------------ -- ------------- ------------- -------------  ------------- ------------- ------------- ------------
    21 Site2_RSXX_Constraint
                    BS    3155.04202    1844.95798          -Inf     2719.61345       -.20588   74553.49262 Product4_Site2
                                             .        5000.00000     3407.56303       3.30000   85614.69875 Site2_CARBON_30_Constraint

    22 Site2_SBRXL_Constraint
                    BS         .         400.00000          -Inf           .             -Inf   75203.06010
                                             .         400.00000      131.57895       4.62667   75203.06010 Product2_Site2

    23 Site2_SLSM_Constraint
                    BS         .        1200.00000          -Inf           .             -Inf   75203.06010
                                             .        1200.00000      512.26891        .17500   75203.06010 Product4_Site2

    24 Site2_STYY_Constraint
                    BS     571.42857    1928.57143          -Inf      318.90756      -3.30000   73317.34581 Site2_CARBON_30_Constraint
                                             .        2500.00000     1467.89916       1.44267   76027.44105 Product3_Site2
```

```
GLPK 5.0  - SENSITIVITY ANALYSIS REPORT                                                         Page   4

Problem:
Objective:  Total_Profit = 75203.0601 (MAXimum)

   No. Column name  St      Activity      Obj coef   Lower bound       Activity      Obj coef  Obj value at Limiting
                                          Marginal   Upper bound          range         range   break point variable
------ ------------ -- ------------ ------------ ------------  ------------ ------------ ------------ ------------
     1 Product1_Site1
                    NL            .      39.90000            .      -40.46499         -Inf   75315.00447 Site1_SLSM_Constraint
                                         -2.76645         +Inf      138.25215     42.66645   74820.59240 Site1_CARBON_10_Constraint

     2 Product1_Site2
                    BS     170.75630     39.90000            .     -201.79272     39.37500   75113.41304 Product4_Site2
                                                .         +Inf      238.09524     52.27500   77316.16934 Site2_CARBON_30_Constraint

     3 Product2_Site1
                    NL            .      26.02000            .      -67.44164         -Inf   76064.57016 Site1_SLSM_Constraint
                                        -12.77416         +Inf       66.66667     38.79416   74351.44971 Site1_SBRXL_Constraint

     4 Product2_Site2
                    NL            .      26.02000            .      -67.33894         -Inf   76137.72452 Site2_CARBON_10_Constraint
                                        -13.88000         +Inf       43.85965     39.90000   74594.28817 Site2_CARBON_60_Constraint

     5 Product3_Site1
                    BS      88.61921     46.18000            .           -Inf            .   71110.62520 Site1_Labor_Constraint
                                                .         +Inf      139.49121     53.69824   75869.32013 Product1_Site1

     6 Product3_Site2
                    NL            .      46.18000            .     -172.19608         -Inf   77066.22167 Site2_RSXX_Constraint
                                        -10.82000         +Inf      119.52941     57.00000   73909.75186 Product1_Site2

     7 Product4_Site1
                    BS     238.68942     39.37500            .       29.21646     37.54914   74767.24732 Product1_Site1
                                                .         +Inf      300.00000     58.44757   79755.48102 Site1_CARBON_30_Constraint

     8 Product4_Site2
                    NL            .      39.37500            .      -67.33894         -Inf   75238.41304 Site2_CARBON_10_Constraint
                                          -.52500         +Inf      170.75630     39.90000   75113.41304 Product1_Site2

     9 Product5_Site1
                    NL            .      26.93000            .      -10.40866         -Inf   75316.47374 Site1_CBAA_Constraint
                                        -10.89608         +Inf       99.54751     37.82608   74118.38223 Site1_CBZZ_Constraint

    10 Product5_Site2
                    NL            .      26.93000            .      -28.65762         -Inf   76395.36025 Product9_Site2
                                        -41.60500         +Inf      128.20513     68.53500   69869.08574 Product7_Site2
```

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GLPK 5.0  - SENSITIVITY ANALYSIS REPORT                                                         Page   5

Problem:
Objective:  Total_Profit = 75203.0601 (MAXimum)

   No. Column name  St      Activity      Obj coef   Lower bound       Activity      Obj coef  Obj value at Limiting
                                          Marginal   Upper bound          range         range   break point variable
------ ------------ -- ------------- ------------- -------------  ------------- ------------- ------------- ------------
    11 Product6_Site1
                    NL          .         51.32500          .          -6.64054          -Inf   75237.03696 Site1_CBAA_Constraint
                                          -5.11658          +Inf       99.54751      56.44158   74693.71728 Site1_CBZZ_Constraint

    12 Product6_Site2
                    NL          .         51.32500          .         -28.65762          -Inf   76186.30293 Product9_Site2
                                         -34.31000          +Inf      119.52941      85.63500   71102.00598 Product1_Site2

    13 Product7_Site1
                    BS     156.86275     126.82500          .        -162.83190     102.27015   71351.31916 Product9_Site1
                                              .             +Inf      156.86275          +Inf          +Inf

    14 Product7_Site2
                    BS     128.20513     126.82500          .        -817.92717      98.19000   71531.90625 Site2_CBZZ_Constraint
                                              .             +Inf      128.20513          +Inf          +Inf

    15 Product8_Site1
                    BS     152.38095      52.27500          .          71.45098      37.82608   73001.32025 Site1_CARBON_30_Constraint
                                              .             +Inf      152.38095          +Inf          +Inf

    16 Product8_Site2
                    BS     152.38095      52.27500          .          85.04202      39.90000   73317.34581 Site2_CARBON_30_Constraint
                                              .             +Inf      152.38095          +Inf          +Inf

    17 Product9_Site1
                    NL          .         98.19000          .         -99.54751          -Inf   77647.43415 Site1_CBZZ_Constraint
                                         -24.55485          +Inf      156.86275     122.74485   71351.31916 Product7_Site1

    18 Product9_Site2
                    BS      28.65762      98.19000          .         -38.68132      39.90000   73532.60761 Site2_CARBON_40_Constraint
                                              .             +Inf      156.86275     126.82500   76023.67096 Site2_CBZZ_Constraint
```