第二讲
重整化群与深度学习

尤亦庄
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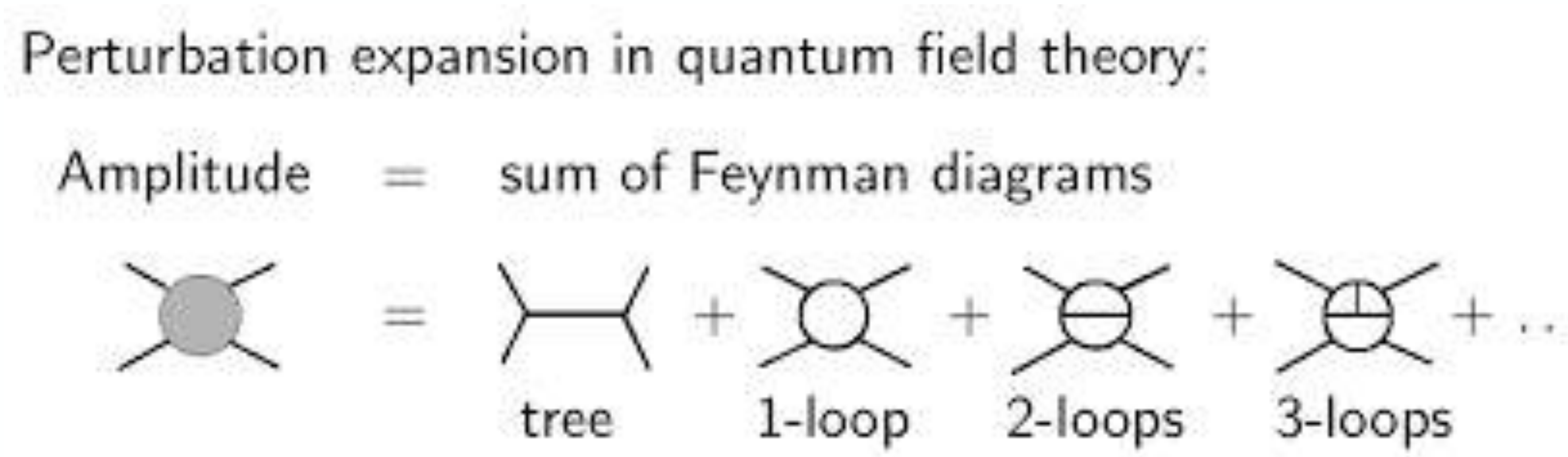
集智因果涌现读书会

2021夏

什么是重整化?

- 早期的黑历史(1950-1970s)
  重整化是试图挽救量子场论圈图发散的奇怪的数学技巧



- 一个(过度简化)的类比:

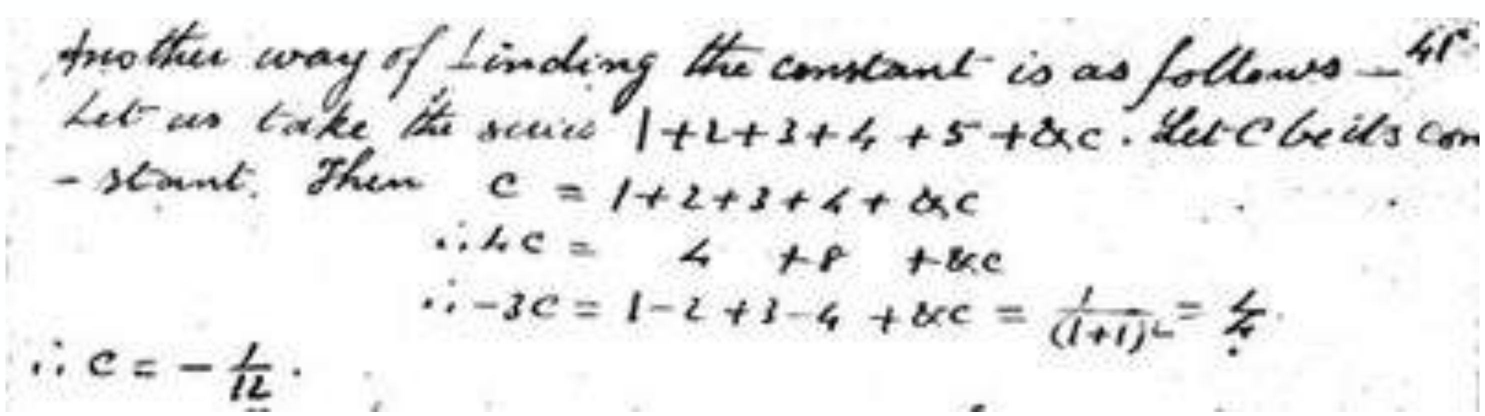
$$1+2+3+4+5+\cdots=-\frac{1}{12}\quad ?!$$

什么是重整化?

$$1+2+3+4+5+\cdots = -\frac{1}{12}$$

- 暴力(错误)的打开方式

$$\begin{array}{rrcccccccccccccc}
 & c & = & 1 & + & 2 & + & 3 & + & 4 & + & 5 & + & 6 & + & \ldots \\
 & 4\,c & = & & & 4 & & & + & 8 & & & + & 12 & + & \ldots \\
-3\,c = & c-4\,c & = & 1 & - & 2 & + & 3 & - & 4 & + & 5 & - & 6 & + & \ldots \\
 & & \overset{x=1}{=} & 1 & - & 2\,x & + & 3\,x^2 & - & 4\,x^3 & + & 5\,x^4 & - & 6\,x^5 & + & \ldots \\
 & & = & & & & & & & (1+x)^{-2} & & & & & & \\
 & & \overset{x=1}{=} & & & & & & & 1/4 & & & & & & \\
\text{so} & c & = & & & & & & & -1/12 & & & & & &
\end{array}$$



S. Ramanujan(1931)

什么是重整化?

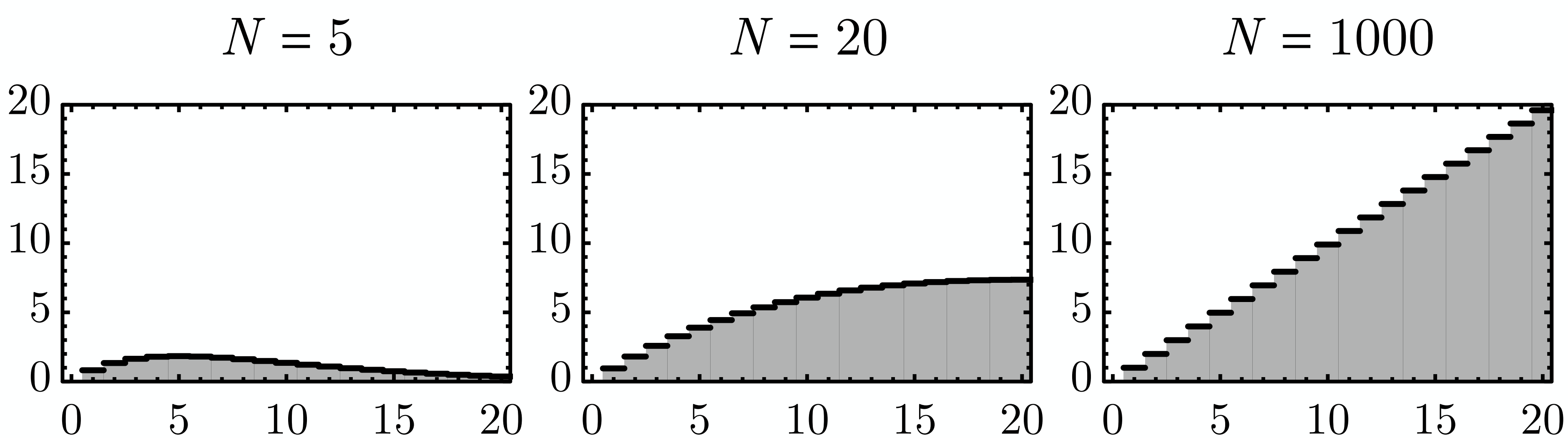
$$1+2+3+4+5+\cdots=-\frac{1}{12}$$

陶哲轩

(2010博客)

- 重整化(正确)的打开方式

引入截断标度 *N*: $$\sum_{n=1}^{\infty} n e^{-n/N} \xrightarrow{N\to\infty} \sum_{n=1}^{\infty} n$$



$$\sum_{n=1}^{\infty} n e^{-n/N} = \frac{1}{2(\cosh\frac{1}{N}+1)} = N^2 - \frac{1}{12} + \frac{1}{240N^2} + \mathcal{O}(\frac{1}{N^4})$$

什么是重整化?

$$1+2+3+4+5+\cdots=-\frac{1}{12}$$

- 重整化(正确)的打开方式

任意的重整化方案

引入截断标度 *N*: $\displaystyle\sum_{n=1}^{\infty} n\eta(n/N) \xrightarrow{N\to\infty} \sum_{n=1}^{\infty} n$

$$\sum_{n=1}^{\infty} ne^{-n/N}=N^2-\frac{1}{12}+\frac{1}{240N^2}+\mathcal{O}(\frac{1}{N^4})$$

$$\sum_{n=1}^{\infty} \frac{n}{((n/N)^2+1)^2}=\frac{N^2}{2}-\frac{1}{12}-\frac{1}{60N^2}+\mathcal{O}(\frac{1}{N^4})$$

$$\sum_{n=1}^{\infty} \frac{n}{(n/N)^4+1}=\frac{\pi N^2}{4}-\frac{1}{12}+\mathcal{O}(\frac{1}{N^4})$$

与重整化方案无关的普适常数

陶哲轩
(2010博客)

什么是重整化?

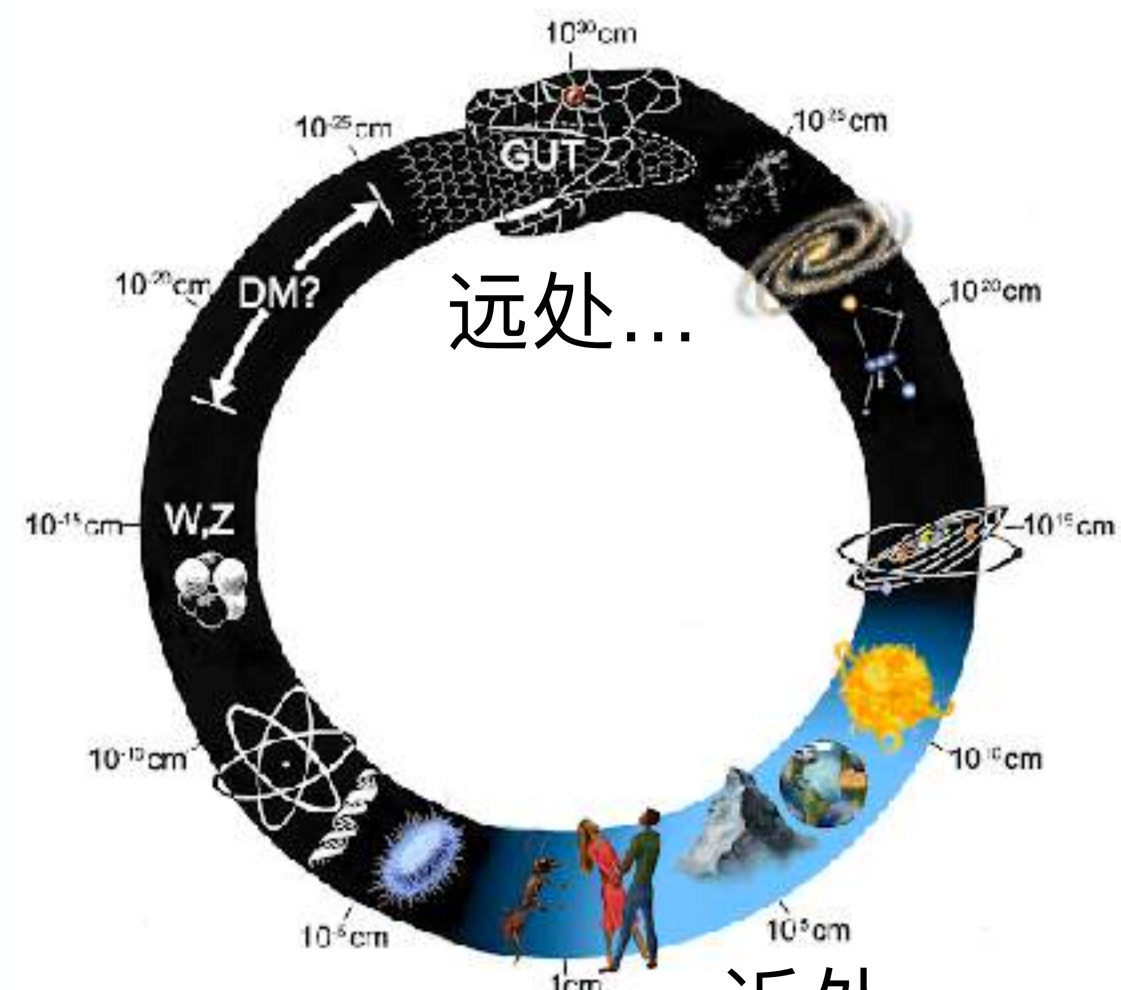
- 为什么需要重整化? - 我们认识世界的能力是有限的
  - 太远的地方我们看不到

$$1+2+3+4+5+\cdots\cdots+1000+1001+\cdots$$

近处　　　　远处…

  - 太小的结构我们看不清
  - 太大的结构我们看不全
- 如何以有限的信息处理能力，试图理解无限信息量的世界?
- 策略：躺平放弃
  - 截断信息
  - 改变标度

} 重整化

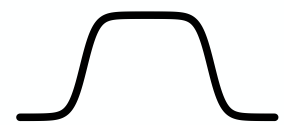


近处

什么是重整化?

