1. Given $X \sim t(n)$, $(n > 1)$, and $Y = \frac{1}{X^2}$, which of the following is correct? (   )

(A) $Y \sim \chi^2(n)$ (B) $Y \sim \chi^2(n-1)$

(C) $Y \sim F(1, n)$ (D) $Y \sim F(n, 1)$

2. Given $X \sim \mathcal{N}(0,1)$ and $\alpha(0 < \alpha < 1)$, we define u_α as $P\{X > u_\alpha\} = \alpha$. If $P\{|X| < x\} = \alpha$, then x equal to? (   )

(A) $u_{\frac{\alpha}{2}}$ (B) $u_{1-\frac{\alpha}{2}}$ (C) $u_{\frac{1-\alpha}{2}}$ (D) $u_{1-\alpha}$

3. Given a sample $X_1, X_2, \dots, X_n$ from the population $X \sim \mathcal{N}(\mu, \sigma^2)$, we define $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$. Then, $D(S^2) = ($ $)$

(A) $\frac{\sigma^4}{n}$ (B) $\frac{2\sigma^4}{n}$ (C) $\frac{2\sigma^4}{n-1}$ (D) $\frac{\sigma^4}{n-1}$

4. Given a sample $X_1, X_2, \dots, X_n$ from the population X, and $E(X) = \mu$, $D(X) = \sigma^2$, we define $\bar{X}$ and S^2 as the sample mean and sample variance, respectively. Thus, (   )

(A) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$ (B) $\bar{X} \sim \mathcal{N}(\mu, \frac{\sigma^2}{n})$

(C) S^2 and $\bar{X}$ are independent (D) S^2 is an unbiased estimator of σ^2

5. Let X and Y be two independent variables that follow normal distribution, then (   )

(A) $X + Y$ follow normal distribution (B) $X^2 + Y^2$ follow χ^2 distribution

(C) X^2 and Y^2 both follow χ^2 distribution (D) $\frac{X^2}{Y^2}$ follow normal distribution

6. Given a sample $X_1, X_2, \dots, X_n$ from the population $\mathcal{N}(0,1)$, we define $\bar{X}$ and S^2 as the sample mean and sample variance, respectively. Thus, (   )

(A) $\bar{X} = \mathcal{N}(0,1)$ (B) $n\bar{X} \sim \mathcal{N}(0,1)$ (C) $\frac{\bar{X}}{S}$ follows t distribution (D) $\sum_{i=1}^{n} X_i^2$ follows χ^2 distribution

7. Given two independent variables $X \sim \mathcal{N}(0,1)$ and $Y \sim \mathcal{N}(0,2)$, which of the following is correct? (   )

(A) $\frac{1}{3}X^2 + \frac{2}{3}Y^2$ follows χ^2 distribution (B) $\frac{1}{2}X^2 + \frac{1}{2}Y^2$ follows χ^2 distribution

(C) $\frac{1}{3}(X+Y)^2$ follows χ^2 distribution (D) $\frac{1}{2}(X+Y)^2$ follows χ^2 distribution

8. Given a sample $X_1, X_2, \dots, X_n$ from the population $\mathcal{N}(0,1)$ and $v = n - 1$, we define $\bar{X}$ and S^2 as the sample mean and sample variance, respectively. Thus, (   )

(A) $\bar{X} \sim \chi^2(v)$ (B) $\frac{nS^2}{\sigma^2} \sim \chi^2(n)$

(C) $\frac{S^2}{\sigma^2} \sim \chi^2(n)$ (D) $\frac{vS^2}{\sigma^2} \sim \chi^2(v)$

9. Given a sample $X_1, X_2, \dots, X_{10}$ from the population $X \sim \mathcal{N}(0,1)$, we know that

$$Y = \frac{4\left(X_1^2 + \cdots + X_i^2\right)}{X_{i+1}^2 + \cdots + X_{10}^2}, (1 < i < 10)$$

follow F distribution, then the value of i is (   )

(A) 4 (B) 2 (C) 3 (D) 5

10. For population $X \sim \mathcal{N}(\mu, \sigma^2)$, the variance σ^2 is known, we assume that the sample size n and confidence level $1-\alpha$ are constant. Then, the length of confidence interval μ will be ( ) for different samples.
(A) Smaller (B) Larger (C) Constant (D) Can not be determined.

