Name: Student ID: Quiz 3

1. Given variables (X, Y) follows uniform distribution in the area enclosed by x axis, y axis and line $y = 2x + 1$, find the probability $P\left\{X < -\frac{1}{8}, Y < \frac{1}{2}\right\} =$_____.

2. Let $F(x, y)$ be the joint CDF of variables (X, Y), please compute the following probability in terms of $F(x, y)$, $P\{0 < Y \leq a\} =$_____.

3. Suppose that X and Y follow the joint PDF $f(x, y) = \begin{cases} e^{-x} & \text{for } 0 \leq y \leq x \leq \infty \\ 0 & \text{otherwise} \end{cases}$, find the conditional PDF of X given Y.

4. Given that the joint PDF of (X, Y) is $f(x, y) = \begin{cases} \frac{21}{4} x^2 y & \text{for } x^2 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$, find the marginal PDF of X and Y.

5. If variables X and Y are independent, and the probability density functions are given as

$$f_X(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}, f_Y(y) = \begin{cases} e^{-y}, & y > 0 \\ 0, & y \leq 0 \end{cases}.$$

The joint PDF of (X, Y) is $f(x, y) =$ _____.

6. Given random variables X and Y satisfying $P\{X \geq 0, Y \geq 0\} = \frac{3}{7}$, $P\{X \geq 0\} = P\{Y \geq 0\} = \frac{4}{7}$, find the following probability $P\{\max(X, Y) \geq 0\} =$_____.

7. Assuming random variables X, Y follow normal distribution $\mathcal{N}(0, \sigma^2)$, and $P\{X \leq 0, Y \geq 0\} = \frac{1}{3}$, find the following probability $P\{X > 0, Y < 0\} =$_____.

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1. Given random variables X and Y with joint PDF $f(x,y)=\begin{cases}\frac{1}{\pi}, & x^2+y^2\le 1\\ 0, & \text{otherwise}\end{cases}$, the relationship between X and Y is ( ).

A. Independent and identically distributed;

B. Independent and not identically distributed;

C. Dependent and identically distributed;

D. Dependent and not identically distributed.

2. Given independent random variables X and Y with CDF $F_X(x)$ and $F_Y(y)$, respectively, the CDF of $Z=\min(X,Y)$ is ( ).

(A) $F_Z(z)=F_X(z)$ (C) $F_Z(z)=\min\{F_X(z),F_Y(z)\}$

(B) $F_Z(z)=F_Y(z)$ (D) $F_Z(z)=1-[1-F_X(z)][1-F_Y(z)]$

3. Given independent random variables X,Y follow uniform distribution in $[0,1]$, which of the following variables follow uniform distribution? ( )

(A) (X,Y) (B) $X+Y$ (C) X^2 (D) $X-Y$

4. Given random variables X,Y with independent and identical distribution, and $P\{X=-1\}=P\{Y=-1\}=\frac{1}{2}$, $P\{X=1\}=P\{Y=1\}=\frac{1}{2}$, which of the following is correct? ( )

(A) $P\{X=Y\}=\frac{1}{2}$ (B) $P\{X=Y\}=1$ (C) $P\{X+Y=0\}=\frac{1}{4}$ (D) $P\{X-Y=0\}=\frac{1}{4}$

5. Given independent random variables $X\sim\mathcal{N}(a_1,\sigma_1^2)$, $Y\sim\mathcal{N}(a_2,\sigma_2^2)$, the sum of X and Y follows normal distribution, which of the following is correct? ( )

(A) $Z\sim\mathcal{N}(a_1,\sigma_1^2+\sigma_2^2)$ (B) $Z\sim\mathcal{N}(a_1+a_2,\sigma_1\sigma_2)$

(C) $Z\sim\mathcal{N}(a_1+a_2,\sigma_1^2\sigma_2^2)$ (D) $Z\sim\mathcal{N}(a_1+a_2,\sigma_1^2+\sigma_2^2)$

6. Given independent random variables $X\sim\mathcal{N}(0,1)$,$Y\sim\mathcal{N}(1,1)$, which of the following is correct? ( )

(A) $P\{X+Y\le 0\}=\frac{1}{2}$ (B) $P\{X+Y\le 1\}=\frac{1}{2}$

(C) $P\{X-Y\le 0\}=\frac{1}{2}$ (D) $P\{X-Y\le 1\}=\frac{1}{2}$