- 现代化的理解(1970s-)
  - Wilson: 动量空间重整化
    - 截断高动量模式

$$\phi(k) \to \phi(k)\eta(k/\Lambda)$$



重整化方案 η
(卷积核)

  - Kadanoff: 实空间重整化
    - 粗粒化细节结构

$$\tilde{\phi}(r) \to (\tilde{\eta} \star \tilde{\phi})(r) = \int_{r'} \tilde{\phi}(r - r')\tilde{\eta}(r'/R)$$

  - 两者等价性：傅立叶变换下
    - 大动量结构 = 小尺度结构
    - 动量空间乘积 = 实空间卷积 （脑洞：卷积神经网络…）

K.G. Wilson

L.P. Kadanoff

什么是重整化?

- 重整化方法的未来(2014-)：机器学习重整化

（以机器而不是人来设计重整化方案）

arXiv:1410.3831 (stat)

[Submitted on 14 Oct 2014]

An exact mapping between the Variational Renormalization Group and Deep Learning

Pankaj Mehta, David J. Schwab

arXiv:1704.06279 (cond-mat)

[Submitted on 20 Apr 2017 (v1), last revised 24 Sep 2018 (this version, v2)]

Mutual Information, Neural Networks and the Renormalization Group

Maciej Koch-Janusz, Zohar Ringel

arXiv:1802.02840 (cond-mat)

[Submitted on 8 Feb 2018 (v1), last revised 19 Dec 2018 (this version, v4)]

Neural Network Renormalization Group

Shuo-Hui Li, Lei Wang

arXiv:1903.00804 (cond-mat)

[Submitted on 3 Mar 2019 (v1), last revised 29 Aug 2019 (this version, v3)]

Machine Learning Holographic Mapping by Neural Network Renormalization Group

Hong-Ye Hu, Shuo-Hui Li, Lei Wang, Yi-Zhuang You

arXiv:2010.00029 (cs)

[Submitted on 30 Sep 2020 (v1), last revised 18 Dec 2020 (this version, v4)]

RG-Flow: A hierarchical and explainable flow model based on renormalization group and sparse prior

Hong-Ye Hu, Dian Wu, Yi-Zhuang You, Bruno Olshausen, Yubei Chen

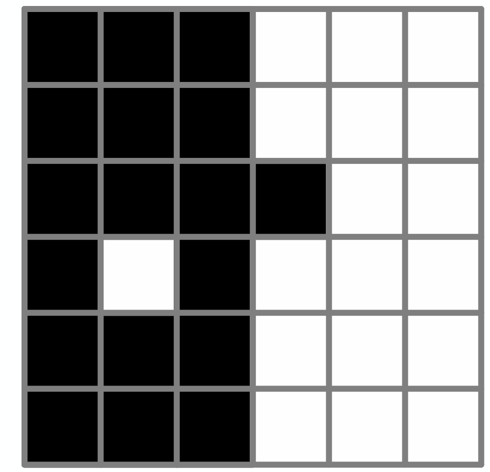
Ising模型重整化

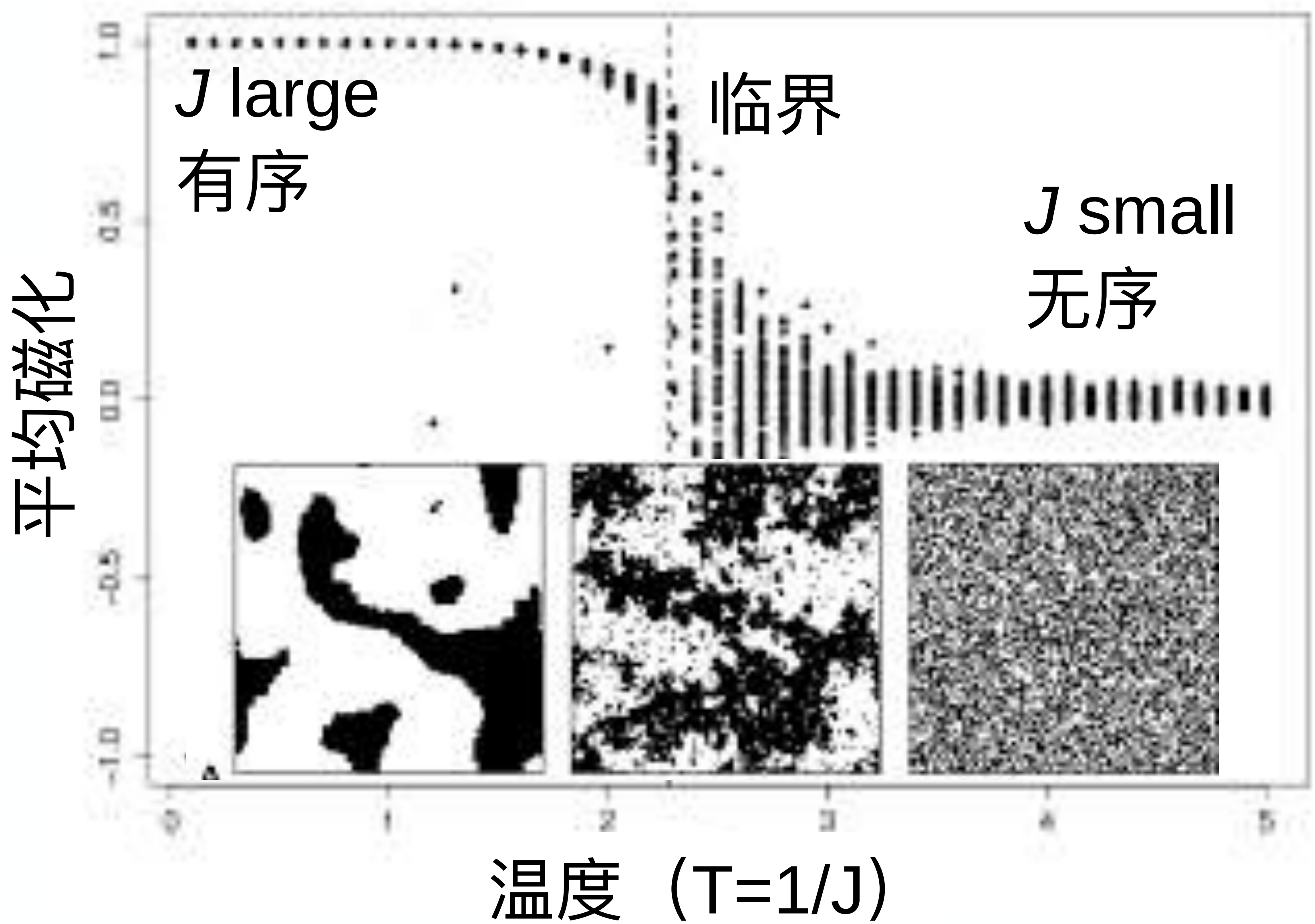
- 2D Ising模型

$$H(\boldsymbol{\sigma}) = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j$$

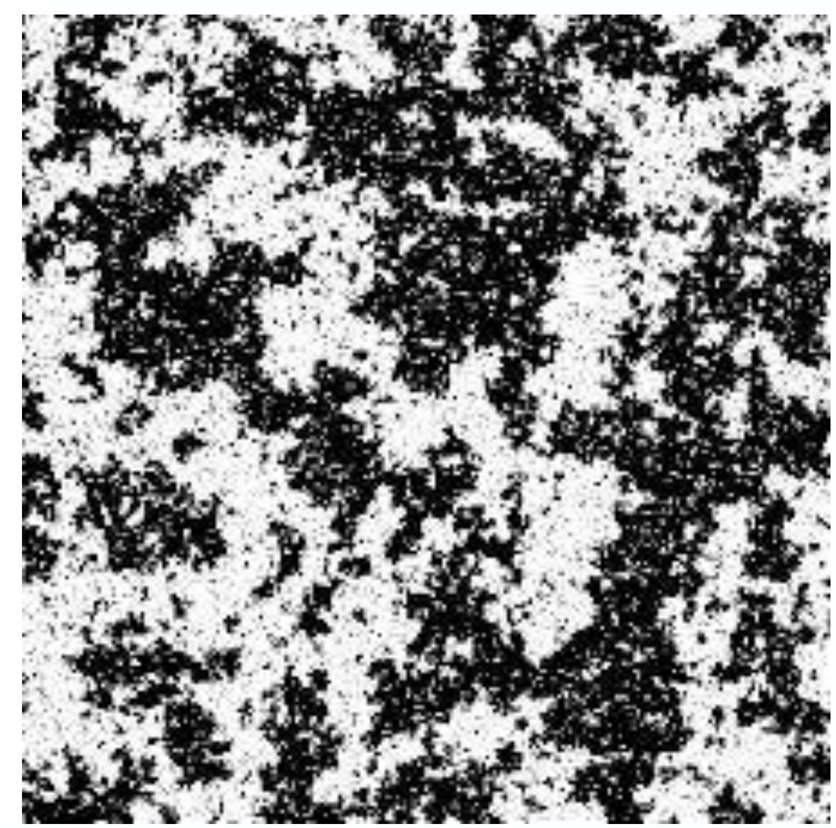
能量函数

Ising自旋
$\sigma_i = \pm 1$





临界：分形，无标度

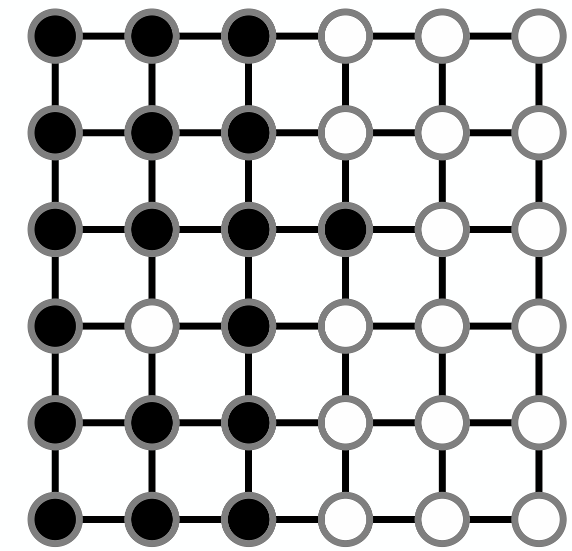


Ising模型重整化

- 概率模型 (Boltzmann概率)

$$\underset{\text{概率分布}}{P(\boldsymbol{\sigma})} = \frac{1}{Z} e^{-H(\boldsymbol{\sigma})} \qquad \underset{\text{配分函数}}{Z} = \sum_{\boldsymbol{\sigma}} e^{-H(\boldsymbol{\sigma})}$$

$$= \frac{1}{Z} e^{J\sum_{\langle ij\rangle}\sigma_i\sigma_j} = \frac{1}{Z}\prod_{\langle ij\rangle} e^{J\sigma_i\sigma_j}$$



- 实空间重整化
  - 第一步：块划分 (2x2 角共享块)

