Chapter 2 homework

1. Point out where the following functions can be differentiated and analytic:
   (a) $f(z) = x^2 + 2yi$;
   (b) $f(z) = 3x^2 + 2y^3 i$;
   (c) $f(z) = z\,|z|^2$.
2. Determine the domains of analyticity and derivatives of the following functions:
   (a) $f(z) = z^3 + iz$;
   (b) $f(z) = \dfrac{z+1}{z^2-1}$;
   (c) $f(z) = \dfrac{z}{z^2+1}$.
3. Let $f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$, find the values of a, b, c and d, so that $f(z)$ is analytic everywhere in the complex plane.
4. Let $f(z)$ be analytic in D. Proved that the necessary and sufficient condition for $f(z)$ to be constant in D is that $\overline{f(z)}$ is analytic in D.
5. If $f(z)$ is analytic in D, proved that the following functions in D are true:
   (a) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0,\ \dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} = 0$;
   (b) $\left(\dfrac{\partial}{\partial x}|f(z)|\right)^2 + \left(\dfrac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$.
6. Solve the following equations:
   (a) $z^3 + 1 = 0$;
   (b) $e^z - 1 = 0$;
   (c) $\sin z = 0$;
   (d) $\sin z - \cos z = 0$.
7. Do the following equations hold?
   (a) $\operatorname{Ln}\dfrac{z_1}{z_2} = \operatorname{Ln} z_1 - \operatorname{Ln} z_2$;

(b) $\operatorname{Ln} z^{\frac{1}{n}} = \dfrac{1}{n} \operatorname{Ln} z$.

8. Find the values of the following formulas:

(a) $e^{2+i\pi}$;

(b) $(1+i)^{1-i}$;

(c) $\operatorname{Ln} 3 + 4i$.