11. In the estimation of a confidence interval, $P\{\hat{\theta}_1 < \theta < \hat{\theta}_2\} = 1-\alpha$ means ( )
(A) θ lies in interval $(\hat{\theta}_1, \hat{\theta}_2)$ with a probability of $1-\alpha$.
(B) θ lies outside interval $(\hat{\theta}_1, \hat{\theta}_2)$ with a probability of α.
(C) θ does not lies in interval $(\hat{\theta}_1, \hat{\theta}_2)$ with a probability of α.
(D) The interval $(\hat{\theta}_1, \hat{\theta}_2)$ cover θ with a probability of $1-\alpha$.

12. Population X follows PDF $f(x) = \begin{cases} |x|, & -1 < x < 1 \\ 0, & \text{others} \end{cases}$. $X_1, X_2, \dots, X_{50}$ is a sample from X. We define $\bar{X}$ and S^2 as the sample mean and sample variance, respectively. Then, $E(\bar{X}) =$ __________ , $D(\bar{X}) =$ __________ , $E(S^2) =$ __________。

13. Let $X_1, X_2, \dots, X_6$ be a sample from the population $\mathcal{N}(0, \sigma^2)$, and $Y = (X_1 + X_2 + X_3)^2 + (X_4 + X_5 + X_6)^2$, please find the value of $c =$ __________ such that $cY \sim \chi^2$.

14. Let $X_1, X_2, \dots, X_n$ be a sample from population $X \sim \mathcal{N}(\mu, \sigma^2)$, then $\sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{\sigma^2} \sim$ __________.

15. Let X_1, X_2, X_3, X_4 be a sample from population $X \sim \mathcal{N}(0, 2^2)$, we define $Y = a(X_1 - 2X_2)^2 + b(3X_3 - 4X_4)^2$. Then, Y follows χ^2 distribution with _______ degree of freedom, when $a =$ ________, $b =$ __________.

16. Let $X_1, X_2, \dots, X_9$ be a sample from population $X \sim \mathcal{N}(\mu, 0.9^2)$ and $\bar{x} = 5$, then a 95% confidence interval for parameter μ is ________________.

17. Let $X_1, X_2, \dots, X_{16}$ be a sample from population $X \sim \mathcal{N}(\mu, \sigma^2)$, we define $\bar{X}$ and S^2 as the sample mean and sample variance, respectively. If $P\{\bar{X} \geq \mu + aS\} = 0.05$, then $a =$ _________. When $c =$ __________, $c\sum_{i=1}^{15}(X_{i+1} - X_i)^2$ is an unbaised estimator of σ^2.

18. Let the population $X \sim \pi(\lambda)$, $\lambda > 0$ is unknown, and $(X_1, X_2, \dots, X_n)$ is a sample from the population X. Then the moment estimator of parameter λ is $\hat{\lambda} =$ _____________.

19. Let population X follows PDF $f(x) = \begin{cases} 2e^{-2(x-\theta)} & x > \theta \\ 0 & x \leq \theta \end{cases}$, where $\theta > 0$ is unknown. $X_1, X_2, \dots, X_n$ is a sample from population X. Find the moment estimator and MLE estimator for parameter θ, and check whether they are unbiased estimators.

20. Given the PMF of X as

X	0	1	2	3
p	θ^2	$2\theta(1-\theta)$	θ^2	$1-2\theta$

where $\theta (0 < \theta < 1/2)$ is unknown. Given a sample from X with the following values 3, 1, 3, 0, 3, 1, 2, 3. Find the MLE and moment estimator of θ.