Probability Theory & Statistics

Instructor: Changsheng CHEN

Course Hour: 8:30am-11:40am, Mon

Q&A Hour: 2:00pm-4:00pm, Mon

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群聊：2024-2025 概率论与数理统计

Please show your name.

About You

- Wenhua Honors Class

文华班旨在构建服务国家战略、对标湾区（粤港澳大湾区）一流、突出深大特色的拔尖创新人才培养新模式，培养人格健全、视野宽广、意识领先、素质优良、能力突出，且电子信息理论及应用基础扎实、熟悉学科前沿领域，能够从事电子信息领域研究的拔尖创新人才。

About Me

Bachelor in SYSU (04-08)

Ph.D. in NTU (08-13)

PostDoc in HKUST (13-15)

Now at SZU (since 16)

Instructor of “概率论与数理统计”

(Top 30% for 7 years straight)

Instructor of “Prob. & Stat.” in Wenhua Honors Class

(Top 4.93% for last year)

About Our Course

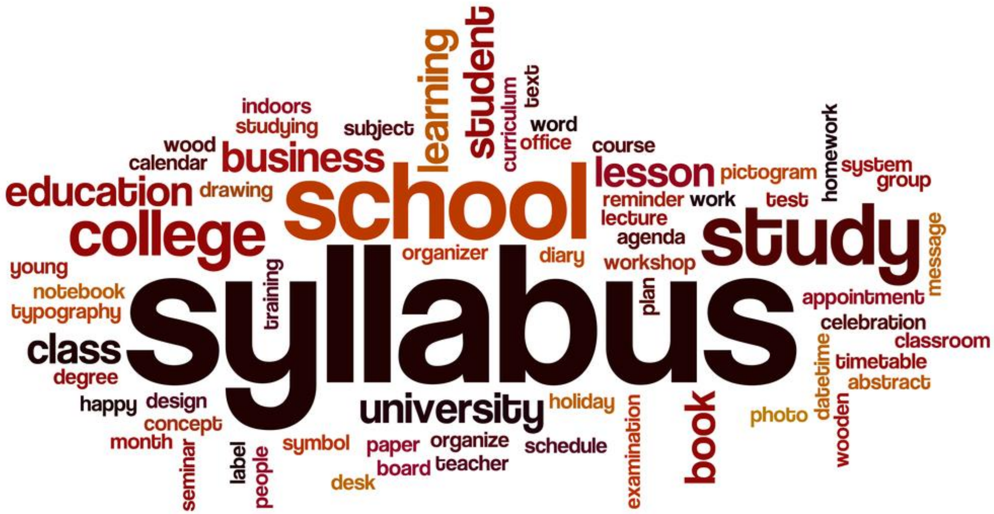
Course evaluation:

- Final exam: 40%
- Quiz: 30% (~every 4 weeks)
- Course projects: 30% (4 small projects in Python)

- Homework?
- Take attendance?

Issues in Course Development

In which way should "Probability Theory & Statistics" differ from "概率论与数理统计"?



- Slides, textbook;
- No experiment (3+0) vs. Python experiments (2+2 or 3+1);
- International syllabus vs. domestic syllabus???

This version of Probability Theory & Statistics

Compared to “概率论与数理统计”

- New slides, New textbook;
- New Python experiments, between 2+2 and 3+1;
- Domestic syllabus with international flavor.

International

Domestic

Probability & Statistics

70:30 in course hours & final credits

Why Probability & Statistics?

- Certainty vs. Uncertainty

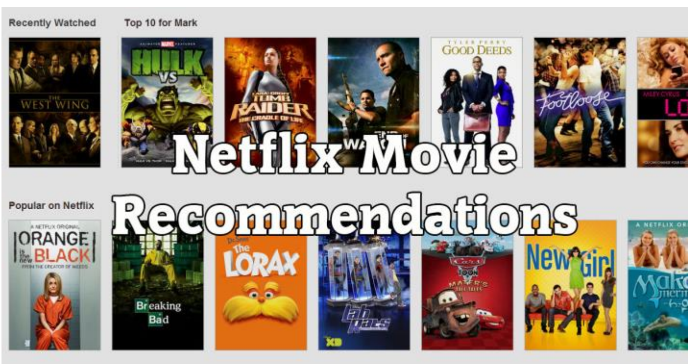
Probability in Navigation

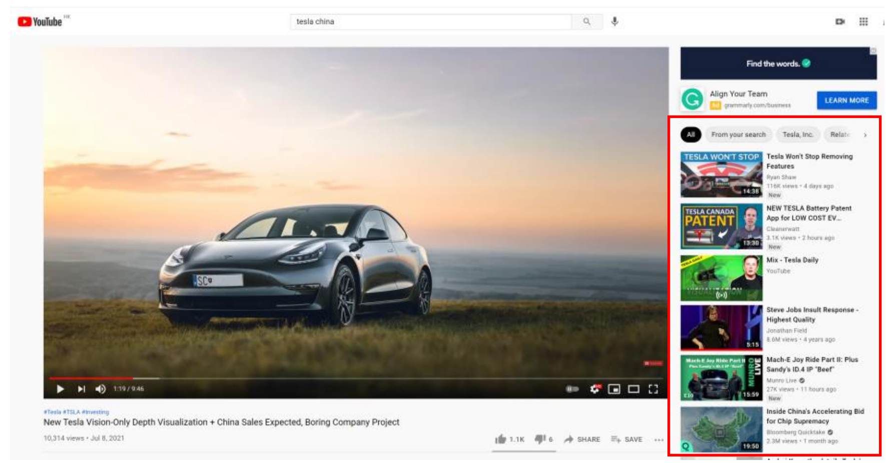
The shortest path

The fastest path

Why Probability & Statistics?

