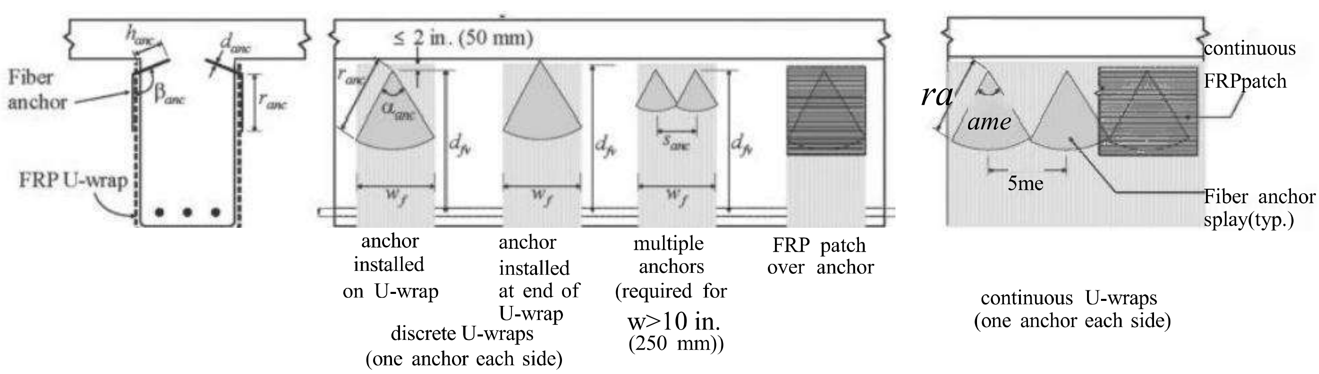


Fig.14.1.4—Details offiber anchorsfor U-wraps.

Table 14.1.4(in.-lb)—Recommendations for fiber anchors for anchoring shear U-wraps

Sane' in.	U-wrap NEt,kip/in.	RA: 90 degrees≤βane ≤125 degrees	RA: 125 degrees<βane ≤180 degrees	Fame, in.	hame, in.: 90 degrees≤Pame ≤110 degrees	hame, in.: 110 degrees<Pame ≤125 degrees	hame, in.: 125 degrees<βame ≤180 degrees
≤4	NEt≤288	1.25	1.00	Sane	Larger of 4.0 and 74mm25	Larger of 6.0 and 74am25	6.0
	288<NEIy≤575	1.25	1.00	San			6.0
	575<NEty≤863	1.25	100	8.0			6.0
	863<NEd≤1150	1.25	1.00	10.0			8.0
4<8San≤6	NEty≤288	1.25	1.00	Sane			6.0
	288<NEty≤575	1.25	1.00	Sae			6.0
	575<NE;A;≤863	1.50	1.25	8.0			8.0
	863<NEA≤1150	1.50	125	10.0			10.0
6<8am≤8	NE,ty≤288	1.25	1.00	Sune			6.0
	288<NEt≤575	1.50	1.25	Sune			8.0
	575<NEIy≤863	1.50	1.25	San			10.0
	863<NEA≤1150	1.75	1.50	10.0			12.0
8<8m≤10	NE,ty≤288	125	100	Sane			6.0
	288<NE;ty≤575	150	1.25	Sa			8.0
	575<NEA≤863	1.75	1.50	San			10.0
	863<NEt≤1150	2.00	2.00	Same			12.0

For discrete U-wraps having width,w≤10 in.anchored with a single anchor,use sSam=F

Increase ra 50%ifno patch is provided.

Embedment should extend at least 2 in.beyond concrete cover.

The length of the splay,rane,should be equal to or exceed that given in Table 14.1.4.The anchor splay should be placed over the U-wrap.A single-layer FRP patch of the same material as that being anchored should be placed over the entire anchor splay with fibers oriented perpendicular to the anchored FRP.In place of providing a perpendicular FRP patch,rane in Table 14.1.4 may be increased by 50%. Although not necessary to achieve the objectives of this section,an additional FRP patch with fibers parallel to the anchored FRP has been shown to further enhance the effectiveness of the anchors(Pudleiner et al.2019).

The angle at which the anchor is embedded,βane,can be between 90 and 180 degrees (Fig.14.1.4).The minimum embedment depth of the anchor,hane,for different installation angles,βanc,is given in Table 14.1.4.Anchors should not be installed such that βane<90 degrees.

Fiber anchors are fabricated to be nominally round to be inserted into drilled holes.The diameter of the drilled hole should be between 0.125 in.(+3 mm)and 0.375 in. (+10 mm)larger than the resulting anchor diameter,dane The depth of the drilled hole should be at least 0.25 in. (6 mm)deeper than hanc.Holes should be located to avoid damaging existing internal reinforcement.To mitigate stress concentrations in the anchor fibers,the edge of the drilled hole should be rounded or chamfered to a distance of one half the hole diameter,but not less than 0.5 in. (13 mm)and should extend over the splay angle,αane(refer to Fig.14.1.4).Anchor installation,including drilling and cleaning of holes,should be in accordance with manufacturer's recommendations.

14.2—Detailing of laps and splices

Splices of FRP laminates should be provided only as permitted in drawings,specifications,or as authorized by the licensed design professional or as recommended by the system manufacturer.

The fibers of FRP systems should be continuous and oriented in the direction of the greatest tensile forces.

Table 14.1.4(SI)—Recommendations for fiber anchors for anchoring shear U-wraps

Sam*, mm	U-wrap NEys, kN/mm	RA		Fane mm+	hame, mm		
		90 degrees≤Pane ≤125 degrees	125 degrees<Pame ≤180 degrees		90 degrees≤ β ane ≤110 degrees	110 degrees<Pame ≤125 degrees	125 degrees<Pane ≤180 degrees
≤100	NEA≤50	1.25	1.00	Sane	Larger of 100 and 74am5	Larger of 150 and 74m05	150
	50<NEt≤100	1.25	100	Same			150
	100<NEty≤150	1.25	100	200			150
	150<NE,I≤200	1.25	1.00	250			200
100<sm≤150	NE, ≤50	1.25	1.00	Sane			150
	50<NEA≤100	1.25	1.00	Same			150
	100<NEy≤150	1.50	1.25	200			200
	150<NE;≤200	1.50	1.25	250			250
150<sm≤200	NE,A≤50	1.25	1.00	Same			150
	50<NEt≤100	1.50	1.25	Same			200
	100<NEt≤150	1.50	1.25	Sam			250
	150<NE,I≤200	1.75	1.50	250			300
200<same≤250	NEA≤50	1.25	1.00	Samce			150
	50<NEA≤100	1.50	125	Saie			200
	100<NEy≤150	1.75	150	Same			250
	150<NEt≤200	2.00	2.00	Sane			300

"For discrete U-wraps having width,w;≤250mm anchored with a single anchor,use Sm=Wp

Increase ram 50%if no patch is provided.

Embedment should extend at least 50mm beyond concrete cover

Fiber continuity can be maintained with a lap splice.For FRP systems,a lap splice should be made by overlapping the fibers along their length.The required overlap,or lap-splice length,depends on the tensile strength and thickness of the FRP system and on the bond strength between adjacent layers of FRP laminates.Sufficient overlap should be provided to promote the failure of the FRP laminate before debonding of the overlapped FRP laminates.The required overlap for an FRP system should be substantiated through representative physical testing.

Jacket-type FRPsystems used for column members should provide appropriate development area at splices,joints,and termination points to ensure failure through the FRP jacket thickness rather than failure of the spliced sections.

For unidirectional FRP laminates,lap splices are required only in the direction of the fibers.Lap splices are not required in the direction transverse to the fibers.FRP laminates consisting of multiple unidirectional sheets oriented in more than one direction or multidirectional fabrics require lap splices in more than one direction to maintain the continuity of the fibers and the overall strength of the FRP laminates.

14.3—Bond of near-surface-mounted(NSM) systems

For NSM systems,the minimum dimension of the grooves should be at least 1.5 times the diameter of the FRP bar(De Lorenzis and Nanni 2001;Hassan and Rizkalla 2003).When a rectangular plate with a large aspect ratio is used,however,the limit may lose significance due to constructability.In such a case,a minimum groove size of 3.0ab×1.5b,as depicted in Fig.14.3a,is suggested, where as is the smaller plate dimension.The minimum clear groove spacing for NSM FRP should be greater than

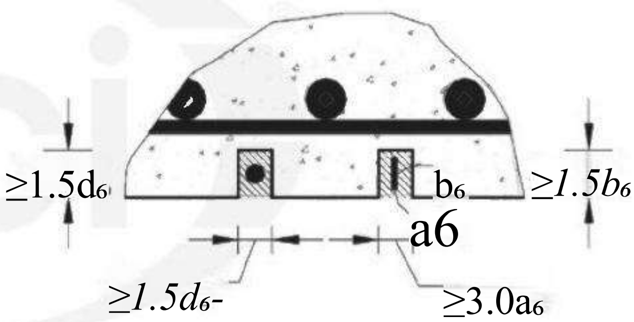


Fig.14.3a—Minimum dimensions ofgrooves.

twice the depth of the NSM groove to avoid overlapping of the tensile stresses around the NSM bars or plates. Furthermore,a clear edge distance of four times the depth of the NSM groove should be provided to minimize edge effects that could accelerate debonding failure(Hassan and Rizkalla 2003).

Bond properties of NSMFRPdepend on many factors such as cross-sectional shape and dimensions and surface properties of the FRP(Hassan and Rizkalla 2003; De Lorenzis et al.2004).Figure 14.3b shows the equilibrium condition of an NSM FRP bar with an embedded length equal to its development length ab having a bond strength ofTmar.Using a triangular stress distribution,the average bond strength can be expressed as ts=0.5tmax.Average bond strength tb for NSM FRP bars in the range of 500 to 3000 psi(3.5 to 20.7MPa)has been reported(Hassan and Rizkalla 2003;De Lorenzis et al.2004);therefore,Tb=1000 psi(6.9 MPa)is recommended for calculating the development length.Using force equilibrium,the following equations for development length can be derived:

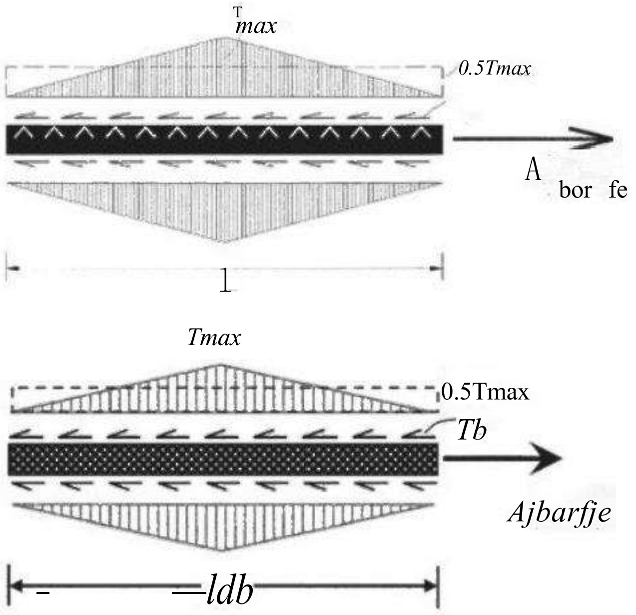


Fig.14.3b—Transfer offorce in NSM FRP bars.

$$\ell_{db} = \frac{d_b}{4(\tau_b)} f_{fd} \text{ for circular FRP} \quad (14.3a)$$

$$\ell_{db} = \frac{a_b b_b}{2(a_b + b_b)(\tau_b)} f_{fd} \text{ for rectangular FRP} \quad (14.3b)$$

In a simply supported beam,NSM FRP should extend a distance at least equal to lab past the point along the span at which the resisted moment falls below the cracking moment, Mcr.For a continuous beam,NSM FRP should extend a distance at least equal to Lapast the inflection point.

CHAPTER 15—DRAWINGS,SPECIFICATIONS, AND SUBMITTALS

15.1—Engineering requirements

Although federal,state,and local codes for the design of externally bonded FRP systems do not exist,other applicable code requirements may influence the selection,design, and installation of the FRP system.For example,code requirements related to fire or potable water may influence the selection of the coatings used with the FRP system.All design work should be performed under the guidance of a licensed design professional familiar with the properties and applications of FRP strengthening systems.

15.2—Drawings and specifications

The licensed design professional should document calculations summarizing the assumptions and parameters used to design the FRP strengthening system and should prepare design drawings and project specifications.The drawings and specifications should show,at a minimum,the following information specific to externally applied FRP systems:

(a)FRP system to be used
(b)Location of the FRP system relative to the existing structure
(c)Dimensions and orientation of each ply,laminate,or NSM FRP
(d)Number of plies and bars and the sequence of installation
(e)Location of splices and lap length
(f)General notes listing design loads and allowable strains in the FRP laminates
(g)Material properties of the FRP laminates and concrete substrate
(h)Concrete surface preparation requirements,including corner preparation,groove dimensions for NSM bars, and maximum irregularity limitations
(i)Protective coatings and sealants,if required
(j)Quality control and inspection procedures,including acceptance criteria
(k)In-place load testing of installed FRP system,if necessary

ACI SPEC-440.5 provides a specification for construction with FRP reinforcing bars. ACI SPEC-440.12 provides a specification for construction using externally bonded wet layup systems.

