1. For random variable X with probability density function $f(x)=\begin{cases}kx^b, & 0<x<1\\ 0, & \text{otherwise}\end{cases}$ $(b>0,k>0)$ and $P\{X>1/2\}=0.75$. Find the following values $k=$__________, $b=$____________.

2. For random variable X with probability mass function

X	0	1	2
p	1/3	1/6	1/2

Find the corresponding cumulative distribution function $F(x)=$________________________.

3. For random variable $X\sim U(1,6)$, find the probability p that the roots for equation $x^2+Xx+1=0$ are real numbers. p=_____________.

4. We know that the probability density function of random variable X is $f(x)=\begin{cases}2x, & 0<x<1\\ 0, & \text{otherwise}\end{cases}$. Denote the number of occurrences of event $\{X\le 1/2\}$ in three independent trials as Y. Find the value of $P\{Y=2\}=$______________.

5. X,Y are two random variables with binomial distribution, $X\sim b(2,p)$, $Y\sim b(3,p)$. Given $P\{X\ge 1\}=5/9$, find $P\{Y\ge 1\}$=____________.

6. If the probability distribution function for random variable X is symmetric, i.e., $f(x)=f(-x)$. Then, $P\{|X|>a\}=($ $)$.

(A) $2[1-F(a)]$ (B) $2F(a)-1$ (C) $2-F(a)$ (D) $1-2F(a)$

7. If the PDF of random variable X is given as $f(x)=\frac{1}{2\sqrt{\pi}}e^{-\frac{(x+3)^2}{4}}$ $(-\infty<x<+\infty)$. Then, ( ) $\sim N(0,1)$.

(A) $\frac{X+3}{2}$ (B) $\frac{X+3}{\sqrt{2}}$ (C) $\frac{X-3}{2}$ (D) $\frac{X-3}{\sqrt{2}}$

8. Given variables $X\sim\mathcal{N}(\mu,4^2)$, $Y\sim\mathcal{N}(\mu,5^2)$. If we define $P(X\le\mu-4)=p_1$, $P(Y\ge\mu+5)=p_2$, which of the following statements is correct? ( )

(A) For arbitrary value of μ, we have $p_1=p_2$

(B) For arbitrary value of μ, we have $p_1<p_2$

(C) For arbitrary value of μ, we have $p_1>p_2$

(D) For some specific values of μ, we have $p_1=p_2$

9. Given that $F(x) = aF_1(x) - bF_2(x)$ is a valid CDF, where $F_1(x)$ and $F_2(x)$ are the CDFs of random variables X_1 and X_2, respectively. Determine the values of a and b ( ).

(A) $a = \frac{3}{5}, b = -\frac{2}{5}$ (B) $a = \frac{2}{3}, b = \frac{2}{3}$ (C) $a = -\frac{1}{2}, b = \frac{3}{2}$ (D) $a = \frac{1}{2}, b = \frac{3}{2}$

10. For random variable $X \sim \mathcal{N}(2, \sigma^2)$ satisfying $P\{2 < X < 4\} = 0.3$, find $P\{X < 0\} =$ ( ).

(A) 0.5 (B) 0.7 (C) 0.3 (D) 0.2

11. For random variable $X \sim \mathcal{N}(\mu, \sigma^2)$, the probability of $P\{|X - \mu| \leq \sigma\}$ ( ) as σ increases.

(A) increases (B) decreases (C) is unchanged (D) none of the above

12. Find the probability density function of $Y = e^X$ when $X \sim \mathcal{N}(\mu, \sigma^2)$.

13. In 10,000 independent tosses of a coin, we know that the number of coins landed on heads follow a normal distribution $\mathcal{N}(5000{,}2500)$. Given that the coin landed on heads 5800 times, is it reasonable to assume that the coin is not fair? Please explain your answer.