

A carbon-based FRP complete wrap is selected to retrofit the columns. The properties of the FRP system, as reported by the manufacturer, are shown in Table 16.8b. The design

calculations to arrive at the number of required complete wraps follow in Table 16.8c.

**Table 16.8a—Column cross section details and material properties**

f	6.5 ksi	45 MPa
f	60 ksi	400 MPa
re	1 in.	25 mm
Bars	12 No. 10	12632
Ag	576 in. <sup>2</sup>	3716 cm <sup>2</sup>
A <sub>u</sub>	15.24 in. <sup>2</sup>	98 cm <sup>2</sup>
Pg%	2.65	2.65
ΦP without FRP	2087 kip	9281 kN
中Paic)	2504 kip	11,138 kN

Note: The column features stel ties for transverse reinforcement.

**Table 16.8b—Manufacturer's reported FRP system properties**

Thickness per plyt	0.013 in.	0.33 mm
Ultimate tensile strengthf	550 ksi	3792 MPa
Rupture strain g <sub>n</sub>	0.0167 in./in.	0.0167 mm/mm
Modulus of elasticity E <sub>1</sub>	33,000 ksi	227,527 MPa

**Table 16.8c—Procedure for strengthening of a noncircular concrete column for axial load increase**

Procedure	Calculation in in.-lb units	Calculation in SI units
<b>Step 1—Compute the design FRP system properties.</b>  The column is located in an interior environment and a CFRP system will be used. Therefore, per Table 9.4, an environmental reduction factor of 0.95 is used.  F <sub>m</sub> =C <sub>ef</sub> m=C <sub>en</sub>	f <sub>m</sub> =(0.95)(550 ksi)=522.5 ksi G <sub>u</sub> =(0.95)(0.0167)=0.0159 in./in	f <sub>m</sub> =(0.95)(3792 MPa)=3603 MPa m=(0.95)(0.0167)=0.0159 mm/mm
<b>Step 2—Determine the required maximum compressive strength of confined concrete f'<sub>c</sub>.</b> f <sub>a</sub> ' can be obtained by reordering Eq(12.1b):  $f'_{c'} = \frac{1}{0.85(A_e - A_a)} \left( \frac{\phi P_{n,req}}{0.80\phi} - f_y A_{vt} \right)$  $f'_{c'} = \frac{1}{0.85 \times (576 \text{ in.}^2 - 15.24 \text{ in.}^2)} \times \left( \frac{2504 \text{ kip}}{0.80 \times 0.65} - 60 \text{ ksi} \times 15.24 \text{ in.}^2 \right)$  f <sub>a</sub> =8.18 ksi		$f'_{c'} = \frac{1}{0.85 \times (371,612 \text{ mm}^2 - 9832 \text{ mm}^2)} \times \left( \frac{11,138 \text{ kN}}{0.80 \times 0.65} - 414 \text{ MPa} \times 9832 \text{ mm}^2 \right)$  $f'_{c'} = 56.4 \text{ MPa}$
<b>Step 3—Determine the maximum confining pressure due to the FRP jacket f<sub>r</sub>.</b>  f <sub>r</sub> can be obtained by reordering Eq. (12.1g):  $f_r = \frac{f'_{c'} - f_c}{3.3\psi_r K_a}$  where $K_a = \frac{A_e}{A_i} \left( \frac{b}{h} \right)^2$  $\frac{A_e}{A_i} = \frac{\frac{1}{1 - \left[ \left( \frac{b}{h} \right) (h - 2r_c)^2 + \left( \frac{b}{h} \right) (b - 2r_c)^2 \right]} - \rho_e}{\frac{3A_x}{1 - \rho_s}}$	$f_r = \frac{8.18 \text{ ksi} - 6.5 \text{ ksi}}{0.95 \times 3.3 \times 0.425} = 1.26 \text{ ksi}$  Ka=0.425 (1) <sup>2</sup> =0.425	$f_r = \frac{56.4 \text{ MPa} - 44.8 \text{ MPa}}{0.95 \times 3.3 \times 0.425} = 8.7 \text{ MPa}$  Ka=0.425 (1) <sup>2</sup> =0.425

Table 16.8c, cont.—Procedure for strengthening of a noncircular concrete column for axial load increase

<b>Step4—Determine the number of plies N.</b> N can be obtained by reordering Eq. (12.1h): $N = \frac{f_i \sqrt{b^2 + h^2}}{2E_i t_f e_{e_e}}$ <p style="text-align: center;"><math>e = KEm</math></p> <p>Checking the minimum confinement ratio:</p> $\frac{f_i}{f'_c} \geq 0.08$	$N = \frac{1.26 \text{ ksi} \sqrt{(24 \text{ in.})^2 + (24 \text{ in.})^2}}{2(33,000 \text{ ksi})(0.013 \text{ in.})(8.8 \times 10^{-3})}$ <p style="text-align: center;">N=5. 7:6 plies</p> $Ee=0.55 \times 0.0159 \text{ in./in.} = 8.8 \times 10^3 \text{ in./in.}$ $\frac{f_i}{f'_c} = \frac{1.26 \text{ ksi}}{6.5 \text{ ksi}} = 0.19 > 0.08 \quad \text{OK}$	$N = \frac{8.7 \text{ MPa} \sqrt{(610 \text{ mm})^2 + (610 \text{ mm})^2}}{2(227,527 \text{ MPa})(0.33 \text{ mm})(8.8 \times 10^{-3})}$ <p style="text-align: center;">N=5.726plies</p> $B\% = 0.55 \times 0.0159 \text{ mm/mm} = 8.8 \times 10^3 \text{ mm/mm}$ $\frac{f_i}{f'_c} = \frac{8.7 \text{ MPa}}{44.8 \text{ MPa}} = 0.19 > 0.08 \quad \text{OK}$
<b>Step5—Verify that the ultimate axial strain of the confined concrete <math>\epsilon_{cu} \leq 0.01</math>.</b> can be obtained using Eq. (12.1j): $\epsilon_{cuy} = \epsilon'_c \left( 1.5 + 12K_b \frac{f_i}{f'_c} \left( \frac{\epsilon_{fc}}{\epsilon'_c} \right)^{0.45} \right)$ <p>where</p> $K_b = \frac{A_c}{A_s} \left( \frac{h}{b} \right)^{0.5}$ <p>If the case that <math>\epsilon_{cuy}</math> was to be greater than 0.01, then <math>f'_c</math> should be recalculated from the stress-strain model using Eq. (12.1c).</p>	$E=(0.002 \text{ in./in.})$ $\times \left( 1.5 + 12 \times 0.425 \times \frac{1.2 \text{ ksi}}{6.5 \text{ ksi}} \left( \frac{8.8 \times 10^{-3} \text{ in./in.}}{0.002 \text{ in./in.}} \right)^{0.45} \right)$ $Ec=0.0067 \text{ in./in.} < 0.01 \quad \text{OK}$ $K_b=0.425(1)^5=0.425$	$= (0.002 \text{ mm/mm})$ $\times \left( 1.5 + 12 \times 0.425 \times \frac{8.3 \text{ MPa}}{44.8 \text{ MPa}} \left( \frac{8.8 \times 10^{-3} \text{ mm/mm}}{0.002 \text{ mm/mm}} \right)^{0.45} \right)$ $Ec=0.0067 \text{ mm/mm} < 0.01 \quad \text{OK}$ $K_b=0.425(1)^5=0.425$

Table 16.9—Procedure for strengthening of a noncircular concrete column for increase in axial and bending forces

Procedure	Calculation in in.-lb units	Calculation in SI units
<b>Step1—Determine the simplified curve for the unstrengthened column (n=0 plies).</b> Points A, B, and C can be obtained by well-known procedures, and by using Eq(C-1) to(C-5) considering $y=1$ , $f_a' = f'$ , $E_z=0$ , and $c_u=8u=0.003$ .	$\phi P_n = 2087 \text{ kip}; \phi M_m(c) = 0 \text{ kip-ft}$ $\phi P_{acB} = 1858 \text{ kip}; \phi M_{ag} = 644 \text{ kip-ft}$ $\phi P_{ao} = 928 \text{ kip}; \phi M_{mc} = 884 \text{ kip-ft}$	$\phi P_{mcu} = 9283 \text{ kN}; \phi M_m = 0 \text{ kN-m}$ $\phi P_{acs} = 8265 \text{ kN}; \phi M_{a(B)} = 873 \text{ kN}$ $\phi P_{ao} = 4128 \text{ kN}; \phi M_{mo} = 1199 \text{ kN-m}$

### 16.9—Strengthening of a noncircular concrete column for increase in axial and bending forces

The column described in 16.8 is subjected to an ultimate axial compressive load  $P=1900$  kip (8451 kN) and an ultimate bending moment  $M=380$  kip-ft (515 kN-m)

( $e=0.1h$ ). It is sought to increase load demands by 30% at constant eccentricity ( $P_u=2470$  kip,  $M=494$  kip-ft). The calculations to determine moment-axial interaction for the FRP confined column follow in Table 16.9.

**Table 16.9,cont.—Procedure for strengthening of a noncircular concrete column for increase in axial and bending forces**

<p><b>Step 2—Determine the simplified curve for a strengthened column.</b></p> <p>A wrapping system composed of six plies will be the starting point to construct the bilinear Curve A-B-C and then be compared with the position of the required <math>P</math>, and <math>M</math>.</p> <p>Points A, B, and C of the curve can be computed using Eq. (12.1b), (C-1), and (C-2):</p> <p><math>\phi P_{ma}\Delta = \phi 0.8(0.85/(4g-Am)+fA)</math></p> <p><math>\phi PaBc = \phi [A(y)^3+Bv.]^2+Cv]</math>  <math>+D+\sum AJa</math></p> <p><math>\phi Ma(Bc) = \phi (E(V)^4+F(v)^3+G(v)^2</math>  <math>+H(v.)+1)+\sum Af_d</math></p>	<p>Point A:</p> <p>Nominal axial capacity:</p> <p><math>\phi P_m(a) = 0.65 \times 0.8(0.85 \times 8.26 \text{ ksi} \times (576 \text{ in.}^2 - 15.24 \text{ in.}) + 60 \text{ ksi} \times 15.24 \text{ in.})</math>  <math>\phi P_m(a) = 2523 \text{ kip}</math></p> <p>where</p> <p><math>f_a' = 6.5 \text{ ksi} + 3.3(0.425)(1.26 \text{ ksi})</math>  <math>f_a' = 8.26 \text{ ksi}</math></p> <p><math>f_i = \frac{0.95 \times 2 \times 33,000 \text{ ksi} \times 6 \times 0.013 \text{ in.} \times \left(0.55 \times 0.0159 \frac{\text{in.}}{\text{in.}}\right)}{\sqrt{(24 \text{ in.})^2 + (24 \text{ in.})^2}}</math>  <math>f_i = 1.26 \text{ ksi}</math></p> <p>Point B:</p> <p>Nominal axial capacity:</p> <p><math>\phi P_B = 0.65[-0.22 \text{ kip/in.}^3 + (15.33 \text{ in.}^3 + 10.17 \text{ ksi})(15.33 \text{ in.}^2 - 156 \text{ kip/in.} (15.33 \text{ in.}) + 3645.2 \text{ kip} + 5.08 \text{ in.}^2 (60 \text{ ksi}) + 2.54 \text{ in.}^2 (60 \text{ ksi}) + 2.54 \text{ in.}^2 (37.21 \text{ ksi}))]</math>  <math>\phi P_B = 2210 \text{ kip}</math></p> <p>where</p> <p><math>A = \frac{-24 \text{ in.}(4595 \text{ ksi} - 190.7 \text{ ksi})^2}{12 \times 6.5 \text{ ksi}} \left(\frac{0.0042 \text{ in./in.}}{22 \text{ in.}}\right)^2</math>  <math>= -0.22 \text{ kip/in.}</math></p> <p><math>B = \frac{24 \text{ in.}(4595 \text{ ksi} - 190.7 \text{ ksi})}{2} \left(\frac{0.0042 \text{ in./in.}}{22 \text{ in.}}\right)</math>  <math>= 10.17 \text{ ksi}</math></p> <p><math>C = -24 \text{ in.} \times 6.5 \text{ ksi} = -156 \text{ kip/in.}</math></p> <p><math>D = 24 \text{ in.} \times 22 \text{ in.} \times 6.5 \text{ ksi}</math>  <math>+ \frac{24 \text{ in.} \times 22 \text{ in.} \times 190.7 \text{ ksi}}{2}</math>  <math>\times (0.0042 \text{ in./in.})</math>  <math>D = 3645.2 \text{ kip}</math></p>	<p>Point A:</p> <p>Nominal axial capacity:</p> <p><math>\phi P_a(a) = 0.65 \times 0.8(0.85 \times 56.96 \text{ MPa} \times (371,612 \text{ mm}^2 - 9832 \text{ mm}^2) + 414 \text{ MPa} \times 9232 \text{ mm}^2)</math>  <math>\phi P_m(a) = 11,223 \text{ kN}</math></p> <p>where</p> <p><math>a' = 44.8 \text{ MPa} + 3.3(0.425)(8.7 \text{ MPa})</math>  <math>a' = 56.96 \text{ MPa}</math></p> <p><math>f_i = \frac{0.95 \times 6 \times 0.33 \text{ mm} \times \left(0.55 \times 0.0159 \frac{\text{mm}}{\text{mm}}\right)}{\sqrt{(610 \text{ mm})^2 + (610 \text{ mm})^2}}</math>  <math>f_i = 8.67 \text{ MPa}</math></p> <p>Point B:</p> <p>Nominal axial capacity:</p> <p><math>\phi P_m(s) = 0.65[-6,003 \times 10^5 \text{ kN/mm}^3 (389 \text{ mm})^3 + 70.14 \times 10^{-3} \text{ kN/mm}^2 (389 \text{ mm})^2 - 27.32 \text{ kN/mm} (389 \text{ mm}) + 16,215 \text{ kN} + 3277 \text{ mm}^2 (414 \text{ MPa}) + 1639 \text{ mm}^2 (414 \text{ MPa}) + 1639 \text{ mm}^2 (257 \text{ MPa})]</math>  <math>\phi P_m(s) = 9892 \text{ kN}</math></p> <p>where</p> <p><math>A = \frac{-610 \text{ mm}(31,685 \text{ MPa} - 1315 \text{ MPa})^2}{12 \times 44.8 \text{ MPa}} \left(\frac{0.0042 \text{ mm/mm}}{559 \text{ mm}}\right)^2</math>  <math>= -6,003 \times 10^3 \text{ kN/mm}^3</math></p> <p><math>B = \frac{600 \text{ mm}(31,685 \text{ MPa} - 1315 \text{ MPa})}{2} \left(\frac{0.0042 \text{ mm/mm}}{559 \text{ mm}}\right)</math>  <math>= 70.14 \times 10^{-3} \text{ kNm/mm}^2</math></p> <p><math>C = -610 \text{ mm} \times 44.84 \text{ MPa} = -27.32 \text{ kN/mm}</math></p> <p><math>D = 610 \text{ mm} \times 559 \text{ mm} \times 44.8 \text{ MPa}</math>  <math>+ \frac{610 \text{ mm} \times 559 \text{ mm} \times 1315 \text{ MPa}}{2} \times (0.0042 \text{ mm/mm})</math>  <math>D = 16,215 \text{ kN}</math></p>
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**Table 16.9,cont.—Procedure for strengthening of a noncircular concrete column for increase in axial and bending forces**