- Certainty vs. Uncertainty



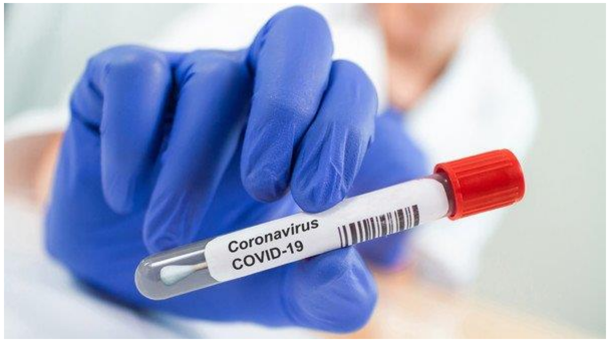


Query of “superman” in movie recommendations:

- Find all related movies (certain);
- Find the 5 most relevant movies (uncertain).

Why Probability & Statistics?

A patient takes a virus test that returns positive. What is the probability that they have the virus?



Correct answer: 0.42%

by Bayes rule

Make decision based on probability prior(先验)!

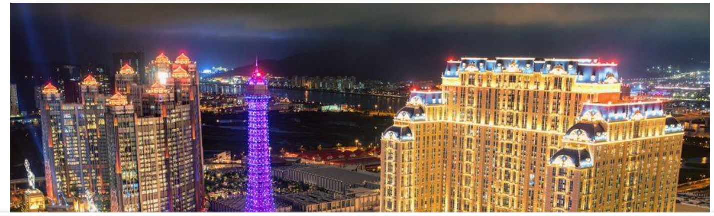
- 0.03% of people have the virus
- Test has a 99% positive rate for people with the virus
- Test has a 7% positive rate for people without the virus

Why Probability & Statistics?

Country/Economy	GDP per capita (Nominal) ($)		Rank (Nominal)		GDP per capita (ppp) (Int. $)		Rank (PPP)		Nominal-PPP
	2018	2019	2019	Change	2018	2019	2019	Change	2019
Luxembourg	116,654	114,705	1	0	116,786	121,293	2	0	-1
Macao SAR, China	87,209	84,096	2	0	135,021	129,103	1	0	+1
Switzerland	82,818	81,994	3	0	69,357	70,989	6	0	-3
Ireland	78,621	78,661	4	1	84,460	88,241	5	0	-1
Norway	81,734	75,420	5	-1	67,640	66,832	8	0	-3
Iceland	72,969	66,945	6	0	57,742	60,061	13	0	-7
United States	62,997	65,281	7	2	62,997	65,281	9	0	-2
Singapore	66,189	65,233	8	0	100,051	101,376	3	0	+5
Qatar	68,794	64,782	9	-2	96,733	96,491	4	0	+5
Denmark	61,391	59,822	10	0	57,218	59,830	14	1	-4
Australia	57,396	54,907	11	0	51,036	53,320	21	1	-10
Netherlands	53,048	52,448	12	1	57,565	59,687	15	-1	-3
Sweden	54,589	51,610	13	-1	53,747	55,815	18	0	-5

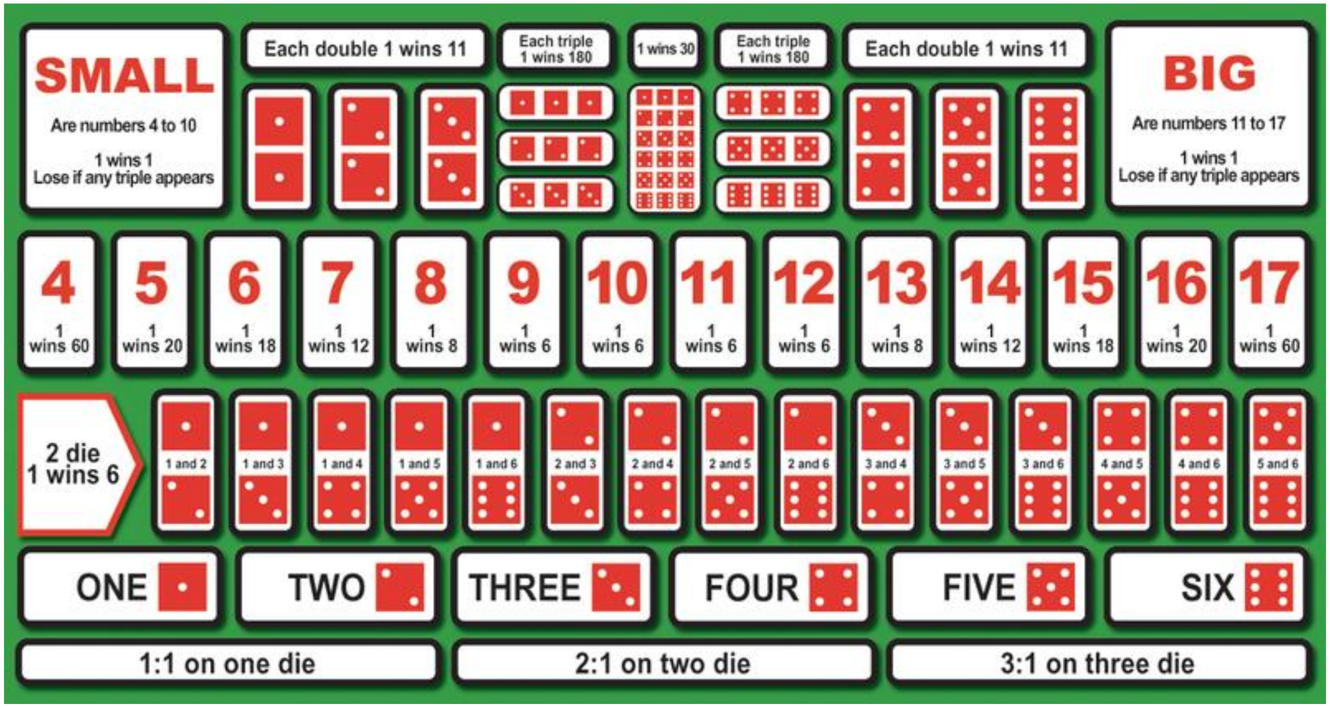
https://statisticstimes.com/economy/world-gdp-capita-ranking.php