15.3—Submittals

Specifications should require the FRP system manufacturer,installation contractor,and inspection agency (if required)to submit product information and evidence of their qualifications and experience to the licensed design professional for review.

15.3.1 *FRP system manufacturer—Submittals required of* the FRP system manufacturer should include:

(a)Indication of compliance with existing specifications (such as ACI 440.8)as applicable
(b)Product data sheets indicating the physical,mechanical,and chemical characteristics of the FRP system and all its constituent materials
(c)Tensile properties of the FRP system,including the method of reporting properties (net fiber,gross laminate,or property per unit width),test methods used, and the statistical basis used for determining the properties(Section 4.3)
(d)FRP system manufacturer's installation and quality control manuals
(e)Manufacturer's safety data sheets(SDSs)for all materials to be used
(f)Durability test data for the FRP system in the types of environments expected
(g)Structural test reports pertinent to the proposed application
(h)Reference projects

15.3.2 FRP contractor—Submittals required of the FRP contractor should include:

(a)Documentation from the FRP system manufacturer of having been trained to install the proposed FRP system
(b)Project references,including installations similar to the proposed installation
(c)Daily log or inspection forms used by the contractor

15.3.3 Inspection agency—If an independent inspection agency is used,submittals required of that agency should include:

(a)A list of inspectors to be used on the project and their qualifications
(b)Sample inspection forms
(c)A list of previous inspection projects

CHAPTER 16—DESIGN EXAMPLES

16.1—Calculation of FRP system tensile properties

The example calculations shown in Table 16.1b illustrate the derivation of material properties based on net-fiber area and those based on gross-laminate area.As described in 4.3.1,both methods of determining material properties are valid.It is important,however,that design calculations consistently use material properties based on only one of the two methods (for example,if the gross-laminate thickness is used in any calculation,the strength based on gross-laminate area should be used in the calculations as well).Reported design properties should be based on a population of 20 or more coupons tested in accordance with ASTM D7565/D7565M,as discussed in 4.3.1.

A test panel is fabricated from two plies of a carbon fiber/resin unidirectional FRP system using the wet layup technique.The areal weight of the fabric reinforcement is 8.76 oz/yd²(297 g/m²)or 0.00676 oz/in.²(0.0298 g/cm²). The density of the constituent carbon fiber is 1.04 oz/in.³(1.8 g/cm³).After the system has cured,five 2 in. (50.8 mm)wide test coupons are cut from the panel.The test coupons are tested in tension to failure in accordance with ASTM D7565/D7565M.Tabulated in Table 16.1a are the results of the tension tests.

Table 16.1a—FRP system tension test results

	Specimen width		Measured coupon thickness		Measured rupture load	
Coupon ID	in.	mm	in.	mm	kip	kN
T-1	2	50.8	0.055	1.40	17.8	79.2
T-2	2	50.8	0.062	1.58	16.4	72.9
T-3	2	50.8	0.069	1.75	16.7	74.3
T-4	2	50.8	0.053	1.35	16.7	74.3
T-5	2	50.8	0.061	1.55	17.4	77.4
Average	2	50.8	0.060	1.52	17.0	75.6

16.2—Comparison of FRP systems'tensile properties

Two FRP systems are being considered for strengthening concrete members;the mechanical properties are available from their respective manufacturers.System A consists of dry,carbon-fiber unidirectional sheets and is installed with an adhesive resin using the wet layup technique.System B consists of precured carbon fiber/resin laminates that are bonded to the concrete surface with an adhesive resin. Excerpts from the data sheets provided by the FRP system manufacturers are given in Table 16.2a.After reviewing the material data sheets sent by the FRP system manufacturers,the licensed design professional compares the tensile strengths ofthe two systems.

Table 16.1b—FRP system net fiber and gross laminate property calculations

Net-fiber area property calculations		Gross-laminate area property calculations	
Calculate net-fiber thickness,t; t=areal weight/density	$t_f=\dfrac{0.00676\ \text{oz./in.}^2}{1.04\ \text{oz./in.}^3}=0.0065\ \text{in.}$ $t_f=\dfrac{0.0298\ \text{g/cm}^2}{1.8\text{g/cm}^3}=0.0165\ \text{cm}$ =0.165mm	Calculate gross-laminate thickness,ty	For the two-ply specimens: $t_f=\dfrac{0.06\ \text{in.}}{2\ \text{plies}}=0.03\ \text{in.}$ $t_f=\dfrac{1.52\text{mm}}{2\text{plies}}=0.76\text{mm}$
Calculate A,using the known, net-fiber area ply thickness: $A_1=Ntwy$	A=(2)(0.0065 in.)(2in.)=0.026 in.² 4=(2)(0.165mm)(50.8 mm)=16.8 mm²	Calculate Ayusing the average, measured laminate thickness: Ay=tw	A,=(2)(0.030 in.)(2 in.)=0.120 in.² A=(2)(0.76 mm)(50.8mm)=77.4 mm²
Calculatethe average FRP system tensile strength based on net-fiber area: $\overline{f_{fu}}=\dfrac{\text{average rupture load}}{A_f}$	$\overline{f_{fu}}=\dfrac{17\ \text{kip}}{0.026\ \text{in.}^2}=650\ \text{ksi}$ $\overline{f_{fu}}=\dfrac{75.62\ \text{kN}}{16.8\ \text{mm}^2}=4.5\ \text{kN/mm}^2$	Calculate the average FRP system tensile strength based on gross-laminate area: $\overline{f_{fu}}=\dfrac{\text{average rupture load}}{A_f}$	$\overline{f_{fu}}=\dfrac{17\ \text{kip}}{0.120\ \text{in.}^2}=140\ \text{ksi}$ $\overline{f_{fu}}=\dfrac{75.62\ \text{kN}}{77.4\ \text{mm}^2}=0.997\ \text{kN/mm}^2$
Calculate the average FRP system tensile strength per unit width based on net-fiber area: $\overline{p_{fu}}=\dfrac{\overline{f_{fu}}A_f}{w_f}$	$\overline{p_{fu}}=\dfrac{(650\ \text{ksi})(0.026\ \text{in.}^2)}{2\ \text{in.}}=8.4\ \text{kip/in.}$ $\overline{p_{fu}}=\dfrac{(4.5\ \text{kN/mm}^2)(16.8\ \text{mm}^2)}{50.8\ \text{mm}}$ =1.49kN/mm	Calculate the average FRP system tensile strength per unit width based on laminate area: $\overline{p_{fu}}=\dfrac{\overline{f_{fu}}A_f}{w_f}$	$\overline{p_{fu}}=\dfrac{(140\ \text{ksi})(0.120\ \text{in.}^2)}{2\ \text{in.}}=8.4\ \text{kip/in.}$ $\overline{p_{fu}}=\dfrac{(0.98\ \text{kN/mm}^2)(77.4\ \text{mm}^2)}{50.8\ \text{mm}}$ =1.49kN/mm

Table 16.2a—Material properties and description of two types of FRP systems

SystemA(excerpts from data sheet)	System B(excerpts from data sheet)
System type:dry,unidirectional sheet Fiber type:high-strength carbon Polymer resin:cpoxy SystemA is installed using a wet layup procedure where the dry carbon-fiber sheets are impregnated and adhered with an epoxy resin on-site	System type:precured,unidirectional laminate Fiber type:high-strength carbon Polymer resin:cpoxy System B's precured laminates are bonded to the concrete substrate using System B's epoxy paste adhesive.
Mechanical properties	Mechanical properties
=0.04 in. (1.02 mm)	t=0.050 in. (1.27 mm)
fG=179ksi(1234N/mm²)	f=380ksi(2620 N/mm²)
B=1.6%	Bm=1.5%
E=10,725ksi(73,946N/mm²)	E=22,000 ksi(151,685N/mm²)

Reported properties are based on a population of20 or more coupons teste in acordance with ASTM D7565D7565M.

Table 16.2b—Procedure comparing two types of FRP systems

Procedure	Calculation in in.-Ib units	Calculation in SI units
Step 1A—Calculate the tensile strength per unit width of SystemA Pm²=fai	P=(179 ksi)(0.04 in.)=7.16 kip/in.	p=(1.234kN/mm²)(1.02 mm)=1.26 kN/mm
Step 1B—Calculate the tensile strength per unit width of System B Pm=fmi	PG'=(380 ksi)(0.050 in.)=19 kip/in.	Pn=(2.62kN/mm²)(1.27mm)=3.33kN/mm
Step 2A—Calculate the tensile modulus per unit width of SystemA k=EA	ky=(10,725ksi)(0.04 in.)=429 kip/in.	ky=(73.946kN/mm²)(1.02 mm)=75.4kN/mm
Step 2B—Calculate the tensile modulus per unit width of System B k;=EA	ky=(22,000 ksi)(0.050 in.)=1100 kip/in.	ky=(151.7kN/mm²)(1.27 mm)=192.7kN/mm
Step 3—Compare the two systems Compare the tensile strengths: Pn(SystemA) Pa(System B)	$\frac{p^*_{fu}(\text{System B})}{p^*_{fu}(\text{System A})} = \frac{19 \text{ kip/in.}}{7.16 \text{ kip/in.}} = 2.65$ 2.three plies of SystemA are required for each ply of System B for an equivalent tensile strength	$\frac{p^*_{fu}(\text{System B})}{p^*_{fu}(\text{System A})} = \frac{3.33 \text{ kN/mm}}{1.26 \text{ kN/mm}} = 2.64$ 2.three plies of System A are required for each ply of System B for an equivalent tensile strength
Compare the stiffinesses: k;(SystemA) k,(System B)	$\frac{k_f(\text{System B})}{k_f(\text{System A})} = \frac{1100 \text{ kip/in.}}{429 \text{ kip/in.}} = 2.56$ 2.three plies of System A are required for each ply of System B for an equivalent stiffness	$\frac{k_f(\text{System A})}{k_f(\text{System B})} = \frac{192.7 \text{ kN/mm}}{75.4 \text{ kN/mm}} = 2.56$ 2.three plies of System A are required for each ply of System B for an equivalent stiffness

Because the data sheets for both systems are reporting statistically based properties,it is possible to directly compare the tensile strength and modulus of both systems, as shown in Table 16.2b.

Because all the design procedures outlined in this document limit the strain in the FRP system,the full nominal strength of the material is not used and should not be the basis of comparison between two material systems.When considering various FRPsystems for a particular application, the FRP systems should be compared based on equivalent stiffness only.In addition,each FRP system under consideration should have the ability to develop the strain associated with the effective strain required by the application without rupturing,eju>εe.

In many instances,it may be possible to vary the width of the FRP strip as opposed to the number of plies(use larger widths for systems with lower thicknesses and vice versa). In such instances,equivalent stiffness calculations typically will not yield equivalent contributions to the strength of a member.In general,thinner (lower Nt)and wider(higher w) FRP systems will provide a greater increment of strength to a member due to the resulting lower bond stresses.The exact equivalency,however,can only be found by performing complete calculations(according to procedures described in Chapters 10,11,and 12 of this guide)for each system.

16.3—Flexural strengthening of an interior reinforced concrete beam with FRP

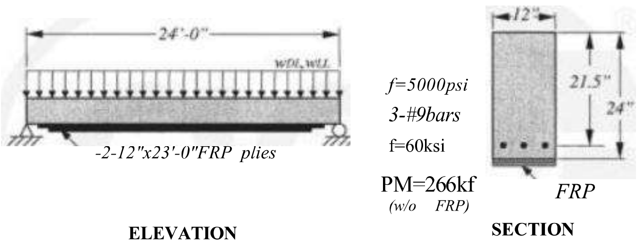
A simply supported concrete beam reinforced with three No.9(No.M29)bars(Fig.16.3)is located in an unoccupied warehouse and is subjected to a 50%increase in its live-load-carrying requirements.An analysis of the existing beam indicates that the beam still has sufficient shear strength to resist the new required shear strength and meets the deflection and crack-control serviceability requirements.Its flex-

Table 16.3a—Loadings and corresponding moments

Loading/moment	Existing loads		Anticipated loads	
Dead loads wou	1.00 kip/ft	14.6 N/mm	1.00 kip/t	14.6 N/mm
Live load wuz	1.20 kip/ft	17.5 N/mm	1.80 kip/ft	26.3 N/mm
Unfactored loads(woL+wz)	2.20 kip/ft	32.1 N/mm	2.80 kip/ft	40.9 N/mm
Unstrengthened load limit(1.1wo+0.75w)	NA	NA	2.50 kip/t	35.8 N/mm
Factored loads(1.2wn+1.6wz)	3.12 kip/ft	45.5 N/mm	4.08 kip/A	59.6N/mm
Dead-load moment Mo	72 kip-ft	98 kN-m	72 kip-ft	98 kN-m
Live-load moment Mu	86 kip-ft	117kN-m	130 kip-ft	176 kN-m
Service-load moment M	158 kip-ft	214kN-m	202 kip-ft	274kN-m
Unstrengthened moment limit(1.1MDL+0.75Mu)	NA	NA	177 kip-ft	240kN-m
Factored moment M	224 kip-ft	304kN-m	294.4 kip-t	399kN-m

Table 16.3b—Manufacturer's reported FRP system properties

Thickness per ply t;	0.040 in.	1.02 mm
Ultimate tensile strengthf	90ksi	621 N/mm²
Rupture strain en	0.015 in./in.	0.015mm/mm
Modulus of elasticity E,	5360ksi	37,000N/mm²



Length of the beaml	24f	7.32m
Width of the beam w	12 in.	305mm
d	21.5in.	546mm
h	24 in.	6096mm
f	5000 psi	34.5Nmm²
f二	60ksi	414N/mm²
M,without FRP	266kip-f	361kN-m
Bars	No.9	ϕ=28.6mm

Fig.16.3—Schematic ofthe idealized simply supported beam with FRP external reinforcement(Note:I in.=25.4 mm; I kip-ft=1.36kN-m;1 ksi=6.9 MPa;I psi=0.0069 MPa.)

ural strength,however,is inadequate to carry the increased live load.