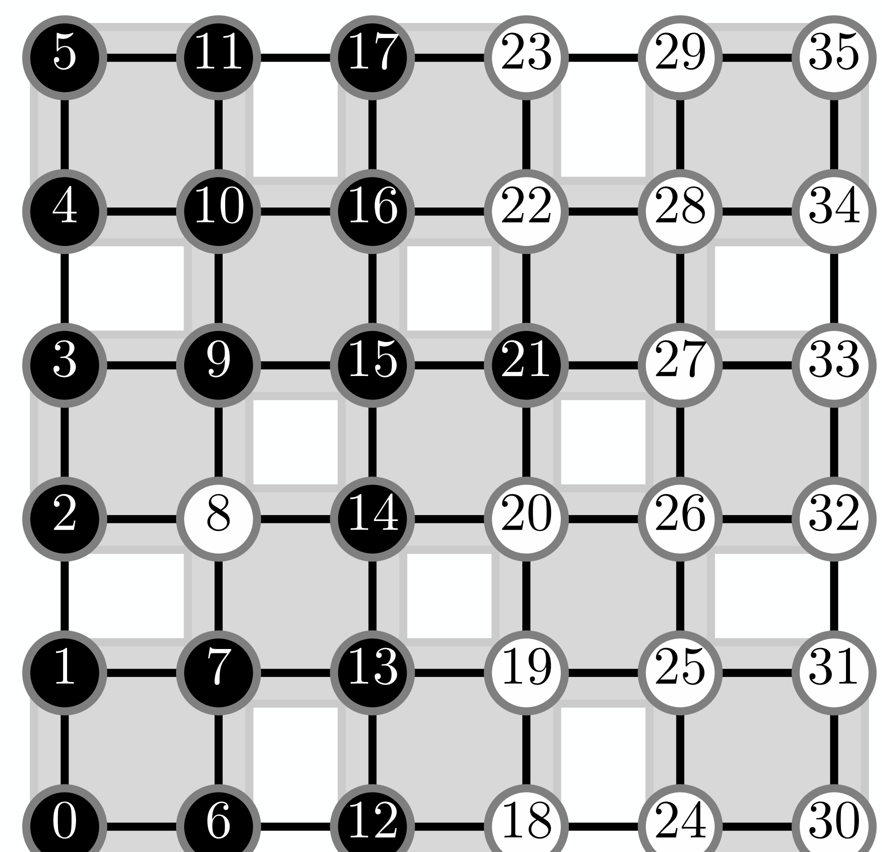
$$P(\boldsymbol{\sigma}) \propto P(\sigma_0,\sigma_1,\sigma_6,\sigma_7)P(\sigma_2,\sigma_3,\sigma_8,\sigma_9)\cdots$$

$$P(\sigma_7,\sigma_8,\sigma_{13},\sigma_{14})\cdots$$ 概率：乘法原理

$$H(\boldsymbol{\sigma}) = H(\sigma_0,\sigma_1,\sigma_6,\sigma_7) + H(\sigma_2,\sigma_3,\sigma_8,\sigma_9)\cdots$$

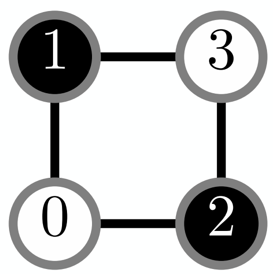
$$+ H(\sigma_7,\sigma_8,\sigma_{13},\sigma_{14})\cdots$$ 能量：加法原理

$$-J(\sigma_0\sigma_1 + \sigma_6\sigma_7 + \sigma_0\sigma_6 + \sigma_1\sigma_7)$$



Ising模型重整化

- 实空间重整化
  - 第二步：隐变量涌现 (表象学习)

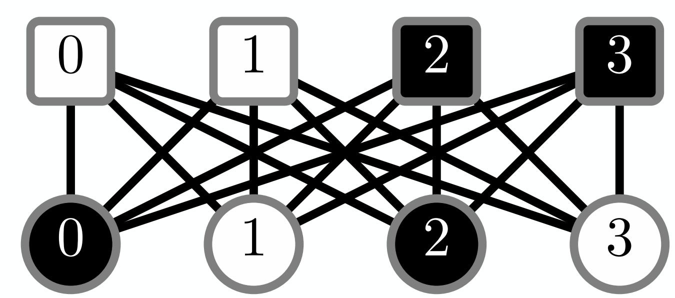


$$H_{\mathrm{loc}}(\boldsymbol{\sigma}) = -J(\sigma_0\sigma_1 + \sigma_2\sigma_3 + \sigma_0\sigma_2 + \sigma_1\sigma_3)$$

$$P_{\mathrm{loc}}(\boldsymbol{\sigma}) = Z_{\mathrm{loc}}^{-1} e^{-H_{\mathrm{loc}}(\boldsymbol{\sigma})}$$

    - 这里可以选择任何具有表象学习能力的生成型模型
    - Restricted Boltzmann Machine (RBM)

隐变量 τ



显变量 σ

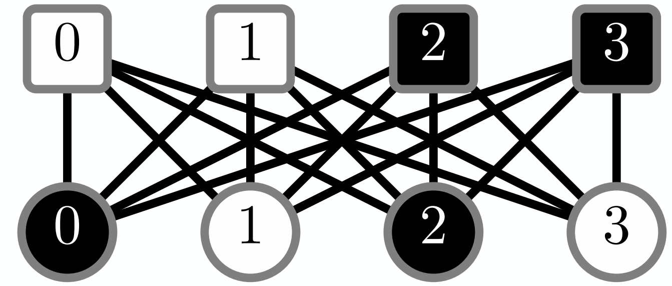
$$H_{\mathrm{RBM}}(\boldsymbol{\sigma}, \boldsymbol{\tau}) = -\sum_{i,j} w_{ij}\sigma_i\tau_j$$

$$P_{\mathrm{RBM}}(\boldsymbol{\sigma}) = Z_{\mathrm{RBM}}^{-1} \sum_{\boldsymbol{\tau}} e^{-H_{\mathrm{RBM}}(\boldsymbol{\sigma},\boldsymbol{\tau})}$$

      - 优化目标：最小化 $D_{\mathrm{KL}}(P_{\mathrm{loc}}||P_{\mathrm{RBM}})$
      - 优化变量：RBM权重 w_{ij}

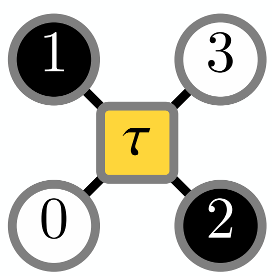
Ising模型重整化

- 实空间重整化
  - 第二步：隐变量涌现 (表象学习)
    - 通过训练RBM，发现最优解为 $\forall i,j: w_{ij} = w(J)$
    - 所有隐变量都学到了一样的特征 → 只用一个隐变量就够

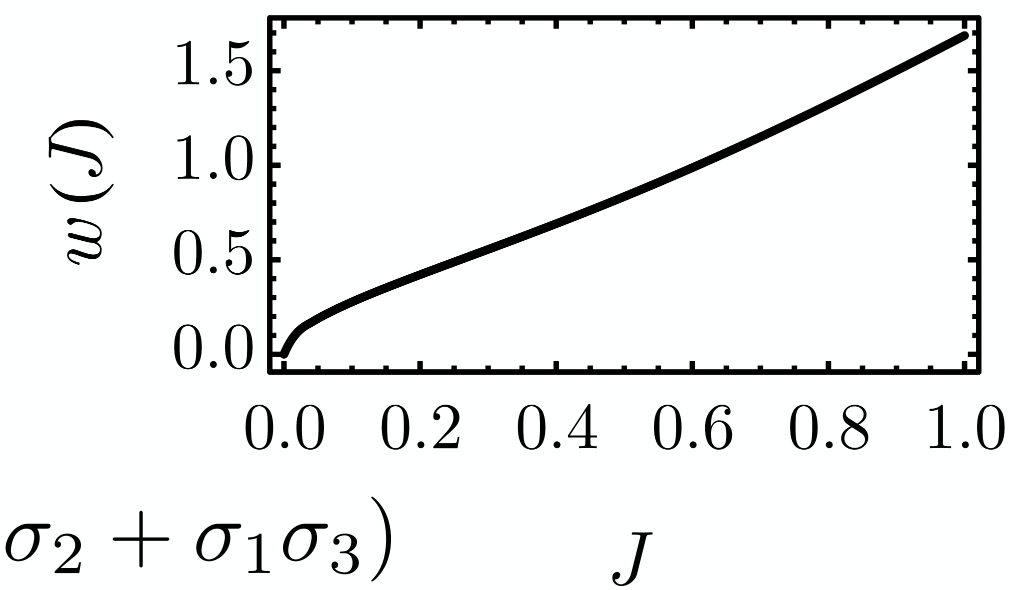


$$H_{\mathrm{RBM}}(\boldsymbol{\sigma}, \tau) = -w(J)\sum_i \sigma_i \tau$$

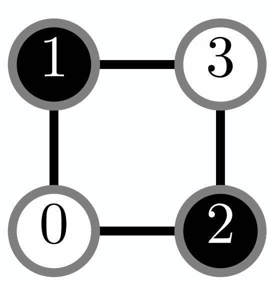
近似



隐变量通过机器学习涌现



$$H_{\mathrm{loc}}(\boldsymbol{\sigma}) = -J(\sigma_0\sigma_1 + \sigma_2\sigma_3 + \sigma_0\sigma_2 + \sigma_1\sigma_3)$$



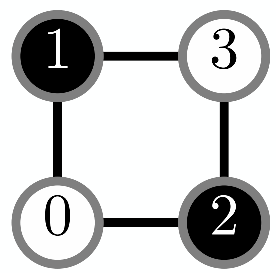
引入隐变量的目的在于改变因果结构

Ising模型重整化

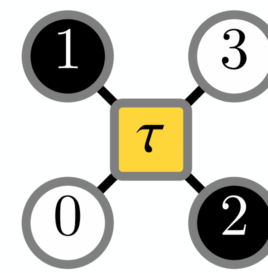
- 实空间重整化
  - 第二步：隐变量涌现 (表象学习)
    - 因果涌现

$$P_{\mathrm{loc}}(\boldsymbol{\sigma}) \propto e^{-H_{\mathrm{loc}}(\boldsymbol{\sigma})} \qquad P_{\mathrm{RBM}}(\boldsymbol{\sigma}, \tau) \propto e^{-H_{\mathrm{RBM}}(\boldsymbol{\sigma},\tau)}$$

概率模型



(近似)等价



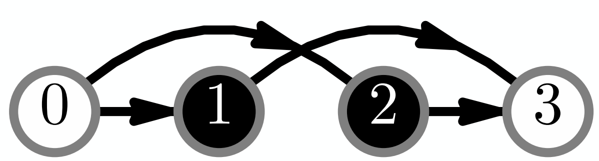
隐变量的涌现

$$P_{\mathrm{loc}}(\boldsymbol{\sigma}) = P(\sigma_0)P(\sigma_1|\sigma_0)P(\sigma_2|\sigma_0) P(\sigma_3|\sigma_1,\sigma_2)$$

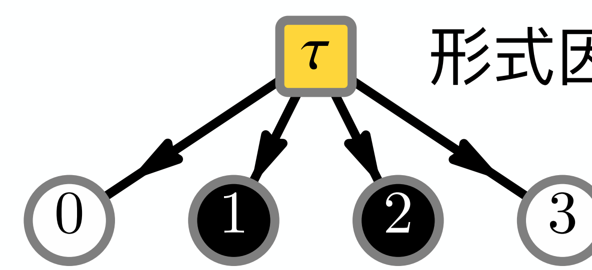
$$P_{\mathrm{RBM}}(\boldsymbol{\sigma},\tau) = P(\tau)P(\sigma_0|\tau)P(\sigma_1|\tau) P(\sigma_2|\tau)P(\sigma_3|\tau)$$

形式因的涌现

因果模型



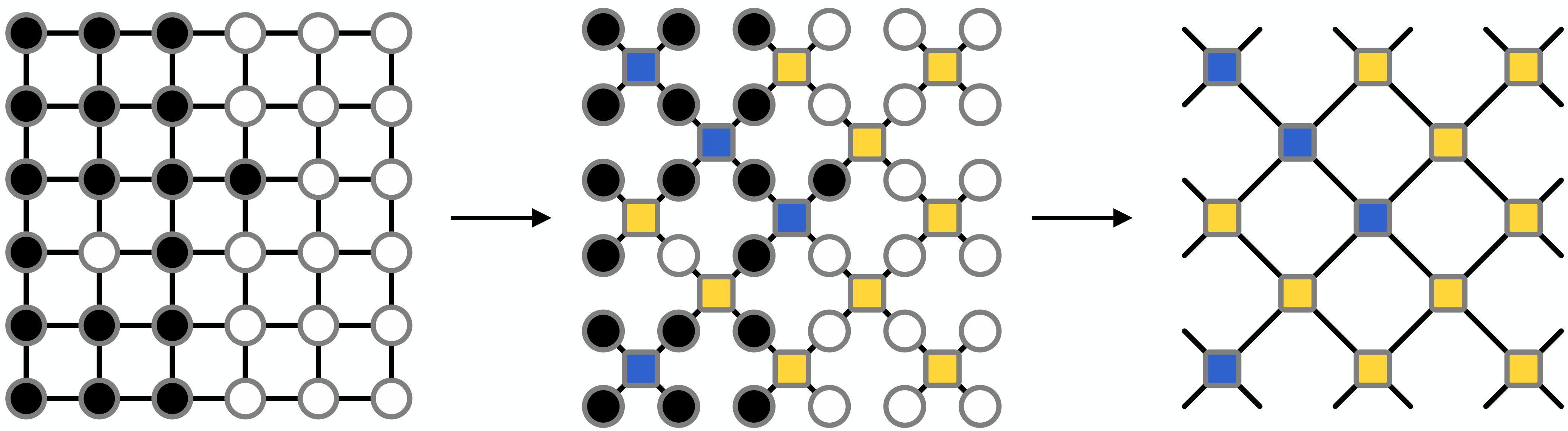
不同的因果结构



$$\forall i,j : (\sigma_i \perp\!\!\!\perp \sigma_j|\tau)_G$$

Ising模型重整化

- 实空间重整化
  - 第三步：显变量边缘化（获取有效模型）



$$P(\boldsymbol{\sigma}) \qquad P(\boldsymbol{\sigma},\boldsymbol{\tau}) \xrightarrow{\text{显变量边缘化}} P(\boldsymbol{\tau}) = \sum_{\boldsymbol{\sigma}} P(\boldsymbol{\sigma},\boldsymbol{\tau})$$

