第十二、十三章 作业参考答案

第十二章 气体动理论 （P224~226）

1. 解：已知 $T=300K$， $P=1.01\times10^{5}Pa$

(1) 由 $P=nkT \Rightarrow n=\dfrac{P}{kT}=\dfrac{1.01\times10^{5}}{1.38\times10^{-23}\times300}=2.44\times10^{25}m^{-3}$

(2) $\rho=\dfrac{M}{V}=nm=\dfrac{nM_{mol}}{N_A}=\dfrac{2.44\times10^{25}\times32\times10^{-3}}{6.02\times10^{23}}=1.30\,kg/m^3$

或 $\rho=\dfrac{P\cdot M_{mol}}{RT}=\dfrac{1.01\times10^{5}\times32\times10^{-3}}{8.31\times300}=1.30\,kg/m^3$

(3) $\varepsilon_t=\dfrac{3}{2}kT=\dfrac{3\times1.38\times10^{-23}\times300}{2}=6.21\times10^{-21}J$

(4) $d=\sqrt[3]{\dfrac{1}{n}}=\sqrt[3]{\dfrac{1}{2.44\times10^{25}}}=3.45\times10^{-9}m$

2. 解：已知 $P_{H_2}=P_{He}$， $V_{H_2}=V_{He}$， $T_{H_2}=T_{O_2}$

由状态方程 $PV=\dfrac{M}{M_{mol}}RT$ 可知 $M=\dfrac{PV\cdot M_{mol}}{RT}$

故 $\dfrac{M_{H_2}}{M_{He}}=\dfrac{M_{mol,H_2}}{M_{mol,He}}=\dfrac{2}{4}=\dfrac{1}{2}$

而内能 $E=\dfrac{i}{2}PV$

故 $\dfrac{E_{H_2}}{E_{He}}=\dfrac{i_{H_2}}{i_{He}}=\dfrac{5}{3}$

3. 解：(1) $S=\int_0^{2v_0}Nf(v)$　　由 $\int_0^{2v_0}f(v)dv=1$ （归一化条件）

可得 $S=N$，即面积为系统的总分子数

(2) 由图 $Nf(v)=\begin{cases}\dfrac{a}{v_0}v & 0<v<v_0\\ a & v_0\le v<2v_0\\ 0 & v>2v_0\end{cases}$

则 $\int_0^{2v_0}Nf(v)dv=\int_0^{v_0}\dfrac{a}{v_0}v+\int_{v_0}^{2v_0}a\,dv=\dfrac{a}{2v_0}v^2\Big|_0^{v_0}+av\Big|_{v_0}^{2v_0}=\dfrac{3}{2}av_0=N$

$\therefore\ a=2N/3v_0$

(3)
$$\Delta N_{\frac{v_0}{2}\sim\frac{3}{2}v_0}=\int_{\frac{v_0}{2}}^{v_0}\frac{av}{v_0}dv+\int_{v_0}^{\frac{3}{2}v_0}a\,dv=\frac{a}{2v_0}v^2\Big|_{\frac{v_0}{2}}^{v_0}+av\Big|_{v_0}^{\frac{3}{2}v_0}$$
$$=\frac{3}{8}av_0+\frac{a}{2}v_0=\frac{7}{8}av_0=\frac{7}{12}N$$

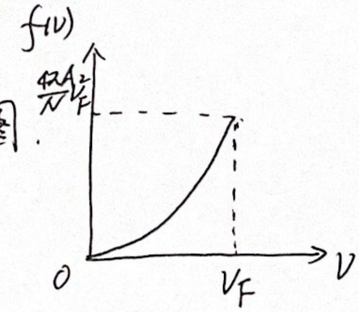
(4) $\overline{v^2}=\int_0^\infty v^2 f(v)dv$ $\qquad f(v)=\begin{cases}\frac{2}{3v_0^2}v & 0<v<v_0\\ \frac{2}{3v_0} & v_0\le v<2v_0\\ 0 & v>2v_0\end{cases}$

此处 $\overline{v^2}=\int_0^{v_0}\frac{2}{3v_0^2}v^3dv+\int_{v_0}^{2v_0}\frac{2}{3v_0}v^2dv$
$$=\frac{1}{6v_0^2}v^4\Big|_0^{v_0}+\frac{2}{9v_0}v^3\Big|_{v_0}^{2v_0}$$
$$=\frac{v_0^2}{6}+\frac{14}{9}v_0^2=\frac{31}{18}v_0^2$$

$\therefore\ \overline{\varepsilon_t}=\frac{1}{2}m\overline{v^2}=\frac{31}{36}mv_0^2$

4. 解：由题设 $f(v)=\frac{dN}{Ndv}$.