Summarized in Table 16.3a are the existing and new loadings and associated midspan moments for the beam.The existing reinforced concrete beam should be strengthened with the FRP system described in Table 16.3b,specifically, two 12 in.(305 mm)widex 23.0 ft(7m)long plies bonded to the soffit of the beam using the wet layup technique.

By inspection,the degree of strengthening is reasonable in that it does not exceed the strengthening limit criteria specified in Eq.(9.2).That is,the existing moment strength without FRP,(φM)wo=266 kip-ft(361 kN-m),is greater than the unstrengthened moment limit,(1.1MDL+0.75ML)new =177 kip-ft(240 kN-m).The design calculations used to verify this configuration follow in Table 16.3b.

In detailing the FRP reinforcement,the FRP should be terminated aminimum oflaj,calculated per Eq.(14.1.3),past the point on the moment diagram that represents cracking. The factored shear force at the termination should also be checked against the shear force that causes FRPend peeling, estimated as two-thirds ofthe concrete shear strength.If the shear force is greater than two-thirds ofthe concrete shear strength,the FRP strips should be extended further toward the supports.U-wraps may also be used to reinforce against cove delamination.

Table 16.3c—Procedure for flexural strengthening of an interior reinforced concrete beam with fiber-reinforced polymer

Procedure	Calculation in in.-Ib units	Calculation in SI units
Step 1—Caleulate the FRPsystem design material properties.		
The beam is located in an interior space and a carbon FRP(CFRP)material will be used. Therefore, per Table 9.4, an environmental reduction factor of0.95 is used.		
Fu=Cef	fm=(0.95)(90ksi)=85ksi	fm=(0.95)(621N/mm²)=590 N/mm²
Em=CeEn	Em=(0.95)(0.015 in./in.)=0.0142 in.in.	Bm=(0.95)(0.015mm/mm)=0.0142mm/mm
Step 2—Preliminary calculations		
Properties of the concrete:		
β:from ACI 318-19(22), Section 22.2.2.4.3.	β_1=1.05–0.0502/1000)=0.80	β 1=1.05–0.05(./6.9)=0.80
E.=57,000 √	E.=57,0005000psi=4,030,000psi	E.=4700 √34.5N/mm²=27,600 N/mm²
Properties of the existing reinforcing steel:	A_2=3(1.00 in.)=3.00 in.²	A_2=3(645mm²)=1935mm²
Properties of the externally bonded FRP reinforcement:		
A,=Nt;w;	4=(2plies)(0.040 in./ply)(12 in.)=0.96 in²	A,=(2plies)(1.02 mm/ply)(305mm)=619mm²
Step 3—Determine the existing state of strain on the soffit.		
The existing state of strain is calculated assuming the beam is cracked and the only loads acting on the beam at the time of the FRPinstallation are dead loads. A cracked section analysis of the existing beam gives k=0.334 andI_4=5937 in.⁴=2471×10⁶mm⁴		
$\varepsilon_{bi}=\frac{M_{DL}(d_f-kd)}{I_{cr}E_c}$	$\varepsilon_{bi}=\frac{(864\text{ kip-in.})[24\text{ in.}-(0.334)(21.5\text{ in.})]}{(5937\text{ in.}^4)(4030\text{ ksi})}$ =0.00061	$\varepsilon_{bi}=\frac{(97.6\text{ kN-mm})[609.6\text{ mm}-(0.334)(546.1\text{ mm})]}{(2471\times10^6\text{ mm}^4)(27.6\text{ kN/mm}^2)}$ =0.00061
Step 4—Determine the design strain of the FRPsystem.		
The design strain of FRPaccounting for debonding failure mode eg is calculated using Eq. (10.1.1).	$\varepsilon_{fd}=0.083\sqrt{\frac{5000\text{ psi}}{2(5,360,000\text{ psi})(0.04\text{ in.})}}$ =0.009≤0.9(0.0142)=0.0128	$\varepsilon_{fd}=0.41\sqrt{\frac{34.5\text{ N/mm}^2}{2(37,000\text{ N/mm}^3)(1.02\text{ mm})}}$ Ea=0.009≤0.9(0.0142)=0.0128
Because the design strain is smaller than the rupture strain, debonding controls the design of the FRPsystem.		
Step 5—Estimate c, the depth to the neutral axis.		
A reasonable initial estimate of cis 0.20d The value of the cis adjusted after checking equilibrium.		
c=0.20d	c=(0.20)(21.5 in.)=4.30 in.	c=(0.20)(546.1 mm)=109mm

Table 16.3c,cont.—Procedure for flexural strengthening of an interior reinforced concrete beam with fiber-reinforced polymer

Step6—Determine the effective level of strain in the FRP reinforcement. The effctive strain level in the FRPmay be found from Eq. (10.2.5). $\varepsilon_{fe} = 0.003\left(\frac{d_f - c}{c}\right) - \varepsilon_{bi} \leq \varepsilon_{fd}$ Note that for the neutral axis depth selected, FRP debonding would be in the failure mode because the second expression in this equation controls. If the first expression govemed, then concrete crushing would be in the failure mode. Because FRP controls the failure of the section, the concrete strain at failure emay be less than 0.003 and can be calculated using similar triangles: $\varepsilon_c = (\varepsilon_{fe} + \varepsilon_{bi})\left(\frac{c}{d_f - c}\right)$	$\varepsilon_{fe} = 0.003\left(\frac{24\text{ in.} - 4.3\text{ in.}}{4.3\text{ in.}}\right) - 0.00061 \leq 0.009$ Be=0.0131>0.009 Be=Ea=0.009 $\varepsilon_c = (0.009 + 0.00061)\left(\frac{4.3\text{ in.}}{24\text{ in.} - 4.3\text{ in.}}\right) = 0.0021$	$\varepsilon_{fe} = 0.003\left(\frac{609.6\text{ mm} - 109.2\text{ mm}}{109.2\text{ mm}}\right) - 0.00061 \leq 0.009$ Bc=0.0131>0.009 B=Bg=0.009 $\varepsilon_c = (0.009 + 0.00061)\left(\frac{109.2\text{ mm}}{609.6\text{ mm} - 109.2\text{ mm}}\right) = 0.0021$
Step 7—Caleulate the strain in the existing reinforcing steel. The strain in the reinforcing steel can be calculated using similar triangles according to Eq. (10.2.6). $\varepsilon_s = (\varepsilon_{fe} + \varepsilon_{bi})\left(\frac{d - c}{d_f - c}\right)$	$\varepsilon_s = (0.009 + 0.00061)\left(\frac{21.5\text{ in.} - 4.3\text{ in.}}{24\text{ in.} - 4.3\text{ in.}}\right) = 0.0084$	$\varepsilon_s = (0.009 + 0.00061)\left(\frac{546.1\text{ mm} - 109.2\text{ mm}}{609.6\text{ mm} - 109.2\text{ mm}}\right)$ =0.0084
Step 8—Calculate the stress level in the reinforcing steel and FRP. The stresses are calculated using Eq. (10.2.10b) and Hooke's Law. . =E, s≤f A=Ee	f=(29,000 ksi)(0.0084)≤60ksi f=244ksi≤60ksi Hence, f=60ksi J=(5360 ksi)(0.009)=48.2 ksi	f_2=(200kN/mm²)(0.0084)≤0.414kN/mm² f_2=1.68kN/mm²≤0.414kN/mm² Hence, f_5=0.414kN/mm² Je=(37kN/mm²)(0.009)=0.33kN/mm²

Table 16.3c,cont.—Procedure for flexural strengthening of an interior reinforced concrete beam with fiber-reinforced polymer

Step 9—Calculate the internal force resultants and check equilibrium. Concrete stress block factors may be calculated using ACI318-19(22). Approximate stress block factors may also be calculated based on the parabolic stress-strain relationship for concrete as follows: $\beta_1 = \frac{4\varepsilon_c' - \varepsilon_c}{6\varepsilon_c' - 2\varepsilon_c}$ $\alpha_1 = \frac{3\varepsilon_c'\varepsilon_c - \varepsilon_c^2}{3\beta_1\varepsilon_c'^2}$ where B/' is strain corresponding tof calculated as $\varepsilon_c' = \frac{1.7f_c'}{E_c}$ Force equilibrium is verified by checking the initial estimate of cwith Eq. (10.3.1.6g) $c = \frac{A_s f_s + A_f f_{fe}}{\alpha_1 f_c' \beta_1 b}$	$\beta_1 = \frac{4(0.0021) - 0.0021}{6(0.0021) - 2(0.0021)} = 0.749$ $\alpha_1 = \frac{3(0.0021)(0.0021) - (0.0021)^2}{3(0.749)(0.0021)^2} = 0.886$ $\varepsilon_c' = \frac{1.7(5000)}{4.03 \times 10^6} = 0.0021$ $c = \frac{(3.00 \text{ in.}^2)(60 \text{ ksi}) + (0.96 \text{ in.}^2)(48.2 \text{ ksi})}{(0.886)(5 \text{ ksi})(0.749)(12 \text{ in.})}$ c=5.68 in. ≠4.30 in NG ∴Revise estimate of c and repeat Steps 6 through 9until equilibrium is achieved.	$\beta_1 = \frac{4(0.0021) - 0.0021}{6(0.0021) - 2(0.0021)} = 0.749$ $\alpha_1 = \frac{3(0.0021)(0.0021) - (0.0021)^2}{3(0.749)(0.0021)^2} = 0.886$ $\aleph \quad \frac{1.7(34.5)}{27,600} \quad 0.0021$ $c = \frac{(1935.48 \text{ mm}^2)(414 \text{ N/mm}^2) + (619 \text{ mm}^2)(330 \text{ N/mm}^2)}{(0.886)(34.5 \text{ N/mm}^2)(0.749)(304.8 \text{ mm})}$ c=149mm≠109mm NG ∴Revise estimate of c and repeat Steps 6 through 9until equilibrium is achieved.
Step 10—Adjust c until force equilibrium is satisfied. Steps 6 through 9 were repeated several times with different values of cuntil equilibrium was achieved. The results of the final iteration are: c=5.17 in.; e_5=0.0083; f=f=60ksi; β_1=0.786; a_1=0.928; andfa=48.2 ksi	$c = \frac{(3.00 \text{ in.}^2)(60 \text{ ksi}) + (0.96 \text{ in.}^2)(48.2 \text{ ksi})}{(0.925)(5 \text{ ksi})(0.786)(12 \text{ in.})}$ c=5.17 in. ∴the value of c selected for the final iteration is correct.	$c = \frac{(1935.5 \text{ mm}^2)(414 \text{ N/mm}^2) + (619 \text{ mm}^2)(330 \text{ N/mm}^2)}{(0.928)(34.5 \text{ N/mm}^2)(0.786)(304.8 \text{ mm})}$ c=131 mm ∴. the value of c sclected for the final iteration is correct.
Step 11—Calculate flexural strength components. The design flexural strength is calculated using Eq. (10.2.10d). An additional reduction factor, v=0.85, is applied to the contribution of the FRP system. Steel contribution to bending: $M_{ns} = A_s f_s \left(d - \frac{\beta_1 c}{2} \right)$ FRPcontribution to bending: $M_{nf} = A_f f_{fe} \left(d_f - \frac{\beta_1 c}{2} \right)$	$M_{ns} = (3.00 \text{ in.}^2)(60 \text{ ksi})\left(21.5 \text{ in.} - \frac{0.786(5.17 \text{ in.})}{2} \right)$ M=3504 kip-in.=292 kip-ft $M_{nf} = (0.96 \text{ in.}^2)(48.2 \text{ ksi})\left(24 \text{ in.} - \frac{0.786(5.17 \text{ in.})}{2} \right)$ M_4=1020kip-in.=85 kip-ft	$M_{ns} = (1935.5 \text{ mm}^2)(414 \text{ N/mm}^2)\left(546.1 \text{ mm} - \frac{0.786(131 \text{ mm})}{2} \right)$ Mm=3.963×10^8N-mm=396.3kN-m $M_{nf} = (619 \text{ mm}^2)(330 \text{ N/mm}^2)\left(609.6 \text{ mm} - \frac{0.786(131 \text{ mm})}{2} \right)$ Mm=1.140×10*N-mm=114kN-m

Table 16.3c,cont.—Procedure for flexural strengthening of an interior reinforced concrete beam with fiber-reinforced polymer

