CSC384 – Assignment 1 Document

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To construct a heuristic for the puzzle, I chose to focus on three constraints for the problem. For a piece to move from A to B, the following must hold:

1. A and B need to be adjacent to one another.
2. B must be empty.
3. The top left quadrant of the 2x2 goal piece must be at location (1,3) on the board to be at the goal state.

For the case of the Manhattan distance heuristic, we relax the second constraint to allow us to measure the x and y axis distances of the 2x2 piece to the goal state. One advanced heuristic is to calculate the Manhattan distance for the 2x2 goal piece and add it to the number of pieces that are below it:

$$h(state) = Manhattan\ Distance + \#\ of\ pieces\ below\ goal\ piece$$

To be specific, we count the number of pieces that are both below the goal piece and are contained (either partially or fully) within the two columns that occupy the 2x2 goal piece. This is based on the third constraint as we can recognize that minimizing the number of pieces that are directly below the goal piece allows us to prioritize moving the goal piece downwards towards the goal state, which is at the bottom of the board.

A heuristic is admissible if it never overestimates the true cost, h*(n), of reaching the goal state from the current state. This advanced heuristic is admissible because it follows the basis of the Manhattan distance heuristic which is also admissible. For instance, in the worst case where the goal piece is at the top of the board, you can have a maximum of 6 pieces that are below it. We know that in this case, the true cost from the successor state to goal is much more than the Manhattan distance + 6. This logic can be extended to when the goal piece is at any level of the board. If the goal piece has a successor that has a true cost of 1, then that means that the successor has to be one of three states: one where the top right of the 2x2 piece is at the (1,3) location, one where the top left of the 2x2 piece is at the (2,3) location, and another where the bottom left of the 2x2 piece is located at the (1,3) spot. For the first two cases, there are no pieces below the goal piece, so the heuristic would be based on the Manhattan distance, which is already admissible. For the third case, for h*(n) to be 1, there must be two empty spots immediately below it, meaning that this case would also depend solely on the Manhattan distance heuristic. Thus, by this logic, this advanced heuristic must also be admissible.

This heuristic dominates the Manhattan distance heuristic because the heuristic value will always be greater than or equal to the Manhattan distance, depending on where the goal piece is (if there are any pieces below it or not). Essentially, since the advanced heuristic is the sum of the Manhattan distance with an extra parameter to add, we can guarantee that the dominance inequality is satisfied. For instance, when the goal piece has no pieces below it, the heuristic values are equal. However, in any other state, the goal piece will have at least 1 piece below it, meaning that the advanced heuristic will be greater than that of the Manhattan distance.