Chapter 3 homework

1. Compute the integral $\int_C (x - iy)\,\mathrm{d}z$ along paths $y = x$ and $y = x^2$ respectively, where z starts at 0 and ends at $1 + i$.

2. Compute the integral $\int_0^{3+4i} z^2\,\mathrm{d}z$ along the following paths:

   (a) Straight line segment from origin to $3 + 4i$;

   (b) Broken line segment from the origin along the real axis to 3 and then vertically upward to $3 + 4i$.

3. Compute $\int_C \mathrm{Re}(z)\,\mathrm{d}z$, along the following paths:

   (a) Straight line segment from origin to $1 + i$;

   (b) Arc segment from origin to $1 + i$ on parabola $y = x^2$.

4. Compute the following integrals:

   (a) $\oint_{|z|=2} \frac{z}{z-1}\,\mathrm{d}z$;

   (b) $\oint_{|z|=2} \frac{\overline{z}}{|z|}\,\mathrm{d}z$;

   (c) $\oint_{|z|=1} \frac{\sin z}{z}\,\mathrm{d}z$;

   (d) $\oint_{|z|=3} \left(\frac{2}{z+i} + \frac{3}{z+2}\right)\mathrm{d}z$.

5. Compute the integral $\oint_C \frac{e^z}{z}\,\mathrm{d}z$, where C consists of a positive circumference $|z| = 2$ and a negative circumference $|z| = 1$.

6. Compute the following integrals:

   (a) $\int_0^{\pi i} e^{3z}\,\mathrm{d}z$;

   (b) $\int_0^{\pi} \cos^2 z\,\mathrm{d}z$;

(c) $\displaystyle\int_0^i (z-1)e^{-iz}\,\mathrm{d}z$.

7. Compute the following integrals:

   (a) $\displaystyle\oint_{|z|=2} \frac{\sin z}{z^4}\,\mathrm{d}z$;

   (b) $\displaystyle\oint_{|z|=1} \frac{e^{-z}\cos z}{z^2}\,\mathrm{d}z$.

8. Let $u(x,y)$ be a harmonic function in the complex plane, prove that $f = u_x - iu_y$ is an analytic function in the complex plane.

9. Prove that $u = e^x \sin y$ is a harmonic function, and find the analytic function $f(z) = u + iv$, where $f(0) = 1$.