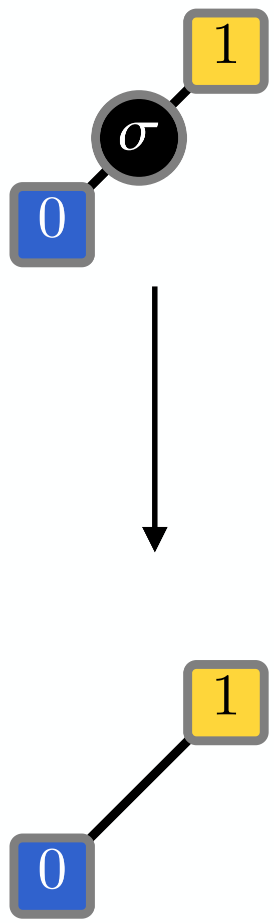
块划分 ↓　　生成 ↑　　↓ 提取有效模型

$$H(\boldsymbol{\sigma}) = \sum_{\text{blocks}} H_{\text{loc}}(\boldsymbol{\sigma}) \qquad H(\boldsymbol{\sigma},\boldsymbol{\tau}) = \sum_{\text{blocks}} H_{\text{RBM}}(\boldsymbol{\sigma},\boldsymbol{\tau}) \qquad H(\boldsymbol{\tau})$$

隐变量涌现(无监督机器学习)

Ising模型重整化

- 实空间重整化
  - 第三步：显变量边缘化（获取有效模型）



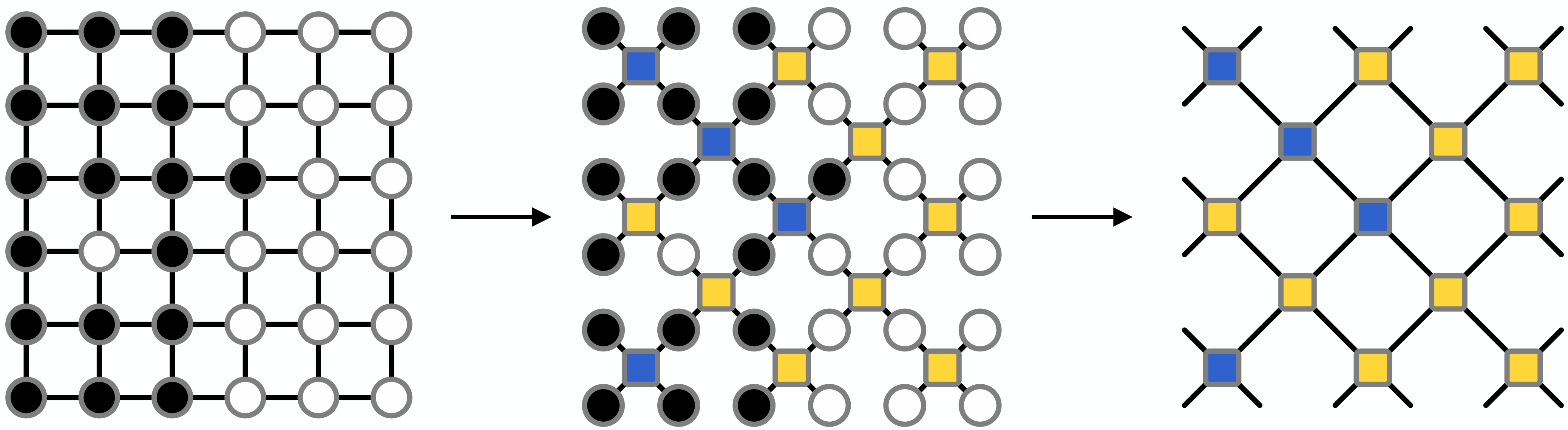
$$H(\sigma,\tau_1,\tau_2) = -w(J)(\sigma\tau_1 + \sigma\tau_2)$$

$$P(\tau_1,\tau_2) = \frac{1}{Z}\sum_{\sigma} e^{-H(\sigma,\tau_1,\tau_2)}$$
$$\propto \cosh(w(J)(\tau_1+\tau_2))$$

$$\begin{aligned}H(\tau_1,\tau_2) &= -\ln P(\tau_1,\tau_2)\\ &= -\ln\cosh\left(w(J)(\tau_1+\tau_2)\right)\\ &= -\frac{1}{2}\ln\cosh(2w(J))\tau_1\tau_2 + \text{const.}\\ &= -J'\tau_1\tau_2 + \text{const.}\end{aligned}$$

Ising模型重整化

- 实空间重整化



$$H(\boldsymbol{\sigma}) = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j \xrightarrow[\text{等效近似}]{} H(\boldsymbol{\tau}) = -J'\sum_{\langle ij\rangle}\tau_i\tau_j$$

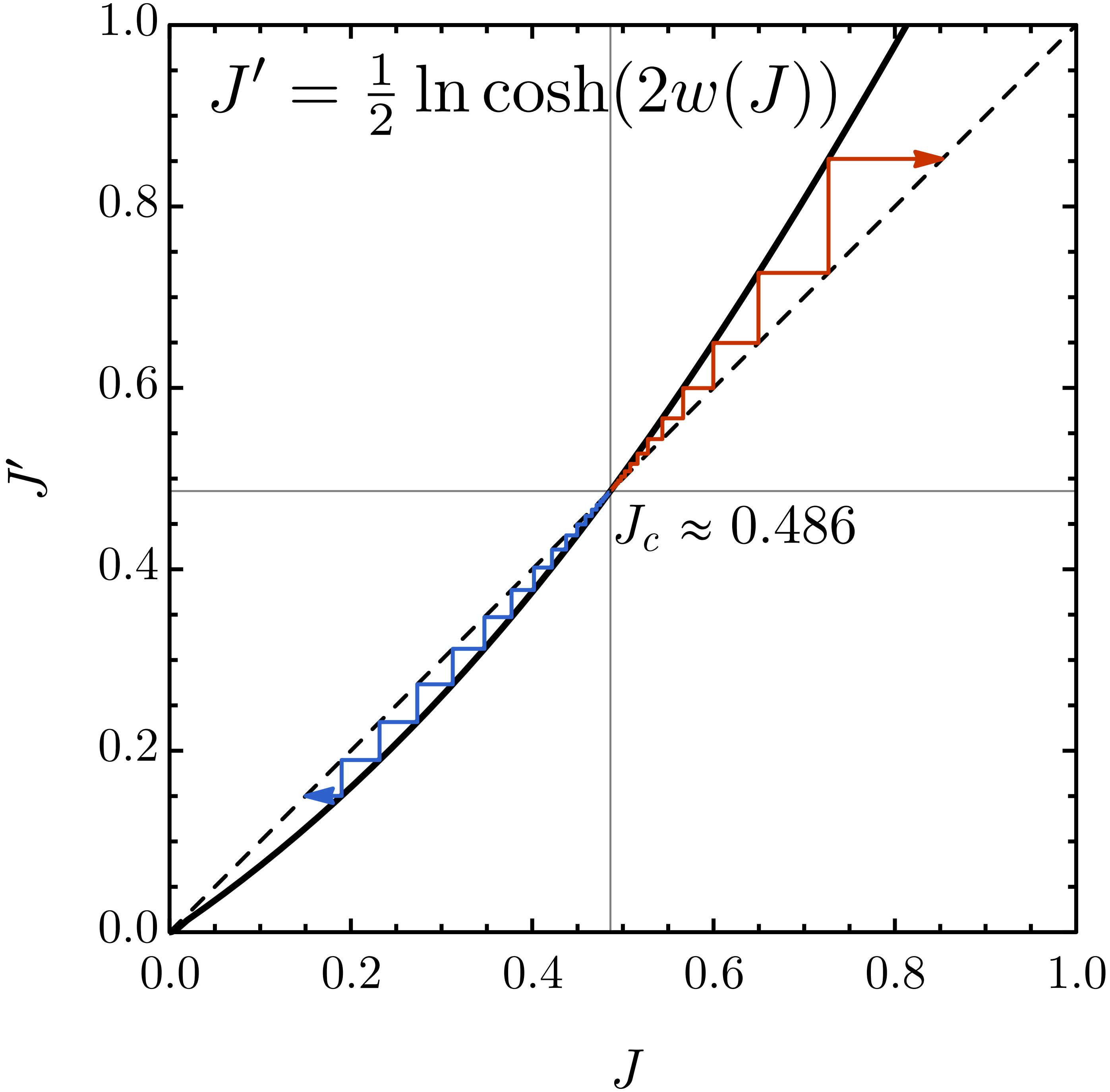
$$J' = \frac{1}{2}\ln\cosh(2w(J))$$ (重整化方程)

方程的一部分可以依靠机器学习
而不需要人来推导一切

- 格点间距扩大 $\sqrt{2}$ 倍
- 耦合常数变化为 $J \to J'$

Ising模型重整化

- 重整化流 - 模型参数随重整化迭代而跑动



- 重整化不动点

$J = 0$ (无序 - 稳定)

$J \approx 0.486$ (临界 - 非稳)

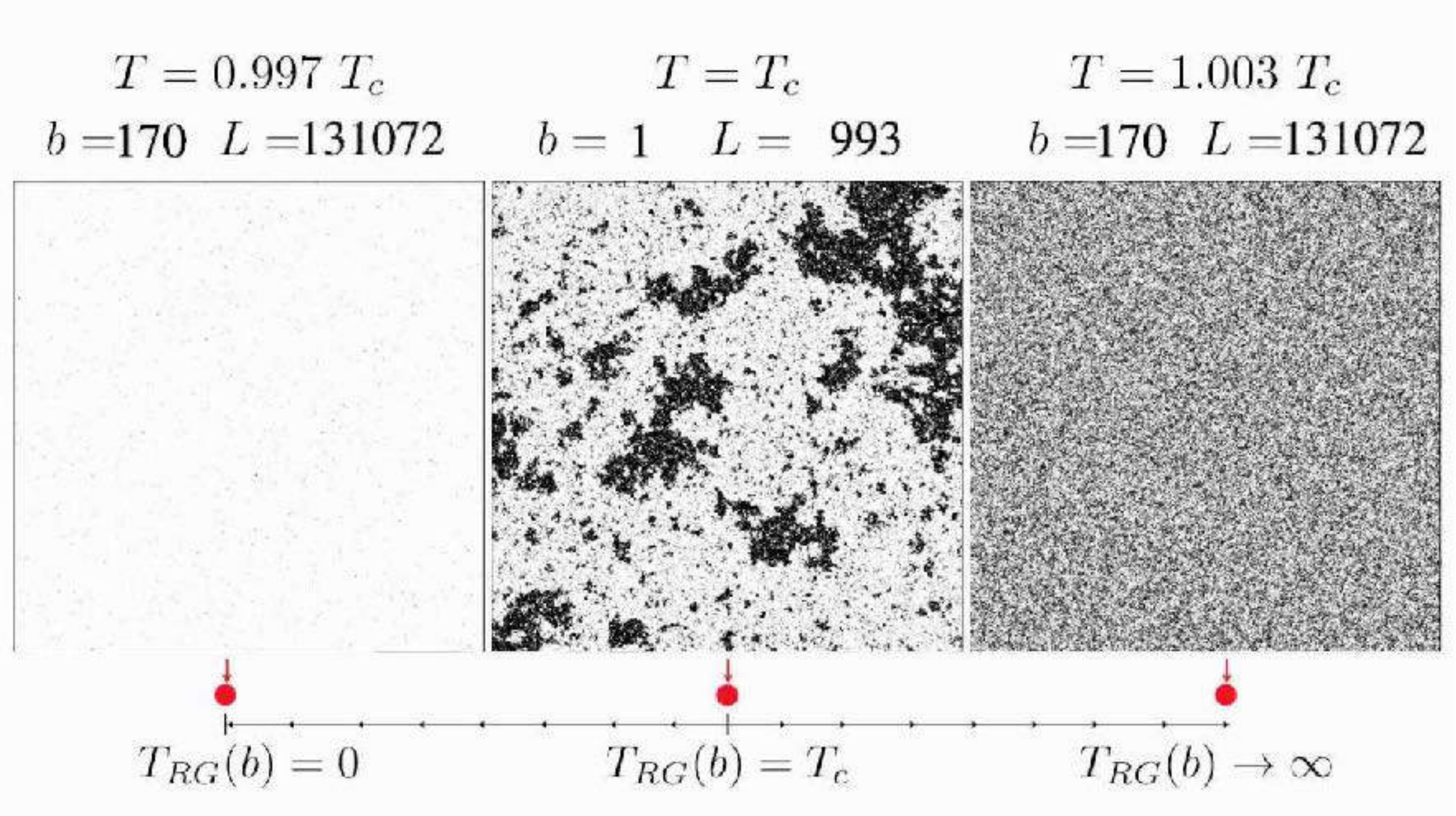
$J = \infty$ (有序 - 稳定)