Why Probability & Statistics?



Probability in gambling

Why Probability & Statistics?



[Big and small](#) is an unequal game of chance of ancient Chinese origin played with three dice.

a.k.a. Sic bo (骰宝), tai sai (大细), dai siu (大小).

Why Probability & Statistics?

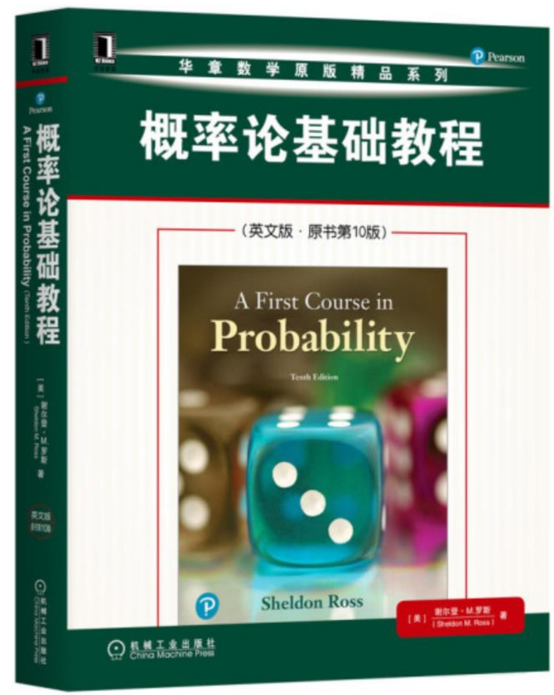
Betting options [edit]

Type	Wager	Probability	United Kingdom		New Zealand		Macau		No House Edge
			Odds	House Edge	Odds	House Edge	Odds	House Edge	Odds
Big (大)	The total score will be from 11 to 17 (inclusive) with the exception of a triple	48.61%	1 to 1	2.78%	1 to 1	2.78%	1 to 1	2.78%	37 to 35
Small (小)	The total score will be from 4 to 10 (inclusive) with the exception of a triple	48.61%	1 to 1	2.78%	1 to 1	2.78%	1 to 1	2.78%	37 to 35
Odd	The total score will be an odd number with the exception of a triple	48.61%	1 to 1	2.78%	1 to 1	2.78%			37 to 35
Even	The total score will be an even number with the exception of a triple	48.61%	1 to 1	2.78%	1 to 1	2.78%			37 to 35

Probability = Important
+ Needs Studying

Textbook

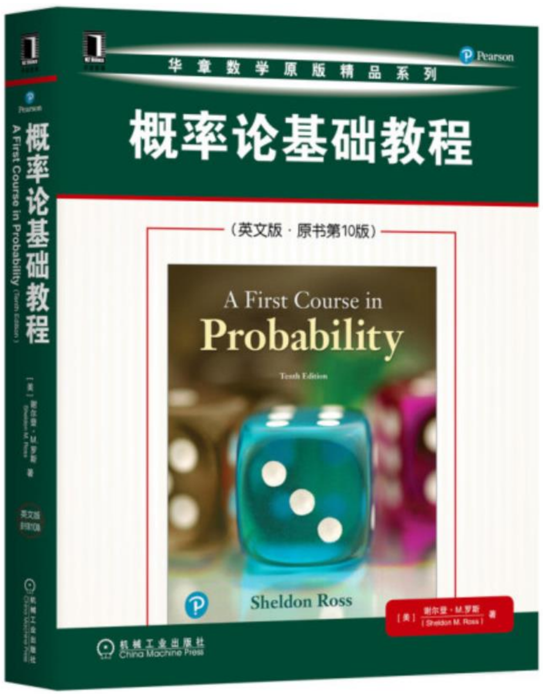
The PDF version is available on blackboard.