The coefficients A, B, C, D, E, F, G, H, and I of the previous expressions are given by Eq. (C-3): $A = \frac{-b(E_c - E_2)^2}{12f'_c} \left( \frac{\varepsilon_{con}}{c} \right)^2$ $B = \frac{b(E_c - E_2)}{2} \left( \frac{\varepsilon_{con}}{c} \right)$ $C = bf'!$ $D = bf'_c + \frac{bcE_2}{2} (\varepsilon_{con})$ $E = \frac{-b(E_c - E_2)^2}{16f'_c} \left( \frac{\varepsilon_{con}}{c} \right)^2$ $F = b \left( c - \frac{h}{2} \right) \frac{(E_c - E_2)^2}{12f'_c} \left( \frac{\varepsilon_{con}}{c} \right)^2 + \frac{b(E_c - E_2)}{3} \left( \frac{\varepsilon_{con}}{c} \right)$ $G = -\left( \frac{b}{2} f'_c + b \left( c - \frac{h}{2} \right) \frac{(E_c - E_2)}{2} \left( \frac{\varepsilon_{con}}{c} \right) \right)$ $H = bf'_c \left( c - \frac{h}{2} \right)$ $I = \frac{bc^2}{2} f'_c - bcf'_c \left( c - \frac{h}{2} \right) + \frac{bc^2 E_2}{3} (\varepsilon_{con})$ $- \frac{bcE_2}{2} \left( c - \frac{h}{2} \right) (\varepsilon_{con})$ $f_r = \frac{\psi_r 2E_r Nt_f \varepsilon_{re}}{\sqrt{b^2 + h^2}}$	For the calculation of the coefficients, it is necessary to compute key parameters from the stress-strain model: $y_t = 22 \text{ in.} \times \frac{0.003 \text{ in./in.}}{0.0042 \text{ in./in.}} = 15.33 \text{ in.}$ $c=22 \text{ in.}$ $\varepsilon'_r = \frac{2 \times 6.5 \text{ ksi}}{4595 \text{ ksi} - 190.7 \text{ ksi}} = 0.003 \text{ in./in.}$ $E_2 = \frac{7.31 \text{ ksi} - 6.5 \text{ ksi}}{0.0042 \text{ in./in.}} = 190.7 \text{ ksi}$ $f'_c = 6.5 \text{ ksi} + 3.3 (0.425) (0.58 \text{ ksi}) = 7.31 \text{ ksi}$ $Bm=0.002 \text{ in./in.}$ $f_r = \frac{0.95 \times 2 \times 33,000 \text{ ksi} \times 6 \times 0.013 \text{ in.} \times (0.004 \text{ in./in.})}{\sqrt{(24 \text{ in.})^2 + (24 \text{ in.})^2}}$ $\text{Checking the minimum confinement ratio:}$ $f'U' = 0.58 \text{ ksi} / 6.5 \text{ ksi} = 0.09 : 0.08 \quad \text{OK}$ $The strains in each layer of steel are determined by similar triangles in the strain distribution. The corresponding stresses are then given by:$ $JA = E_1 E_5 = 0.0038 \text{ in./in.} \times 29,000 \text{ ksi} = 60 \text{ ksi}$ $Ja = E_2 E_5 = 0.0026 \text{ in./in.} \times 29,000 \text{ ksi} = 260 \text{ ksi}$ $fs = E_3 E_5 = 0.0013 \text{ in./in.} \times 29,000 \text{ ksi} = 37.2 \text{ ksi}$ $Ja = E_4 E_5 = 0 \text{ in./in.} \times 29,000 \text{ ksi} = 0 \text{ ksi}$	For the calculation of the coefficients, it is necessary to compute key parameters from the stress-strain model: $y_t = 559 \text{ mm} \times \frac{0.003 \text{ mm/mm}}{0.0042 \text{ mm/mm}} = 389 \text{ mm}$ $c=559 \text{ mm}$ $\varepsilon'_r = \frac{2 \times 44.8 \text{ MPa}}{31,685 \text{ MPa} - 1315 \text{ MPa}} = 0.003 \text{ mm/mm}$ $E_2 = \frac{50.4 \text{ MPa} - 44.8 \text{ MPa}}{0.0042 \text{ mm/mm}} = 1315 \text{ MPa}$ $fa' = 44.8 \text{ MPa} + 3.3 (0.425)(3.97 \text{ MPa}) = 50.4 \text{ MPa}$ $Ec = 0.0042 \text{ mm/mm}$ $Ka = KA = 0.425$ $f_r = \frac{0.95 \times 2 \times 227,527 \text{ MPa} \times 6 \times 0.33 \text{ mm} \times (0.004 \text{ mm/mm})}{\sqrt{(610 \text{ mm})^2 + (610 \text{ mm})^2}}$ $\text{Checking the minimum confinement ratio:}$ $f'fc' = 3.97 \text{ MPa} / 44.8 \text{ MPa} = 0.09 : 0.08 \quad \text{OK}$ $The strains in each layer of steel are determined by similar triangles in the strain distribution. The corresponding stresses are then given by:$ $f = E_1 E_2 = 0.0038 \text{ mm/mm} \times 200,000 \text{ MPa} = 414 \text{ MPa}$ $Jz = B_2 E_2 = 0.0026 \text{ mm/mm} \times 200,000 \text{ MPa} = 414 \text{ MPa}$ $fs = E_3 E_4 = 0.0013 \text{ mm/mm} \times 200,000 \text{ MPa} = 257 \text{ MPa}$ $Ja = E_4 E_2 = 0 \text{ mm/mm} \times 200,000 \text{ MPa} = 0 \text{ MPa}$
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**Table 16.9,cont.—Procedure for strengthening of a noncircular concrete column for increase in axial and bending forces**

Key parameters of the stress-strain model:	Nominal bending moment:	Nominal bending moment:
$y_s = c \frac{\varepsilon'_s}{\varepsilon_{cov}}$ $c = \begin{cases} d & \text{for Point B} \\ d \frac{\varepsilon_{cov}}{\varepsilon_{cv} + \varepsilon_{cov}} & \text{for Point C} \end{cases}$ $\varepsilon'_s = \frac{2f'_s}{E_s - E_2}$ $E_2 = \frac{f'_s - f'_v}{\varepsilon_{cov}}$ $a' = +3.3K$ fi $\varepsilon_{cov} = \varepsilon'_c \left( 1.5 + 12\kappa_b \frac{f_v}{f'_v} \left( \frac{\varepsilon_{fv}}{\varepsilon'_v} \right)^{0.45} \right)$ $E_c = \min(0.004, K_e \cdot \varepsilon'_c)$ $\kappa_a = \frac{A_e}{A_c} \left( \frac{b}{h} \right)^2$ $\kappa_b = \frac{A_e}{A_c} \left( \frac{h}{b} \right)^{0.5}$	Nominal bending moment: $\phi M_{nB} = 0.65 [-0.166 \text{ kip/in. (15.33 in.)}^4 + 8.99 \text{ ksi (15.33 in.)}^3 - 179.73 \text{ kip/in. (15.33 in.)}^2 + 1560 \text{ kip (15.33 in.)} + 4427 \text{ kip-in.} + 5.08 \text{ in.}^2 (60 \text{ ksi}) (10 \text{ in.}) + 2.54 \text{ in. (60 ksi) (3.3 in.)} - 2.54 \text{ in.}^2 (37.21 \text{ ksi}) (3.3 in.)]$ $\phi M_s = 682 \text{ kip-ft}$ where $E = \frac{-24 \text{ in.} (4595 \text{ ksi} - 190.7 \text{ ksi})^2}{16 \times 6.5 \text{ ksi}} \left( \frac{0.0042 \text{ in./in.}}{22 \text{ in.}} \right)^2 = -0.166 \text{ kip/in.}^3$ $F = 24 \text{ in. (22 in. - 12 in.)} \times \frac{(4595 \text{ ksi} - 190.7 \text{ ksi})^2}{12 \times 6.5 \text{ ksi}} \times \left( \frac{0.0042 \text{ in./in.}}{22 \text{ in.}} \right)^2 + \frac{24 \text{ in.} (4595 \text{ ksi} - 190.7 \text{ ksi})}{3} \times \left( \frac{0.0042 \text{ in./in.}}{22 \text{ in.}} \right)^2 = 8.99 \text{ ksi}$ Notes: The designer should bear in mind that, for the case of pure compression, the effective strain in the FRP, $\varepsilon$ , is limited by $K_n$ and, in the case of combined axial and bending, by $E_e = \min(0.004, K_E \cdot \varepsilon'_c)$ . $G = -6.5 \text{ ksi} \times 12 \text{ in.} + 24 \text{ in. (22 in. - 12 in.)} \times 0.5 \times (4595 \text{ ksi} - 190.7 \text{ ksi}) \times (0.0042 \text{ in./in.} / 22 \text{ in.}) = -179.73 \text{ kip/in.}$ $H = 6.5 \text{ ksi} \times 24 \text{ in. (22 in. - 12 in.)} = 1560 \text{ kip}$ $I = 6.5 \text{ ksi} \times 24 \text{ in.} \times ([22 \text{ in.}]^2 / 2) - 6.5 \text{ ksi} (22 \text{ in. - 12 in.}) \times 22 \text{ in.} \times 24 \text{ in.} + 190.7 \text{ ksi} \times 24 \text{ in.} \times ([22 \text{ in.}]^2 / 3) (0.0042 \text{ in./in.}) - 190.7 \text{ ksi} \times 24 \text{ in.} \times (22 \text{ in.} / 3) (22 \text{ in. - 12 in.}) (0.0042 \text{ in./in.}) = 4427 \text{ kip-in.}$	Nominal bending moment: $\phi M_{nB} = 0.65 [-4.502 \times 10^5 \text{ kN/mm}^3 (389 \text{ mm})^4 + 62.01 \times 10^{-3} \text{ kN/mm}^3 (389 \text{ mm})^3 - 31.48 \text{ kN/mm} (389 \text{ mm})^2 + 6939 \text{ kN (389 mm)} + 500,162 \text{ kN-mm} + 3277 \text{ mm}^2 (414 \text{ MPa}) (254 \text{ mm}) + 1639 \text{ mm}^2 (414 \text{ MPa}) (85 \text{ mm}) - 1639 \text{ mm}^2 (257 \text{ MPa}) (85 \text{ mm})]$ $\phi M_s = 924 \text{ kN-mm}$ where $E = \frac{-610 \text{ mm} (31,685 \text{ MPa} - 1315 \text{ MPa})^2}{16 \times 44.8 \text{ MPa}} \times \left( \frac{0.0042 \text{ mm/mm}}{559 \text{ mm}} \right)^2 = -0.452 \times 10^{-5} \text{ kN/mm}^5$ $F = 610 \text{ mm} (559 \text{ mm} - 305 \text{ mm}) \times \frac{(31,685 \text{ MPa} - 1315 \text{ MPa})^2}{12 \times 44.8 \text{ MPa}} \times \left( \frac{0.0042 \text{ mm/mm}}{559 \text{ mm}} \right)^2 + \frac{610 \text{ mm} (31,685 \text{ MPa} - 1315 \text{ MPa})}{3} \times \left( \frac{0.0042 \text{ mm/mm}}{559 \text{ mm}} \right)^2 = 62.01 \times 10^{-5} \text{ kN/mm}^5$ $G = -44.8 \text{ MPa} \times 305 \text{ mm} + 610 \text{ mm} (559 \text{ mm} - 305 \text{ mm}) \times 0.5 \times (31,685 \text{ MPa} - 1315 \text{ MPa}) \times (0.0042 \text{ mm/mm} / 559 \text{ mm}) = -31.48 \text{ kN/mm}$ $H = 44.8 \text{ MPa} \times 610 \text{ mm} (559 \text{ mm} - 305 \text{ mm}) = 6939 \text{ kN}$ $I = 44.8 \text{ MPa} \times 610 \text{ mm} \times ([559 \text{ mm}]^2 / 2) - 44.8 \text{ MPa} (559 \text{ mm} - 305 \text{ mm}) \times (559 \text{ mm}) (610 \text{ mm}) + 1315 \text{ MPa} \times 610 \text{ mm} \times ([559 \text{ mm}] / 3) (0.0042 \text{ mm/mm}) - 1315 \text{ MPa} \times 610 \text{ mm} \times (559 \text{ mm} / 3) (559 \text{ mm} - 305 \text{ mm}) (0.0042 \text{ mm/mm}) = 500,162 \text{ kN-mm}$

**Table 16.9,cont.—Procedure for strengthening of a noncircular concrete column for increase in axial and bending forces**

