MATH 165
Linear Algebra & Diff. Equation
Final
Notes with Examples

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Chapter 1

Linear Transformations

1.1 Lecture 18 & 19: Kernel, Range, Eigenvalues

This lecture covers:

- *6.1 Definition of Linear Transformations*
- *6.2 Transformations of $\mathbb{R}^2$*
- *6.3 The Kernel and Range of a Linear Transformation*

1.1.1 Definition of Linear Transformations

Definition 1.1.1: Mapping

Let V and W be vector spaces. A **mapping** T from V to W is a rule that assigns to each vector $\vec{v}$ in V precisely one vector $\vec{w} = T(\vec{v})$. We write $T : V \to W$.

Definition 1.1.2: Linear Transformation

Let V and W be vector spaces over the same field. A mapping $T : V \to W$ is a **linear transformation** if for all $\vec{v}_1, \vec{v}_2 \in V$ and all scalars c:

1. $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$ for all $\vec{u}, \vec{v} \in V$
2. $T(c\vec{v}) = cT(\vec{v})$ for all $\vec{v} \in V$

In the above equations, the operations on the left of the equal signs are the ones defined in the domain V and the ones on the right of the equal signs are the ones defined in the codomain W.

Theorem 1.1.3

Let V, W be vector spaces over field F. A mapping $T : V \to W$ is a linear transformation if and only if for all $\lambda_1, \lambda_2 \in F$ and all $\vec{v}_1, \vec{v}_2 \in V$:

$$T(\lambda_1 \vec{v}_1 + \lambda_2 \vec{v}_2) = \lambda_1 T(\vec{v}_1) + \lambda_2 T(\vec{v}_2)$$

Example.

> Show $T : P_2 \to P_4$ given by $T(p) = x^2 p(x)$ is linear.