- 概率论基础教程（英文版·原书第10版）
出版社： 机械工业出版社
- A First Course in Probability,
出版社： Pearson

Textbook

Only the first part (probability theory) is covered in our textbook.



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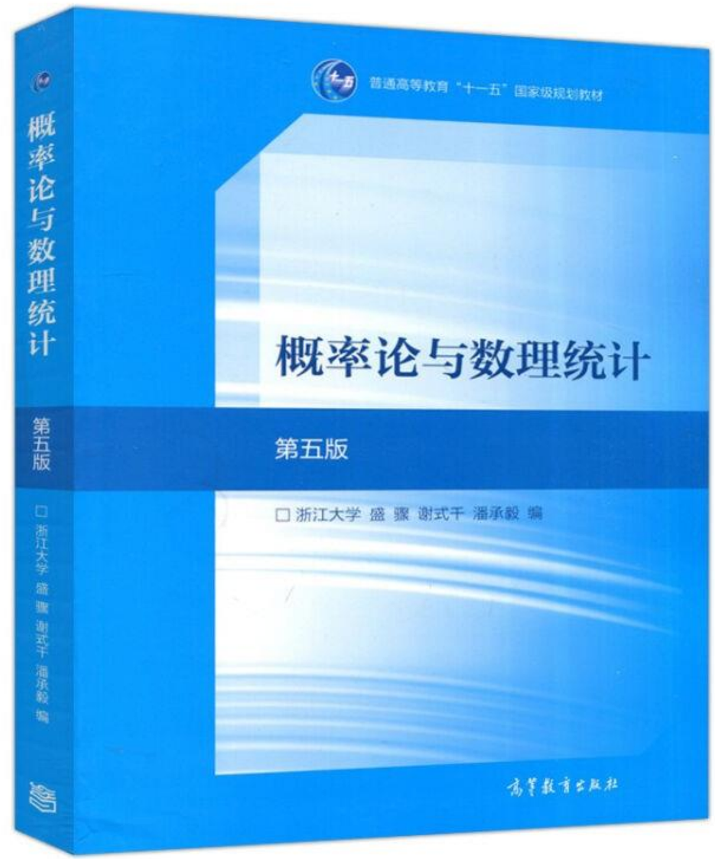
Skill with English textbook?

- Textbook is only for your reference.
- Trust yourself. We all begin with 2 pages/hour.
- English textbook is much clearer and more intuitive!
- You may use some reference materials in Chinese.

Other reference materials



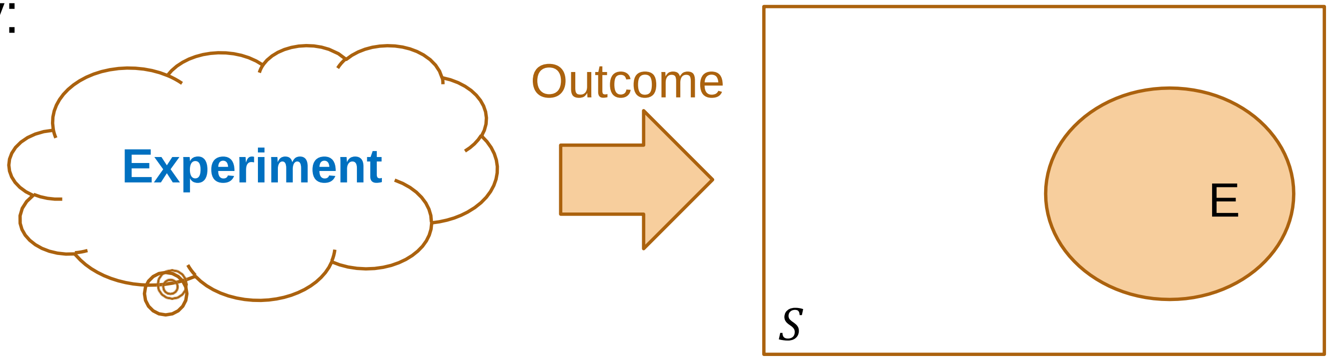
PDF versions are available on blackboard.

Lecture 01: Intro. to Probability

- Sample space and event
- What is probability?
- Set operations between events
- Laws of set operations
- Simple propositions of probability

Key definitions

An experiment in probability:



Sample Space, S: The set of all possible **outcomes** of an experiment

样本空间

Event, E: Some subset of S ($E \subseteq S$).

随机事件

Scientific experiment: Repeatability, Observability, Uncertainty

Sample Space and Event

Sample Space, S

- Coin flip
  $S = \{\text{Heads}, \text{Tails}\}$
- Flipping two coins
  $S = \{(H,H),(H,T),(T,H),(T,T)\}$
- Roll of a 6-sided die (骰子)
  $S = \{1,2,3,4,5,6\}$
- # emails in a day
  $S = \{x \mid x \in \mathbb{Z}, x \geq 0\}$
- TikTok hours in a day
  $S = \{x \mid x \in \mathbb{R}, 0 \leq x \leq 24\}$

Event, E

- Flip lands heads
  $E = \{\text{Heads}\}$
- ≥ 1 head on 2 coin flips
  $E = \{(H,H),(H,T),(T,H)\}$
- Roll is 3 or less:
  $E = \{1,2,3\}$
- Low email day (≤ 20 emails)
  $E = \{x \mid x \in \mathbb{Z}, 0 \leq x \leq 20\}$
- Wasted day (≥ 5 TT hours):
  $E = \{x \mid x \in \mathbb{R}, 5 \leq x \leq 24\}$

Event

- An event (denoted by letter A, B, or C…) is a subset of the sample space.