<p>The distances from each layer of steel reinforcement to the geometric centroid of the cross section are:</p> $d_1=10 \text{ in.}$ $d_2=d_3=3.3 \text{ in.}$ <p style="text-align: center;">Point C: Nominal axial capacity:</p> $\begin{aligned} P_{no}= & 0.65[-0.49 \text{ kip/in. } (10.3 \text{ in.}^3 \\ & +15.14 \text{ ksi } (10.3 \text{ in.}^2 -156 \text{ kip-in. } (10.3 \text{ in.}) \\ & +2448.71 \text{ kip+5.08 in. } (60 \text{ ksi}) \\ & +2.54 \text{ in. } ?(50.79 \text{ ksi})+2.54 \text{ in. } ?(-4.61 \text{ ksi}) \\ & +5.08 \text{ in. } ?(-60 \text{ ksi})] \\ & \phi P_{ao}=1320 \text{ kip} \end{aligned}$ <p>where</p> $A = \frac{-24 \text{ in. } (4595 \text{ ksi} - 190.7 \text{ ksi})^2}{12 \times 6.5 \text{ ksi}} \left( \frac{0.0042 \text{ in./in.}}{14.78 \text{ in.}} \right)^2$ $= -0.49 \text{ kip/in.}$ $B = \frac{-24 \text{ in. } (4595 \text{ ksi} - 190.7 \text{ ksi})}{2} \left( \frac{0.0042 \text{ in./in.}}{14.78 \text{ in.}} \right)$ $= 15.14 \text{ ksi}$ $C = -24 \text{ in. } \times 6.5 \text{ ksi} = -156 \text{ kip/in.}$ $D = 24 \text{ in. } \times 14.78 \text{ in. } \times 6.5 \text{ ksi}$ $+ \frac{24 \text{ in. } \times 14.78 \text{ in. } \times 190.7 \text{ ksi}}{2}$ $\times (0.0042 \text{ in./in.}) = 2448.71 \text{ kip}$ <p>For the calculation of the coefficients, it is necessary to compute key parameters from the stress-strain model:</p> $c = 22 \text{ in. } \times \left( \frac{0.0042 \text{ in./in.}}{0.0021 \text{ in./in.} + 0.0042 \text{ in./in.}} \right)$ $= 14.67 \text{ in.}$ $y_c = 14.64 \text{ in. } \frac{0.003 \text{ in./in.}}{0.0042 \text{ in./in.}} = 10.5 \text{ in.}$	<p>The distances from each layer of steel reinforcement to the geometric centroid of the cross section are:</p> $d_1=254 \text{ mm}$ $d_2=d_3=85 \text{ mm}$ <p style="text-align: center;">Point C: Nominal axial capacity:</p> $\begin{aligned} \phi P_{mc}= & 0.65[-1.33 \times 10^4 \text{ kN/mm}^3 (262 \text{ mm})^3 + \\ & 104.41 \times 10^3 \text{ kN/mm}^2 \times (262 \text{ mm})^2 \\ & -27.32 \text{ kN/mm } (262 \text{ mm}) \\ & +10,892 \text{ kN} + 3277 \text{ mm}^2 (414 \text{ MPa}) \\ & +1315 \text{ mm}^2 (350 \text{ MPa}) + 1315 \text{ mm}^2 (-31.8 \text{ MPa}) \\ & +3277 \text{ mm}^2 (-414 \text{ MPa})] \\ & \phi P_{mc}=5870 \text{ kN} \end{aligned}$ <p>where</p> $A = \frac{-610 \text{ mm} (31,681 \text{ MPa} - 1315 \text{ MPa})^2}{12 \times 44.8 \text{ MPa}}$ $\times \left( \frac{0.0042 \text{ mm/mm}}{375 \text{ mm}} \right)^2$ $= 1.33 \times 10^4 \text{ kN/mm}^2$ $B = \frac{-610 \text{ mm} (31,681 \text{ MPa} - 1315 \text{ MPa})}{2}$ $\times \left( \frac{0.0042 \text{ mm/mm}}{375 \text{ mm}} \right)$ $= -104.41 \times 10^3 \text{ kN/mm}^2$ $C = -610 \text{ mm} \times 44.8 \text{ MPa} = -27.32 \text{ kN/mm}$ $D = 610 \text{ mm} \times 375 \text{ mm} \times 44.8 \text{ MPa}$ $+ \frac{610 \text{ mm} \times 375 \text{ mm} \times 1315 \text{ MPa}}{2}$ $\times (0.0042 \text{ mm/mm}) = 10,892 \text{ kN}$ <p>For the calculation of the coefficients, it is necessary to compute key parameters from the stress-strain model:</p> $c = 560 \text{ mm}$ $\times \left( \frac{0.0042 \text{ mm/mm}}{0.0021 \text{ mm/mm} + 0.0042 \text{ mm/mm}} \right)$ $= 373 \text{ mm}$ $y_c = 373 \text{ mm } \frac{0.003 \text{ in./in.}}{0.0042 \text{ in./in.}} = 266 \text{ mm}$
<p>The strains in each layer of steel are determined by similar triangles in the strain distribution. The corresponding stresses are then given by:</p> $f_a=E_aE_3=0.0037 \text{ in./in. } \times 29,000 \text{ ksi} = 60 \text{ ksi}$ $f=EE_4=0.0018 \text{ in./in. } \times 29,000 \text{ ksi} = 50.78 \text{ ksi}$ $f_a=E_aE_3=-1.59 \times 10^4 \text{ in./in. } \times 29,000 \text{ ksi} = -4.61 \text{ ksi}$ $f_a=e_aE_2=-0.0021 \text{ in./in. } \times 29,000 \text{ ksi} = -60 \text{ ksi}$	<p>The strains in each layer of steel are determined by similar triangles in the strain distribution. The corresponding stresses are then given by:</p> $f_i=E_iE_i=0.0037 \text{ mm/mm } \times 200,000 \text{ MPa} = 414 \text{ MPa}$ $f_2=E_2E_2=0.0018 \text{ mm/mm } \times 200,000 \text{ MPa} = 350 \text{ MPa}$ $f_s=E_3E_4=-1.59 \times 10^4 \text{ mm/mm } \times 200,000 \text{ MPa} = -31.8 \text{ MPa}$ $f_4=\epsilon_4E_i=-0.0021 \text{ mm/mm } \times 200,000 \text{ MPa} = 414 \text{ MPa}$

**Table 16.9, cont.—Procedure for strengthening of a noncircular concrete column for increase in axial and bending forces**

	<p>Nominal bending moment:</p> $\Phi M_c = 0.65[-0.37 \text{ kip/in.}^3 (10.3 \text{ in.})^4 + 11.46 \text{ ksi} (10.3 \text{ in.})^3 - 120.08 \text{ kip/in.} (10.3 \text{ in.})^2 + 433.5 \text{ kip}(10.3 \text{ in.}) + 11,643 \text{ kip-in.} + 5.08 \text{ in.}^2 (60 \text{ ksi}) (10 \text{ in.}) + 2.54 \text{ in.}^2 (50.79 \text{ ksi}) (3.33 \text{ in.}) - 2.54 \text{ in.}^2 (4.61 \text{ ksi}) (3.33 \text{ in.}) - 5.08 \text{ in.}^2 (-60 \text{ ksi}) (10 \text{ in.})]$ $\Phi M_c = 992 \text{ kip-ft}$ <p>where</p> $E = \frac{-24 \text{ in.} (4595 \text{ ksi} - 190.7 \text{ ksi})^2}{16 \times 6.5 \text{ ksi}} \left( \frac{0.0042 \text{ in./in.}}{14.78 \text{ in.}} \right)^2 = -0.37 \text{ kip/in.}^3$ $F = 24 \text{ in.} (14.78 \text{ in.} - 12 \text{ in.}) \times \frac{(4595 \text{ ksi} - 190.7 \text{ ksi})^2}{12 \times 6.5 \text{ ksi}} \left( \frac{0.0042 \text{ in./in.}}{14.78 \text{ in.}} \right)^2 + \frac{24 \text{ in.} (4595 \text{ ksi} - 190.7 \text{ ksi})}{3} \times \left( \frac{0.0042 \text{ in./in.}}{14.78 \text{ in.}} \right) = 11.46 \text{ ksi}$ $G = -6.5 \text{ ksi} \times 12 \text{ in.} + 24 \text{ in.} (14.78 \text{ in.} - 12 \text{ in.}) \times \left( \frac{4595 \text{ ksi} - 190.7 \text{ ksi}}{2} \right) \left( \frac{0.0042 \text{ in./in.}}{14.78 \text{ in.}} \right)$ $H = 6.5 \text{ ksi} \times 24 \text{ in.} (14.78 \text{ in.} - 12 \text{ in.}) = 433.5 \text{ kip}$ $I = 6.5 \text{ ksi} \times 24 \text{ in.} \times ([14.78 \text{ in.}] / 2) - 6.5 \text{ ksi} (14.78 \text{ in.} - 12 \text{ in.}) (14.78 \text{ in.}) (24 \text{ in.}) + 190.7 \text{ ksi} \times 24 \text{ in.} \times ([14.78 \text{ in.}]^2 / 3) (0.0042 \text{ in./in.}) - 190.7 \text{ ksi} \times 24 \text{ in.} \times (14.78 \text{ in.}) / 3 (14.78 \text{ in.} - 12 \text{ in.}) (0.0042 \text{ in./in.}) = 11,643 \text{ kip-in.}$	<p>Nominal bending moment:</p> $\Phi M_{nc} = 0.65[-9.98 \times 10^5 \text{ kN/mm}^4 + 79 \times 10^{-3} \text{ kN/mm}^2 (262 \text{ mm})^3 - 21.03 \text{ kN/mm} (262 \text{ mm})^2 + 1928 \text{ kN} (262 \text{ mm}) + 1,315,453 \text{ kN-mm} + 3277 \text{ mm}^2 (414 \text{ MPa}) (254 \text{ mm}) + 1639 \text{ mm}^2 (350 \text{ MPa}) (85 \text{ mm}) - 1639 \text{ mm}^2 (-31.8 \text{ MPa}) (85 \text{ mm}) - 3277 \text{ mm}^2 (414 \text{ MPa}) (254 \text{ mm})]$ $\Phi M_n(o=1345 \text{ kN-mm})$ <p>where</p> $E = \frac{-610 \text{ mm} (31,681 \text{ MPa} - 1315 \text{ MPa})^2}{16 \times 44.8 \text{ MPa}} \left( \frac{0.0042 \text{ mm/mm}}{375 \text{ mm}} \right)^2 = 9.98 \times 10^{-3} \text{ kN/mm}^3$ $F = 610 \text{ mm} (375 \text{ mm} - 305 \text{ mm}) \times \frac{(31,681 \text{ MPa} - 1315 \text{ MPa})^2}{12 \times 44.8 \text{ MPa}} \left( \frac{0.0042 \text{ mm/mm}}{375 \text{ mm}} \right)^2 + \frac{610 \text{ mm} (31,681 \text{ MPa} - 1315 \text{ MPa})}{3} \times \left( \frac{0.0042 \text{ mm/mm}}{375 \text{ mm}} \right) = 79 \times 10^{-3} \text{ kN/mm}^3$ $G = 44.8 \text{ MPa} \times 305 \text{ mm} + 610 \text{ mm} (375 \text{ mm} - 305 \text{ mm}) \times \frac{(31,681 \text{ MPa} - 1315 \text{ MPa})^2}{2} \left( \frac{0.0042 \text{ mm/mm}}{375 \text{ mm}} \right)$ $H = -21.03 \text{ kN/mm}$ $I = 44.8 \text{ MPa} \times 610 \text{ mm} (375 \text{ mm} - 305 \text{ mm}) = 1928 \text{ kN}$ $I = 44.8 \text{ MPa} \times 610 \text{ mm} \times (1375 \text{ mm})^2 / 2 - 44.8 \text{ MPa} (375 \text{ mm} - 305 \text{ mm}) \times (375 \text{ mm}) (610 \text{ mm}) + 1315 \text{ MPa} \times 610 \text{ mm} \times ([375 \text{ mm}]^2 / 3) (0.0042 \text{ mm/mm}) - 1315 \text{ MPa} \times 610 \text{ mm} \times (375 \text{ mm}) / 2 (375 \text{ mm} - 305 \text{ mm}) (0.0042 \text{ mm/mm}) = 1,315,453 \text{ kN-mm}$																														
<p><b>Step 3—Comparison of simplified partial interaction diagram with required <math>P</math>, and <math>M</math>.</b></p>	<p>The following table summarizes the axial and bending nominal capacities (unstrengthened and strengthened) for Points A, B, and C. These points are plotted in the following figure:</p> <table border="1"> <thead> <tr> <th>Point</th> <th>Unstressed Capacity (<math>\mu_u</math>)</th> <th>Stressed Capacity (<math>\mu_s</math>)</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>~2500 kip</td> <td>~2500 kip</td> </tr> <tr> <td>A'</td> <td>~2000 kip</td> <td>~2000 kip</td> </tr> <tr> <td>B</td> <td>~1500 kip</td> <td>~1500 kip</td> </tr> <tr> <td>C</td> <td>~1000 kip</td> <td>~1000 kip</td> </tr> </tbody> </table>	Point	Unstressed Capacity ( $\mu_u$ )	Stressed Capacity ( $\mu_s$ )	A	~2500 kip	~2500 kip	A'	~2000 kip	~2000 kip	B	~1500 kip	~1500 kip	C	~1000 kip	~1000 kip	<p>The following table summarizes the axial and bending nominal capacities (unstrengthened and strengthened) for Points A, B, and C. These points are plotted in the following figure.</p> <table border="1"> <thead> <tr> <th>Point</th> <th>Unstressed Capacity (<math>\mu_u</math>)</th> <th>Stressed Capacity (<math>\mu_s</math>)</th> </tr> </thead> <tbody> <tr> <td>aA</td> <td>~12000 kN</td> <td>~12000 kN</td> </tr> <tr> <td>A'</td> <td>~9000 kN</td> <td>~9000 kN</td> </tr> <tr> <td>oB</td> <td>~6000 kN</td> <td>~6000 kN</td> </tr> <tr> <td>C</td> <td>~3000 kN</td> <td>~3000 kN</td> </tr> </tbody> </table>	Point	Unstressed Capacity ( $\mu_u$ )	Stressed Capacity ( $\mu_s$ )	aA	~12000 kN	~12000 kN	A'	~9000 kN	~9000 kN	oB	~6000 kN	~6000 kN	C	~3000 kN	~3000 kN
Point	Unstressed Capacity ( $\mu_u$ )	Stressed Capacity ( $\mu_s$ )																														
A	~2500 kip	~2500 kip																														
A'	~2000 kip	~2000 kip																														
B	~1500 kip	~1500 kip																														
C	~1000 kip	~1000 kip																														
Point	Unstressed Capacity ( $\mu_u$ )	Stressed Capacity ( $\mu_s$ )																														
aA	~12000 kN	~12000 kN																														
A'	~9000 kN	~9000 kN																														
oB	~6000 kN	~6000 kN																														
C	~3000 kN	~3000 kN																														

### 16.10—Plastic hinge confinement for seismic strengthening

This example illustrates the design of an FRP retrofit to enhance the plastic rotation capacity of a nonductile reinforced concrete column. In this example, the column cannot efficiently resist seismic loads for two reasons: 1) the tie spacing does not conform to current seismic design codes; and 2) the ties do not project into the core and under seismic loads may open once the cover concrete begins to spall. The aforementioned deficiencies indicate that the column may have inadequate shear strength, inadequate confinement of the plastic hinge region, and inadequate clamping of lap splices. In addition, the designer should ensure adequate resistance against buckling of the main longitudinal reinforcement. In this example, the deficiency under consideration is inadequate plastic rotation capacity, which can be enhanced by confinement of the plastic hinge region with FRP. A seismic analysis has already determined that the column is capable of resisting the required seismic moments. Thus, there is no need to increase the flexural capacity of the section. This example is limited in scope to the FRP design requirements and does not cover the seismic analysis. ASCE/SEI 41 is used as the base standard for this example.

The column, which is to be part of a lateral-load-resisting system, is illustrated in Fig. 16.10a. Expected material properties and other relevant information are listed in Table 16.10a. The axial load on the column, including gravity plus seismic loads, is  $P_u = 75$  kip (334 kN). From a seismic analysis, the column should be capable of developing a plastic rotation  $\theta_p = 0.025$  rad. This plastic rotation demand exceeds the limiting value of 0.015 stipulated in ASCE/SEI 41 numerical acceptance criteria for reinforced concrete columns that do not conform to current seismic design codes. The concrete and reinforcing steel strain limitations of ASCE/SEI 41 as listed in Table 16.10b should not be exceeded.

The column is strengthened with CFRP laminates having the composite properties listed in Table 16.10c and bonded around the column using the wet layup technique. Glass FRP (GFRP), however, can similarly be used if desired. The design process was initiated by considering a wrapping system composed of three plies. After two iterations, the final wrapping system was found to require five plies. Only the calculations used to verify the final configuration are provided. These calculations are shown in Table 16.10d.

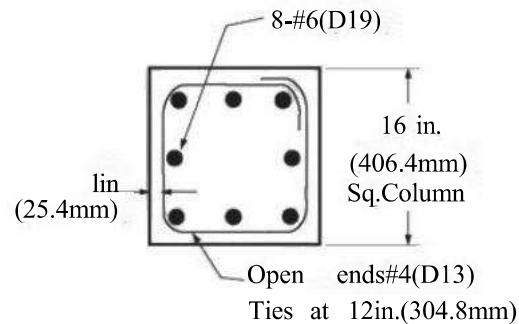


Fig. 16.10a—As-built column details.