2D Ising模型严格解

$$J_c = \frac{1}{2}\ln(1+\sqrt{2}) \approx 0.4407$$

Ising模型重整化

- 重整化不动点 - 系统在尺度变换下保持不变 (自相似，无标度)



Douglas Ashton
YouTube

Ising模型重整化

- 标度律与临界指数
  - 在不动点附近线性化重整化方程

$$J' = \mathcal{R}(J) \xrightarrow{J', J \to J_c} (J' - J_c) = \partial_J \mathcal{R}(J_c)(J - J_c) + \cdots$$

$$= \frac{1}{2} \ln \cosh(2w(J))$$

  - 经过 *n* 步迭代

$$(J - J_c) = (\partial_J \mathcal{R}(J_c))^n (J_0 - J_c)$$

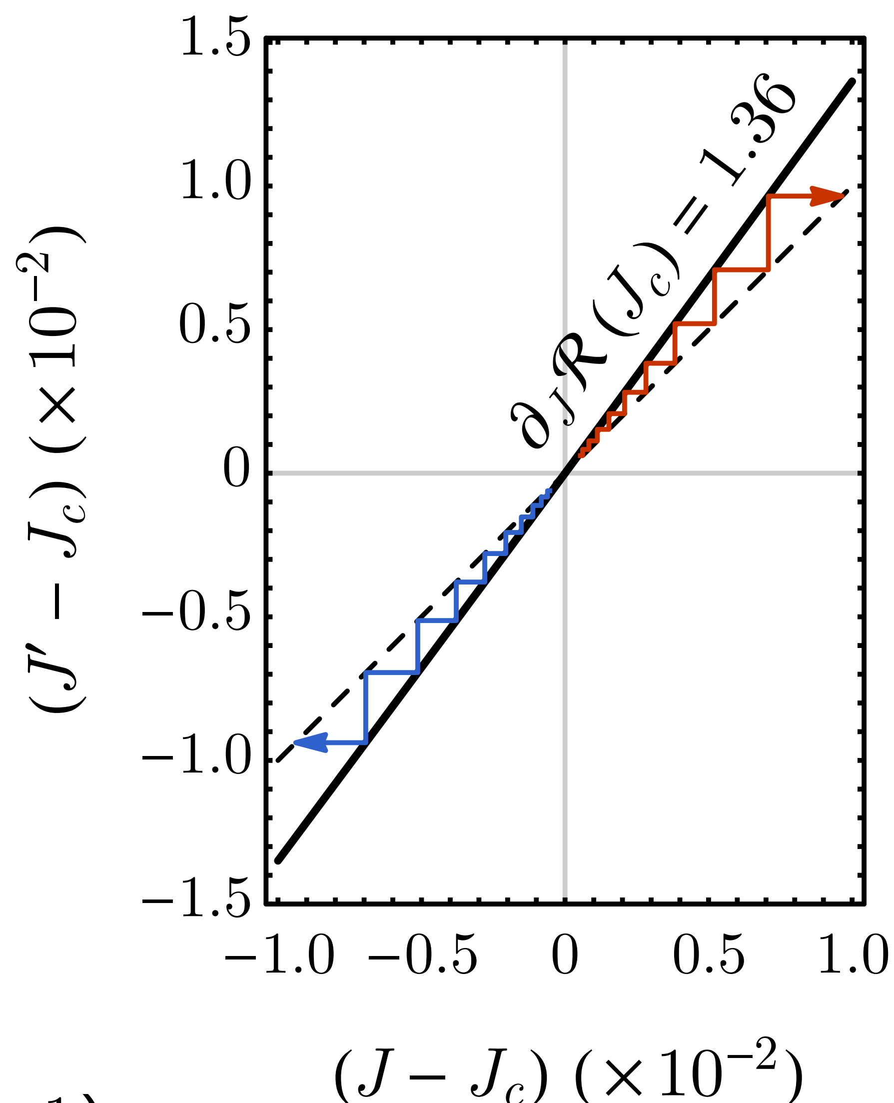
$$\xi = \xi_0 / (\sqrt{2})^n$$ (关联长度 - 磁畴之特征尺度)

  - 联立方程消去 *n*

标度律 $\xi \propto (J - J_c)^{-\nu}$

临界指数 $\nu = \dfrac{\ln \sqrt{2}}{\ln \partial_J \mathcal{R}(J_c)} \approx 1.1$

(严格解 $\nu = 1$)



重整化与生成型模型

- 重整化的一般思路
  - 第一步：显变量划分

    $$X = X_1 \cup X_2 \cup X_3 \cdots$$

    $$P(X) \propto \prod_k P(X_k)$$ (Not disjoint)

  - 第二步：隐变量涌现

    $$P(X_k) \to P'(X_k) = \sum_{Y_k} P'(X_k, Y_k)$$

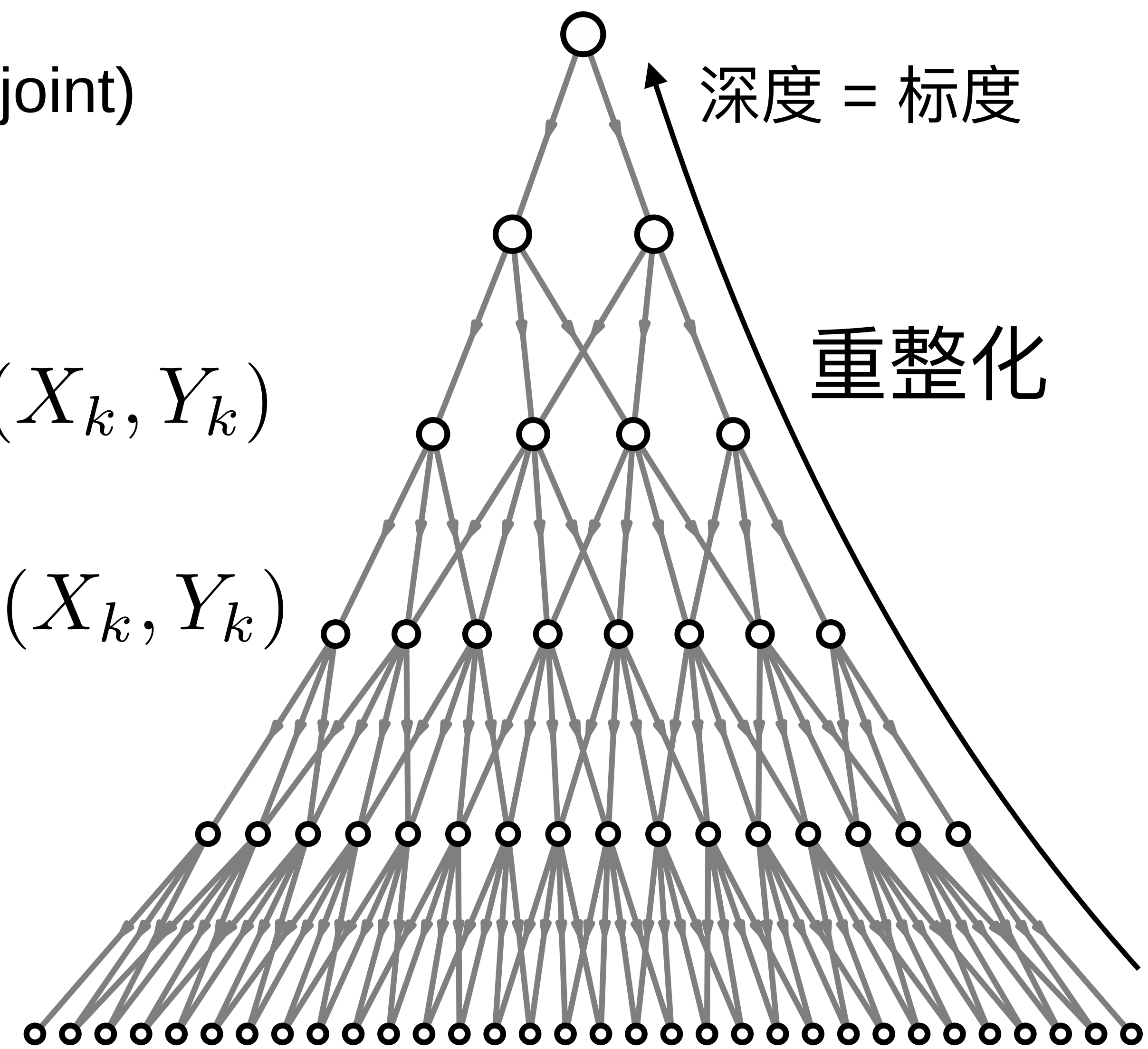
    $$P(X) \to P'(X, Y) \propto \prod_k P'(X_k, Y_k)$$

  - 第三步：显变量边缘化

    $$P'(Y) = \sum_X P'(X, Y)$$

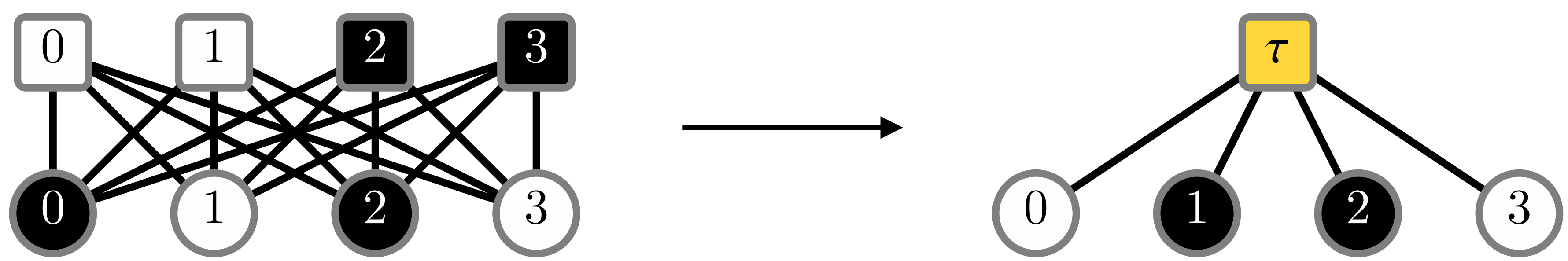
深度生成型模型

(Deep Generative Model)



重整化与生成型模型

- 在Ising模型例子中，表象学习只发现了一个特征（即铁磁模式），这使得隐变量得以继续保持单个Ising变量的形式



- 然而，对于更加一般的系统，未必能够在重整化过程中维持体系的自由度与模型的形式不变（事实上很多复杂系统的集体行为与个体行为的描述是完全不同的）
- 这就要求我们放弃重整化必须保模型形式简洁不变的幻想
- 一般意义的重整化需要依赖不断迭代的机器学习，在不同尺度上发掘新的特征变量并构建新的概率模型

重整化方案的设计原则

- 机器应该如何自动设计重整化方案？重整化的目标是什么？
- 基于信息论的原则
  - 最大环境互信息原则（传统重整化）

arXiv:1704.06279 (cond-mat)

[Submitted on 20 Apr 2017 (v1), last revised 24 Sep 2018 (this version, v2)]

Mutual Information, Neural Networks and the Renormalization Group

Maciej Koch-Janusz, Zohar Ringel

  - 最小全息互信息原则（全息重整化）

arXiv:1903.00804 (cond-mat)

[Submitted on 3 Mar 2019 (v1), last revised 29 Aug 2019 (this version, v3)]

Machine Learning Holographic Mapping by Neural Network Renormalization Group

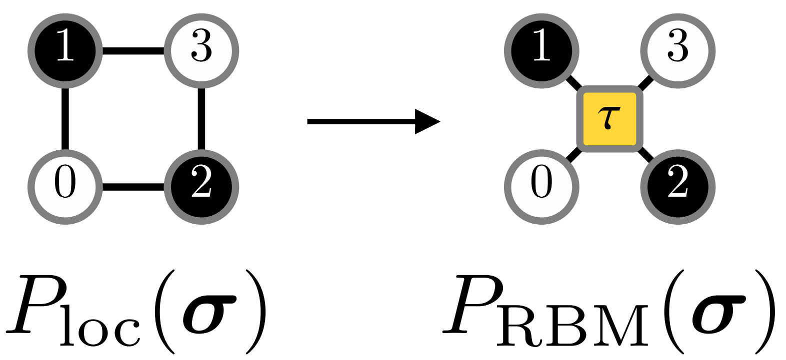
Hong-Ye Hu, Shuo-Hui Li, Lei Wang, Yi-Zhuang You