- An event occurs/happens: one sample in an event (set) occurs.

- Different types of event:
  1) Elementary/fundamental event (基本事件).
  2) Certain event (必然事件), S.
  3) Impossible event (不可能事件), ∅.

What is probability?

- The quality or state of being probable; the extent to which something is likely to happen or be the case.

--------Definitions from Oxford Dictionary

- Def. Probability is the *long run* relative frequency:

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

n: # total trials, $n(E)$: # occurrences of event E in n trials.

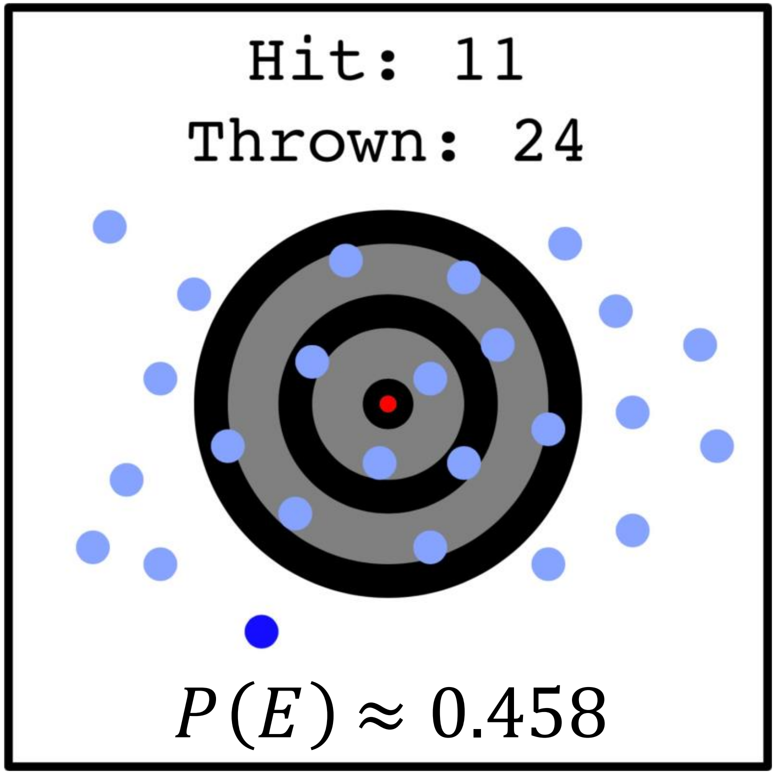
What is probability?

Let E = the set of outcomes where you hit the target.

$$P(E) = \lim_{n\to\infty} \frac{n(E)}{n}$$

n: # total trials,

$n(E)$: # occurrences of event E in n trials.



Lecture 01: Intro. to Probability

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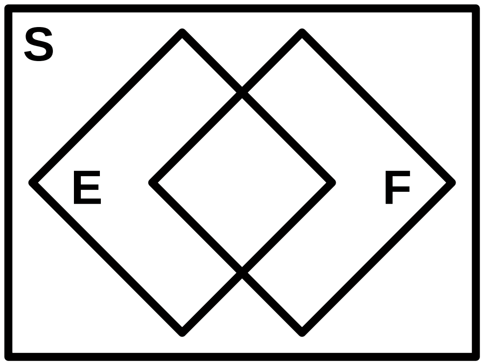
- Set operations between events
- Laws of set operations
- Simple propositions of probability

Set operations between Events



A die-rolling experiment.

$S = \{1, 2, 3, 4, 5, 6\}$,
let $E = \{1, 2\}$, $F = \{2, 3\}$,
$H = \{1, 2\}$.

- $E, F \subset S$ (子集), E or F occurs $\Rightarrow S$ occurs
- $\phi \subset E, F, S$
- $E \subset H$ and $E \supset H$ (事件的相等, $E = H$)