Table 16.10a—Column material properties

Concrete strength <sup>f</sup>	4000 psi	27.6 MPa
Concrete elastic modulus $E = 57,000 \sqrt{f_c}$ (psi) $E = 4700 \sqrt{f_c}$ (MPa)	3605 ksi	25 GPa
Longitudinal reinforcing steel: yield strength <sup>f</sup> , Modulus of elasticity of steel E,	44,000 psi	303 MPa
Longitudinal reinforcing steel: yield strain $\epsilon_y$ ,	0.0015	0.0015
Column height between plastic hinges L	10 ft	3.05 m
Distance to extreme tension steel d	14.625 in.	371 mm

Table 16.10b—Maximum usable strain levels (ASCE/SEI 41)

	Strain limits	
	Unconfined sections	Confined sections per 13.3.1
Compressive (concrete)	0.003	$\epsilon_w$
Tensile (steel)	0.005	0.05
Compressive (steel)	Limited by the concrete	0.02

Table 16.10c—Manufacturer's reported FRP System properties

Thickness per ply <sup>j</sup>	0.023 in.	0.584 mm
Ultimate tensile strength <sup>j</sup>	155 ksi	1072 MPa
Rupture strain $\epsilon_u$	0.015	0.015
Modulus of elasticity, E	9326 ksi	64.3 GPa

Table 16.10d—Procedure for designing plastic hinge confinement for seismic strengthening

Procedure	Calculation in in.-lb units	Calculation in SI units
<b>Step1—FRP-system design material properties:</b>  En=Cen Cg=0.95  The column is located in an interior space. Therefore, per Table 9.4, an environmental-reduction factor of 0.95 is used.  $\phi = 0.004 \leq K_E m$	$E_m \leq 0.95 \times 0.015 = 0.014$ $K_0 = 0.55$  $E_e = 0.004 \leq 0.55 \times 0.014 = 0.0078 E_e = 0.04$	$\% = 0.04$
<b>Step 2—Plastic hinge length:</b> In FRP-jacketed columns, the plastic hinge length is:  $L_p = g + 0.0003 f, \text{ in. (psi and in.)}$ $L_p = g + 0.044 / d_s, \text{ (MPa and mm)}$	$L_p = 2 + 0.0003 \times 44,000 \times 0.75 = 12 \text{ in.}$  Note: Use a maximum gap between the FRP and the column base of 2 in. (50.80 mm)	$L_p = 50.8 + 0.044 \times 303.4 \times 19 = 305 \text{ in.}$
<b>Step 3—Preliminary calculations:</b>  $b = h = 16 \text{ in. (406mm)}$  From 12.1.2:  $D = \sqrt{h^2 + b^2}$  $K_a = \frac{A_c}{A_e} \left( \frac{b}{h} \right)^2$  $K_b = \frac{A_c}{A_e} \sqrt{\frac{h}{b}}$	$D = \sqrt{16^2 + 16^2} = 22.63 \text{ in.}$  $AJ/A = 0.62$ $N_a = 0.62 \left( \frac{A_c}{A_e} \right)^2 = 0.62$  $N_b = 0.62 \sqrt{\frac{16}{16}} = 0.62$  $\rho_s' = 0.002$	$DN = \sqrt{406.4^2 + 406.4^2} = 575 \text{ mm}$  $AJA = 0.62$ $K_n = 0.62$  $K_b = 0.62$  $B' = 0.002$
where AJ/A is calculated as:  $\frac{A_c}{A_e} = \frac{1 - \left[ \frac{\left( \frac{b}{h} \right) (h - 2r_i)^2 + \left( \frac{h}{b} \right) (b - 2r_i)^2}{3A_e} \right] - \rho_s'}{1 - \rho_s'}$	$h/b = 1.00$  Note: For rectangular members, plastic hinge confinement by jacketing is not recommended for members featuring side aspect ratios, $h/b$ , greater than 1.5, or face dimensions, $b$ or $h$ , exceeding 36 in. (900 mm) unless testing demonstrates their effectiveness (13.3.1).	
<b>Step4—Confining concrete model variables:</b>  Using the trial design of five plies, compute the confined concrete model parameters listed in the following:  $f_t = \frac{2E_f N f_t \epsilon_{t_e}}{D}$  $J_a' = y, 3.3k_f f_t$  $\epsilon_{con} = \epsilon'_c \left( 1.50 + 12K_h \frac{f_t}{f'_c} \left( \frac{\epsilon_{t_e}}{\epsilon'_c} \right)^{0.45} \right)$	$f_t = \frac{2 \times 9,326,000 \times 0.023 \times 5 \times 0.004}{22.63} = 380 \text{ psi}$  $= 0.95 \text{ for fully wrapped sections}$  $f_a' = 4000 + 0.95 \times 3.3 \times 0.62 \times 380 = 4740 \text{ psi}$  $\epsilon_{con} = 0.002 \left( 1.50 + 12 \times 0.62 \times \frac{380}{4000} \left( \frac{0.004}{0.002} \right)^{0.45} \right)$  $= 0.0049 \leq 0.01$	$f_t = \frac{2 \times 64,300 \times 0.584 \times 5 \times 0.004}{575} = 2.61 \text{ MPa}$  $= 0.95$  $f_a' = 27.6 + 0.95 \times 3.3 \times 0.62 \times 2.61 = 32.66 \text{ MPa}$  $\epsilon_{con} = 0.002 \left( 1.50 + 12 \times 0.62 \times \frac{2.61}{27.6} \left( \frac{0.004}{0.002} \right)^{0.45} \right)$  $= 0.0049 \leq 0.01$
	B <u>c</u> is limited to 0.01 to prevent excessive cracking and the resulting loss of concrete integrity	
Note: The expressions presented previously are used in conjunction with a moment-curvature ( $M-\Phi$ ) analysis program to obtain the yield and ultimate curvature. These curvatures are presented in Steps 5 and 6.		
<b>Step5—Output from M-Φ analysis program obtain the neutral axis at yielding of longitudinal reinforcement, <math>G_{xr}</math></b>	Neutral axis: $C_{si} = 5.34 \text{ in.}$	Neutral axis: $C_{scm} = 136 \text{ mm}$
	Note: Yield strain of longitudinal reinforcement: $\epsilon_y = 0.00152$	

**Table 16.10d, cont.—Procedure for designing plastic hinge confinement for seismic strengthening**

Step6—Output from M-p analysis program obtain the neutral axis at ultimate, $C_u, p$	Neutral axis: $C_{up}=1.95$ in.	Neutral axis: $C_{ui}=50\text{mm}$
1. Concrete compression strain: $C_c=0.0049$	Note: The ultimate concrete compressive strain is computed per Step 4: $\epsilon_{cu}=0.0049$ .	
2. Steel tension strain: $\epsilon_s = \epsilon_{cu} \left( \frac{d}{c_u} - 1 \right)$	$\epsilon_s = 0.0049 \left( \frac{14.625}{5.34} - 1 \right) = 0.032 < 0.05$	$\epsilon_s = 0.0049 \left( \frac{371}{136} - 1 \right) = 0.032 < 0.05$
<b>Step7—Compute the yield curvature:</b> $\phi_{y,frp} = \frac{\epsilon_y}{d - c_{y,frp}}$	From an M-o analysis the yield curvature is: $\phi_{y,frp} = \frac{0.0015}{14.625 - 5.34} = 0.000163/\text{in.}$	From an M-p analysis the yield curvature is: $\phi_{y,frp} = \frac{0.0015}{371 - 136} = 0.0064/\text{m}$
<b>Step8—Compute the ultimate curvature capacity:</b> $\phi_{u,frp} = \frac{\epsilon_{cu}}{C_{u,FRP}}$	From an M-analysis the ultimate curvature is: $\phi_{u,frp} = \frac{0.0049}{1.95} = 0.0025/\text{in.}$	From an M-p analysis the ultimate curvature is: $\phi_{u,frp} = \frac{0.0049}{50} = 0.099/\text{m}$
<b>Step9—Compute the ultimate curvature demand:</b> $\phi_D = \frac{\theta_p}{L_p} + \phi_{y,frp}$	$\phi_D = \frac{0.025}{12} + 0.000163 = 0.0022/\text{in.}$	$\phi_D = \frac{0.025}{0.305} + 0.0064 = 0.084/\text{m}$
<b>Step 10—Verify design:</b> $D < u_i$	$D = 0.0022/\text{in.} < 0.0025/\text{in. OK}$	$u_i = 0.084/\text{m} < 0.099/\text{m OK}$
<b>Step 11—Length of confined region, (<math>\ell_o</math>)</b> Transverse reinforcement as specified per Section 18.7 of ACI 318-19(22) shall be provided over a length $l_o$ .	$\ell_o \geq \begin{cases} 16 \text{ in.} \\ \frac{120/2}{6} = 10 \text{ in.} \\ 18 \text{ in.} \end{cases}$	$\ell_o \geq \begin{cases} 406.4 \text{ mm} \\ \frac{3050/2}{6} = 254 \text{ mm} \\ 457 \text{ mm} \end{cases}$ $t > L_p = 18 \text{ in. (457mm)}$
Design summary: Completely wrap the section with five transverse plies. Confining jacket should extend at least 18 in. (457mm) beyond the joint interface.		

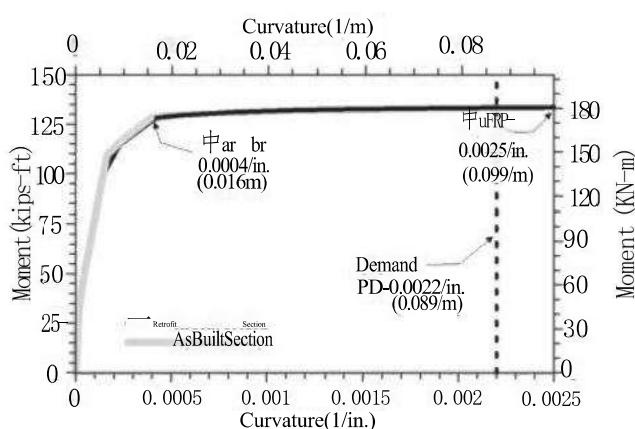


Fig. 16.10b—Moment-curvature analysis.

Figure 16.10b shows the moment curvature analysis of the as-built and retrofit rectangular sections. The moment curvature analysis results show the as-built ultimate curvature capacity is significantly lower than the required curvature demand.

**Table 16.11a—Column material properties**

Concrete strength	4000 psi	27.6 MPa
Concrete elastic modulus $E = 57,000 \sqrt{\psi} (\text{psi})$ $E = 4700 \sqrt{\psi} (\text{MPa})$	3605 ksi	25 GPa
Longitudinal reinforcing steel:yield strength	44,000 psi	303.4 MPa
Modulus of elasticity of steel $E$ ,	29,000 ksi	200 GPa
Longitudinal reinforcing steel:yield strain $\epsilon_y$ ,	0.0015	0.0015
Column height between plastic hinges, $L$	10 f	3.05m
Distance to extreme tension steel, $d$	14.625 in.	371mm

#### 16.11—Lap-splice clamping for seismic strengthening

This example illustrates the design of an FRP retrofit to improve the seismic performance of a reinforced concrete column that is constructed with a lap splice in a region of plastic rotations. Material properties, details, and other relevant information are provided in Table 16.11a and Fig. 16.11. The column is strengthened with CFRP laminates

having the composite properties listed in Table 16.11b and bonded around the column using the wet layup technique.

The final wrapping system was found to require five plies. Only the calculations used to verify the final configuration are provided. These calculations are shown in Table 16.11c.

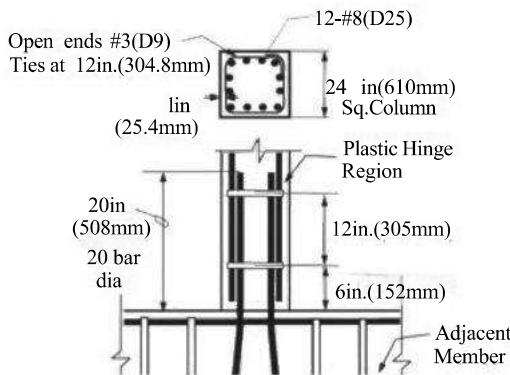


Fig. 16.11—Detail of lap splice in a plastic hinge region.

Table 16.11b—Manufacturer's reported FRP system properties

Thickness per ply $t$	0.08 in.	2 mm
Ultimate tensile strength	143 ksi	986 MPa
Rupture strain	0.010	0.010
Modulus of elasticity $E_y$	13,900 ksi	95.8 GPa

Table 16.11c—Procedure for designing lap-splice clamping for seismic strengthening

Procedure	Calculation in in.-lb units	Calculation in SI units
Step 1—Compute the tensile stress that existing splice can develop: (ACI 318-19 (22) Section 25.4)		
$f_t = \frac{40 \ell_a \lambda \sqrt{f'_c} \left( \frac{c_b K_{cr}}{d_b} \right)}{3 d_b \psi_i \psi_r \psi_s}$ (in. and psi)	$s = 12 \text{ in.}$ $n = 1$	$s = 305 \text{ mm}$ $n = 1$
$f_t = \frac{3.33 \ell_a \lambda \sqrt{f'_c} \left( \frac{c_b K_{cr}}{d_b} \right)}{3 d_b \psi_i \psi_r \psi_s}$ (mm and MPa)		
$K_{cr} = \frac{40 A_p}{sn}$ $\gamma_c = \gamma_e = \gamma_r = 1.00$ $A = 1.00 \text{ (normalweight concrete)}$	$K_{cr} = \frac{40 \times 1 \times 0.11}{12 \times 1} = 0.37 \text{ in.}$ $\frac{c_b + K_{cr}}{d_b} = \frac{(1.00 + 0.375 + 0.5) + 0.37}{1.00} = 2.24 \leq 2.50$	$K_{cr} = \frac{40 \times 1 \times 71}{305 \times 1} = 9.3 \text{ mm}$ $\frac{c_b + K_{cr}}{d_b} = \frac{(25.4 + 9.53 + 12.7) + 9.3}{25.4} = 2.24 \leq 2.50$
	Per Eq. (25.4.2.4a) of ACI 318-19 (22) the value	
	$\frac{c_b + K_{cr}}{d_b} \text{ cannot be greater than 2.5.}$ $f_t = \frac{40 \times 20.0 \times 1.0 \sqrt{4000} \times 2.24}{3 \times 1.0 \times 1.0 \times 1.0 \times 1.0} = 37,779 \text{ psi} \leq 44,000 \text{ psi}$	$f_t = \frac{3.33 \times 508 \times 1.0 \sqrt{27.6} \times 2.24}{3 \times 25.4 \times 1.0 \times 1.0 \times 1.0} = 260 \text{ MPa} \leq 303 \text{ MPa}$
	Note: Computed stress $f_t$ does not reach $f_y$ , longitudinal bar yield strength, and as such lap splice must be clamped.	

Table 16.11c, cont.—Procedure for designing lap-splice clamping for seismic strengthening

Step 2—Stress corresponding to pullout capacity of splice $f_s \leq \frac{33f_d\lambda\sqrt{f'_c}}{d_{bs}\Psi_t\Psi_e\Psi_s}$ (in. and psi) $f_s \leq \frac{2.75f_d\lambda\sqrt{f'_c}}{d_{bs}\Psi_t\Psi_e\Psi_s}$ (mm and MPa)	$p=20d_a = 20 \times 1.0 = 20$ in. Pullout capacity of splice: $f_p = \frac{33 \times 20.0 \times \sqrt{4000}}{1.0 \times 1.0 \times 1.0 \times 1.0} = 41,742$ psi	$= 20du = 20 \times 25.4 = 508$ mm Pullout capacity of splice: $f_p = \frac{2.75 \times 508 \times \sqrt{27.6}}{25.4 \times 1.0 \times 1.0 \times 1.0} = 289$ MPa
Step 3—Compute the required jacket thickness, $Nt_f = 218 \times \frac{D}{E_f}$ (ksi and in.) $Nt_f = 1500 \times \frac{D}{E_f}$ (MPa and mm)	D=24 in. (maximum dimension in a rectangular member) E=13,900 ksi $t_j = 218 \times \frac{24}{13,900} = 0.38$ in.	d=610 mm E <sub>s</sub> =95,800 MPa $t_j = 1500 \times \frac{610}{95,800} = 9.6$ mm
Step 4—Compute the required number of plies, N $N=t/1$	$t=0.08$ in. N=0.38/0.08=five plies	ty=2 mm N=9.60/2=five plies

**Design summary:**  
Completely wrap the section with five transverse plies.  
Confining jacket should extend at least full height of lap splice—that is, 20 in. (508 mm)  
Using five plies of the FRP—specified results in a splice capacity of 41,742 psi (289 MPa), which is controlled by the maximum pullout capacity calculated in Step 2.