$\therefore\ f(v)=\begin{cases}\frac{4\pi A}{N}v^2 & 0<v<v_F\\ 0 & v>v_F\end{cases}$ 分布曲线如图.



(2) 由归一化条件 $\int_0^{v_F}\frac{4\pi A}{N}v^2dv=1$

$$\int_0^{v_F}\frac{4\pi A}{N}v^2dv=\frac{4\pi A}{3N}v^3\Big|_0^{v_F}=\frac{4\pi A}{3N}v_F^3=1\Rightarrow A=\frac{3N}{4\pi v_F^3}$$

(3) $\overline{v^2}=\int_0^{v_F}v^2f(v)dv=\int_0^{v_F}\frac{3}{v_F^3}v^4dv=\frac{3v^5}{5v_F^3}\Big|_0^{v_F}=\frac{3}{5}v_F^2$

$\therefore\ \overline{\varepsilon}=\frac{1}{2}m\overline{v^2}=\frac{3}{5}\cdot\frac{1}{2}mv_F^2=\frac{3}{5}\varepsilon_F$ 得证

选做

B-1, 解：已知 $V=1m^3$. $N_1=N_2=1.0\times10^{25}$. $T=400K$.

(1) 由 $\overline{\varepsilon_t}=\frac{3}{2}kT$.

$\overline{E_t}=N.\frac{3}{2}kT=(N_1+N_2)\times\frac{3}{2}kT=2.0\times10^{25}\times\frac{3}{2}\times1.38\times10^{-23}\times400=1.66\times10^5J$

(2) $p=nkT=\frac{N}{V}kT=\frac{(N_1+N_2)}{V}kT=\frac{2.0\times10^{25}}{1}\times1.38\times10^{-23}\times400=1.10\times10^5Pa$

B-3. 解：已知 $T=273K$，$\rho=1.25\times10^{-3}kg/m^3$，$P=1.0\times10^{-3}\times1.01\times10^{5}Pa$

(1) 由 $\rho=\frac{M}{V}=\frac{P\cdot M_{mol}}{RT}$

$\therefore M_{mol}=\frac{\rho RT}{P}=\frac{1.25\times10^{-3}\times8.31\times273}{1.0\times10^{-3}\times1.01\times10^{5}}=28\times10^{-3}kg$，该气体为 N_2 或 CO

（气体二分子也可视为固有参数）

(2) N_2、CO，其 $t=3$，$r=2$，$i=5$

$\therefore \overline{\varepsilon_t}=\frac{3}{2}kT=\frac{3}{2}\times1.38\times10^{-23}\times273=5.65\times10^{-21}J$

$\overline{\varepsilon_r}=\frac{2}{2}kT=1.38\times10^{-23}\times273=3.77\times10^{-21}J$

(3) $\frac{\overline{E_t}}{V}=n\overline{\varepsilon_t}=\frac{P}{kT}\overline{\varepsilon_t}=\frac{P}{kT}\cdot\frac{3}{2}kT=\frac{3}{2}P=\frac{3}{2}\times10^{-3}\times1.01\times10^{5}=152J$

(4) $E=\frac{i}{2}\nu RT=\frac{5}{2}\times0.3\times8.31\times273=1.70\times10^{3}J$

第十三章 热力学（P275~280）

1. 解：此处可直接由面积法求解

$W_{AB}=\frac{1}{2}(P_B+P_A)\times(V_B-V_A)=\frac{1}{2}\times(2+1)\times10^{5}\times(3-2)\times10^{-3}=150J$

2. 解：已知 $Q_{ABC}=326J$，$W_{ABC}=126J$，$W_{CA}=-52J$

由热力学第一定律，$Q=\Delta E+W$

即 $Q_{ABC}=\Delta E_{ABC}+W_{ABC}$，$\therefore\Delta E_{ABC}=200J$

$\Delta E_{CA}=-\Delta E_{AC}=-200J$

$Q_{CA}=\Delta E_{CA}+W_{CA}=-200-52=-252J$

$\therefore$ 热量为 252J，系统放热。

3. 解：$p_A V_A = p_B V_B$ ∴ $T_A = T_B$ （记号ACB并非等温过程）

由热力学第一定律，$Q = \Delta E + W$

即 $Q_{ACB} = \Delta E_{AB} + W_{AB}$ $\Delta E_{AB} = 0$

∴ $Q_{ACB} = W_{ACB} = 700\,J$

对BDA过程，$\Delta E_{BA} = 0$ $W_{BD} = 0$（等体）

$W_{DA} = p_A(V_A - V_D) = 4.0\times10^5 \times (1-4)\times10^{-3} = -1.2\times10^3\,J$