- …基于因果科学的原则？？？

最大环境互信息原则

- 通过设计目标函数来指导机器发掘相关(relevant)的隐变量
  - 局部目标：最小化KL散度

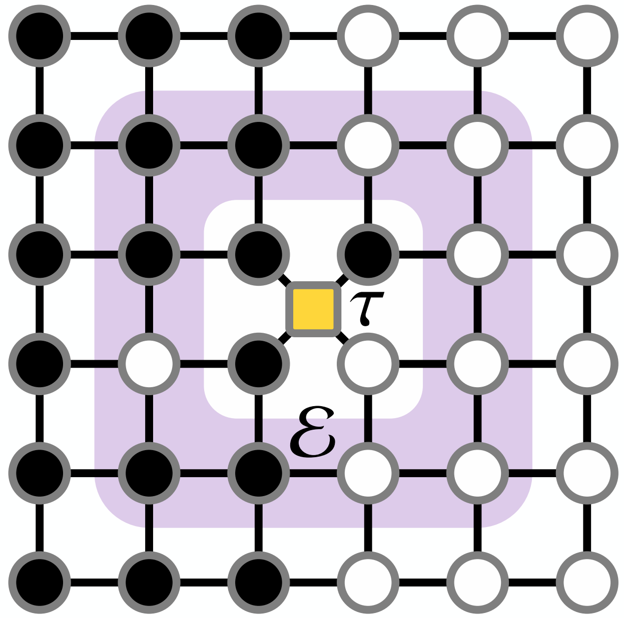
$$D_{\mathrm{KL}}(P_{\mathrm{loc}}||P_{\mathrm{RBM}})$$



  - 环境目标：最大化隐变量与环境显变量的互信息

$$I(\boldsymbol{\sigma}_{\mathcal{E}}:\boldsymbol{\tau})=\sum_{\boldsymbol{\tau},\boldsymbol{\sigma}_{\mathcal{E}}}P(\boldsymbol{\sigma}_{\mathcal{E}},\boldsymbol{\tau})\ln\frac{P(\boldsymbol{\sigma}_{\mathcal{E}},\boldsymbol{\tau})}{P(\boldsymbol{\tau})P(\boldsymbol{\sigma}_{\mathcal{E}})}$$

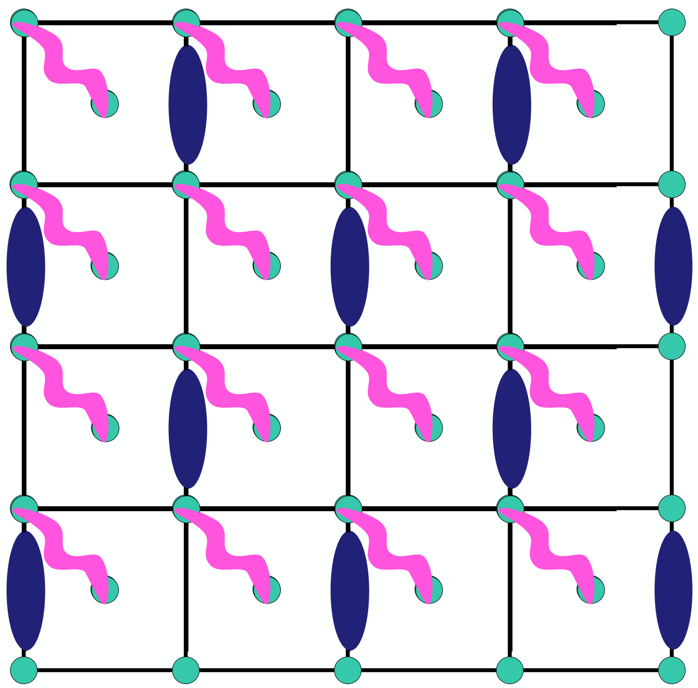
$$P(\boldsymbol{\sigma},\boldsymbol{\tau})\propto P_{\mathrm{RBM}}(\boldsymbol{\sigma}_0,\boldsymbol{\tau})\prod_{k\neq 0}P_{\mathrm{loc}}(\boldsymbol{\sigma}_k)$$

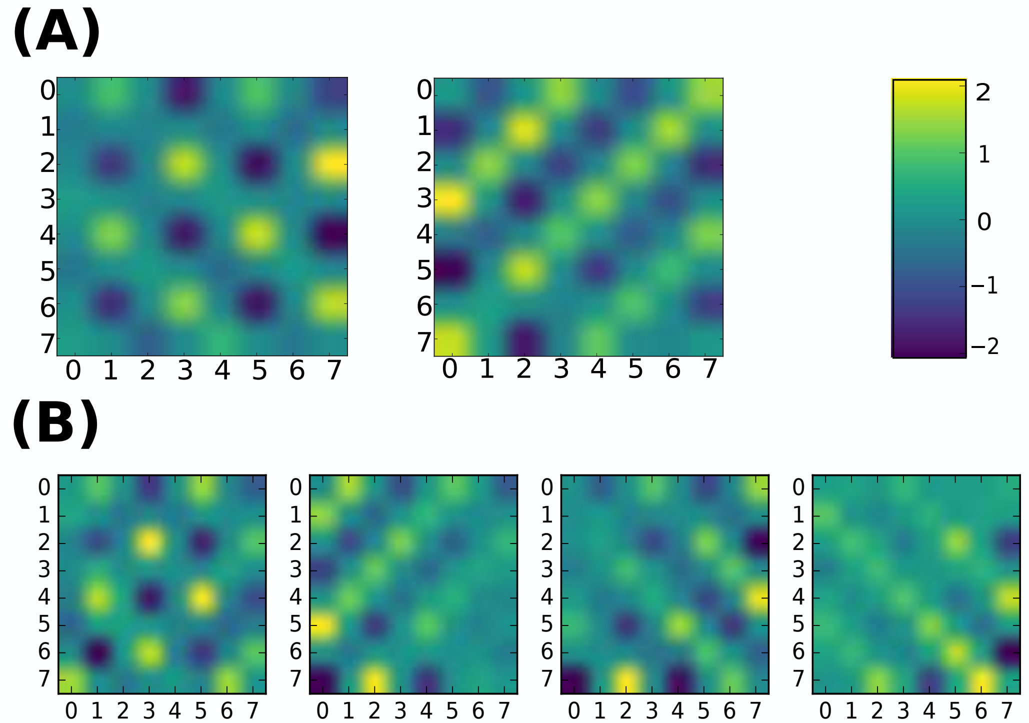


  - 总体目标：$\mathcal{L}=D_{\mathrm{KL}}(P_{\mathrm{loc}}||P_{\mathrm{RBM}})-\beta I(\boldsymbol{\sigma}_{\mathcal{E}}:\boldsymbol{\tau})$

最大环境互信息原则

- 优势：防止机器被无关(irrelevant)变量欺骗



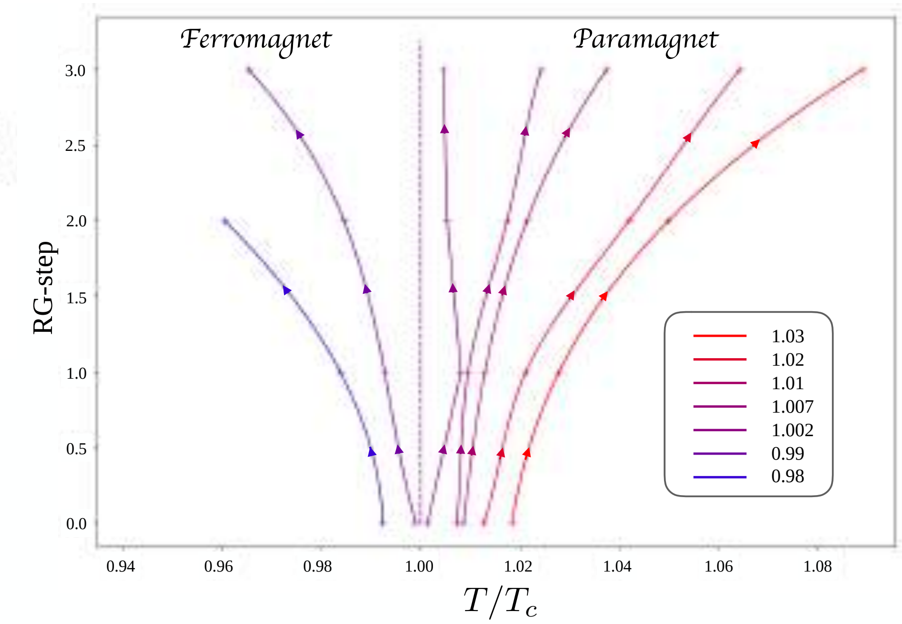


arXiv:1704.06279 (cond-mat)

[Submitted on 20 Apr 2017 (v1), last revised 24 Sep 2018 (this version, v2)]

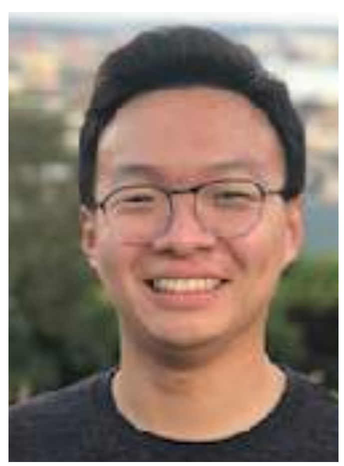
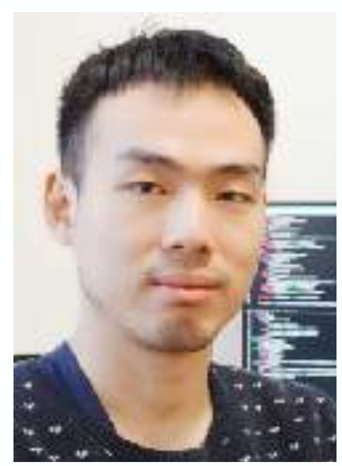
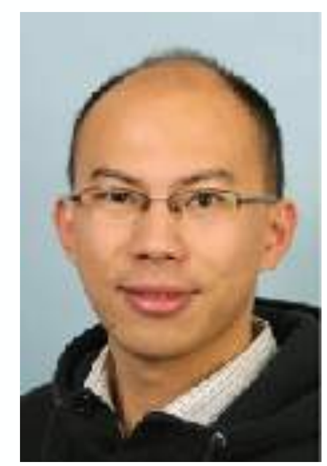
Mutual Information, Neural Networks and the Renormalization Group

Maciej Koch-Janusz, Zohar Ringel



Machine Learning Holographic Mapping by Neural Network Renormalization Group

H Hu, S-H Li, L Wang, Y-Z You. arXiv: 1903.00804

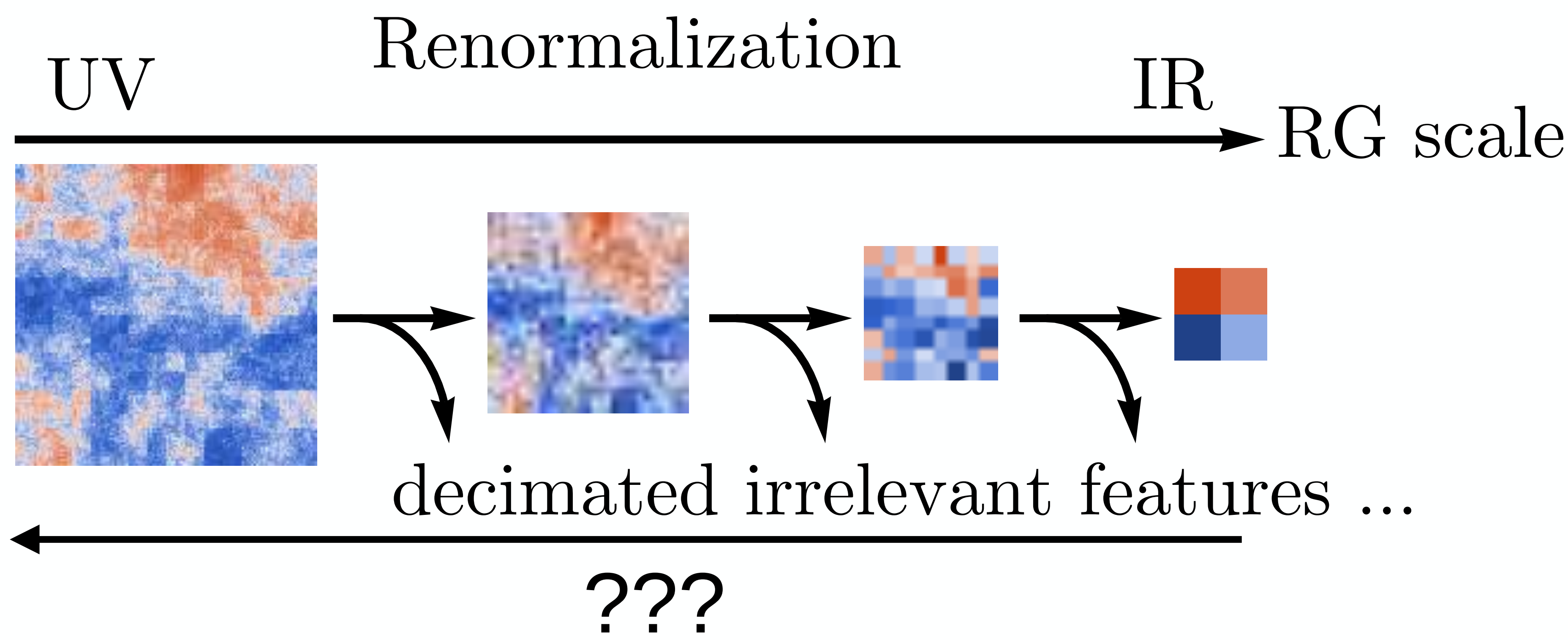




Hong-Ye Hu Shuo-Hui Li Lei Wang

(UCSD) (IOP, CAS)