### 16.12—Seismic shear strengthening

This example illustrates the main steps in calculating the amount of FRP required for the shear strengthening of a reinforced concrete member. The member used in this example is illustrated in Fig. 16.12. Column material properties are shown in Table 16.12a, the configuration of the FRP shear reinforcement is described in Table 16.12b, and the CFRP laminate material properties are listed in Table 16.12c. ASCE/SEI 41 is used as the standard for this example. Calculations for determining the FRP required are shown in Table 16.12d.

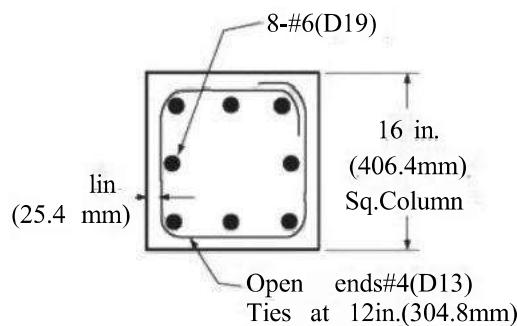


Fig. 16.12—As-built column details.

Table 16.12a—Column material properties

Concrete strengthf	4000 psi	27.6 MPa
Concrete elastic modulus E=57,000 √ : (psi)	3605 ksi	25 GPa
$E_c = 4700\sqrt{f'_c}$ (MPa)		
Longitudinal reinforcing steel:yield strengthf, 44,000 psi	303.4 MPa	
Modulus of elasticity of steel E,	29,000 ksi	200 GPa
Longitudinal reinforcing steel:yield strain e,	0.0015	0.0015
Column height between plastic hinges, L	10f	3.05m
Distance to extreme tension steel, d	14.625 in.	371 mm
Ultimate axial load P.	75 kip	333.62kN

Table 16.12b—Configuration of supplemental FRP shear reinforcement

Minimum section dimension	16 in.	406 mm
$d_3$ (governed by minimum section dimension)	16 in.	406 mm
Width w	8 in.	203mm
Spacing s(full coverage)	8 in.	203mm

**Table 16.12c—Manufacturer's reported FRP system properties**

Thickness per plyt	0.023 in.	0.584 mm
Ultimate tensile strengthf*	155ksi	1072 MPa
Rupture strain em*	0.015	0.015
Modulus of elasticity E	9326 ksi	64.3 GPa

**Table 16.12d—Procedure for seismic shear strengthening**

Procedure	Calculation in in.-lb units	Calculation in SI units
<b>Step1—Compute the probable moment capacity of the member, M<sub>m</sub>, per 13.5.1; in calculating the probable moment <math>\phi=1</math> and <math>y=1</math> are used.</b>	From a M-analysis the probable moment capacity is: $M_m=322 \text{ kip}\cdot\text{ft}$ Notes: 1) In seismic applications, the shear design process initiates with the calculation of the probable moment capacity of the section; and 2) moment M <sub>m</sub> is computed at both top and bottom ends of the member.	From aM-analysis the probable moment capacity: $M_m=437\text{kN}\cdot\text{m}$
<b>Step 2—Obtain the yield and ultimate curvature from an M-analysis:</b>	Yield curvature: $\phi_{y,yp}=0.000163/\text{in.}$ Ultimate curvature: $\phi_{usp}=0.0019/\text{in.}$	Yield curvature: $\phi_{y,yp}=0.0064/\text{m}$ Ultimate curvature: $\phi_{usp}=0.074/\text{m}$
<b>Step 3—Compute the displacement ductility and reduction factor, k. In FRPjacketed columns, the plastic hinge length is (Eq. (1)):</b>  $L_y = g + 0.0003f$ f, is in ksi and d, is in inches  $L_p = g + 0.044d$ f is in MPa and d, is in mm	Plastic hinge length: $L_p = 2 + 0.0003 \times 4,000 \times 0.75 = 12 \text{ in.}$ Note: Use a maximum gap between the FRP and the column base of 2 in.	Plastic hinge length: $L_p = 50.8 + 0.044 \times 303.4 \times 19 = 305 \text{ mm}$ Note: Use a maximum gap between the FRP and the column base of 50.80 mm.
In this example:	Yield deflection: $\Delta_{y,yp} = \frac{\phi_{y,yp} L_p^2}{3} = 0.20 \text{ in.}$	Yield deflection: $\Delta_{y,yp} = \frac{0.000163 \times 60^2}{3} = 0.20 \text{ in.}$
<b>Yield deflection<math>\Delta_y</math>:</b> $\Delta_{y,yp} = \frac{\phi_{y,yp} L_p^2}{3}$		Yield deflection: $\Delta_y = \frac{0.0065 \times 1524^2}{3 \times 1000} = 5.1 \text{ mm}$
<b>Plastic deflection<math>\Delta_p</math>:</b> $\Delta_{p,yp} = (\phi_y - \phi_u)L_p \left( L_{eff} - \frac{L_p}{2} \right)$	Plastic deflection: $\Delta_{p,yp} = (0.0019 - 0.000163) \times 12 \left( 60 - \frac{12}{2} \right) = 1.13 \text{ in.}$	Plastic deflection: $\Delta_{p,yp} = \frac{(0.074 - 0.0064) 305}{1000} \left( 1524 - \frac{305}{2} \right) = 28.6 \text{ mm}$
<b>Displacement ductility A:</b> $\mu_A = 1 + \frac{\Delta_p}{\Delta_y}$	Displacement ductility: $\mu_A = 1 + \frac{1.13}{0.20} = 6.7$	Displacement ductility: $\mu_A = 1 + \frac{28.6}{5.1} = 6.7$
<b>Shear reduction factor per ASCE/SEI41</b>	Reduction factor: $k = 0.70$	Reduction factor: $k = 0.70$
$\begin{cases} \mu_A \leq 2; & k = 1.0 \\ 2 \leq \mu_A \leq 6; & k = 0.7 + \frac{0.30 \times (6 - \mu_A)}{4} \\ \mu_A > 6; & k = 0.7 \end{cases}$		

**Table 16.12d, cont.—Procedure for seismic shear strengthening**

<b>Step4—Compute the shear design force, <math>V_u</math></b> Per ACI 318-19(22)Section 18.7.6: $V_u = \frac{M_{pr,Ap} + M_{pr,As}}{L} \pm \frac{w_y L}{2}$	In this example, with the exception of the earthquake forces, there are no applied uniform loads on the member, as such: $w=0.00$ From Step 1, the probable moment capacity is: $M_{mop}=M_{mcbo}=322 \text{ kip-ft (437kN-m)}$	
	Design shear force: $V_u = \frac{322 + 322}{120/12} \pm 0 = 64.4 \text{ kip}$	
	Design shear force: $V_u = \frac{437 + 437}{3.05} \pm 0 = 286 \text{ kN}$	
<b>Step 5—Calculate the concrete, <math>V_c</math>, and steel contribution, <math>V_s</math></b> Concrete contribution, $V_c$ : Per ACI 318-19(22)Section 18.7.6.2.1, and because: $\frac{M_{pr,c} + M_{pr,s}}{\ell_{eff}} \geq \left( \frac{w_y \ell_{eff}}{2} \right) \frac{1}{2} \Rightarrow V_c = 0$ Steel contribution $V_s$ : $V_s = A_{sh} f_y \frac{d}{s}$	Concrete contribution to shear capacity: $V_c=0$ Concrete contribution to shear capacity: $V_c=0$ Steel contribution to shear capacity: $V_s = 2 \times 0.20 \times 44 \times \frac{14.625}{12} = 21.4 \text{ kip}$ Combined concrete and steel contribution: $k(V_c+V_s)=0.70(0+21.4)=14.98 \text{ kip}$	
	Steel contribution to shear capacity: $V_s = 2 \times 129 \times 303.4 \times \frac{371.475}{305} = 95.2 \text{ kN}$ Combined concrete and steel contribution: $k(V_c+V_s)=0.70(0+95.2)=66.6 \text{kN}$	
<b>Step6—Calculate the required FRP force, <math>V_f</math>:</b> $V_{f,R} = \frac{[V_u - k(V_c + V_s)]}{\Psi_f}$ Per ASCE/SEI41, $\phi=1.00$ For completely wrapped members, Table 11.3 recommends $y=0.95$ .	Required FRP force: $V_{f,R} = \frac{[64.4 - 14.98]}{0.95} = 45.26 \text{ kip}$	Required FRP force: $V_{f,R} = \frac{[286 - 66.6]}{0.95} = 201.3 \text{ kN}$
<b>Step 7—Calculate the effective stress, <math>f_{je}</math></b> For fully wrapped members, the effective strain is computed using Eq. (11.4.1.1); $\epsilon_e=0.75C=0.90.004$ The column is located in an interior space. Therefore, per Table 9.4, an environmental-reduction factor of 0.95 is used The effective FRP stress can be computed from Hooke's law: $\sigma = E \epsilon$	Effective strain: $\epsilon_e=0.75 \times 0.95 \times 0.015=0.11 \leq 0.004$	Effective strain: $\epsilon_e=0.75 \times 0.95 \times 0.015=0.11 \leq 0.004$ Effective stress: $f_o=0.004 \times 9326=37.3 \text{ ksi}$
<b>Step 8—Calculate the number of plies, <math>N</math>.</b> Area of a single ply for a fully wrapped member(Eq. (11.4a)) $A_p=2tw;$ The shear contribution of the FRP can be then calculated from Eq. (11.4a). $V_f = A_{sp} f_{je} \frac{(\sin \alpha + \cos \alpha)d_{je}}{s_f}$	Area per ply: $A_p=2 \times 0.023 \times 8=0.37 \text{ in.}^2$ Force per ply: $V_f = 0.37 \times 37.3 \frac{(1)16}{8} = 27.6 \text{ kip}$ Number of plies: $N = \frac{V_{f,R}}{V_f} = \frac{45.26}{27.6} = 2 \text{ (minimum)}$	Area per ply: $A_p=2 \times 0.584 \times 203=237 \text{ mm}^2$ Force per ply: $V_f = 237 \times 257 \frac{(1)406}{203} = 122 \text{ kN}$ Number of plies: $N = \frac{V_{f,R}}{V_f} = \frac{201.3}{122} = 2 \text{ (minimum)}$
Design summary: Completely wrap the section with two transverse plies.		

### 16.13—Flexural and shear seismic strengthening of shear walls

This example illustrates the use of an FRP retrofit to increase the shear and flexural strength of an existing cantilevered concrete wall. The shear strengthening is achieved by installing horizontally oriented FRP on one face of the wall. The flexural strengthening is achieved by installing multiple layers of vertically oriented (longitudinal)FRP on both faces of the wall near the wall ends. Details for anchorage of the FRP to the foundation are not addressed. Carbon FRP(CFRP)is used in this example.

The example incorporates the following two major phases:

- 1.Determine the shear and flexural capacity of the existing wall
- 2.Design the FRP to achieve the required strength

Details of the wall and relevant information are provided in Fig.16.13a and Table 16.13a. The wall is assumed to be an ordinary shear wall,that is, the wall is at risk of a brittle

shear failure. Plastic hinging or inelastic deformations are not expected. The wall is strengthened with FRP having the composite properties listed in Table 16.13b. A factored axial load,P,of 12 kip(53 kN)is assumed in addition to the lateral force. ASCE/SEI 41 is assumed to be the standard used as the basis for the rehabilitation and shear is considered a force-controlled action.

This example illustrates a manual calculation approach for the design of FRP strengthening of a shear wall. A moment curvature analysis of the existing and repaired wall,shown in Fig.16.13b, is used to assess the accuracy of the manual approach. The reasonably good agreement of the flexural results from the design example with those from a moment-curvature analysis validates the illustrative example. The design calculations are shown in Table 16.13c.

Per 13.7.2.2, the longitudinal FRP should be anchored to the foundations to ensure load path continuity and,where feasible,confined or anchored (refer to 13.7.2.2).

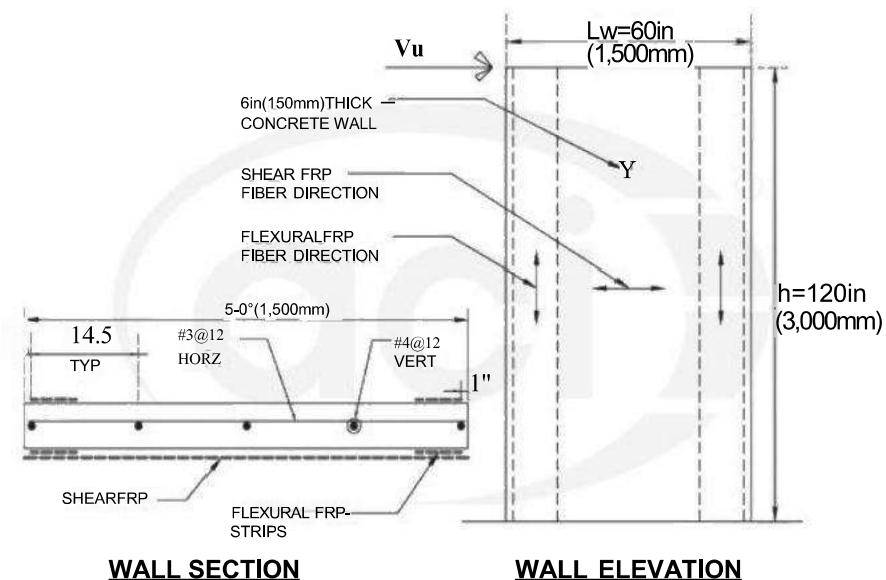


Fig.16.13a—Concrete wall details.