Three Axioms (公理) of Probability

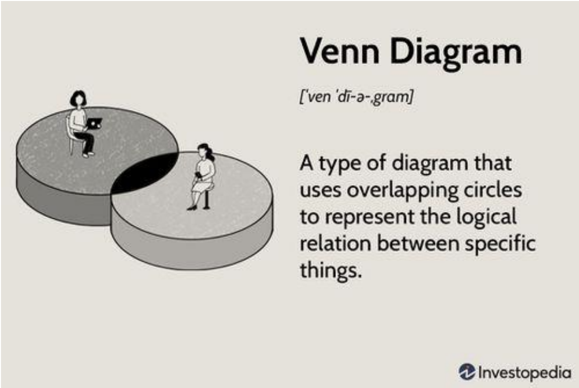
- Axiom 1: $0 \leq P(E) \leq 1$

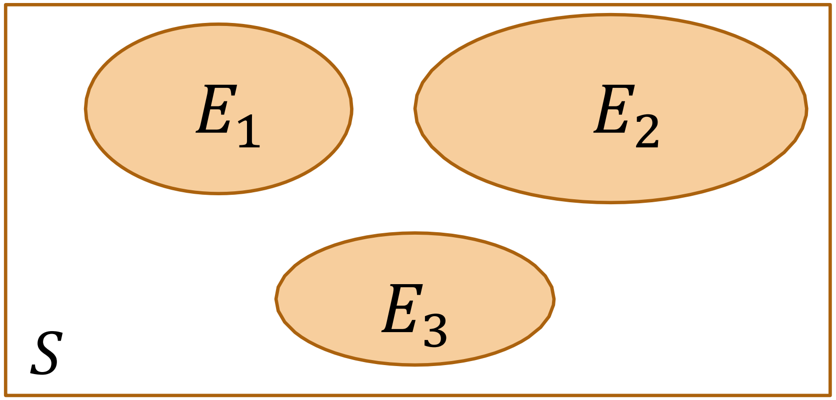
- Axiom 2: $P(S) = 1$

- Axiom 3:

$P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$,

for any mutually exclusive (互斥) sets E_i .



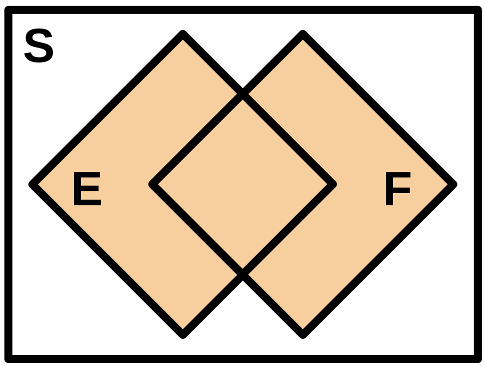


Venn diagram (文氏图)

Math Terminology

- Axiom： 公理. Given as it is, no proof is required.
- Theorem： 定理. Need to be proved.
- Lemma： 引理. Critical step in proving Theorem.
- Corollary： 推论. Provable from Theorem.
- Proposition： 命题. Similar to Theorem. Regarded as a theorem of less importance.

Set operations between Events



A die-rolling experiment.

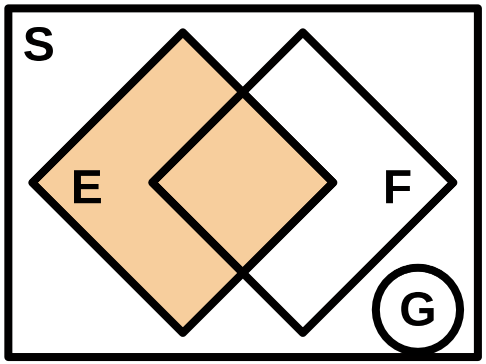
$S = \{1, 2, 3, 4, 5, 6\}$,
let $E = \{1, 2\}$, $F = \{2, 3\}$.

Def. Union of events(和事件)

$$E \cup F = \{1,2,3\}$$

The event containing all outcomes in E or F.

Set operations between Events



A die-rolling experiment.

$S = \{1, 2, 3, 4, 5, 6\}$,
let $E = \{1, 2\}$, $F = \{2, 3\}$,
and $G = \{4, 5\}$.

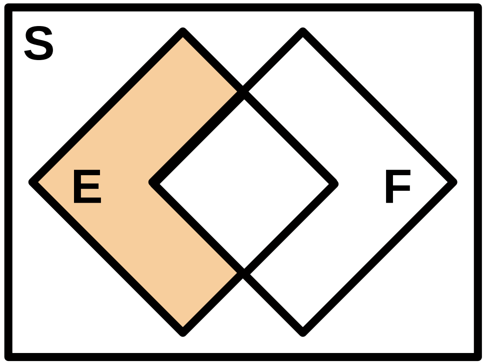
Def. Intersection of events (积事件).
The event containing all outcomes in E and F.

$$E \cap F = EF = \{2\}$$

Def. Mutually exclusive events (互斥事件) do not happen simultaneously.

$$F \cap G = \emptyset$$

Set operations between Events



A die-rolling experiment.

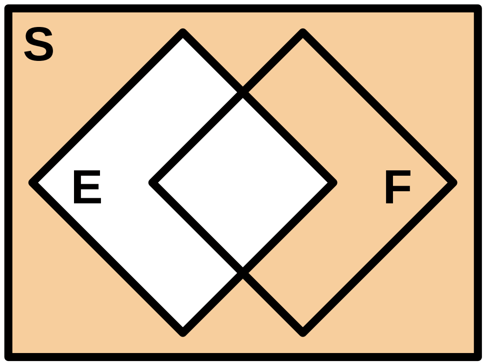
$S = \{1, 2, 3, 4, 5, 6\}$,
let $E = \{1, 2\}$, $F = \{2, 3\}$.

Def. Difference of events(差事件).

$$E - F = \{1\}$$

The event containing all outcomes in E but not in F.

Set operations between Events



A die-rolling experiment.

$S = \{1, 2, 3, 4, 5, 6\}$,
let $E = \{1, 2\}$, $F = \{2, 3\}$.