重整化与全息编码

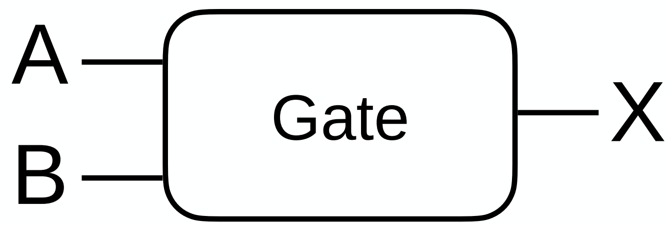
- 传统意义上的重整化：不断地丢信息，不可逆，不成群

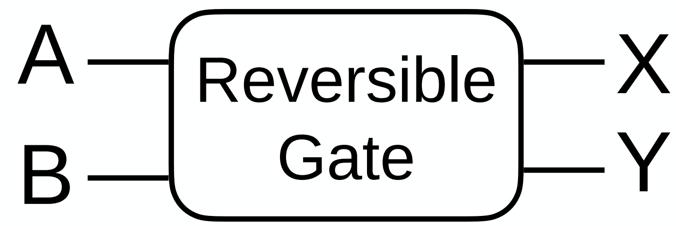


Traditional RG is not invertible…

重整化与全息编码

- 可逆计算





A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR

A	B	X	Y
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

CNOT

A	B	C	X	Y	Z
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	0
1	1	1	0	1	1

Toffoli (CCNOT)
Universal

- 可逆计算可以没有熵产生，原则上消除计算机发热
- 可逆元件构成的计算线路/神经网络也是可逆的

R.Landauer

重整化与全息编码

- 传统重整化
  - 压缩映射
  - 提取相关变量，同时丢弃无关变量
  - 目标：信息压缩
- 全息重整化
  - 可逆映射（编码-解码）
  - 提取相关变量，同时保留无关变量
  - 目标：全息编码

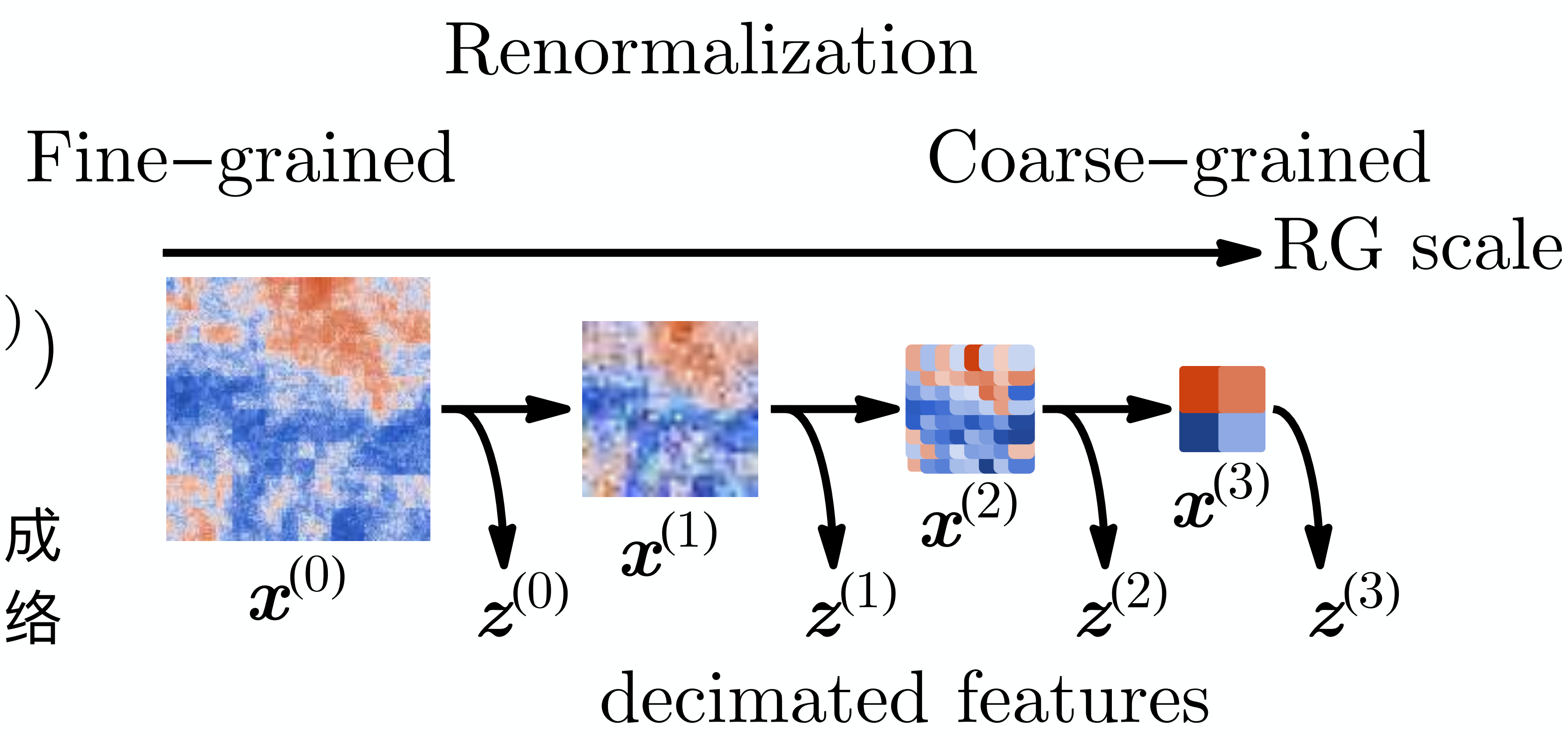
全息重整化变换

$$\boldsymbol{x}^{(h+1)}, \boldsymbol{z}^{(h)} = R_h(\boldsymbol{x}^{(h)})$$

相关变量

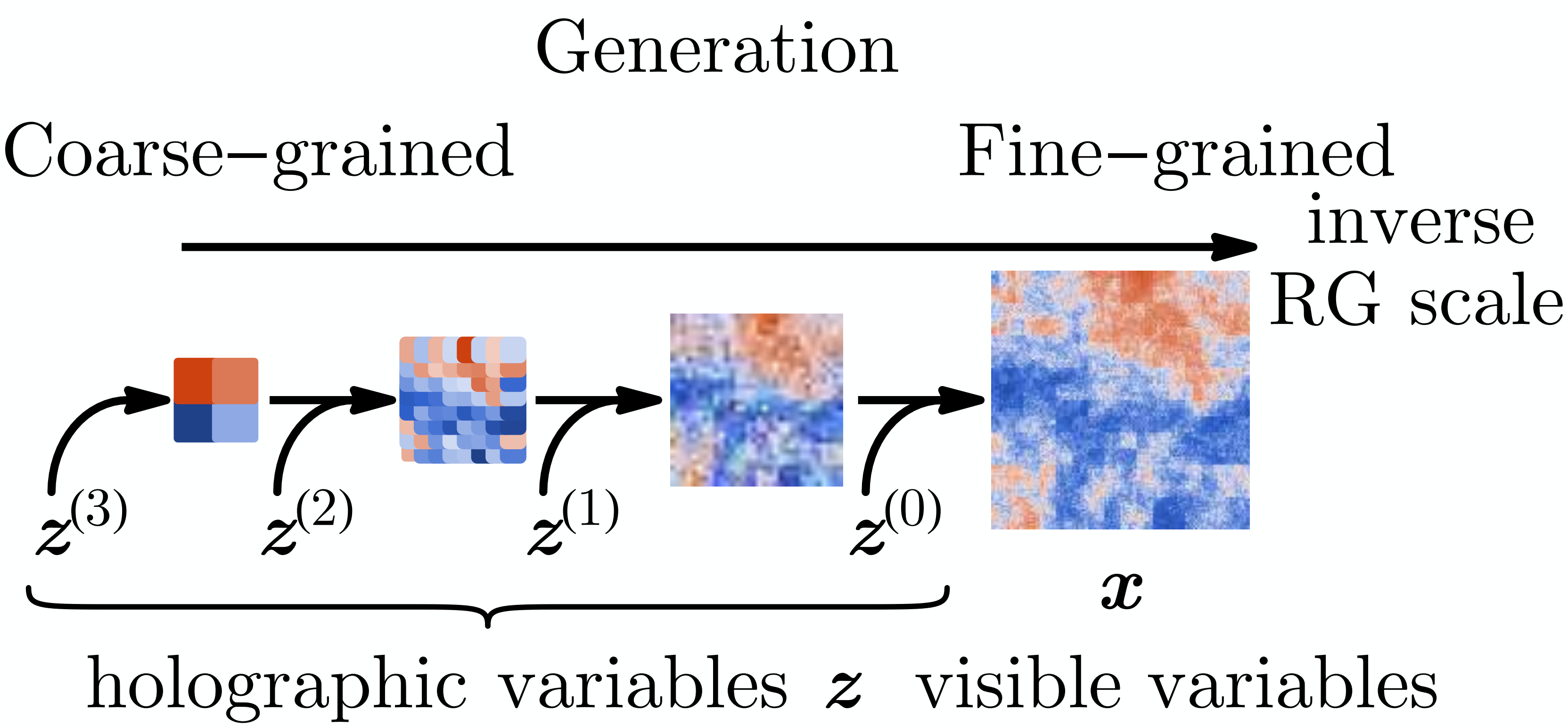
无关变量

由可逆元件构成的可逆神经网络



重整化与全息编码

- 不压缩信息还要重整化什么用? - 考虑问题不要太急功近利
- **Generation (生成)** ←inverse→ **Renormalization (重整)**



$$\boldsymbol{x}^{(h)} = R_h^{-1}(\boldsymbol{x}^{(h+1)}, \boldsymbol{z}^{(h)}) = G_h(\boldsymbol{x}^{(h+1)}, \boldsymbol{z}^{(h)})$$

逆重整化迭代 → 生成型模型的生成映射

$$\boldsymbol{x} \equiv \boldsymbol{x}^{(0)} = G_0(G_1(G_2(\cdots, \boldsymbol{z}^{(2)}), \boldsymbol{z}^{(1)}), \boldsymbol{z}^{(0)}) \equiv G(\boldsymbol{z})$$

重整化与全息编码

- 流基生成型模型(Flow-Based Generative Model)
  - 概率模型：生成映射诱导概率分布的变换

$$P_{\mathrm{mdl}}(\boldsymbol{x}) = P_{\mathrm{prior}}(\boldsymbol{z}) \det\left(\frac{\partial G(\boldsymbol{z})}{\partial \boldsymbol{z}}\right)^{-1}$$

  - 目标概率： $P_{\mathrm{tgt}}(\boldsymbol{x}) = Z^{-1} e^{-H(\boldsymbol{x})}$ (来自于物理模型)
  - 优化目标：最小化KL散度

$$D_{KL}(P_{\mathrm{mdl}} || P_{\mathrm{tgt}}) = \mathop{\mathbb{E}}_{\boldsymbol{x} \sim P_{\mathrm{mdl}}} (H(\boldsymbol{x}) + \ln P_{\mathrm{mdl}}(\boldsymbol{x})) + \mathrm{const.}$$