Table 16.13a—As-built shear wall properties and demands

Concrete strength	2500 psi	17.23 MPa
Longitudinal reinforcing steel yield strength	40.0 ksi	275.8 MPa
Modulus of elasticity of steel	29,000 ksi	200 GPa
Longitudinal reinforcing yield strain	0.0014 in./in.	0.0014 mm/mm
Shear wall height h	10f	3000mm
Shear wall length	60.0 in.	1500 mm
Shear wall thickness	6 in.	150 mm
Existing wall reinforcement		
Horizontal:No. 3 at 12 in. (305mm) on center=>p <sub>z</sub> =0.0015	—	—
Longitudinal:No. 4 at 12 in. (305mm) on center=>p <sub>e</sub> =P <sub>b</sub> =0.0027		
Axial factored load P	12 kip	53.4kN
Design shear demand V(force-controlled action)	52 kip	232kN
Design moment demand at wall base M	260 kip-ft	348 kN-m
A <sub>w</sub> =area of wall web steel=>area of four No. 4(No. M13)bars (neglecting one bar within the compression zone)	0.8 in. <sup>2</sup>	500 mm <sup>2</sup>

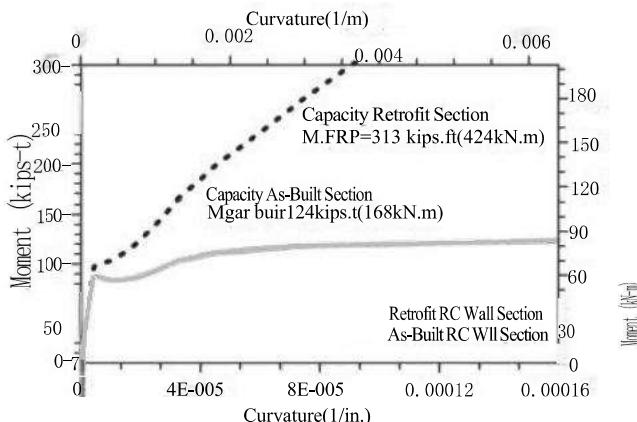


Fig. 16.13b—Moment-curvature analysis for shear wall example.

Table 16.13c—Procedure for flexural and shear seismic strengthening of shear walls

Procedure	Calculation in in.-lb units	Calculation in SI units
<b>Step 1—Compute existing wall capacity</b> Shear capacity: Shear capacity per ACI 318-14:		
$V_s = 2 \lambda J \gamma ad$ (in-lb)	$V_c = \frac{2(6 \text{ in.} \times 0.8 \times 60 \text{ in.})\sqrt{2500 \text{ psi}}}{1000}$	$V_c = \frac{(0.167 \times 150 \text{ mm} \times 0.8 \times 1500 \text{ mm})\sqrt{17.23 \text{ MPa}}}{1000}$
$V_s = \frac{0.167 b_u d \sqrt{f'_c}}{1000}$ (SI)	$V_s = 28.8 \text{ kip}$	$V_s = 124.8 \text{ kN}$
$V_{su} = \frac{A_c f_v d_{sf}}{s}$	$V_{su} = 0.11 \text{ in.}^2 \times 40,000 \text{ psi} \times \frac{0.8 \times 60 \text{ in.}}{12 \text{ in.}}$ $V_{su} = 17.6 \text{ kip}$	$V_{su} = \frac{71 \text{ mm}^2 \times 275.8 \text{ MPa} \times 0.8 \times \frac{1500 \text{ mm}}{305 \text{ mm}}}{1000}$ $V_{su} = 77.0 \text{kN}$
$V_s = V_{su} + V_a$	$V_a = 28.8 \text{ kip} + 17.6 \text{ kip}$ $V_a = 46.4 \text{ kip}$ $\phi V_n = 46.4 \text{ kip} < V_a = 52 \text{ kip}$	$V_a = 124.8 \text{ kN} + 77.0 \text{ kN}$ $V_a = 201.8 \text{ kip}$ $\phi V_n = 201.8 \text{ kN} < V = 232 \text{ kN}$
Using $\phi = 1.0$ for shear (per ASCE/SEI 41): Flexural capacity: A quick method to assess the flexural capacity is shown in the following.		
Assume that all web steel in the wall yields This includes all the longitudinal reinforcement except for the one bar that is adjacent to the compression face, that is, four No. 4 bars can be considered to yield.	$a = \frac{A_{sv} f_y + P_u}{0.85 f_y I_w}$	$a = \frac{0.8 \text{ in.}^2 \times 40 \text{ ksi} + 12 \text{ kip}}{0.85 \times 2.5 \text{ ksi} \times 6 \text{ in.}} = 3.45 \text{ in.}$ $a = \frac{\left(\frac{500 \text{ mm}^2 \times 275.8 \text{ MPa}}{1000}\right) + 53.4 \text{ kN}}{\left(0.85 \times 17.23 \text{ MPa} \times 150 \text{ mm}\right)} = 87.1 \text{ mm}$
$M = (Af, +P)(d - a/2)$ where $d = LW/2$	$d = L/2 = 30 \text{ in.}$ $M = 104 \text{ kip-ft}$	$d = 750 \text{ mm}$ $M_n = 135.1 \text{ kN-m}$
Using $\phi = 1.0$ : It can be observed that the wall does not have adequate capacity.	$\phi M = 104 \text{ kip-ft} < M = 260 \text{ kip-ft}$	$\phi M = 135.1 \text{ kN-m} < M = 348 \text{ kN-m}$

**Table 16.13c,cont.—Procedure for flexural and shear seismic strengthening of shear walls**

Step 2—FRP design material properties For exterior exposure for carbon FRP:Cg=0.85

Use environmental reduction factors from Table 9.4.

Per 9.4:

$$f_u = C_e f$$

$$B = C_E B_m$$

$$f_m = 199 \text{ ksi}$$

$$E_m = 0.0102 \text{ in./in.}$$

$$f_m = 820.5 \text{ MPa}$$

$$E = 0.0102 \text{ mm/mm}$$

Step 3—Flexural strengthening

Step 3a—Compute the debonding strain limit,  $\epsilon_{eu}$ 

This is the limit for the effective strain in the FRP.

Per Eq.(10.1.1):

In in.-lb units:

$$\epsilon_{eu} = 0.083 \sqrt{\frac{f'_c}{NE_f t_f}} \leq 0.9 \epsilon_{eu}$$

For one layer of FRP,  $\epsilon_{eu}=0.0088$ For two layers of FRP,  $\epsilon_{eu}=0.0062$ For three layers of FRP,  $\epsilon_{eu}=0.0051$ For one layer of FRP,  $\epsilon_{eu}=0.0088$ For two layers of FRP,  $\epsilon_{eu}=0.0062$ For three layers of FRP,  $\epsilon_{eu}=0.0051$ 

In SI units:

$$\epsilon_{eu} = 0.41 \sqrt{\frac{f'_c}{NE_f t_f}} \leq 0.9 \epsilon_{eu}$$

Flexural design involves iteration to achieve equilibrium across the section. Usually, the first step is to assume the depth of the neutral axis c.

Assume that the flexural strengthening will require three layers of 8 in.(200 mm)wide strips on each side of the wall at each end. Assume that the effective strain is at the centroid of the FRP area.Knowing the maximum effective strain in the FRP,compute the force in the FRP:

$$Ty = C_a A_s E$$

Use the available information to assist with an assumption for the depth of the neutral axis c.

$$\begin{aligned} w &= 8 & N &= 3 \\ T &= 0.023 \text{ in.} & & \\ Ay &= \text{two sides} \times 3(0.023 \text{ in.})(8 \text{ in.}) & & \\ &Ay = 1.104 \text{ in.}^2 & & \end{aligned}$$

$$\begin{aligned} w &= 200 \text{ mm} & N &= 3 \\ T &= 0.575 \text{ mm} & & \\ A &= 690 \text{ mm}^2 & & \end{aligned}$$

$$T = 0.85(1.104 \text{ in.})(0.0051)(9600 \text{ ksi}) = 45.94 \text{ kip} \quad T_f = \frac{0.85(690 \text{ mm}^2)(0.0051)(66,200 \text{ MPa})}{1000} = 198.0 \text{ kN}$$

Depth of corresponding compression block:

$$a = \frac{A_w f_c + P_u + T_f}{0.85 f'_c t_w}$$

$$a = 7.05 \text{ in.}$$

$$a = 177.2 \text{ mm}$$

Compute an estimate of the depth of the neutral axis,c:

$$c = a / 0.85 = 8.3 \text{ in.}$$

$$c = a / 0.85 = 208.5 \text{ mm}$$

Check actual strain at centroid ofFRP area and corresponding force in the FRP:

$$\epsilon_{fr,CG} = \epsilon_{eu} \left( 1 + \frac{W_f / 2}{c + 1 - L_n} \right) = 0.00469$$

$$\epsilon_{fr,CG} = \epsilon_{eu} \left( 1 + \frac{W_f / 2}{c + 25.4 - L_o} \right) = 0.00469$$

It is observed that the force in the FRP does not agree with that based on theinitial assumption. However,the above steps provide a reasonable starting point for an assumption for c.

Corresponding force in FRP,  
 $T_f = CAeccE = 42.32 \text{ kip}$   
Assume  $c = 8.0 \text{ in.}$ 

$$T_f = CeAeycGE = 182.1 \text{ kN}$$
  
Assume  $c = 200 \text{ mm}$

**Table 16.13c, cont.—Procedure for flexural and shear seismic strengthening of shear walls**

Compute concrete strain at extreme compression surface per Eq. (13.7.2.1): $\varepsilon_c = \varepsilon_{st} \left( \frac{1}{(L_u/c) - 1} \right) \leq \varepsilon_{cu}$	$\varepsilon_c = 0.0008$ $\& \Delta B_m = 0.003 \text{ OK}$	$E := 0.0008$ $B \mu = 0.003 \text{ OK}$
Compute strain in the bar in the compression zone: $\varepsilon_w = \left( \frac{\varepsilon_c}{c} \right) (c - d')$	$E_{se} = 0.007$ where $d = 1\text{in.}$ $e < e_c = 0.0014$	$B \mu = 0.007$ where $d = 25\text{mm}$ $E_n < E_c = 0.0014$
Compute strain in the bars in the tension zone:	$\varepsilon_{st} = \left( \frac{\varepsilon_c}{c} \right) (14.5\text{ in.} + 1\text{ in.} - 8.0\text{ in.})$ $E_n = 0.0075 < 2,$	$\varepsilon_{st} = \left( \frac{\varepsilon_c}{c} \right) (362.5\text{ mm} + 25\text{ mm} - 200\text{ mm})$ $E_n = 0.0075 < 8y$
Compute strain at centroid of FRP area:	Similarly, $B_{xz} = 0.0022 > 8,$ $E_{s3} = 0.0037 > 8y$ $E_{x4} = 0.0051 > E$ $E_{jecc} = 0.0047$	Similarly, $E_{u2} = 0.0022 > 8y$ $E_{u3} = 0.0037 > 8y$ $E_4 = 0.0051 > E$ $B_{ecc} = 0.0047$
Recompute total tensile force components at the above determined strain levels:		$T = 182.5\text{kN}$ $T_{sw} = 125\text{mm}^2 \times 0.00075 \times 200,000 \text{ MPa} = 3 \times 125\text{mm}^2 \times 275.8 \text{ MPa} = 122,175\text{kN}$ $T_m = 122.2\text{kN}$
Recalculate depth of compression block and depth to neutral axis:	$T_m = 0.2 \text{ in.}^2 \times 0.00075 \times 29,000 \text{ ksi} + 3 \times 0.2 \text{ in.}^2 \times 40 \text{ ksi} = 28.35 \text{ kip}$	$a = 6.48 \text{ in.}$ $c = 7.63 \text{ in.}$
Iterate as required to reach convergence:		$a = 163\text{mm}$ $c = 191.8 \text{ mm}$
<b>Step 3b—Calculate the strains and force components in concrete, FRP, and reinforcing steel</b>		
Force in the bar in the compression zone:	Final value of the depth of the neutral axis is achieved after iteration. $c = 7.648 \text{ in.}$ $a = 6.5 \text{ in.}$	$c = 191 \text{ mm}$ $a = 162.4 \text{ mm}$
Strains and forces in bars in tensile zone:	$B = 0.00076 < C_c$ $= 0.00066 < 8,$ $C_{rc} = 0.2 \times 0.00066 \times 29,000 = 3.83 \text{ kip}$	$s = 0.00076 < E_c$ $E_b = 0.00066 < 8,$ $C = 125 \times 0.00066 \times 200 = 16.5 \text{ kN}$
Strain and force in FRP:	$B_{nri} = 0.0078 < e$ $E_{s2} = 0.0022 > 6$ $E_{s3} = 0.0037 > 8$ $E_{s4} = 0.0051 > E$	$B_{ai} = 0.0078 < E_y$ $E_{ua} = 0.0022 > 8y$ $E_{us} = 0.0037 > 8,$ $E_{m4} = 0.0051 > E_y$
From equilibrium, compressive force in concrete:	$T_n = 4.52 \text{ kip}$ $T_2 = T_3 = T_4 = 8.0 \text{ kip}$ $\therefore T_{3r} = 28.52 \text{ kip}$	$T_n = 19.5 \text{ kN}$ $T_{n2} = T_s = T_n = 34.48 \text{ kN}$ $\therefore T_r = 122.94 \text{ kN}$

Table 16.13c, cont.—Procedure for flexural and shear seismic strengthening of shear walls

<p><b>Step 3c—Calculate the moment capacity of the section</b></p> <p>Compute lever arm for different force components:</p> <p><math>C = (T_a + T_y + P_a + C_a) = 79.06 \text{ kip}</math></p> <p>Bar in compression: <math>d_t = c - l_{in} = 7.648 - 1 = 6.65 \text{ in.}</math></p> <p>Concrete compression: <math>\left(c - \frac{a}{2}\right) = 7.648 \text{ in.} - \frac{6.5 \text{ in.}}{2} = 4.4 \text{ in.}</math></p> <p>First bar in tension: <math>(1 \text{ in.} + 14.5 \text{ in.} - c) = 7.85 \text{ in.}</math></p> <p>Second bar in tension: 22.35 in. Third bar in tension: 36.85 in. Fourth bar in tension: 51.35 in.</p> <p><math>L_w - c - W/2 - 1 \text{ in.} = 47.35 \text{ in.}</math></p> <p><math>\frac{P_u}{L_w/2 - c} = 22.35 \text{ in.}</math></p> <p>Nominal moment capacity, <math>M</math>: Per Section 10.2.10, include reduction factor for FRP contribution, <math>\psi = 0.85</math>.</p>	<p><math>E_{co} = 0.0047</math> <math>T = 42.37 \text{ kip}</math></p> <p><math>J_{co} = 0.0047</math> <math>T = 182.5 \text{kN}</math></p> <p><math>C = (T_a + T_y + P_u + C_a) = 342.4 \text{ kN}</math></p> <p>Bar in compression: <math>d_t = c - 25 \text{ mm} = 166 \text{ mm}</math></p> <p><math>\left(c - \frac{a}{2}\right) = 109.8 \text{ mm}</math></p> <p>First bar in tension: <math>(25 \text{ mm} + 362.5 \text{ mm} - c) = 196.5 \text{ mm}</math></p> <p>Second bar in tension: 559 mm Third bar in tension: 921.5 mm Fourth bar in tension: 1284 mm</p> <p><b>FRP:</b> <math>L_w - c - W/2 - 25 \text{ mm} = 1184 \text{ mm}</math></p> <p><math>\frac{P_u}{L_w/2 - c} = 559 \text{ mm}</math></p> <p><math>M_n = 353 \text{kNm}</math></p> <p><math>M_a = 79.06 \text{ kip} \times 4.4 \text{ in.} + 3.83 \text{ kip} \times 6.65 \text{ in.} + 4.52 \text{ kip} \times 7.85 \text{ in.} + 8 \text{ kip} \times 22.35 \text{ in.} + 8 \text{ kip} \times 36.85 \text{ in.} + 8 \text{ kip} \times 51.35 \text{ in.} + 0.85 \times 42.37 \text{ kip} \times 47.35 \text{ in.} + 12 \text{ kip} \times 22.35 \text{ in.}</math> <math>M_n = 3266.7 \text{ kip-in.}</math> <math>M_n = 272.2 \text{ kip-ft}</math></p> <p><math>\phi M_a = 272.2 \text{ kip-ft} &gt; M = 260 \text{ kip-ft}</math> <math>\phi M_a &gt; M \therefore \text{OK}</math></p>	<p><math>\phi M = 353 \text{kN} \cdot \text{m} &gt; M_n = 348 \text{kNm}</math></p> <p><math>\phi M_a &gt; M \therefore \text{OK}</math></p>
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Table 16.13c, cont.—Procedure for flexural and shear seismic strengthening of shear walls