Def. Complement of event (逆事件) E,

The event containing all outcomes in the sample space S that are *not* in E.

$$E^C = \overline{E}$$
$$= S - E$$
$$= \{3,4,5,6\}$$

Ex. Consider a product inspection procedure for a large number of products. Let A_k denotes the event "a qualified product is obtained for the k-th inspection test"($k = 1,2,3$). Write the following events in terms of A_k:

(1) A: "Qualified products were obtained for all three inspections";

(2) B: "Qualified products were obtained at least once in three inspections";

(3) C: "Qualified products were obtained exactly twice out of the three inspections".

(4) D: "Qualified products were obtained at most once in three inspections".

(1) A: "Qualified products were obtained for all three inspections";

(2) B: "Qualified products were obtained at least once in three inspections";

(3) C: "Qualified products were obtained exactly twice out of the three inspections".

(4) D: "Qualified products were obtained at most once in three inspections".

Sol.

(1) $A = A_1 A_2 A_3$

(2) $B = A_1 \cup A_2 \cup A_3 \quad \left(B = \overline{\overline{A_1}\,\overline{A_2}\,\overline{A_3}}\right)$

(3) $C = \overline{A_1}\, A_2\, A_3 \cup A_1\, \overline{A_2}\, A_3 \cup A_1\, A_2\, \overline{A_3}$

(4) $D = \overline{A_1}\,\overline{A_2}\,\overline{A_3} \cup \overline{A_1}\,\overline{A_2}\, A_3 \cup A_1\, \overline{A_2}\,\overline{A_3} \cup \overline{A_1}\, A_2\, \overline{A_3}$

$$\left(D = \overline{A_1}\,\overline{A_2} \cup \overline{A_1}\,\overline{A_3} \cup \overline{A_2}\,\overline{A_3}\right)$$

Answer is not unique.

Laws (法则) of set operations

1. Commutative laws (交换律):

$$A \cup B = B \cup A, A \cap B = B \cap A$$

2. Associative laws (结合律):

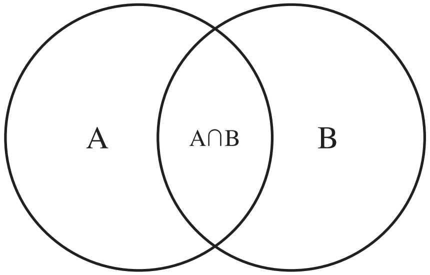
$$A \cup (B \cup C) = (A \cup B) \cup C$$

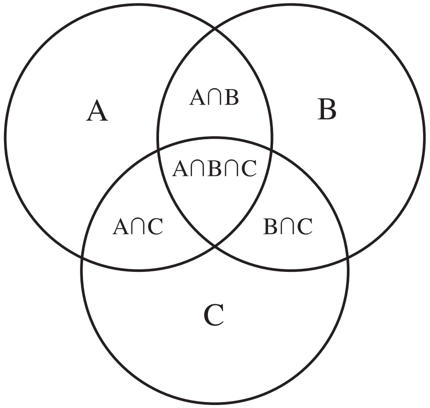
$$A \cap (B \cap C) = (A \cap B) \cap C$$

3. Distributive laws (分配律):

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Understand with Venn diagrams