Step 12—Calculate design flexural strength of the section. The design flexural strength is calculated using Eq. (10.1)and(10.2.10d). Because e,=0.0083 >0.005, a strength reduction factor of Φ=0.90 is appropriate per Eq. (10.2.7). ΦM=Φ[M+vMd	ΦMn=0.9[292k-ft+0.85(85k-ft) ΦM=327k-ft≥M=294k-ft 2. the strengthened section is capable of sustaining the new required moment strength.	ΦMn=0.9[396.3kN-m+0.85(114kN-m)] ΦMn=443kN-m≥M=399kN-m . the strengthened scction is capable of sustaining the new required moment strength.
Step 13—Check service stresses in the reinforcing steel and the FRP. Calculate theelastic depth to the cracked neutral axis. This can be simplified for a rectangular beam without compression reinforcement as follows: $k=\sqrt{\left(\rho_s\frac{E_s}{E_c}+\rho_f\frac{E_f}{E_c}\right)^2+2\left(\rho_s\frac{E_s}{E_c}+\rho_f\frac{E_f}{E_c}\left(\frac{d_f}{d}\right)\right)}-\left(\rho_s\frac{E_s}{E_c}+\rho_s\frac{E_f}{E_c}\right)$ Calculate the stress level in the reinforeing steel using Eq. (10.2.10.1)and verify that it is less than the recommended limit per Eq. (10.2.8a). 1. $\frac{\left[M_s+\varepsilon_{bi}A_fE_f\left(d_f-\frac{kd}{3}\right)\right](d-kd)E_s}{A_sE_s\left(d-\frac{kd}{3}\right)(d-kd)+A_fE_f\left(d_f-\frac{kd}{3}\right)(d_f-kd)}$ Jso≤0.80f,	* $k=0.343$ $kd=(0.343)(21.5\text{ in.})=7.37\text{ in.}$ Lx=40.4ksi≤(0.80)(60 ksi)=48 ksi ∴. the stress level in the reinforcing steel is within the recommended limit.	‡ $k=0.343$ $kd=(0.343)(546.1\text{ mm})=187\text{ mm}$ fs=279 N/mm² ≤(0.80)(410N/mm²)-330N/mm² ∴the stress level in the reinforcing steel is within the recommended limit
Step 14—Check creep rupture limit at service of the FRP. Calculate the stress level in the FRPusing Eq. (10.2.10.2)and verify that it is less than creep-rupture stress limit given in Table 10.2.9. Assume that the full service load is sustained $f_{f,s}=f_{s,s}\left(\frac{E_f}{E_s}\right)\left(\frac{d_f-kd}{d-kd}\right)-\varepsilon_{bi}E_f$ For a carbon FRPsystem, the sustained plus cyclic stresslimit is obtained from Table 10.2.9: Sustained plus cyclic stress limit=0.55fu	$f_{f,s}=40.4\text{ ksi}\left(\frac{5360\text{ ksi}}{29{,}000\text{ ksi}}\right)\left(\frac{24\text{ in.}-7.37\text{ in.}}{21.5\text{ in.}-7.37\text{ in.}}\right)$ -(0.00061)(5360ksi) f.=5.60 ksi≤(0.55)(85ksi)=47ksi ∴the stress level in the FRPis within the recommended sustained plus cyclic stresslimit	$f_{f,s}=0.278\text{ kN/mm}^2\left(\frac{37\text{ kN/mm}^2}{200\text{ kN/mm}^2}\right)\left(\frac{609.6\text{ mm}-187\text{ mm}}{546\text{ mm}-187\text{ mm}}\right)$ -(0.00061)(38 N/mm²) J=38 N/mm² ≤(0.55)(590 N/mm²)=324N/mm² ∴the stress level in the FRP is within the recommended sustained plus cyclic stress limit.

* $k=\sqrt{\left[0.0116\left(\frac{29{,}000}{4030}\right)+0.00372\left(\frac{5360}{4030}\right)\right]^2+2\left[0.0116\left(\frac{29{,}000}{4030}\right)+0.00372\left(\frac{5360}{4030}\right)\left(\frac{24\text{ in.}}{21.5\text{ in.}}\right)\right]}-\left[0.0116\left(\frac{29{,}000}{4030}\right)+0.00372\left(\frac{5360}{4030}\right)\right]$

† $$f_{s,s}=\frac{\left(\left[2424\text{ kip-in.}+\left[(0.00061)(0.96\text{ in.}^2)\times(5360\text{ ksi})\left(24\text{ in.}-\frac{7.37\text{ in.}}{3}\right)\right]\right]\times[(21.5\text{ in.}-7.37\text{ in.})(29{,}000\text{ ksi})]\right)}{\left(\left[(3.00\text{ in.}^2)(29{,}000\text{ ksi})\times\left(21.5\text{ in.}-\frac{7.37\text{ in.}}{3}\right)(21.5\text{ in.}-7.37\text{ in.})\right]+\left[(0.96\text{ in.}^2)(5360\text{ ksi})\left(24\text{ in.}-\frac{7.37\text{ in.}}{3}\right)(24\text{ in.}-7.37\text{ in.})\right]\right)}$$

‡ $k=\sqrt{\left[0.0116\left(\frac{200}{27.6}\right)+0.00372\left(\frac{37}{27.6}\right)\right]^2+2\left[0.0116\left(\frac{200}{27.6}\right)+0.00372\left(\frac{37}{27.6}\right)\left(\frac{609.6\text{ mm}}{546\text{ mm}}\right)\right]}-\left[0.0116\left(\frac{200}{27.6}\right)+0.00372\left(\frac{37}{27.6}\right)\right]$

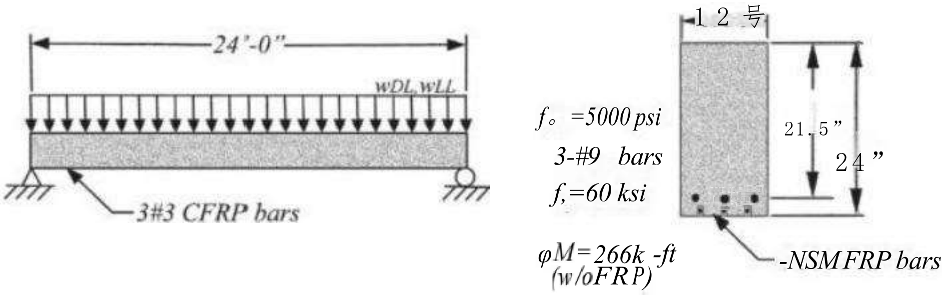
§ $$f_{s,s}=\frac{\left(\left[273{,}912\text{ kN-mm}+\left[(0.00061)(619\text{ mm}^2)(37\text{ kN/mm}^2)\times\left(609.6\text{ mm}-\frac{187\text{ mm}}{3}\right)\right]\right]\times[(546\text{ mm}-187\text{ mm})(200\text{ kN/mm}^2)]\right)}{\left(\left[(1935\text{ mm}^2)(200\text{ kN/mm}^2)\times\left(546\text{ mm}-\frac{187\text{ mm}}{3}\right)(546\text{ mm}-187\text{ mm})\right]+\left[(619\text{ mm}^2)(37\text{ kN/mm}^2)\left(607\text{ mm}-\frac{187\text{ mm}}{3}\right)(607\text{ mm}-187\text{ mm})\right]\right)}$$

16.4—Flexural strengthening of an interior reinforced concrete beam with near-surface-mounted (NSM)FRP bars

An existing reinforced concrete beam(Fig.16.4)is to be strengthened using the loads given in Table 16.3a and the NSM FRP system described in Table 16.4.Specifically,three No.3(No.M10)carbon FRP(CFRP)bars are to be used at a distance 23.7 in.(602.1 mm)from the extreme top fiber of the beam.

By inspection,the degree of strengthening is reasonable in that it does not exceed the strengthening limit criteria put forth in Eq.(10.1).That is,the existing flexural strength without FRP,(φMa)wo=266k-ft(361kN-m),is greater than the unstrengthened moment limit,(1.1MDL+0.75MLnew= 177 k-ft(240 kN-m).The design calculations used to verify this configuration follow.

In detailing the FRP reinforcement,FRP bars should be terminated at a distance equal to the bar development length past the point on the moment diagram that represents cracking.



ELEVATION SECTION

Fig.16.4—Schematic of the idealized simply supported beam with FRP external reinforcement.(Note:I in.=25.4 mm; 1 kip-ft=1.36kN-m;I ksi=6.9 MPa;1 psi=0.0069 MPa.)

Table 16.4a—Manufacturer's reported NSM FRP system properties

Area per No.3(No.M10)bar	0.10 in.²	64.5 mm²
Ultimate tensile strengthfm*	250 ksi	1725N/mm²
Rupture strain en	0.013 in./in.	0.013 mm/mm
Modulus of elasticity of FRPE,	19,230ksi	132,700 N/mm²

Table 16.4b—Procedure for flexural strengthening of an interior reinforced concrete beam with NSM FRP bars

Procedure	Calculation in in.-Ib units	Calculation in SI units
Step1—Calculate the FRPsystem design material properties. The beam is located in an interior space and a CFRP system will be used.Therefore,per Table 9.4,an environmental reduction factor of0.95 is used.		
Fm=Cfp*	Jm=(0.95)(250 ksi)=237.5ksi	Ju=(0.95)(1725 N/mm²)=1639 N/mm²
m=Cem°	Em=(0.95)(0.013 in./in.)=0.0123 in.in.	E吻l=(0.95)(0.013mm/mm)=0.0123 mm/mm
Step 2—Preliminary calculations. Properties of the concrete:		
β:fromACI 318-19(22),Section 22.2.2.4.3	$\beta_1 = 1.05 - 0.05\frac{f'_c}{1000} = 0.85$	$\beta_1 = 1.05 - 0.05\frac{f'_c}{6.9} = 0.85$
E。=57,000J	E.=57,000J5000 psi=4,030,00 psi	E.=4700 √34.5N/mm²=27,600N/mm²
	A₂=3(1.00 in.?)=3.00 in.²	A₃=3(645.2mm²)=1935mm²
	A;=(3 bars)(0.01 in.?bar)=0.3 in²	A₂=(3 bars)(64.5 mm²/bar)=194 mm²

Table 16.4b,cont.—Procedure for flexural strengthening of an interior reinforced concrete beam with NSM FRP bars

Step 3—Determine the existing state of strain on the soffit. The existing state of strain is calculated assuming the beam is cracked and the only loads acting on the beam at the time of the FRPinstallation are dead loads.A cracked section analysis of the existing beam gives k=0.334 and l=5937 in.4 =2471×10⁶mm⁴ → $k=0.334$ and $I=5937\ in.^4 = 2471\times10^6 mm^4$ $\varepsilon_{bi}=\dfrac{M_{DL}(d_f-kd)}{I_{cr}E_c}$	$\varepsilon_{bi}=\dfrac{(864\text{ kip-in.})[23.7\text{ in.}-(0.334)(21.5\text{ in.})]}{(5937\text{ in.}^4)(4030\text{ ksi})}$ =0.00061	$\varepsilon_{bi}=\dfrac{(97.6\text{ kN-mm})[602\text{ mm}-(0.334)(546\text{ mm})]}{(2471\times10^6\text{ mm})(27.6\text{ kN/mm})}$ =0.00061
Step4—Determine the design strain of the FRPsystem. The design strain of NSM FRP,em is calculated using 10.1.1 as 0.7em.	Ea=0.7(0.0123)=0.00865	a=0.7(0.0123)=0.00865
Step 5—Estimate c,the depth to the neutral axis. A reasonable initial estimate ofcis 0.20d.The value of the cis adjusted after checking equilibrium. c=0.20d	c=(0.20)(21.5 in.)=4.30 in.	c=(0.20)(546 mm)=109mm
Step 6—Determine the efective level of strain in the FRP reinforcement. The effective strain level in the FRP may be found from Eq. (10.2.5). $\varepsilon_{fe}=\left(\dfrac{d_f-c}{c}\right)-\varepsilon_{bi}\le\varepsilon_{fd}$ Note that for the neutral axis depth selected,FRP dcbonding would be the failure mode because the second expres-sion in this equation controls.If the first expression governed,then concrete crushing would be the failure mode. Because FRP controls the failure of the section,the concrete strain at failure, 8.,may be less than 0.003 and can be calculated using similar triangles: $\varepsilon_c=(\varepsilon_{fd}+\varepsilon_{bi})\left(\dfrac{c}{d_f-c}\right)$	$\varepsilon_{fe}=0.003\left(\dfrac{23.7\text{ in.}-4.3\text{ in.}}{4.3\text{ in.}}\right)-0.00061=0.0129$ Hence,Ee=0.00865 (Mode of failure is FRPdebonding) $\varepsilon_c=(0.00865+0.00061)\left(\dfrac{4.3}{23.7-4.3}\right)=0.0020$	$\varepsilon_{fe}=0.003\left(\dfrac{602\text{ mm}-109\text{ mm}}{109\text{ mm}}\right)-0.00061=0.0129$ Hence,En=0.00865 (Mode of failure is FRP debonding) $\varepsilon_c=(0.00865+0.00061)\left(\dfrac{109}{602-109}\right)=0.0020$
Step 7—Caleulate the strain in the existing reinforcing steel. The strain in the reinforcing steel can be calculated using similar triangles according to Eq. (10.2.10a). $\varepsilon_s=(\varepsilon_{fe}+\varepsilon_{bi})\left(\dfrac{d-c}{d_f-c}\right)$	$\varepsilon_s=(0.00865+0.00061)\left(\dfrac{21.5-4.3}{23.7-4.3}\right)=0.0082$	$\varepsilon_s=(0.00865+0.00061)\left(\dfrac{546-109}{602-109}\right)=0.0082$

Table 16.4b,cont.—Procedure for flexural strengthening of an interior reinforced concrete beam with NSM FRP bars