<b>Step4—Shear strengthening</b>		
Per 13.7.4, Eq. (13.7.2.4b):	V <sub>n</sub> =V+w, V	$V_n = V_n^* + \psi_f V_f$
V is the shear contribution of FRP and is computed in accordance with Chapter 11.	From Step 1, V=46.4 kip	From Step 1, V*=201.8 kN
<b>Step 4a—Calculate V;</b>	For one layer of the FRP:	For one layer of the FRP:
Because the FRP will be face-bonded to the wall, the effective FRP strain will be:	L <sub>e</sub> =1.99 in. k <sub>1</sub> =0.731 k <sub>2</sub> =0.91 K <sub>s</sub> =0.277	L=51.36 mm k <sub>1</sub> =0.741 k <sub>2</sub> =0.91 K <sub>s</sub> =0.285
Be=K <sub>s</sub> m≤0.004	B% $=0.277 \times 0.0102 = 0.00283$	% $=0.285 \times 0.0102 = 0.0029$
Use Eq. (11.4.1.2b) to compute K <sub>s</sub> :	$K_s = \frac{k_1 k_2 L_e}{468 \epsilon_{f_y}} \leq 0.75$ (in.-lb)	
	$K_s = \frac{k_1 k_2 L_e}{11,900 \epsilon_{f_y}} \leq 0.75$ (SD)	
L <sub>ay</sub> and k <sub>z</sub> are calculated from Eq. (11.4.1.2c), (11.4.1.2d), and (11.4.1.2e), respectively		
Note: The small difference in k <sub>s</sub> is due to soft conversions in the FRP system thickness and in Eq. (11.4.1.2d) for k <sub>s</sub> .		
Corresponding tensile stress in the FRP:	d=0.8L=48 in.	d <sub>n</sub> =0.8L <sub>w</sub> =1200 mm
Per 13.7.4.1, d, is taken as 0.8L	V=0.75×0.023 in. × 48 in. × 0.00283×9600 ksi	V=0.75×0.575 mm × 1200 mm × 0.0029×66.2 GPa
Compute shear capacity of FRP strengthened wall using $\phi=1.0$ and Eq. (13.7.2.4c) for one-sided strengthening:	$=22.49$ kip	=99.35 kN
V <sub>s</sub> =0.75A <sub>ne</sub> E <sub>dy</sub> /S;	Reduction factor for FRP shear contribution, v=0.85	W <sub>V</sub> =84.45 kN
FRP is installed over the full height of the wall and not in discrete strips.	V <sub>y</sub> =19.11 kip	φ V <sub>s</sub> =1.0(201.8 kN+81.5 kN)
A <sub>m</sub> =N <sub>id</sub> , N=1	Shear capacity of the retrofitted wall:	φ V <sub>a</sub> =283.3 kN>V=232 kN
d/s <sub>y</sub> =1.0	φ V <sub>s</sub> >V: OK	o V <sub>n</sub> >V: OK
Step 4b—Compute the shear corresponding to the nominal flexural strength. Refer to Section 13.7.1.	Ma=272.2 kip-ft	M=353 kNm
Note: The FRP for shear strengthening could be optimized by using horizontal strips rather than full coverage.	Shear corresponding to the nominal flexural strength:	VM <sub>nom</sub> =353 kNm/3m=117.1 kN
	V <sub>umom</sub> =272.2 kip·ft/10ft=27.2 kip	The shear strength of the wall is:
	The shear strength of the wall is:	Va=284.6 kN
	V <sub>n</sub> =65.3 kip	V <sub>n</sub> <VM <sub>nom</sub> OK
	V△V <sub>umom</sub> . OK	



### 16.14—Flexural strengthening of continuous unbonded prestressed concrete slab with FRP laminates

Figure 16.14a shows a design example of a 10 in.(250 mm) thick one-way continuous unbonded post-tensioned slab. The tendons are 1/2 in.(12.7 mm)seven-wire stress-relieved strands, Grade 270 ksi(1860 MPa)having area/strand of 0.153 in.<sup>2</sup>(98.7 mm<sup>2</sup>)spaced at 10 in.(250 mm)apart ( $A_p=0.184$  in.<sup>2</sup>/ft [390 mm<sup>2</sup>/m]). Ordinary steel reinforcement consisting of No.4(No.M13)bars spaced at 10 in.(250 mm) ( $A_r=0.236$  in.<sup>2</sup>/ft [500 mm<sup>2</sup>/m])and placed at a depth  $d=8.75$  in.(222 mm)is provided at the top and bottom of the slab throughout the span length as minimum bonded reinforcement required in members with unbonded tendons by ACI 318. The prestressing steel,reinforcing steel, and concrete material design properties are given in Table 16.14a. This example is intended only to illustrate application offlexural design provisions covered in Section 10.3.2;other criteria,such as serviceability,might govern the overall design of the upgraded slab.

The slab was originally designed for a superimposed dead load of 92 lb/f<sup>2</sup>(4.4kN/m<sup>2</sup>)and live load of 40.91b/f<sup>2</sup>(2.0 kN/m<sup>2</sup>) of slab surface area. The effective prestressing force  $P_e=26.5$  kip/ft(388 kN/m)and the depths  $d_{p1},d_{p2}$ ,and  $d_{p3}$ (or eccentricities) of the prestressing steel at critical sections 1,2, and 3,respectively (Fig.16.14a),were selected such that the prestressing force at service balances the self-weight of the slab. It is required to renovate the floor and raise the load capacity of the slab by using carbon FRP laminates at the critical positive and negative

**Table 16.14a—Concrete,prestressing steel, and reinforcing steel properties**

	5000 psi	34.5 MPa
A	0.184 in. <sup>2</sup> /ft	390 mm <sup>2</sup> /m
f <sub>u</sub>	270 ksi	1860 MPa
J <sub>eu</sub>	230 ksi	1585 MPa
E <sub>p</sub>	28,500ksi	196,500 MPa
f	145ksi	1000 MPa
A <sub>s</sub>	0.236 in. <sup>2</sup> /ft	500mm <sup>2</sup> /m
f <sub>s</sub>	60ksi	414MPa
E <sub>c</sub>	29,000 ksi	200,000 MPa

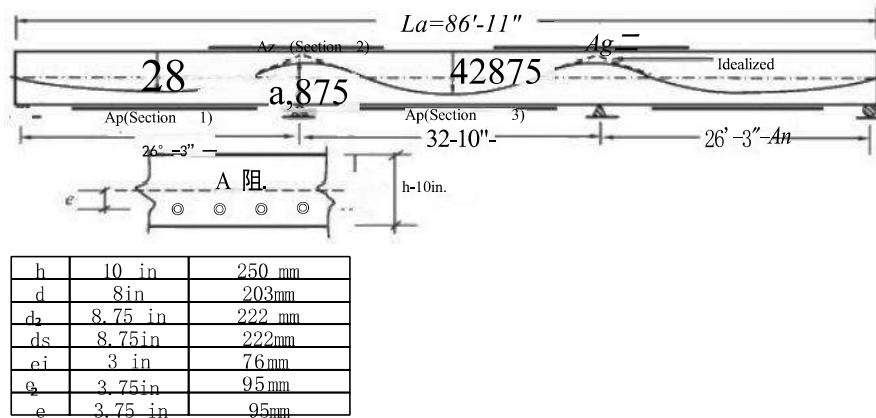
moment regions of the slab for resisting an upgraded live load of 160 Ib/f<sup>2</sup>(7.7 kN/m<sup>2</sup>).At the same time,to compensate partly for the increased live load,renovation measures were taken to reduce the design superimposed dead load from 921b/f<sup>2</sup>(4.4 kN/m<sup>2</sup>)to 701b/f<sup>2</sup>(3.4 kN/m<sup>2</sup>).The manufacturer's reported FRP properties are given in Table 16.14b.

A summary of moments produced by self-weight of the slab,reduced superimposed dead load, and upgraded live-load(for different loading patterns to produce maximum live-load moment M)is provided in Table 16.14c.Included also in Table 16.14c is the secondary moment due to prestressing  $M_o$  corresponding to  $P_e=26.5$  kip/ft(388 kN/m). $M_o$  is calculated as the difference between the moment produced by the upward equivalent uniform load of the prestressing force(equal to the self-weight of the slab of 0.125 k/f<sup>2</sup> [6.0 kN/m<sup>2</sup>])and the internal moment caused by the effective prestressing force of  $P_e$ .Without FRP strengthening, the ultimate moment capacity of the slab  $\phi M$  is calculated at 24.8 kip-f/ft(110.0kN-m/m),27.6 kip-ft/t(123.0 kN-m/m),and 26.7 kip-f/ft(119.0kN-m/m)at sections 1,2, and 3,respectively,which are all lower than the applied ultimate moment Mat the respective sections(Table 16.14c).

Starting with one FRP layer,a trial and error procedure should be used for estimating the width b,of the FRP sheets, or the area  $A=bjy$  per unit width of the slab such that  $\phi M \geq M$  at every critical section,where  $\phi M$ ,is the ultimate load capacity of the FRP-strengthened slab per unit width at the section under consideration.It was decided to use a single layer of 600 mm (23.6 in.)wide strips per meter(3.28 ft) width of the slab at the critical negative moment section (section 2)[( $A_2=0.284$  in.<sup>2</sup>/ft [600 mm<sup>2</sup>/m]),and 300 mm (11.8 in.)wide strips per meter(3.28 ft)width of slab at each of the critical positive moment sections(sections 1 and 3) [ $A_1=A_3=0.142$  in.<sup>2</sup>/ft(300 mm<sup>2</sup>/m)].

**Table 16.14b—Manufacturer's reported FRP system properties**

Thickness per ply, t	0.0394in.	1 mm
Ultimate tensile strength, f <sub>a</sub>	140ksi	965 MPa
Rupture strain, E	0.01 in./in	0.01 mm/mm
Modulus of elasticity of FRP laminates, E	13,895ksi	95,800 MPa



*Fig.16.14a—Schematic of idealized multi-span unbonded one-way slab with FRP external reinforcement. (Note:I in.=25.4 mm.)*

Table 16.14c—Summary of applied moments per unit width of slab

Loading/moment	Positive moment section in the exterior span (section 1)		Negative moment section at the interior support (section 2)		Positive moment section in the interior span (section 3)	
Slab moment	5.9 kip-f/ft	26.3 kN-m/m	-11.1 kip-f/ft	49.5 kN-m/m	5.8 kip-ft/ft	25.9 kN-m/m
Superimposed dead-load moment	3.3 kip-f/f	14.7 kN-m/m	-6.2 kip-ft/t	-27.6 kN-m/m	3.2 kip-ft/f	14.3 kN-m/m
Total dead-load moment, $M_0$	9.2 kip-f/f	41.0 kN-m/m	-17.3 kip-ft/t	-77.1 kN-m/m	9.0 kip-ft/f	40.2 kN-m/m
Maximum live-load moment, $M$	11.5 kip-f/t	51.3 kN-m/m	-16.0 kip-f/t	-71.3 kN-m/m	12.2 kip-f/t	54.4 kN-m/m
Span(s) loaded to produce maximum live-load moment	Two exterior spans		Two adjacent spans		Interior span	
Secondary moment due to prestressing, $M_p$	1.2 kip-f/t	5.4 kN-m/m	2.8 kip-f/ft	12.5 kN-m/m	2.8 kip-ft/ft	12.5 kN-m/m
Ultimate moment, $M_u$ *	30.6 kip-ft/ft	137.0 kN-m/m	-43.6 kip-ft/t	-194.0 kN-m/m	33.1 kip-t/ft	148.0 kN-m/m

$$M_2 = 1.2M_{ou} + 1.6M_u + M_0$$

Table 16.14d—Procedure for flexural strengthening of an interior unbonded prestressed concrete slab with FRP

Procedure	Calculations in in.-lb units	Calculations in SI units
<b>Step 1—Preliminary calculations</b>		
Concrete modulus of elasticity:		
$E_c = 57,000\sqrt{f'_c}$	$E=57,000\sqrt{5000 \text{ psi}}=4,030,000 \text{ psi}$	$E_c = 4700\sqrt{34.5 \text{ MPa}}=27,600 \text{ MPa}$
Effective prestressing force:		
$P_p = A_p d_e$	$P_p = (0.184 \text{ in.}^2/\text{f}) (145 \text{ ksi})=26.7 \text{ kip/t}$	$P_p = (388 \text{ mm}^2/\text{m}) (1000 \text{ MPa}/1000)=388 \text{ kN/m}$
Effective strain in the unbonded prestressing steel:		
$\epsilon_{ps} = \frac{f_{ps}}{E_p}$	$\epsilon_{ps} = \frac{145 \text{ ksi}}{28,500 \text{ ksi}}=0.0051 \text{ in./in.}$	$\epsilon_{ps} = \frac{1000 \text{ MPa}}{196,500 \text{ MPa}}=0.0051 \text{ mm/mm}$
<b>Step 2—Calculate the FRP system design properties.</b>		
The slab is located in an interior space (interior exposure) and CFRP material will be used. Therefore, per Table 9.4, an environmental reduction factor of 0.95 is suggested		
$f_{ps} = C_f f_{ps}^*$ $\epsilon_{ps} = C_f \epsilon_{ps}^*$	$f_a = (0.95) (140 \text{ ksi})=133 \text{ ksi}$ $\epsilon_a = (0.95) (0.01 \text{ in./in.})=0.0095 \text{ in./in.}$	$J = (0.95) (965 \text{ MPa})=917 \text{ MPa}$ $\epsilon_u = (0.95) (0.01 \text{ mm/mm})=0.0095 \text{ mm/mm}$
<b>Step 3—Determine the design strain of the FRP system</b>		
The design strain of FRP accounting for debonding failure is calculated using Eq. (10.1.1)		
$\epsilon_{sf} = 0.083 \sqrt{\frac{f'_c}{NE_f t_f}} \leq 0.9 \epsilon_{su} \text{ (in.-lb)}$ $\epsilon_{sf} = 0.41 \sqrt{\frac{f'_c}{NE_f t_f}} \leq 0.9 \epsilon_{su} \text{ (SD)}$	$\epsilon_{sf} = 0.083 \sqrt{\frac{5000 \text{ psi}}{(1)(13,895,000 \text{ psi})(0.0394 \text{ in.})}}=0.0079 \leq 0.9(0.0095)=0.0086$	$\epsilon_{sf} = 0.41 \sqrt{\frac{34.5 \text{ MPa}}{(1)(95,800)(1)}}=0.0079 \leq 0.9(0.0095)=0.0086$
<b>Step 4—Determine the existing state of strain at the different critical sections.</b>		
The only external load acting on the beam at the time of the FRP application is the self-weight of the slab. Because the self-weight of the slab is balanced by the effect of the prestressing force, the initial strain in the soffit at all sections is calculated as:		
$\epsilon_{bi} = \frac{-P_e}{E_c A_{sy}}$	$\epsilon_{bi} = \frac{-(26.5 \text{ kip/ft})(1000)}{(4,030,000 \text{ psi})(120 \text{ in.}^2/\text{ft})}=-5.5 \times 10^{-5}$	$\epsilon_{bi} = \frac{-(388 \text{ kN/m})(1000)}{(27,600 \text{ MPa})(250,000 \text{ mm}^2/\text{m})}=-5.5 \times 10^{-5}$

