Chapter 5 homework

1. Find the singularities of the following functions. If they are poles, indicate their order.
   (a) $\dfrac{\ln(z+1)}{z}$;
   (b) $\dfrac{1}{e^{z-1}}$;
   (c) $\dfrac{\sin z}{z^3}$;
   (d) $\dfrac{z^2}{(z+1)^3}$;
   (e) $\dfrac{z-2}{z(z^2+4)^2}$.
2. Find the residues of the following functions at every singularity except infinity.
   (a) $\dfrac{z+1}{z^2-2z}$;
   (b) $\dfrac{z^2}{(z-2)(z^2+1)}$;
   (c) $e^{\frac{1}{z}} \sin \dfrac{1}{z}$;
   (d) $\dfrac{1}{z^2 \sin z}$.
3. Use the residue theorem to compute the following integrals, where each circle is counterclockwise.
   (a) $\displaystyle\oint_{|z|=1} \frac{1}{z \sin z}\, \mathrm{d}z$;
   (b) $\displaystyle\oint_{|z|=2} \frac{e^{2z}}{(z-1)^2}\, \mathrm{d}z$;
   (c) $\displaystyle\oint_{|z|=4} \frac{1}{(z-1)(z-2)(z-3)}\, \mathrm{d}z$.
4. Use the residue theorem to compute the following integrals, where each circle is counterclockwise.

(a) $\displaystyle\oint_C \frac{1}{(z-1)^2(z^2+1)}\,\mathrm{d}z, \quad C : x^2+y^2-2x-2y=0;$

(b) $\displaystyle\oint_C \frac{z^3}{1+z}e^{\frac{1}{z}}\,\mathrm{d}z, \quad C : |z|=2.$