Step 8—Calculate the stress level in the reinforcing steel and FRP. The stresses are calculated using Eq. (10.2.10b)and Hooke's Law. f=Ee≤f 乐=Ee	 A=(29,000 ksi)(0.0082)≤60ksi f.=238ksi>60ksi therefore, f_5=60ksi J=(19,230ksi)(0.00865)=166ksi	 f=(200kN/mm²)(0.0082)≤0.414kN/mm² f_5=164kN/mm²≤0.414kN/mm² therefore, f=0.414kN/mm² C%e=(132,700 N/mm²)(0.00865)=1147N/mm²
Step 9—Calculate the internal force resultants and check equilibrium. Concrete stress block factors may be calculated usingAC1318-19(22). Approximate stress block factors may also be calculated based on the parabolic stress-strain relationship for concrete as follows: $\beta_1 = \frac{4\varepsilon_c' - \varepsilon_c}{6\varepsilon_c' - 2\varepsilon_c}$ $\alpha_1 = \frac{3\varepsilon_c'\varepsilon_c - \varepsilon_c^2}{3\beta_1\varepsilon_c'^2}$ where s,'is strain corresponding tof' calculated as $\varepsilon_c' = \frac{1.7f_c'}{E_c}$ Force equilibrium is verified by checking the initial estimate ofcwith Eq.(10.3.1.6g) $c = \frac{A_s f_s + A_f f_{fe}}{\alpha_1 f_c' \beta_1 b}$	$\varepsilon_c' = \frac{1.7(5000)}{4030 \times 10^6} = 0.0021$ $\beta_1 = \frac{4(0.0021) - 0.002}{6(0.0021) - 2(0.002)} = 0.743$ $\alpha_1 = \frac{3(0.0021)(0.002) - (0.002)^2}{3(0.743)(0.0021)^2} = 0.870$ $c = \frac{(3.00\text{ in.}^2)(60\text{ ksi}) + (0.3\text{ in.}^2)(166\text{ ksi})}{(0.87)(5\text{ ksi})(0.743)(12\text{ in.})}$ c=5.92 in.≠4.30 in**NG** .revise estimate of c and repeat Steps 6 through 9 until equilibrium is achieved.	$\varepsilon_c' = \frac{1.7(34.5)}{27,606} = 0.0021$ $\beta_1 = \frac{4(0.0021) - 0.002}{6(0.0021) - 2(0.002)} = 0.743$ $\alpha_1 = \frac{3(0.0021)(0.002) - (0.002)^2}{3(0.743)(0.0021)^2} = 0.870$ $c = \frac{(1935\text{ mm}^2)(414\text{ N/mm}^2) + (194\text{ mm}^2)(1147\text{ N/mm}^2)}{(0.87)(34.5\text{ N/mm}^2)(0.743)(305\text{ mm})}$ c=150mm≠109 in**NG** 2 revise estimate of c and repeat Steps 6 through 9until equilibrium is achieved.
Step 10—Adjust cuntil force equilib-rium is satisfied. Steps 6 through 9 were repeated several times with different values of cuntil cquilibrium was achieved.The results of the final iteration are: c=5.26 in.;eg=0.0082;f=f=60ksi; Ee=0.00865;e=0.0027;β_1=0.786; a_1=0.928;andfa=166ksi	$c = \frac{(3.00\text{ in.}^2)(60\text{ ksi}) + (0.3\text{ in.}^2)(166\text{ ksi})}{(0.928)(5\text{ ksi})(0.786)(12\text{ in.})}$ c=5.25 in.≈5.26 in. ∴.the value ofc selected for the final iteration is correct	=95692 $\frac{414\text{ N/mm}^2) + (193\text{ mm}^2)(1147\text{ N/mm}^2)}{8)(34.5\text{ N/mm}^2)(0.786)(305\text{ mm})}$ $c = 133\text{ mm} \simeq 134\text{ mm}$ .the value of c selected for the final iteration is correct.

Table 16.4b,cont.—Procedure for flexural strengthening of an interior reinforced concrete beam with NSM FRP bars

Step 11—Caleulate flexural strength components. The design flexural strength is calculated using Eq. (10.2.10d). An additional reduction factor, w=0.85, is applied to the contribution of the FRPsystem. Steel contribution to bending: $M_{ns} = A_s f_s \left(d - \frac{\beta_1 c}{2} \right)$ FRPcontribution to bending: $M_{nf} = A_f f_{fe} \left(d_f - \frac{\beta_1 c}{2} \right)$	$M_{ns} = (3.0 \text{ in.}^2)(60 \text{ ksi})\left(21.5 \text{ in.} - \frac{0.786(5.25 \text{ in.})}{2} \right)$ Mm=3498 kip-in.=291 kip-t $M_{nf} = (0.3 \text{ in.}^2)(166 \text{ ksi})\left(23.7 \text{ in.} - \frac{0.786(5.25 \text{ in.})}{2} \right)$ M_u=1077 kip-in.=90 kip-ft	$M_{ns} = (1935 \text{ mm}^2)(414 \text{ N/mm}^2)\left(546 \text{ mm} - \frac{0.786(133 \text{ mm})}{2} \right)$ Mm=394kN-m M=(194 mm²)(1147N/mm²) $\left(602.1 \text{ mm} - \frac{0.786(133 \text{ mm})}{2} \right)$ Mm=122 kN-m
Step 12—Calculate design flexural strength of the section. The design flexural strength is calculated using Eq. (10.1)and(10.2.10d). Because Bg=0.0082>0.005, a strength reduction factor of Φ=0.90 is appropriate per Eq. (10.2.7). ΦMn=Φ[Mm+M]	ΦM=0.9[291 kip-ft+0.85(90 kip-ft)] ΦM=331 kip-ft2M=294 kip-ft ∴the strengthened section is capable of sustaining the new required flexural strength.	ΦMn=0.9[394kN-m+0.85(122 kN-m)] ΦM=448kN-m∴M=398 kN-m ∴. the strengthened section is capable of sustaining the new required flexural strength.
Step 13—Check service stresses in the reinforcing steel and the FRP. Calculate the elastic depth to the cracked neutral axis. This can be simplificd for a rectangular beam without compression reinforcement as follows:		
$k = \sqrt{\left(\rho_s \frac{E_s}{E_c} + \rho_f \frac{E_f}{E_c} \right)^2 + 2\left(\rho_s \frac{E_s}{E_c} + \rho_f \frac{E_f}{E_c}\left(\frac{d_f}{d} \right) \right)} - \left(\rho_s \frac{E_s}{E_c} + \rho_f \frac{E_f}{E_c} \right)$ Calculate the stress level in the reinforcing steel using Eq. (10.2.10.1)and verify that it is less than the recommended limit per Eq. (10.2.8a). a= $\frac{\left[M_s + \varepsilon_{bi} A_f E_f \left(d_f - \frac{kd}{3} \right) \right](d - kd)E_s}{A_s E_s \left(d - \frac{kd}{3} \right)(d - kd) + A_f E_f \left(d_f - \frac{kd}{3} \right)(d_f - kd)}$ fss≤0.80f	k=0.345 kd=(0.345)(21.5in.)=7.4 in. fa=40.3 ksi≤(0.80)(60 ksi)=48 ksi ∴. the stress level in the reinforcing steel is within the recommended limit.	4 k=0.345 kd=(0.345)(546 mm)=188mm 8 fs=278 N/mm²≤(0.80)(410N/mm²)=330 N/mm² ∴the stress level in the reinforcing steel is within the recommended limit.

Table 16.4b,cont.—Procedure for flexural strengthening of an interior reinforced concrete beam with NSM FRP bars

Step 14—Check creep rupture limit at service of the FRP. Calculate the stress level in the FRP using Eq. (10.2.10.2)and verify that it is less than creep-rupture stress limit given in Table 10.2.9.Assume that the full-service load is sustained. $f_{f,s} = f_{s,s}\left(\frac{E_f}{E_s}\right)\left(\frac{d_f - kd}{d - kd}\right) - \varepsilon_{bi}E_f$ For a carbon FRP system,the sustained plus cyclic stress limit is obtained from Table 10.2.9: Sustained plus cyclic stress limit=0.55fu	$f_{f,s} = 40.3\text{ ksi}\left(\frac{19,230\text{ ksi}}{29,000\text{ ksi}}\right)\left(\frac{23.7\text{ in.} - 7.4\text{ in.}}{21.5\text{ in.} - 7.4\text{ in.}}\right)$ $-(0.00061)(19,230\text{ksi})$ Ja=19ksi≤(0.55)(85ksi)=50 ksi ∴the stress level in the FRP is within the recommended sustained plus cyclic stress limit	$f_{f,s} = 0.278\text{ kN/mm}^2\left(\frac{133\text{ kN/mm}^2}{200\text{ kN/mm}^2}\right)\left(\frac{602\text{ mm} - 188\text{ mm}}{546\text{ mm} - 188\text{ mm}}\right)$ $-(0.00061)(133\text{N/mm}^2)$ Jo=134N/mm² ≤(0.55)(590 N/mm² =324.5 N/mm² A.the stress level in the FRP is within the recommended sustained plus cyclic stress limit.

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$$^{\dagger}\ f_{s,s} = \frac{\left(\left[2424\text{ kip-in.} + \left[(0.00061)(0.3\text{ in.}^2)\times(19,230\text{ ksi})\left(23.7\text{ in.} - \frac{7.4\text{ in.}}{3}\right)\right]\right]\times\left[(21.5\text{ in.} - 7.4\text{ in.})(29,000\text{ ksi})\right]\right)}{\left(\left[(3.00\text{ in.}^2)(29,000\text{ ksi})\left(21.5\text{ in.} - \frac{7.4\text{ in.}}{3}\right)\times(21.5\text{ in.} - 7.4\text{ in.})\right] + \left[(0.3\text{ in.}^2)(19,230\text{ ksi})\left(23.7\text{ in.} - \frac{7.4\text{ in.}}{3}\right)\times(23.7\text{ in.} - 7.4\text{ in.})\right]\right)}$$

$$^{\ddagger}\ k = \sqrt{\left[0.0116\left(\frac{200}{27.6}\right) + 0.0012\left(\frac{133}{27.6}\right)\right]^2 + 2\left[0.0116\left(\frac{200}{27.6}\right) + 0.0012\left(\frac{133}{27.6}\right)\left(\frac{602\text{ mm}}{546\text{ mm}}\right)\right]} - \left[0.0116\left(\frac{200}{27.6}\right) + 0.0012\left(\frac{133}{27.6}\right)\right]$$

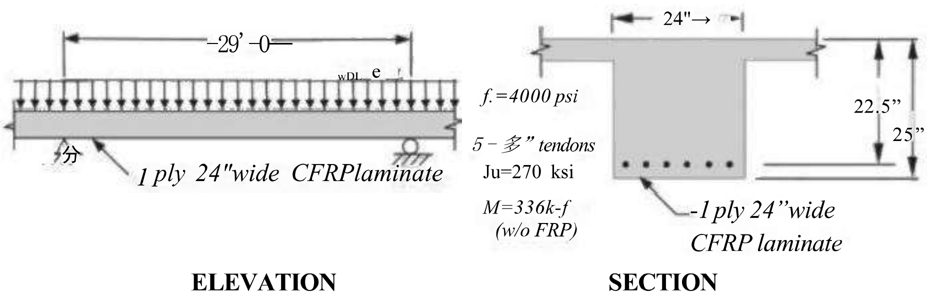
$$^{\S}\ f_{s,s} = \frac{\left(\left[273,912\text{ kN-mm} + \left[(0.00061)(194\text{ mm}^2)\times(132.7\text{ kN/mm}^2)\times\left(602\text{ mm} - \frac{188\text{ mm}}{3}\right)\right]\right]\times\left[(546\text{ mm} - 188\text{ mm})(200\text{ kN/mm}^2)\right]\right)}{\left(\left[(1935\text{ mm}^2)(200\text{ kN/mm}^2)\times\left(546\text{ mm} - \frac{188\text{ mm}}{3}\right)(546\text{ mm} - 188\text{ mm})\right] + \left[(194\text{ mm}^2)(132.7\text{ kN/mm}^2)\times\left(602\text{ mm} - \frac{188\text{ mm}}{3}\right)(602\text{ mm} - 188\text{ mm})\right]\right)}$$

16.5—Flexural strengthening of an interior prestressed(bonded strands)concrete beam with FRP

A number of continuous prestressed concrete beams with five 1/2 in.(12.7 mm)diameter bonded strands(Fig.16.5) are located in a parking garage that is being converted to an office space.All prestressing strands are Grade 270 ksi (1860 N/mm²)low-relaxation seven-wire strands.The beams require an increase in their live-load-carrying capacity from 50 to 751b/f²(244 to 366 kg/m²).The beams are also required to support an additional dead load of 101b/ft²(49 kg/m²).Analysis indicates that each existing beam has adequate flexural capacity to carry the new loads in the negative moment region at the supports but is deficient in flexure at midspan and in shear at the supports.The beam meets the deflection and crack control serviceability requirements.The cast-in-place beams support a 4 in.(100 mm) slab.For bending at midspan,beams should be treated as T-sections.Summarized in Table 16.5a are the existing and new loads and associated midspan moments for the beam. FRP system properties are shown in Table 16.3b.