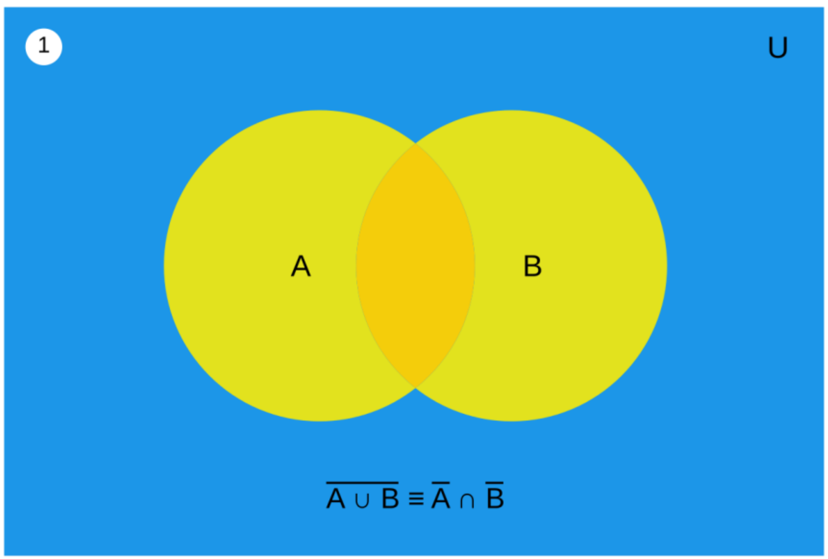


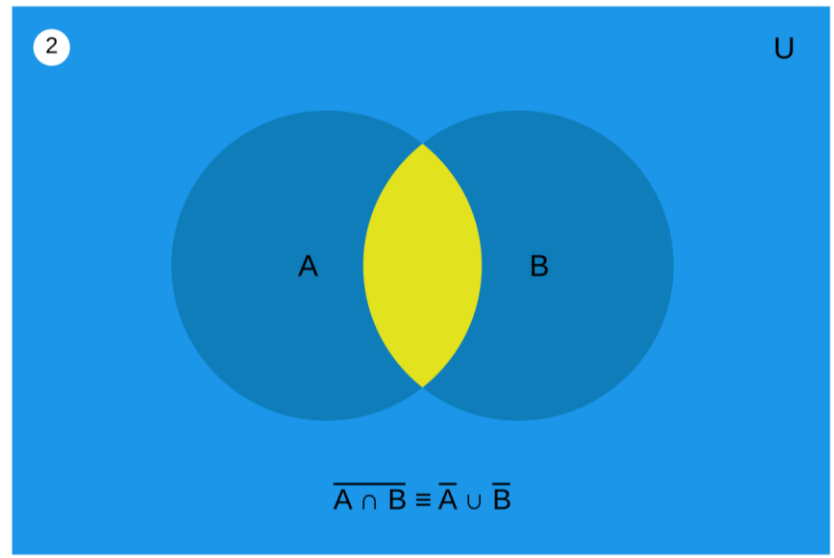


Laws (法则) of set operations

4. DeMorgan's laws (德·摩根律，对偶律)：

$$\overline{A \cup B} = \bar{A} \cap \bar{B}, \qquad \overline{A \cap B} = \bar{A} \cup \bar{B}.$$



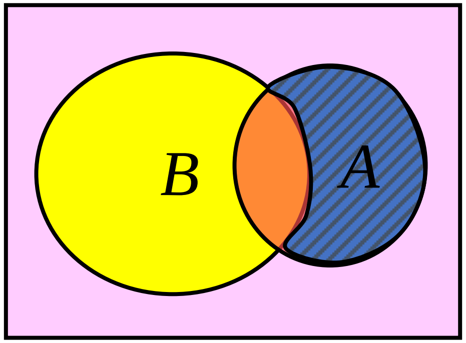


Understand with Venn diagrams.

Ex. Simplify the following set operations.

(1) $(\bar{A} \cup \bar{B})(\bar{A} \cup B)$ (2) $A\bar{B} \cup \bar{A}B \cup \bar{A}\bar{B}$

Sol. (1) $(\bar{A} \cup \bar{B})(\bar{A} \cup B) \xRightarrow{\text{Distributive laws}} \bar{A} \cup (\bar{B}B) = \bar{A} \cup \emptyset = \bar{A}$



(2) $A\bar{B} \cup \bar{A}B \cup \bar{A}\bar{B}$

$= A\bar{B} \cup \bar{A}B \cup \bar{A}\bar{B} \cup \bar{A}\bar{B}$

$= (A\bar{B} \cup \bar{A}\bar{B}) \cup (\bar{A}B \cup \bar{A}\bar{B})$

$= (A \cup \bar{A})\bar{B} \cup \bar{A}(B \cup \bar{B})$ (Commutative & Associative Laws)

$= \bar{B} \cup \bar{A} = \overline{AB}$ (DeMorgan's laws)

Lecture 01: Intro. to Probability

- Sample space and event
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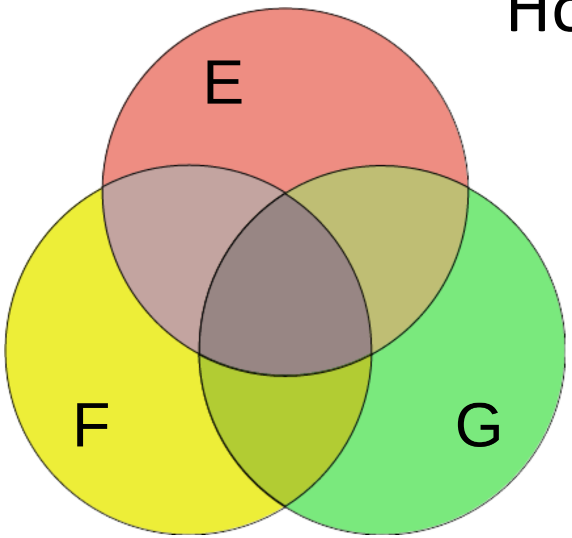
Simple propositions

- **Prop. 4.1** $P(\bar{E}) = 1 - P(E)$.
- **Prop. 4.2** If $E \subset F$, then $P(E) \leq P(F)$.
- **Prop. 4.3** $P(E \cup F) = P(E) + P(F) - P(EF)$



How about $P(E \cup F \cup G)$?

Check Prop. 4.4 in P31

Proofs are available in P29-31 of textbook.

Ex. (1) Given $P(AB) = 0,$ find $P(ABC)$;

(2) Given $P(A) = 0.4,\ P(B) = 0.3, P(A \cup B) = 0.6,$

find $P(A\bar{B})$;

(3) Given $P(\bar{A}\bar{B}) = P(AB), P(A) = p,$ find $P(B)$.

Sol. (1) $ABC \subset AB$, from **Prop. 4.2**,

$0 \leq P(ABC) \leq P(AB) = 0$, thus $P(ABC) = 0$

(2) $P(A\bar{B}) = P(A) - P(AB) = P(A \cup B) - P(B) = 0.3$

(3) $P(\bar{A}\bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$$= 1 - P(A) - P(B) + P(AB)$$

$\Longrightarrow P(B) = 1 - P(A) = 1 - p$