**Table 16.14d,cont.—Procedure for flexural strengthening of an interior unbonded prestressed concrete slab with FRP**

<b>Step 5—Consider the most critical section at the interior support (Section 2)</b>  Estimate the depth of the neutral axis at this section. A reasonable initial estimate of $c$ is $0.15h$ . The value of $c$ is adjusted after checking equilibrium.  $c=0.15h$	$c=(0.15)(10.0 \text{ in.})=1.5 \text{ in.}$	$c=(0.15)(250 \text{ mm})=38 \text{ mm}$
<b>Step 6—Determine the effective level of strain in the FRP reinforcement</b>  The effective strain in the FRP reinforcement may be found from Eq. (10.3)  $\varepsilon_{fr} = \varepsilon_{ru} \left( \frac{d_f - c}{c} \right) - \varepsilon_{fu} \leq \varepsilon_{sf}$  Failure is controlled by FRP debonding, and therefore the concrete strain at debonding failure is calculated from;  $\varepsilon_c = (\varepsilon_{sf} + \varepsilon_{fr}) \left( \frac{c}{d_f - c} \right)$	$\varepsilon_{fr} = 0.003 \left( \frac{10 \text{ in.} - 1.5 \text{ in.}}{1.5 \text{ in.}} \right) - (-5.5 \times 10^{-5}) \leq 0.079$ $=0.017 > 0.0079$ $\varepsilon_c = (0.0079 - 5.5 \times 10^{-5}) \left( \frac{1.5 \text{ in.}}{10 \text{ in.} - 1.5 \text{ in.}} \right) = 0.0014$	$\varepsilon_{fr} = 0.003 \left( \frac{250 \text{ mm} - 38 \text{ mm}}{38 \text{ mm}} \right) - (-5.5 \times 10^{-5}) \leq 0.079$ $c=0.017 > 0.0079$ $\varepsilon_c = (0.0079 - 5.5 \times 10^{-5}) \left( \frac{38 \text{ mm}}{250 \text{ mm} - 38 \text{ mm}} \right) = 0.0014$
<b>Step 7—Calculate the strain in the existing reinforcing steel</b>  $\varepsilon_s = \varepsilon_c \left( \frac{d - c}{c} \right)$	$\varepsilon_s = 0.0014 \left( \frac{8.75 \text{ in.} - 1.5 \text{ in.}}{1.5 \text{ in.}} \right) = 0.0068$	$\varepsilon_s = 0.0014 \left( \frac{222 \text{ mm} - 38 \text{ mm}}{38 \text{ mm}} \right) = 0.0068$
<b>Step 8—Calculate the strain in the unbonded prestressing steel from Eq. (10.3.2.2b)</b>  $\varepsilon_{ps} = \varepsilon_{pe} + \eta \varepsilon_c \left( \frac{d - c}{L_s} \right)$  For the section under consideration (negative moment at the interior support), $\eta=43.5$	$\varepsilon_{ps} = 0.0051 + (43.5)(0.0014) \left( \frac{8.75 \text{ in.} - 1.5 \text{ in.}}{1043 \text{ in.}} \right)$ $=0.0055$	$\varepsilon_{ps} = 0.0051 + (43.5)(0.0014) \left( \frac{222 \text{ mm} - 38 \text{ mm}}{26,490 \text{ mm}} \right)$ $=0.0055$
<b>Step 9—Calculate the stress level in the FRP, prestressing steel and reinforcing steel from Eq. (10.2.6), (10.2.10b), and (10.3.2.2c)</b>  $f=Eee = E, B, \leq f$ $f=Eem \leq 0.95f_m$	$J_e = (13,895 \text{ ksi}) (0.0079) = 109.8 \text{ ksi}$ $f = (29,000 \text{ ksi}) (0.0068) = 197.2 \text{ ksi} \leq 60 \text{ ksi}$ $f_s = 60 \text{ ksi}$ $J_m = (28,500 \text{ ksi}) (0.0055) = 156.8 \text{ ksi} \leq (0.95)$ $(230 \text{ ksi})$ $= 218.5 \text{ ksi}$	$f_o = (95,800 \text{ MPa}) (0.0079) = 757 \text{ MPa}$ $f = (200,000 \text{ MPa}) (0.0068) = 1360 \text{ MPa} \leq 414 \text{ MPa}$ $f_s = 414 \text{ MPa}$ $f = (196,500 \text{ MPa}) (0.0055) = 1081 \text{ MPa} \leq (0.95) (1585 \text{ MPa}) = 1506 \text{ MPa}$

**Table 16.14d, cont.—Procedure for flexural strengthening of an interior unbonded prestressed concrete slab with FRP**

<b>Step 10—Calculate the internal force resultants and check equilibrium</b>	<p>Concrete stress block factors may be calculated using ACI 318. Approximate stress factors may also be calculated based on the parabolic stress-strain relationship for concrete as follows:</p> $\beta_i = \frac{4\epsilon'_c - \epsilon_c}{6\epsilon'_c - 2\epsilon_c}$ $\alpha_i = \frac{3\epsilon'_c \epsilon_c - (\epsilon_c)^2}{3\beta_i (\epsilon'_c)^2}$ <p>where <math>\epsilon_c' = 1.76/E_s</math></p> <p>Force equilibrium is verified by checking the initial estimate of <math>c</math> using Eq. (10.3.1.6f):</p> $c = \frac{A_{ps}f_{ps} + A_cf_c + A_tf_{tc}}{\alpha_i f'_c \beta_i b}$	$\beta_i = \frac{4(0.0021) - 0.0014}{6(0.0021) - 2(0.0014)} = 0.714$ $\alpha_i = \frac{3(0.0021)(0.0014) - (0.0014)^2}{3(0.714)(0.0021)^2} = 0.726$ $\epsilon_c' = \frac{1.7(5000)}{4030 \times 10^3} = 0.0021$ $\beta_i = \frac{4(0.0021) - 0.0014}{6(0.0021) - 2(0.0014)} = 0.714$ $\alpha_i = \frac{3(0.0021)(0.0014) - (0.0014)^2}{3(0.714)(0.0021)^2} = 0.726$ $\epsilon_c' = \frac{1.7(34.5)}{27,600} = 0.0021$ <p><math>c = [(0.184 \text{ in.}^2/\text{ft})(156.8 \text{ ksi}) + (0.236 \text{ in.}^2/\text{ft})(60 \text{ ksi}) + (0.284 \text{ in.}^2/\text{ft})(109.8 \text{ ksi})]/[0.726(5 \text{ ksi})(0.714)(12 \text{ in.}/\text{ft})] = 2.39 \text{ in.} \neq 1.5 \text{ in. NG}</math></p> <p>Revise estimate of <math>c</math> and repeat Steps 5 through 10 until force equilibrium is achieved</p>
<b>Step 11—Adjust <math>c</math> until force equilibrium is achieved</b>	Steps 5 through 10 were repeated with different values of $c$ until force equilibrium is achieved. The results of the final iteration are:	$c = [(0.184 \text{ in.}^2/\text{ft})(160.6 \text{ ksi}) + (0.236 \text{ in.}^2/\text{ft})(60 \text{ ksi}) + (0.284 \text{ in.}^2/\text{ft})(109.8 \text{ ksi})]/[0.86(5 \text{ ksi})(0.74)(12 \text{ in.}/\text{ft})] = 1.96 \text{ in.} = 1.96 \text{ in. OK}$ $c = [(390 \text{ mm}^2/\text{m})(1081 \text{ MPa}) + (500 \text{ mm}^2/\text{m})(414 \text{ MPa}) + (600 \text{ mm}^2/\text{m})(757 \text{ MPa})]/[0.726(34.5 \text{ MPa})(0.714)(1000)] = 61 \text{ mm} \neq 38 \text{ mm NG}$ <p>Revise estimate of <math>c</math> and repeat Steps 5 through 10 until force equilibrium is achieved</p>
<b>Step 12—Estimate the moment capacity by considering the contribution of prestressing steel, reinforcing steel, and FRP to the flexural strength (Eq. 10.3.1.6g)</b>	$M_a = A_{ps}f_{ps}\left(d_p - \frac{\beta_i c}{2}\right) + A_cf_c\left(d - \frac{\beta_i c}{2}\right) + \psi_f A_tf_{tc}\left(d_f - \frac{\beta_i c}{2}\right)$ $M_a = (0.184 \text{ in.}^2)(160.6 \text{ ksi})\left(8.75 \text{ in.} - \frac{0.74 \times 1.96 \text{ in.}}{2}\right) + (0.236 \text{ in.}^2)(60 \text{ ksi})\left(8.75 \text{ in.} - \frac{0.74 \times 1.96 \text{ in.}}{2}\right) + 0.85(0.284 \text{ in.}^2)(109.8 \text{ ksi})\left(10 \text{ in.} - \frac{0.74 \times 1.96 \text{ in.}}{2}\right) = 597.0 \text{ kip-in/t} = 49.8 \text{ kip-ft/t}$	$M_a = (390 \text{ mm}^2)(1107 \text{ MPa})\left(222 \text{ mm} - \frac{0.74 \times 50 \text{ mm}}{2}\right) + (500 \text{ mm}^2)(414 \text{ MPa})\left(222 \text{ mm} - \frac{0.74 \times 50 \text{ mm}}{2}\right) + 0.85(600 \text{ mm}^2)(757 \text{ MPa})\left(250 \text{ mm} - \frac{0.74 \times 50 \text{ mm}}{2}\right) = 219,357,000 \text{ N-mm/m} = 219.0 \text{ kN-m/m}$
<b>Step 13—Determine the strength reduction factor <math>\phi</math></b> and, hence, calculate the design flexural strength of the section $\phi M$	<p>The strain in concrete <math>\epsilon_c</math> at the level of prestressing steel is expressed as:</p> $\epsilon_{cr} = \epsilon_c \left( \frac{d_p - c}{c} \right)$ $\epsilon_{cr} = 0.0019 \left( \frac{8.75 \text{ in.} - 1.96 \text{ in.}}{1.96 \text{ in.}} \right) = 0.0066 \geq 0.005$ <p>Hence, <math>\phi = 0.9</math></p> $\phi M_n = 0.9(49.8) = 44.8 \text{ kip-ft/t} \geq M = 43.6 \text{ kip-ft/t}$ <p>The strengthened section (Section 2) is capable of sustaining the new required moment strength.</p>	$\epsilon_{cr} = 0.0019 \left( \frac{222 \text{ mm} - 50 \text{ mm}}{50 \text{ mm}} \right) = 0.0066 \geq 0.005$ <p>Hence, <math>\phi = 0.9</math></p> $\phi M_d = 0.9(219.0) = 197.0 \text{ kN-m/m} \geq M = 194.0 \text{ kN-m/m}$ <p>The strengthened section (Section 2) is capable of sustaining the new required moment strength.</p>

**Table 16.14d, cont.—Procedure for flexural strengthening of an interior unbonded prestressed concrete slab with FRP**

<p><b>Step 14—Carry out similar calculations (Steps 5 through 13) for Section 1, noting that:<math>A_n=0.142 \text{ in}^2/(300 \text{ mm}^2/\text{m})</math> and <math>N_s=19.0</math> for this section.</b></p> <p>Following is a summary of the results:</p> $c=1.68 \text{ in.}; g=0.0016; e\mu=0.0053; e_3=0.0067; f_m=150.3 \text{ ksi}; \beta_r=0.723; a_r=0.786; f=109.8 \text{ ksi}; E_a=0.006; \phi=0.9$	$M_s = (0.184 \text{ in}^2)(150.3 \text{ ksi})\left(8.0 \text{ in.} - \frac{0.723 \times 1.68 \text{ in.}}{2}\right) + (0.236 \text{ in}^2)(60 \text{ ksi})\left(8.75 \text{ in.} - \frac{0.723 \times 1.68 \text{ in.}}{2}\right) + 0.85(0.142 \text{ in}^2)(109.8 \text{ ksi})\left(10 \text{ in.} - \frac{0.723 \times 1.68 \text{ in.}}{2}\right)$ $=444.2 \text{ kip-in}/\text{ft}=37.0 \text{ kip-f/f}$ $\phi M_n=0.9(37.0)=33.3 \text{ kip-f/t} \geq M=30.6 \text{ kip-f/ft}$	$M_s = (390 \text{ mm}^2)(1041 \text{ MPa})\left(203 \text{ mm} - \frac{0.723 \times 43 \text{ mm}}{2}\right) + (500 \text{ mm}^2)(414 \text{ MPa})\left(222 \text{ mm} - \frac{0.723 \times 43 \text{ mm}}{2}\right) + 0.85(300 \text{ mm}^2)(757 \text{ MPa})\left(250 \text{ mm} - \frac{0.723 \times 43 \text{ mm}}{2}\right)$ $=164,099,000 \text{ N-m/m}=164.0 \text{ kN-m/m}$ $\phi M_n=0.9(164.0)=148.0 \text{ kN-m/m} \geq M=137.0 \text{ kN-m/m}$
<p><b>Step 15—Carry out similar calculations (Steps 5 through 13) for Section 3, noting that:<math>A_s=0.142 \text{ in}^2/(300 \text{ mm}^2/\text{m})</math> and <math>N_s=24.5</math> for this section.</b></p> <p>Following is a summary of the results:</p> $c=1.69 \text{ in.}; g=0.0016; e\mu=0.0054; e_3=0.0067; f=152.6 \text{ ksi}; \beta_r=0.724; a_r=0.788; f=109.8 \text{ ksi}; E_a=0.0067; \phi=0.9$	$M_s = (0.184 \text{ in}^2)(152.6 \text{ ksi})\left(8.75 \text{ in.} - \frac{0.724 \times 1.69 \text{ in.}}{2}\right) + (0.236 \text{ in}^2)(60 \text{ ksi})\left(8.75 \text{ in.} - \frac{0.724 \times 1.69 \text{ in.}}{2}\right) + 0.85(0.142 \text{ in}^2)(109.8 \text{ ksi})\left(10 \text{ in.} - \frac{0.724 \times 1.69 \text{ in.}}{2}\right)$ $=468.0 \text{ kip-in/t}=39.0 \text{ kip-ft/}$ $\phi M_n=0.9(39.0)=35.1 \text{ kip-f/t} \geq M=33.1 \text{ kip-f/ft}$	$M_s = (390 \text{ mm}^2)(1053 \text{ MPa})\left(222 \text{ mm} - \frac{0.724 \times 43 \text{ mm}}{2}\right) + (500 \text{ mm}^2)(414 \text{ MPa})\left(222 \text{ mm} - \frac{0.724 \times 43 \text{ mm}}{2}\right) + 0.85(300 \text{ mm}^2)(757 \text{ MPa})\left(250 \text{ mm} - \frac{0.724 \times 43 \text{ mm}}{2}\right)$ $=172,760,000 \text{ N-mm}=173.0 \text{ kN-m/m}$ $\phi M_n=0.9(173.0)=156.0 \text{ kN-m/m} \geq M=148.0 \text{ kN-m/m}$

## CHAPTER 17—REFERENCES

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