By inspection,the degree of strengthening is reasonable in that it does not exceed the strengthening limit criteria put forth in Eq.(10.1).That is,the existing flexural strength without FRP,(φMn)wo=336 kip-ft(455 kN-m),is greater than the unstrengthened moment limit,(1.1MoL+0.75M)new =273 k-ft(370 kN-m).The design calculations used to verify this configuration follow.The beam is to be strengthened using the FRP system described in Table 16.3b.A one-ply,24 in.(610 mm)wide strip of FRPis considered for this evaluation.

In detailing the FRP reinforcement,the FRP should be terminated a minimum oflj,calculated per Eq.(14.1.3),past the point on the moment diagram that represents cracking. The factored shear force at the termination should also be checked against the shear force that causes FRPend peeling, estimated as two-thirds of the concrete shear strength.If the shear force is greater than two-thirds of the concrete shear strength,FRP strips should be extended further toward the supports.U-wraps may also be used to reinforce against cover delamination.



Length of the beam	29ft	8.84m
Bay widthl₂	30ft	9.14m
Width of beamw	24 in.	610mm
d,	22.5in.	571 mm
h	25in.	635mm
Effective flange width by	87 in.	2210mm
Flange thicknessh	4 in.	102mm
f.	4000 psi	27.6 N/mm²
Strand diameter	1/2in.	12.7mm
J	165ksi	1138 N/mm²
fn	230ksi	1586 N/mm²
fm	270ksi	1860 N/mm²
E	28,500 ksi	196000 N/mm²
Mawithout FRP	336 kip-ft	455kN-m

Fig.16.5—Schematic of the idealized continuous prestressed beam with FRPexternal reinforcement.(Note:I in.=25.4 mm; I kip-ft=1.36kN-m;I ksi=6.9 MPa;I psi=0.0069 MPa.)

Table 16.5a—Loadings and corresponding moments

Loading/moment	Existing loads		Anticipated loads	
Dead loads wDL	2.77 kip/ft	40.4 N/mm	3.09 kip/ft	45.1 N/mm
Live load wu	1.60 kip/ft	23.3 N/mm	2.4 kip/f	35N/mm
Unfactored loads(woL+wz)	4.37 kip/ft	63.8 N/mm	5.49kip/t	80.2 N/mm
Unstrengthened load limit(1.1wo+0.75w)	N/A	NA	5.2 kip/f	75.9 N/mm
Factored loads(1.2wo+1.6w)	5.88 kip/ft	85.9 N/mm	7.55 kip/t	110.2 N/mm
Dead-load moment Mou	147kip-ft	199kN-m	162 kip-ft	220.2 kN-m
Live-load moment Mu	85 kip-ft	115kN-m	126 kip-ft	171.1 kN-m
Service-load moment M,	232 kip-ft	314kN-m	289 kip-ft	391.3kN-m
Unstrengthened moment limit(1.1MpL+0.75M)e	NA	N/A	273 kip-ft	371kN-m
Factored moment M	312kip-ft	423kN-m	397kip-ft	538kN-m

Table 16.5b—Procedure for flexural strengthening of an interior prestressed concrete beam with fiber-reinforced polymer

Procedure	Calculation in in.-Ib units	Calculation in SI units
Step 1—Calculate the FRP-system design material properties. The beam is located in an interior space and a CFRPsystem will be used. Therefore, per Table 9.4, an environmental reduction factor of 0.95 is used.		
Fm=Cef*	fa=(0.95)(90ksi)=85ksi	fm=(0.95)(621N/mm²)=590 N/mm²
n=Cem	咖=(0.95)(0.015 in./in.)=0.0142 in./in.	n=(0.95)(0.015mm/mm)=0.0142mm/mm
Step 2—Preliminary calculations. **Properties of the concrete:**		
β:fromACI 318-19(22), Section 22.2.2.4.3	$\beta_1 = 1.05 - 0.05\frac{f'_c}{1000} = 0.85$	$\beta_1 = 1.05 - 0.05\frac{f'_c}{6.9} = 0.85$
E. =57,000 √	E。=57,000 √4000 psi=3,605,000 si	E. =470027.6 N/mm² =24,700N/mm²
Properties of the existing prestressing steel:	Am=5(0.153in.?)=0.765 in.²	Am=5(99 mm²)=495mm²
Area of FRP reinforcement: Ay=Nimy	A=(1ply)(0.040in.ply)(24 in.)=0.96 in²	A=(1ply)(1.0mm/ply)(610mm)=610mm²
Cross-sectional area: Acg=bh, +b(h-h)	Ag=(87in.)(4in.)+(24in.)(25in.-4in.)=852 in.?	Agg=(2210 mm)(102 mm) +(610mm)(635mm-102 mm)=5.5×10⁵mm²
Distance from the top fiber to the section centroid: $y_t = \frac{b_t\frac{h_f^2}{2} + b_w(h-h_f)\left(h_f + \frac{(h+h_f)}{2}\right)}{A_{cg}}$	$y_t = \frac{87\text{ in.}\times\frac{4\text{ in.}^2}{2} + 24\text{ in.}\times 21\times 14.5}{852} = 9.39\text{ in.}$	$y_t = \frac{2210\text{ mm}\times\frac{102\text{ mm}^2}{2} + 610\text{ mm}\times 533\times 368}{5.5\times 10^5}$ =238mm
Gross moment of inertia; $I_g = \frac{b_f h_f^3}{12} + b_f h_f\left(y_t - \frac{h_f}{2}\right)^2 + \frac{b_w(h-h_f)^3}{12} + b_w(h-h_f)\left(y_t - h_f - \frac{h}{2}\right)^2$	$I_g = \frac{87\text{ in.}\times 4\text{ in.}^3}{12}$ +87 in. ×4 in. (9.39 in.-2)² $+\frac{24\text{ in.}\times 21^3}{12} + 24\text{ in.}\times 21(9.39-4-10.5)^2$ =51,150 in.	$I_g = \frac{2210\text{ mm}\times 102\text{ mm}^3}{12}$ +2210mm×102mm(238-51² $+\frac{610\text{mm}\times 533^3}{12} + 610\text{ mm}\times 533(238-102-266.7)^2$ =2.13×10° mm⁴
Radius of gyration: $r = \sqrt{\frac{I_g}{A_{cg}}}$	$r = \sqrt{\frac{51,150}{852}} = 7.75\text{ in.}$	$r = \sqrt{\frac{2.13\times 10^{10}}{5.5\times 10^5}} = 197\text{ mm}$
Effective prestressing strain: $\varepsilon_{pe} = \frac{f_{pe}}{E_p}$	$\varepsilon_{pe} = \frac{165}{28,500} = 0.00579$	ε_{pe} $\frac{1138}{1.96\times 10^5}$ 0.00579
Effective prestressing force: $P_e = A_{ps}f_{pe}$	Pe=0.765×165=126.2 kip	P. =495×1138=563,310N
Eccentricity of prestressing force: e=dp-y	e=22.5-9.39=13.1 in.	e₃=571-238=333 mm

Table 16.5b,cont.—Procedure for flexural strengthening of an interior prestressed concrete beam with fiber-reinforced polymer

Step 3—Determine the existing state of strain on the soffit. The existing state of strain is calculated assuming the beam is uncracked and the only loads acting on the beam at the time of the FRP installation are dead loads. Distance from extreme bottom fiber to the section centroid: yb=h-y; Initial strain in the beam soffit: $\varepsilon_{bi} = \frac{-P_e}{E_c A_{cg}}\left(1+\frac{e y_b}{r^2}\right)+\frac{M_{DL} y_b}{E_c I_g}$	y%=25-9.39=15.61 in. $\varepsilon_{bi} = \frac{-126.2}{3605\times852}\left(1+\frac{13.1\times15.6}{7.75^2}\right) + \frac{147\times12\times15.6}{3605\times51{,}150}$ Bm=-3.2×10^{5}	yo=635-238=397mm $\varepsilon_{bi} = \frac{-563{,}310}{24{,}700\times5.5\times10^5}\left(1+\frac{333\times397}{171^2}\right) + \frac{199\times10^6\times397}{24{,}700\times2.13\times10^{10}}$ BA=-3.2×10^{-5}
Step4—Determine the design strain of the FRP system. The design strain ofFRPaccounting for debonding failure mode ea is calculated using Eq. (10.1.1). Because the design strain is smaller than the rupture strain,debonding controls the design of the FRPsystem.	$\varepsilon_{fd} = 0.083\sqrt{\frac{4000\ \text{psi}}{1(5{,}360{,}000\ \text{psi})(0.04\ \text{in.})}}$ =0.0113≤0.9(0.0142)=0.0128	$\varepsilon_{fd} = 0.042\sqrt{\frac{27.6\ \text{N/mm}^2}{1(37{,}000\ \text{N/mm}^2)(1.016\ \text{mm})}}$ =0.0113≤0.9(0.0142)=0.0128
Step5—Estimate c,the depth to the neutral axis.Areasonable initial estimate of cis 0.1h, The value of the c is adjusted after checking equilibrium. c=0.1h	c=(0.1)(25 in.)=2.50 in.	c=(0.1)(635mm)=63.5mm
Step 6—Determine the effective level of strain in the FRP reinforcement. The effective strain level in the FRPmay be found from Eq. (10.3.1.6c). $\varepsilon_{fe} = 0.003\left(\frac{d_f - c}{c}\right) - \varepsilon_{bi} \le \varepsilon_{fd}$ Note that for the neutral axis depth selected,FRP debonding would be the failure mode because the second expression in this equation controls If the first(limiting)expression govened,then FRP rupture would be the failure mode.	$\varepsilon_{fe} = 0.003\left(\frac{25-2.5}{2.5}\right)+0.00003 = 0.027$ >E=0.0113 Failure is governed by FRP debonding e=Ea=0.0113	$\varepsilon_{fe} = 0.003\left(\frac{635-63.5}{63.5}\right)+0.00003 = 0.027$ >Em=0.0113 Failure is governed by FRP debonding Ee=Eu=0.0113
Step 7—Calculate the strain in the existing prestressing steel. The strain in the prestressing steel can be calculated using Eq. (10.3.1.6c)and(10.3.1.6a). $\varepsilon_{pnet} = (\varepsilon_{fe} + \varepsilon_{bi})\left(\frac{d_p - c}{d_f - c}\right)$ $\varepsilon_{ps} = \varepsilon_{pe} + \frac{P_e}{A_{cg} E_c}\left(1+\frac{e^2}{r^2}\right) + \varepsilon_{pnet} \le 0.035$	$\varepsilon_{pnet} = (0.0113 - 0.00003)\left(\frac{22.5-2.5}{25-2.5}\right)$ Ema=0.01 $\varepsilon_{ps} = 0.00579 + \frac{126.2}{852\times3605}\left(1+\frac{13.1^2}{7.75^2}\right)+0.01$ Em=0.016≤0.035	$\varepsilon_{pnet} = (0.0113 - 0.00003)\left(\frac{571-63.5}{635-63.5}\right)$ E,me=0.01 $\varepsilon_{ps} = 0.00579 + \frac{563{,}310}{5.5\times10^5\times24{,}700}\left(1+\frac{333^2}{197^2}\right)+0.01$ Em=0.016≤0.035

Table 16.5b,cont.—Procedure for flexural strengthening of an interior prestressed concrete beam with fiber-reinforced polymer

Step 8—Calculate the stress level in the prestressing steel and FRP. The stresses are calculated using Eq. (10.3.1.6e) and Hooke's Law. $f_{ps} = \begin{cases} 28,500\varepsilon_{ps} & \text{for } \varepsilon_{ps} \le 0.0086 \\ 270 - \dfrac{0.04}{\varepsilon_{ps} - 0.007} & \text{for } \varepsilon_{ps} > 0.0086 \end{cases}$ f=Een	$f_{ps} = 270 - \dfrac{0.04}{0.016 - 0.007} = 265.6$ ksi Je=(5360 ksi)(0.0113)=60.6ksi	$f_{ps} = 1860 - \dfrac{0.276}{0.016 - 0.007} = 1831$ N/mm² J=(37,000N/mm²)(0.0113)=418N/mm²
Step 9—Calculate the equivalent concrete compressive stress block parameters t; and β i.The strain in conerete at failure can be calculated from strain compatibility as follows: $\varepsilon_c = (\varepsilon_{fe} + \varepsilon_{bi})\left(\dfrac{c}{d_f - c}\right)$ The strain g'corresponding tof'is calculated as $\varepsilon_c' = \dfrac{\ddot{u}\ f_c'}{E_c}$ Concrete stress block factors can be estimated using ACI 318.Approximate stress block factors may be calculated from the parabolic stress-strain relationship for concrete and is expressed as follows: $\beta_1 = \dfrac{4\varepsilon_c' \ \ _c}{6\varepsilon_c' \ \ 2\ _c}$ $\alpha_1 = \dfrac{3\varepsilon_c'\varepsilon_c - \varepsilon_c^2}{3\beta_1\varepsilon_c'^2}$	$\varepsilon_c = (0.0113 - 0.00003)\left(\dfrac{2.5}{25 - 2.5}\right) = 0.0013$ $\varepsilon_c' = \dfrac{1.7(4000)}{3605 \times 10^6} = 0.0019$ $\alpha_1 = \dfrac{3(0.0019)(0.0013) - (0.0013)^2}{3(0.716)(0.0019)^2} = 0.738$	$\varepsilon_c = (0.0113 - 0.00003)\left(\dfrac{63.5}{635 - 63.5}\right) = 0.0013$ $\varepsilon_c' = \dfrac{1.7(27.6)}{24,700} = 0.0019$ $\beta_1 = \dfrac{4(0.0019) - 0.0013}{6(0.0019) - 2(0.0013)} = 0.716$ $\alpha_1 = \dfrac{3(0.0019)(0.0013) - (0.0013)^2}{3(0.716)(0.0019)^2} = 0.738$
Step 10—Calculate the internal force resultants and check equilibrium. Force equilibrium is verified by checking the initial estimate of c with Eq. (10.3.1.6f).	c=1.42 in.22.50 in.NG .revise estimate of c and repeat Steps 6 through 10 until equilibrium is achieved.	c=36 mm≠63.5 in.NG ∴revise estimate ofc and repeat Steps 6 through 10 until equilibrium is achieved.
Step 11—Adjust cuntil force equilibrium is satisfied. Steps 6 through 10 were repeated several times with different values of c until equilibrium was achieved.The results of the final iteration are: c=1.86 in.;Em=0.016;fm=f=265.6 ksi;E_6= 0.0113;fe=60.6ksi;e=0.00091; a_1=0.577;and β_1=0.698.	c=1.86 in. ∴the value of c selected for the final iteration is correct.	c=47mm ∴the value of c sclected for the final iteration is correct.
Step 12—Calculate flexural strength components. The design flexural strength is calculated using Eq. (10.3.1.6g).An additional reduction factor, 步=0.85,is applied to the contribution of the FRPsystem. Prestressing steel contribution to bending: FRPcontribution to bending:	M=4440kip-in.=370 kip-ft My=1417 kip-in.=118kip-t	Mm=501.6×10⁶N-mm=501.6kN-m M=160.1×10⁵N-mm=160.1 kN-m

