Chapter 4 homework

1. Show whether the following series are divergent or convergent or absolutely convergent.

   (a) $\sum_{n=1}^{\infty} \frac{i^n}{n}$;

   (b) $\sum_{n=1}^{\infty} \frac{(3+5i)^n}{n!}$.

2. Is the following statements correct? Why?

   (a) Every power series converges everywhere on its circle of convergence;

   (b) The sum function of each power series may have singularity inside the circle of convergence;

   (c) Every function that is continuous at z_0 must be expanded into a Taylor series in the neighborhood of z_0.

3. Find the radius of convergence of the following series.

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[n]{n!}} z^n$;

   (b) $\sum_{n=1}^{\infty} \frac{z^n}{n^2 2^n}$;

   (c) $\sum_{n=1}^{\infty} e^{i\frac{\pi}{n}} z^n$.

4. Let $\sum_{n=0}^{\infty} c_n$ converges and $\sum_{n=0}^{\infty} |c_n|$ diverges, and prove that the radius of convergence of $\sum_{n=0}^{\infty} c_n z^n$ is 1.

5. If $\sum_{n=0}^{\infty} c_n z^n$ is absolutely convergent at a point z_0 on the circumference of its convergence circle, proved that it is absolutely convergent in the domain surrounded by the convergence circle.

6. Expand the following functions into power series of z and determine the radius of convergence.

   (a) $\dfrac{1}{1+z^3}$;

   (b) $\dfrac{1}{(1-z)^2}$;

   (c) $\cos z^2$;

   (d) $\sin \dfrac{1}{1-z}$.

7. Find the Taylor series of the following functions at the point z_0 and determine the radius of convergence.

   (a) $\dfrac{z-1}{z+1}, \quad z_0 = 1$;

   (b) $\dfrac{z}{(z+1)(z+2)}, \quad z_0 = 2$;

   (c) $\dfrac{1}{z^2-z-2}, \quad z_0 = 0$;

   (d) $\dfrac{z^2}{(1+z)^2}, \quad z_0 = 1$.

8. Find the Laurent series of the following functions in the specified annulus domain.

   (a) $\dfrac{1}{1+z} e^{\frac{1}{1+z}}, \quad 1 < |z+1| < \infty$;

   (b) $\dfrac{1}{(z+i)(z-2)}, \quad 1 < |z| < 2; 2 < |z| < \infty$;

   (c) $z^2 e^{\frac{1}{z}}, \quad 0 < |z| < \infty$.

9. Find the Laurent series of $f(z) = \dfrac{1}{z^2-3z+2}$ in every annulus centered on the origin.