$Q_{BDA} = \Delta E_{BA} + W_{BD} + W_{DA} = -1.2\times10^3\,J$

∴ $Q_{ACBDA} = Q_{ACB} + Q_{BDA} = -1200 + 700 = -500\,J$

$Q<0$，表示系统对外放热500J.

4. 解：已知 $p_A = 2.0\times10^5\,Pa$，$V_A = V_C = 2.0\times10^{-2}\,m^3$

$p_B = p_C = 1.0\times10^5\,Pa$，$V_B = 4.0\times10^{-2}\,m^3$ $\ln 2 = 0.693$

(1) AB等温膨胀 $T_A = T_B$.

∴ $W_{AB} = \nu RT\ln\frac{V_B}{V_A} = p_A V_A \ln\frac{V_B}{V_A} = 2.0\times10^5 \times 2.0\times10^{-2} \times \ln 2 \doteq 2.8\times10^3\,J$

$\Delta E_{AB} = 0$.

∴ $Q_{AB} = W_{AB} = 2.8\times10^3\,J$

(2) $W_{AC} = 0$ $W_{CB} = p_B(V_B - V_C) = 10^5 \times (4.0-2.0)\times10^{-2} = 2.0\times10^3\,J$

$\Delta E_{AB} = 0$

∴ $Q_{ACB} = W_{AC} + W_{CB} = 2.0\times10^3\,J$

5. 解：已知 $\nu = \frac{0.32}{32\times10^{-3}} = 10\,mol$. $V_2 = 2V_1$. $T_1 = 300K$. $T_2 = 200K$

由于BC和DA均为等体过程，系统做功为0，故计算净功和总吸热更为方便.

AB等温膨胀，$Q_{AB} = W_{AB} = \nu RT_1 \ln\frac{V_2}{V_1} = 10\times8.31\times300\ln 2 = 1.73\times10^4\,J$

CD等温压缩 $Q_{CD} = W_{CD} = -\nu RT_2 \ln\frac{V_2}{V_1} = -10\times8.31\times200\ln 2 = 1.15\times10^4\,J$

而DA等体增压升温也为吸热过程

$Q_{DA} = \Delta E_{DA} = \nu C_V(T_1 - T_2) = 10\times\frac{5}{2}\times8.31\times100 = 2.08\times10^4\,J$

$\therefore \eta = \frac{W_{净}}{Q_1} = \frac{W_{AB}+W_{CD}}{Q_{AB}+Q_{DA}} = \frac{(1.73-1.15)\times10^4}{(1.73+2.08)\times10^4} = 15\%$

6. 解：（解法二）不需转换状态图，直接求出净功 $W_{净}>0$ 为热机

V-T图中 $i=3$，$C_V=\frac{3}{2}R$，$C_p=\frac{5}{2}R$ $W_{净}<0$，为制冷机

AB为等压膨胀，系统对外做功，吸热。

BC为等体降温，系统做功为0，内能减少，放热。

CA为等温压缩，外界对系统做功，系统放热。

$W_{AB}=\nu RT_A$

$W_{CA}=\nu RT_A\ln\frac{V_A}{V_C}=-\nu RT_A\ln2$ $(\ln2=0.693)$

$W_{AB}+W_{CA}=(1-\ln2)\nu RT_A>0$ 故该循环为热机。

$\eta=\frac{W_{净}}{Q_{AB}}=\frac{(1-\ln2)\nu RT_A}{\nu C_p(T_B-T_A)}=\frac{(1-\ln2)\nu RT_A}{\frac{5}{2}\nu RT_A}=\frac{2-2\ln2}{5}=12.3\%$.