Table 16.5b,cont.—Procedure for flexural strengthening of an interior prestressed concrete beam with fiber-reinforced polymer

Step 13—Calculate design flexural strength of the section. The design ffexural strength is calculated using Eq. (10.1)and(10.3.1.6g).Because E=0.016 >0.015,a strength reduction factor of Φ=0.90 should be used per Eq. (10.2.7).An additional reduction factor y=0.85 is used to calculate the FRP contribution to nominal capacity. 中 M_0= Φ [M + y . M]	ΦMa=0.9[370 kip-ft+0.85(118 kip-f)] ΦM。=423 kip-ftM=397 kip-ft ∴. the strengthened section is capable of sustaining the new required flexural strength.	ΦM=0.9[506.1 kN-m+0.85(160.1kN-m)] ΦM=573kN-maM=538kN-m 2the strengthened section is capable ofsustaining the new required flexural strength.
Step14—Check service condition of the section. Calculate the cracking moment and compare the service moment: $M_{cr} = \frac{f_r I_g}{y_b} + P_e\left(e + \frac{r^2}{y_b}\right)$	f,=7.5 √4000=474 psi=0.474 ksi $M_{cr} = \frac{0.474\times 51,150}{15.61} + 126.2\left(13.1+\frac{7.75^2}{15.61}\right)$ M=3693kip-in.=308 kip-ft >M_3=289 kip-ft 2the strengthened section is uncracked at service,2	f,=0.6 √27.6=3.15N/mm^2 $M_{cr} = \frac{3.15\times 2.13\times 10^{10}}{397} + 563,310\left(333+\frac{197^2}{397}\right)$ M_6=411,654,013 N-mm=412kN-mm >M,=391.3kN-m the strengthened section is uncracked at service.
Step 15—Check stress in prestressing steel at service condition. $\varepsilon_{ps,s} = \varepsilon_{pe} + \frac{P_e}{A_c E_c}\left(1+\frac{e^2}{r^2}\right) + \frac{M_s e}{E_c I_g}$ Calculate the steel stress using Eq.(10.3.1.6d) $f_{ps,s} = \begin{cases} 28,500\varepsilon_{ps,s} & \text{for } \varepsilon_{ps,s} \le 0.0086 \\ 270 - \frac{0.04}{\varepsilon_{ps,s}-0.07} & \text{for } \varepsilon_{ps,s} \le 0.0086 \end{cases}$ Check the service stress limits of Eq.(10.3.1.4a) and(10.3.1.4b): Jms≤0.82/ ms≤0.74/m	ns=0.0063≤0.0086 Jms=28,500(0.0063)=180ksi Jss=180ksi<0.82(230)=189ksi **OK** fms=180ksi<0.74(270)=200 ksi **OK**	Ess=0.0063≤0.0086 fm.=1.96×10^5(0.0063)=1238 N/mm^2 fos=1238 N/mm^2<0.82(1586)=1300 N/mm^2 **OK** fx=1238 Nmm^2<0.74(1860)=1376N/mm^2 **OK**
Step 16—Check stress in concrete at service condition. Calculate the cracking moment and compare to service moment: $\varepsilon_{c,s} = \frac{-P_e}{A_{cg}E_c}\left(1-\frac{y_t e_s}{r^2}\right) - \frac{M_s y_t}{E_c I_g}$ Ls=E& fes≤0.45	$\varepsilon_{c,s} = \frac{-126.2}{852\times 3605}\left(1-\frac{9.39\times 13.1}{7.75^2}\right) - \frac{289\times 12\times 9.39}{3605\times 51,150}$ &s=-0.000134 fea=3,605,000 psi(-0.000134)=-483 psi 0.45f'=0.45(4000)=1800 psi Jes=-483psi<0.45'=1800 psi **OK**	$\varepsilon_{cs} = \frac{-563,310}{5.5\times 10^5\times 24,700}\left(1-\frac{238\times 333}{197^2}\right) - \frac{391.3\times 10^6\times 238}{24,700\times 2.13\times 10^{10}}$ 8=-0.000134 Lx=24,700 N/mm^2 (-0.000134)=-3.31N/mm^2 0.45'=0.45(27.6)=12.42 N/mm^2 La=-3.31 N/mm^2<0.45%'=12.42N/mm^2 **OK**

Table 16.5b,cont.—Procedure for flexural strengthening of an interior prestressed concrete beam with fiber-reinforced polymer

Step 17—Check service stresses in the FRP reinforcement. The stress in the FRPat service condition can be calculated using Eq. (10.3.1.8): $f_{f,s}=\left(\frac{E_f}{E_c}\right)\frac{M_s y_b}{I}-\varepsilon_{bi}E_f$ Because the section is uncracked at servicethe gross moment of inertia of the section must be used. The calculated stress in FRP should be checked against the limits in Table 10.2.9.For carbon FRP: L≤0.55m	$f_{f,s}=\left(\frac{5360}{3605}\right)\frac{289\times12\times15.61\text{ in.}}{51,150}$ +0.00003×5360 fa=1.74 ksi 0.55f=0.55(85)=47ksi f=1.74ksi<0.55fm=47 ksi **OK**	$f_{f,s}=\left(\frac{37,000}{24,700}\right)\frac{391.3\times10^6\times397}{2.13\times10^{10}}$ +0.00003×37,000 fs=12.10 N/mm² 0.55/m=0.55(586)=322 N/mm² f_3=12.10 N/mm²<0.55fm=322 N/mm² **OK**

16.6—Shear strengthening of an interior T-beam

A reinforced concrete T-beam(f.=3000 psi [20.7 N/mm²])located inside of an office building is subjected to an increase in its live-load-carrying requirements.An analysis of the existing beam indicates that the beam is still satisfactory for flexural strength;however,its shear strength is inadequate to carry the increased live load.Based on the analysis, the nominal shear strength provided by the concrete is V_c =

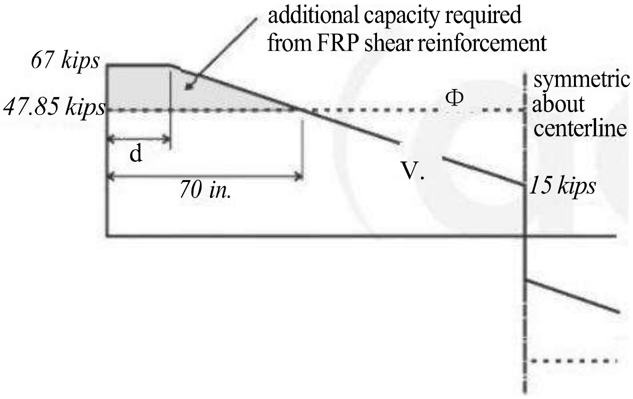


Fig.16.6a—Shear diagram showing demand versus existing strength.The FRP reinforcement should correct the deficiency shown shaded.(Note:1 in.=25.4 mm;1 kip=4.45kN.)

Supplemental FRP shear reinforcement is designed as shown in Fig.16.6b and summarized in Table 16.6a.Each FRP strip consists of three plies(N=3)of a carbon fabric reinforcement installed by wet layup.The FRP system manufacturer's reported material properties are shown in Table 16.6b.

The design calculations used to arrive at this configuration follow in Table 16.6c.

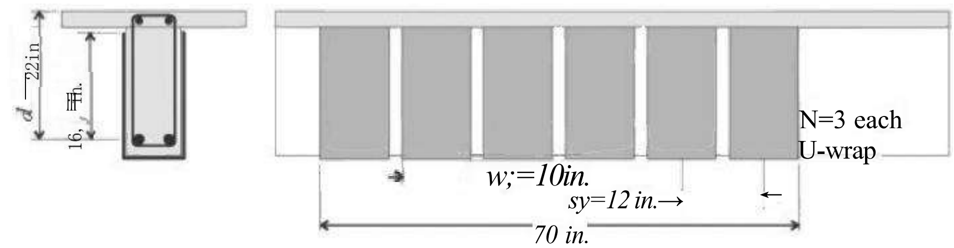


Fig.16.6b—Configuration of the supplemental FRP shear reinforcement without FRPanchors.(Note:1 in.=25.4 mm.)

Table 16.6a—Configuration of the supplemental FRP shear reinforcement

d	22 in.	559 mm
d	16in.	406 mm
Width of each sheet wy;	10 in.	254mm
Span between each sheet s	12 in.	305 mm

Table 16.6b—Manufacturer's reported FRP system properties

Thickness per plyt	0.02 in.	0.51mm
Ultimate tensile strengthf*	179,000 psi	1234N/mm²
Rupture strain em	0.017 in./in.	0.017mm/mm
Modulus of elasticity E,	10,725,000psi	73,946 N/mm²

Table 16.6c—Procedure for shear strengthening of an interior T-beam

Procedure	Calculation in in.-1b units	Calculation in SI units
Step 1—Compute the design material properties. The beam is located in an enclosed and conditioned space and a CFRP system will be used. Therefore, per Table 9.4, an environmental-reduction factor of 0.95 is used. Fm=Cea* m=CeGn	fm=(0.95)(179 ksi)=170ksi En=(0.95)(0.017)=0.016	fu=(0.95)(1.23kN/mm²)=1.17kN/mm² Em=(0.95)(0.017)=0.016
Step 2—Caleulate the effective strain level in the FRP shear reinforcement. The effective strain in FRPU-wraps should be determined using the bond-reduction coefficient k,. This coefficient can be computed using Eq. (11.4.1.2b) through (11.4.2.1e). $L_e = \frac{2500}{(NE_f t_f)^{0.58}}$ $k_1 = \left(\frac{f'_c}{4000}\right)^{2/3}$ $k_2 = \left(\frac{d_{fv} - L_e}{d_{fv}}\right)$ $\kappa_v = \frac{k_1 k_2 L_e}{468\varepsilon_{fu}} \leq 0.75$ The effective strain can then be computed using Eq. (11.4.1.2a) as follows: e=K, En≤0.004	$L_e = \frac{2500}{[(3)(0.02\text{ in.})(10.75\times10^6\text{psi})]^{0.58}} = 1.07\text{ in.}$ $k_1 = \left(\frac{3000\text{ psi}}{4000}\right)^{ii} = 0.825$ $k_2 = \left(\frac{16\text{ in.} - 1.07\text{ in.}}{16\text{ in.}}\right) = 0.933$ $\kappa_v = \frac{(0.825)(0.933)(1.07\text{ in.})}{468(0.016)} = 0.109 \leq 0.75$ Ec=0.109(0.016)=0.00176≤0.004	$L_e = \frac{23,300}{[(3)(0.51\text{ mm})(73.9\times10^3\text{ kN/mm}^2)]^{0.58}} = 27.3\text{ mm}$ $k_1 = \left(\frac{20.7\text{ kN/mm}^2}{27}\right)^{2/3} = 0.837$ $k_2 = \left(\frac{406\text{ mm} - 27.3\text{ mm}}{406\text{ mm}}\right) = 0.933$ $\kappa_v = \frac{(0.837)(0.933)(27.3\text{ mm})}{11,900(0.016)} = 0.112 \leq 0.75$ c=0.112(0.016)=0.00180≤0.004
Step 3—Calculate the contribution of the FRP reinforcement to the shear strength. The area of FRP shcar reinforcement can be computed as: Am=2Nzwy The effective stress in the FRP can be computed from Eq. 11.4d. =EE The shear contribution of the FRPcan be then calculated from Eq. (11.4a): $V_f = \frac{A_{fv} f_{fe}(\sin\alpha + \cos\alpha) d_{fv}}{s_f}$	Ap=2(3)(0.02 in.)(10 in.)=1.2 in.² fe=(0.00176)(10,750 ksi)=18.9ksi $V_f = \frac{(1.2\text{ in.}^2)(18.9\text{ ksi})(1)(16\text{ in.})}{(12\text{ in.})}$ V,=30.2 kip	An=2(3)(0.51mm)(254 mm)=777.2 mm² fe=(0.0018)(73.9kN/mm²)=0.133kN/mm² $V_f = \frac{(777.2\text{ mm}^2)(0.133\text{ kN/mm}^2)(1)(406\text{ mm})}{(304.8\text{ mm})}$ V,=137.7kN

Table 16.6c, cont.—Procedure for shear strengthening of an interior T-beam

Step4—Calculate the shear strength of the section. The design shear strength can be computed from Eq. (11.3b) with w,=0.85 for U-wraps. ΦVa=Φ(Ve+V3+yY)	ΦV,=0.75[44.2+19.6+(0.85)(30.2)] 中Vn=67.1 kip>V=67kip .the strengthened section is capable of sustaining the required shear strength.	ΦV,=0.75[196.6+87.2+(0.85)(137.7] ΦVn=300.3kN>Va=298.2kN .the strengthened section is capable of sustaining the required shear strength.