7. (1) 证明：由于BC、DA为绝热过程，AB等压膨胀，吸热，CD等压压缩，放热。

故 $\eta=1-\frac{|Q_{CD}|}{Q_{AB}}=1-\frac{\nu C_p(T_C-T_D)}{\nu C_p(T_B-T_C)}=1-\frac{T_C-T_B}{T_B-T_A}=1-\frac{T_C(1-\frac{T_D}{T_C})}{T_B(1-\frac{T_A}{T_B})}$

$\frac{T_A}{T_B}=\frac{V_A}{V_B}$. $\frac{T_D}{T_C}=\frac{V_D}{V_C}$ $V_C^{\gamma-1}T_C=V_B^{\gamma-1}T_B$ (1)

$V_D^{\gamma-1}T_D=V_A^{\gamma-1}T_A$ (2)

(2)/(1) $\frac{T_D}{T_C}\cdot(\frac{V_D}{V_C})^{\gamma-1}=\frac{T_A}{T_B}\cdot(\frac{V_A}{V_B})^{\gamma-1}$

$\therefore \frac{V_D}{V_C}=\frac{V_A}{V_B}$. 故 $\frac{T_A}{T_B}=\frac{T_D}{T_C}$ $\cdot 1-\frac{T_D}{T_C}=1-\frac{T_A}{T_B}$

$\therefore \eta=1-\frac{T_C}{T_B}=1-\frac{T_2}{T_1}$ 得证.

(2) 卡诺循环指由两个等温和两个绝热过程构成的循环，该循环中等压过程取代等温过程，故并非卡诺循环。

8. 证：该循环中 AB为绝热膨胀，$Q_{AB}=0$.

BC为等压压缩，系统对外放热，CA为等体增压，系统从外界吸热.

故 $\eta = 1-\dfrac{Q_2}{Q_1} = 1-\dfrac{|Q_{BC}|}{Q_{CA}}$

$Q_{BC} = \nu C_p(T_C-T_B) = -\nu C_p(T_B-T_C)$

$Q_{CA} = \nu C_V(T_A-T_C)$

$\therefore \eta = 1-\dfrac{C_p(T_B-T_C)}{C_V(T_A-T_C)} = 1-\gamma\dfrac{(\frac{T_B}{T_C}-1)T_C}{(\frac{T_A}{T_C}-1)T_C} = 1-\gamma\cdot\dfrac{T_B/T_C-1}{T_A/T_C-1}$

BC过程 $\dfrac{T_B}{T_C}=\dfrac{V_1}{V_2}$ CA过程中 $\dfrac{T_A}{T_C}=\dfrac{p_1}{p_2}$

$\therefore \eta = 1-\gamma\cdot\dfrac{\frac{V_1}{V_2}-1}{\frac{p_1}{p_2}-1}$ 得证.

选做.

B-1. 解：已知 $i=5$. $\nu=1$

(1) $W_I = \dfrac{p_1+p_2}{2}(V_2-V_1) = \dfrac{5.05\times10^5}{2}(2-1)\times10^{-3} = 253\text{ J}$

$\Delta E_I = \dfrac{i}{2}(p_2V_2-p_1V_1) = \dfrac{5}{2}(4.04\times2\times10^2-1.01\times10^2) = 1768\text{ J}$

$\therefore Q_I = \Delta E_I + W_I = 253+1768 = 2.02\times10^3\text{ J}$

(2) 由 $p_2V_2^{\frac{1}{2}} = p_3V_3^{\frac{1}{2}}$，$p_3=p_1$ 可得 $V_3 = (\dfrac{p_2}{p_3})^2V_2 = 32\times10^{-3}\text{ m}^3$

$W_{II} = \int_{V_2}^{V_3}p\,dV = \int_{V_2}^{V_3}\dfrac{C}{V^{\frac{1}{2}}}dV = 2p_2V_2^{\frac{1}{2}}(V_3^{\frac{1}{2}}-V_2^{\frac{1}{2}}) = 2\times4.04\times10^5\times\sqrt{2\times10^{-3}}(\sqrt{32\times10^{-3}}-\sqrt{2\times10^{-3}})$

$= 2\times4.04\times10^5\times(8-2)\times10^{-3} = 4.85\times10^3\text{ J}$

$\Delta E_{II} = \dfrac{i}{2}(p_3V_3-p_2V_2) = \dfrac{5}{2}(1.01\times10^5\times32\times10^{-3}-4.04\times10^5\times2\times10^{-3}) = 6.06\times10^3\text{ J}$

$\therefore Q_{II} = W_{II}+\Delta E_{II} = 1.09\times10^4\text{ J}$

$Q_总 = Q_I + Q_{II} = 1.29\times10^4\text{ J}$