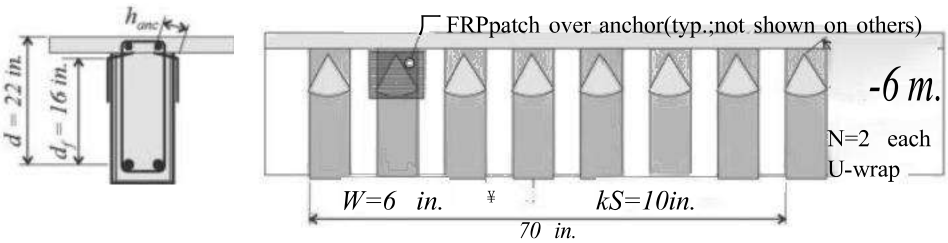


Fig.16.6c—Configuration of the supplemental FRP shear reinforcement using FRPanchors.(Note:I in.=25.4 mm.)

It is possible to reduce the amount of supplemental shear FRP reinforcement by anchoring the U-wraps near the slab underside with fiber anchors.To achieve the desired shear strength,the FRP shear strengthening comprises anchored U-wraps of two plies of 6 in.(152.4 mm)wide FRP at a spacing of 10 in.(254 mm).Table 16.6d shows the shear FRP configuration and Table 16.6e demonstrates the associated calculations.The resulting reinforcement is shown in Fig.16.6c.

Table 16.6d—Configuration of the supplemental FRP shear reinforcement

d	22 in.	559mm
dn	16 in.	406mm
Width of each sheet w;	6 in.	152.4 mm
Span between each sheet s	10 in.	254 mm

Table 16.6e—Calculations for plies of U-wrap with fiber anchors

Procedure	Calculation in in.-Ib units	Calculation in SI units
Step1—Calculate the effective strain level in the anchored FRP shear reinforcement. When FRPanchors are used according to Section 11.4.1.2, the effective strain is computer according to Eq. (11.4.1.1) as follows: E%=0.004≤0.75cn	%=0.004	Be=0.004
Step 2—Calculate the contribution of the FRP reinforcement to the shear strength. The area of FRP shear reinforcement can be computed as: Am=2N, wy The effective stress in the FRP can be computed from Hooke's law. $J_e=e_6E$ The shear contribution of the FRP can be then calculated from Eq. (11.4a): $V_f = \frac{A_{fv} f_{fe} (\sin\alpha + \cos\alpha) d_{fv}}{s_f}$	An=2(2)(0.02 in.)(6 in.)=0.48 in.? fe=(0.004)(10,750ksi)=43ksi $V_f = \frac{(0.48\ in.^2)(43\ ksi)(1)(16\ in.)}{(10\ in.)}$ V_f=33.0 kip	Ap=2(2)(0.51 mm)(152.4 mm)=310.9mm² fe=(0.004)(73.9kN/mm²)=0.296kN/mm² $V_f = \frac{(310.9\ mm^2)(0.296\ kN/mm^2)(1)(406\ mm)}{(254\ mm)}$ V,=147.1 kN
Step 3—Calculate the shear strength of the section. The design shear strength can be computed from Eq. (11.3b) with y=0.85 for U-wraps. $\phi V_o=\phi(Ve+V+yY)$	ΦV,=0.75[44.2+19.6+(0.85)(33.0)] ϕV_o=68.9 kip>V=67kip ∴the strengthened section is capable of sustaining the required shear strength.	ΦV=0.75[196.6+87.2+(0.85)(147.1)] ΦV.=306.6kN>V=298.2kN 2. the strengthened section is capable of sustaining the required shear strength.
Step 4—Fiber anchor design. The size of the fiber anchor is selected from Section 14.1.4. For each supplemental FRPshcar reinforcing U-wrap calculate: NiE Assume that the anchors are installed at β=110 degrees. For discrete U-wraps, w=Sam=6 in. (152.4 mm) Enter Table 14.1.4 and for 4 in.<w;≤6 in. determine the anchor parameters R, ran and h Calculate Amm per Eq. (14.1.4). Aame≥RA(Nisamc) Refer to Section 14.1.4 for anchor detailing	(2)(0.02 in.)(10,750ksi)=430kip/in. RA=1.25 Aam≥(1.25)(2)(0.02 in.)(6 in.) Aance≥0.3 in.² Fane=6 in. ham=max(4 in.,7Aame) hame=max(4 in.,3.83 in.)=4 in.	(2)(0.51mm)(73.9kN/mm²)=75kN/mm RA=1.25 Aam≥(1.25)(2)(0.51mm)(152.4 mm) Aane≥194.3 mm² Fae=150mm he=max(100 mm,7Am) hmc=max(100 mm,97.5 mm)=100mm

16.7—Shear strengthening of an exterior column

A24x24 in.(610x610 mm)square column requires an additional 60 kip(267 kN)of shear strength (△V=60 kip [267kN]).The column is located in an unenclosed parking garage and experiences a wide variation in temperature and climate.A method of strengthening the column using FRP is sought.

Table 16.7a—Manufacturer's reported FRP system properties*

Thickness per plyt	0.051 in.	1.3 mm
Guaranteed ultimate tensile strengthfi*	80,000 psi	552N/mm²
Guaranteed rupture strain en	0.020 in./in.	0.020 mm/mm
Modulus of elasticity E	4,000,000 psi	27,600 N/mm²

The reported properties arelaminate properties.

An E-glass-based FRP complete wrap is selected to retrofit the column.The properties of the FRP system,as reported by the manufacturer,are shown in Table 16.7.The design calculations to arrive at the number of complete wraps required follow.

16.8—Strengthening of a noncircular concrete column for axial load increase

A24x 24 in.(610x610 mm)square column requires an additional20%of axial load-carrying capacity.Concrete and steel reinforcement material properties as well as details of the cross section of the column are shown in Table 16.8a. The column is located in an interior environment,and a CFRP system will be used.A method of strengthening the column is sought.

Table 16.7b—Procedure for shear strengthening of an exterior column

Procedure	Caleulation in in.-1b units	Calculation in SI units
Step1—Compute the design material properties The column is located in an exterior environment and a glass FRP(GFRP)material will be used.Therefore,per Table 9.4,an environmental reduction factor of 0.65 is used. Fm=Caf* m=Cem	m=(0.65)(80ksi)=52ksi u=(0.65(0.020)=0.013	fm=(0.65)(552 N/mm²)=358.5 N/mm² m=(0.65)(0.020)=0.013
Step 2—Calculate the effective strain level in the FRP shear reinforcement. The effective strain in a complete FRP wrap can be determined from Eq. (11.4.1.1): Be=0.004≤0.75em	Ee=0.004≤0.75(0.013)=0.010 Ause an effective strain of e=0.004	Bc=0.004≤0.75(0.013)=0.010 .use an effective strain of e=0.004
Step 3—Determine the area of FRPreinforcement required. The required shear contribution of the FRP reinforcement can be computed based on the increase in strength needed,the strength reduction factor for shear,and a partial-reduction factor v;=0.95 for completely wrapped scctions in shear. $V_{f,reqd} = \frac{\Delta V_n}{\phi(\psi_f)}$ The required area of FRPcan be determined by reorganizing Eq. (11.4a).The required area is left in terms of the spacing. $A_{fv,reqd} = \frac{V_{f,reqd} s_f}{\varepsilon_{fe} E_f (\sin\alpha + \cos\alpha) d_f}$	$V_{f,reqd} = \frac{60 \text{ kip}}{0.85(0.95)} = 74.3 \text{ kip}$ $A_{fv,reqd} = \frac{(74.3 \text{ kip}) s_f}{(0.004)(4000 \text{ ksi})(1)(24 \text{ in.})} = 0.194 s_f$	$V_{f,reqd} = \frac{266.9 \text{ kN}}{0.85(0.95)} = 330.5 \text{ kN}$ $A_{fv,reqd} = \frac{(330.5 \text{ kN}) s_f}{(0.004)(27.6 \text{ kN/mm}^2)(1)(610 \text{ mm})} = 4.91 s_f$
Step4—Determine the number of plies,and strip width and spacing. The number of plies can be determined in terms of the strip width and spacing as follows: $N = \frac{A_{f,reqd}}{2 t_f w_f}$	$N = \frac{0.194 s_f}{2(0.051 \text{ in.}) w_f} = 1.90 \frac{s_f}{w_f}$ ∴use two plies(N=2)continuously along the height of the column(S=w)	$N = \frac{4.91 s_f}{2(1.3 \text{ mm}) w_f} = 1.90 \frac{s_f}{w_f}$ ∴use two plies(N=2)continuously along the height of the column(sy=w).

A carbon-based FRP complete wrap is selected to retrofit the columns.The properties of the FRP system,as reported by the manufacturer,are shown in Table 16.8b.The design calculations to arrive at the number of required complete wraps follow in Table 16.8c.

Table 16.8a—Column cross section details and material properties

f	6.5 ksi	45 MPa
f	60ksi	400 MPa
re	1 in.	25 mm
Bars	12 No.10	12632
Ag	576 $in.^2$	3716 cm^2
Aμ	15.24in.?	98 cm^2
Pg%	2.65	2.65
ΦP.without FRP	2087 kip	9281kN
中Paic)	2504 kip	11,138kN

Note:The column features stel ties for transverse reinforcement.

Table 16.8b—Manufacturer's reported FRP system properties

Thickness per plyt	0.013 in.	0.33 mm
Ultimate tensile strengthf	550 ksi	3792 MPa
Rupture strain gn	0.0167 in./in.	0.0167mm/mm
Modulus of elasticity E_f	33,000 ksi	227,527 MPa

Table 16.8c—Procedure for strengthening of a noncircular concrete column for axial load increase

Procedure	Calculation in in.-1b units	Calculation in SI units
Step 1—Compute the design FRPsystem properties. The column is located in an interior environment and a CFRPsystem will be used.Therefore,per Table 9.4,an environmental reduction factor of 0.95 is used. Fm=Cef m=Ceen	fm=(0.95)(550ksi)=522.5ksi Gu=(0.95)(0.0167)=0.0159 in./in	fm=(0.95)(3792 MPa)=3603 MPa m=(0.95)(0.0167)=0.0159 mm/mm
Step 2—Determine the required maximum compressive strength of confined concretefa'. fa'can be obtained by reordering Eq(12.1b): $f'_{cc}=\frac{1}{0.85(A_g-A_{st})}\left(\frac{\phi P_{n,req}}{0.80\phi}-f_yA_{st}\right)$	$f'_{cc}=\frac{1}{0.85\times(576\text{ in.}^2-15.24\text{ in.}^2)}\times\left(\frac{2504\text{ kip}}{0.80\times0.65}-60\text{ ksi}\times15.24\text{ in.}^2\right)$ fa=8.18ksi	$f'_{cc}=\frac{1}{0.85\times(371{,}612\text{ mm}^2-9832\text{ mm}^2)}\times\left(\frac{11{,}138\text{ kN}}{0.80\times0.65}-414\text{ MPa}\times9832\text{ mm}^2\right)$ $f_{cc}'=56.4$ MPa
Step 3—Determine the maximum confining pressure due to the FRP jacketf. fi can be obtained by reordering Eq. (12.1g): $f_l=\frac{f'_{cc}-f'_c}{3.3\psi_f\kappa_a}$ where $\kappa_a=\frac{A_e}{A_c}\left(\frac{b}{h}\right)^2$ $\frac{A_e}{A_c}=\frac{1-\frac{\left[\left(\frac{b}{h}\right)(h-2r_c)^2+\left(\frac{h}{b}\right)(b-2r_c)^2\right]}{3A_g}-\rho_g}{1-\rho_g}$	$f_l=\frac{8.18\text{ ksi}-6.5\text{ ksi}}{0.95\times3.3\times0.425}=1.26\text{ ksi}$ Ka=0.425$(1)^2$=0.425 $\frac{A_e}{A_c}=\frac{1-\frac{[2\times(1)(24\text{ in.}-2\times1\text{ in.})^2]}{3\times576\text{ in.}^2}-0.0265}{1-0.0265}$ $\frac{A_e}{A_c}=0.425$	$f_l=\frac{56.4\text{ MPa}-44.8\text{ MPa}}{0.95\times3.3\times0.425}=8.7\text{ MPa}$ Ka=0.425$(1)^2$=0.425 $\frac{A_e}{A_c}=\frac{1-\frac{[2\times(1)(610\text{ mm}-2\times25\text{ mm})^2]}{3\times371{,}612\text{ mm}^2}-0.0265}{1-0.0265}$ $\frac{A_e}{A_c}=0.425$