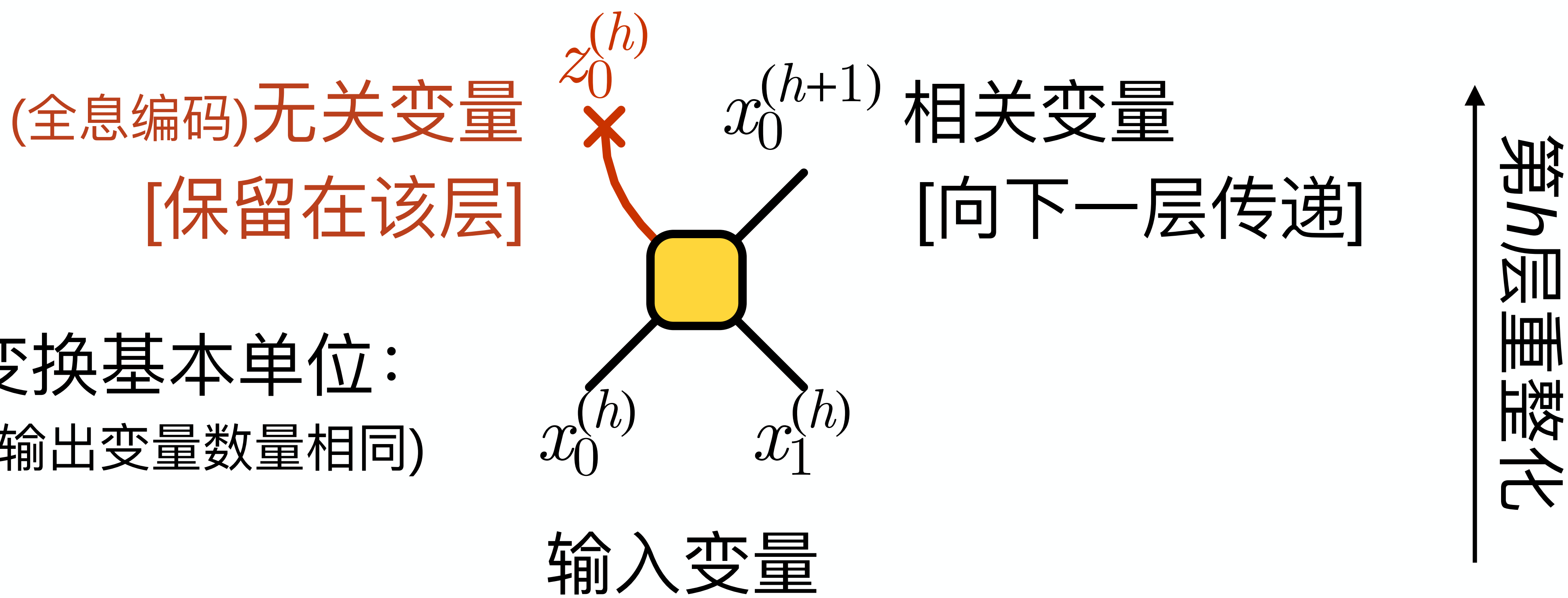
  - 优化变量：生成映射 G (亦或重整映射 $R = G^{-1}$) 中的参数
  - 训练后可获得对物理模型 H 的最优重整化方案 R
  - 待定问题：如何选择 $P_{\mathrm{prior}}(\boldsymbol{z})$？ - 最小全息互信息原理

最小全息互信息原理

- 全息重整化由许多局部的可逆变换组成

可逆变换基本单位：

(输入与输出变量数量相同)

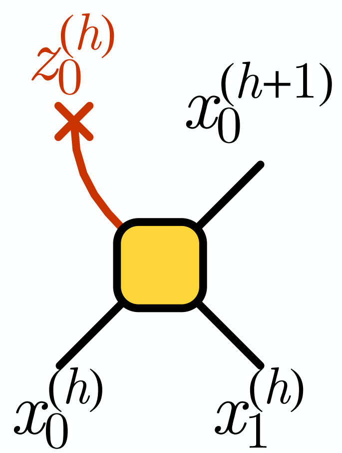


$$x_i^{(h+1)}, z_i^{(h)} = R_h(x_{2i}^{(h)}, x_{2i+1}^{(h)}) \quad \text{(局部变换)}$$

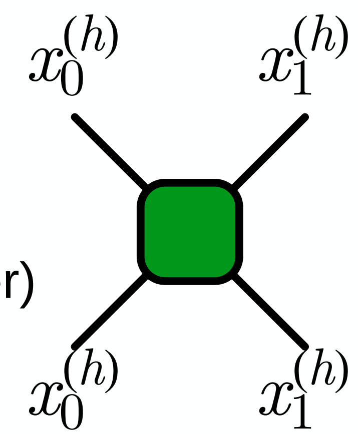
最小全息互信息原理

- 全息重整化由许多局部的可逆变换组成

抽取器
(decimator)



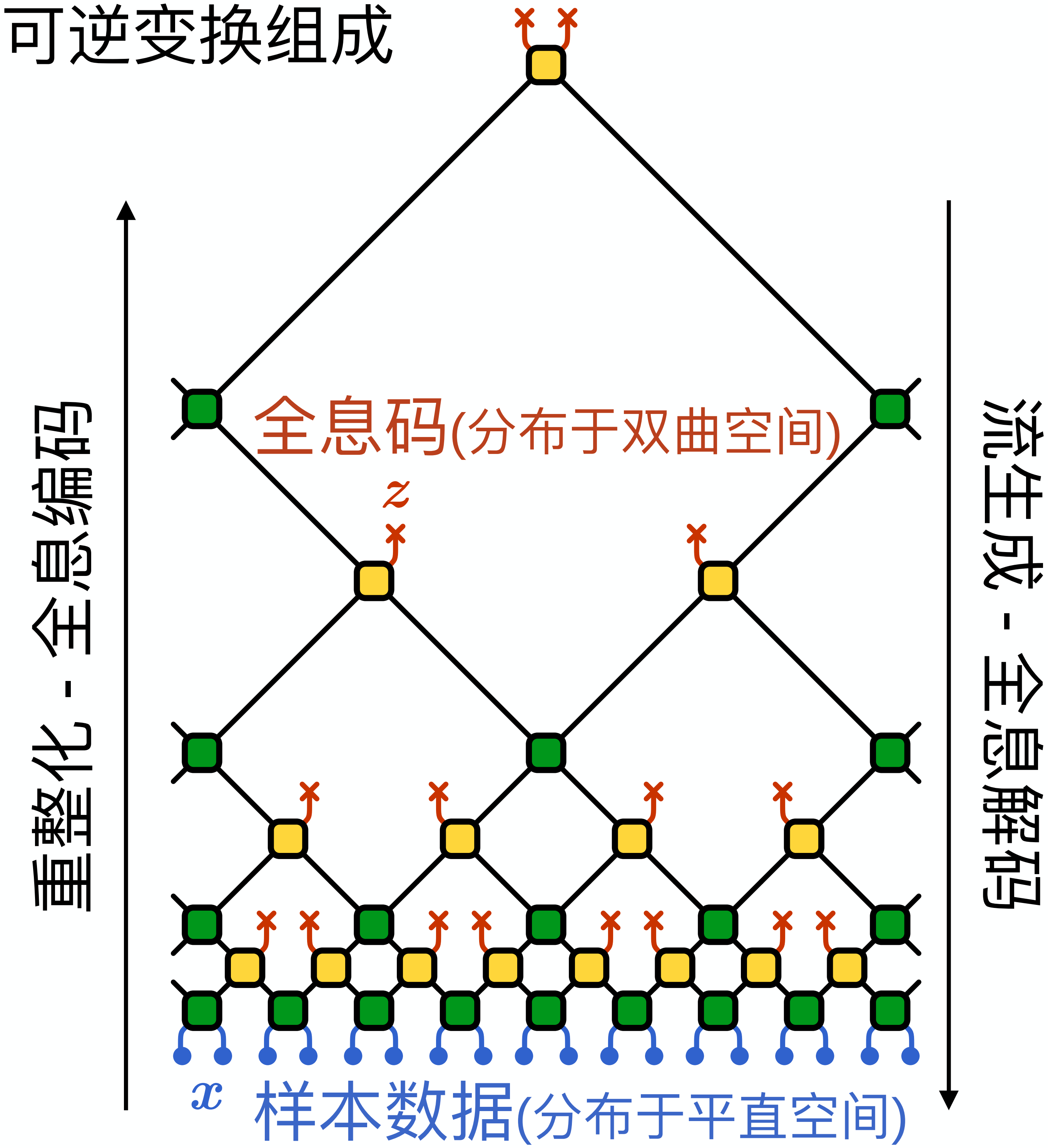
解耦器
(disentangler)



$$\boldsymbol{x}^{(h+1)}, \boldsymbol{z}^{(h)} = R_h(\boldsymbol{x}^{(h)})$$

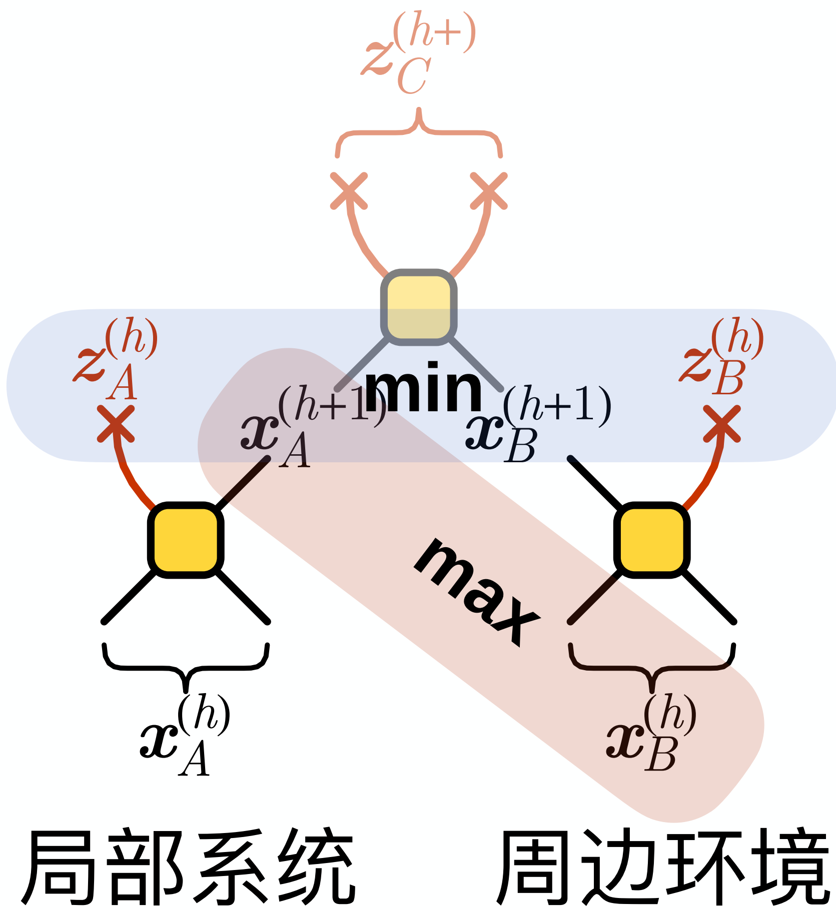
$$\Big\updownarrow \; G_h = R_h^{-1}$$

$$\boldsymbol{x}^{(h)} = G_h(\boldsymbol{x}^{(h+1)}, \boldsymbol{z}^{(h)})$$



最小全息互信息原理

- 最小全息互信息原理：无关变量(全息编码)之间应相互独立



$$I(\boldsymbol{z}_A^{(h)} : \boldsymbol{z}_B^{(h)}) = \mathop{\mathbb{E}}_{\boldsymbol{z}\sim P_{\mathrm{prior}}} \ln \frac{P_{\mathrm{prior}}(\boldsymbol{z}_A, \boldsymbol{z}_B)}{P_{\mathrm{prior}}(\boldsymbol{z}_A) P_{\mathrm{prior}}(\boldsymbol{z}_B)}$$

最小全息互信息　　　最大环境互信息

$$\min I(\boldsymbol{z}_A^{(h)} : \boldsymbol{z}_B^{(h)}) \Rightarrow \max I(\boldsymbol{x}_A^{(h+1)} : \boldsymbol{x}_B^{(h)})$$

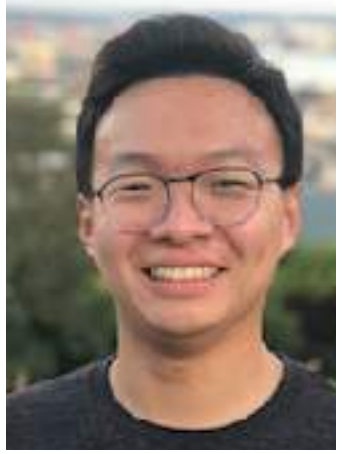
(在相同层级内)

- 除了同层级内解耦，亦要求层级间解耦
- 可通过设定 $P_{\mathrm{prior}}(\boldsymbol{z})$ 在模型上直接满足 (不用与KL散度竞争)

$$P_{\mathrm{prior}}(\boldsymbol{z}) = \prod_{i,h} P_{\mathrm{prior}}(z_i^{(h)})$$

RG-Flow: A hierarchical and explainable flow model

H Hu, D Wu, Y-Z You, B Olshausen, Y Chen. arXiv: 2010.00029

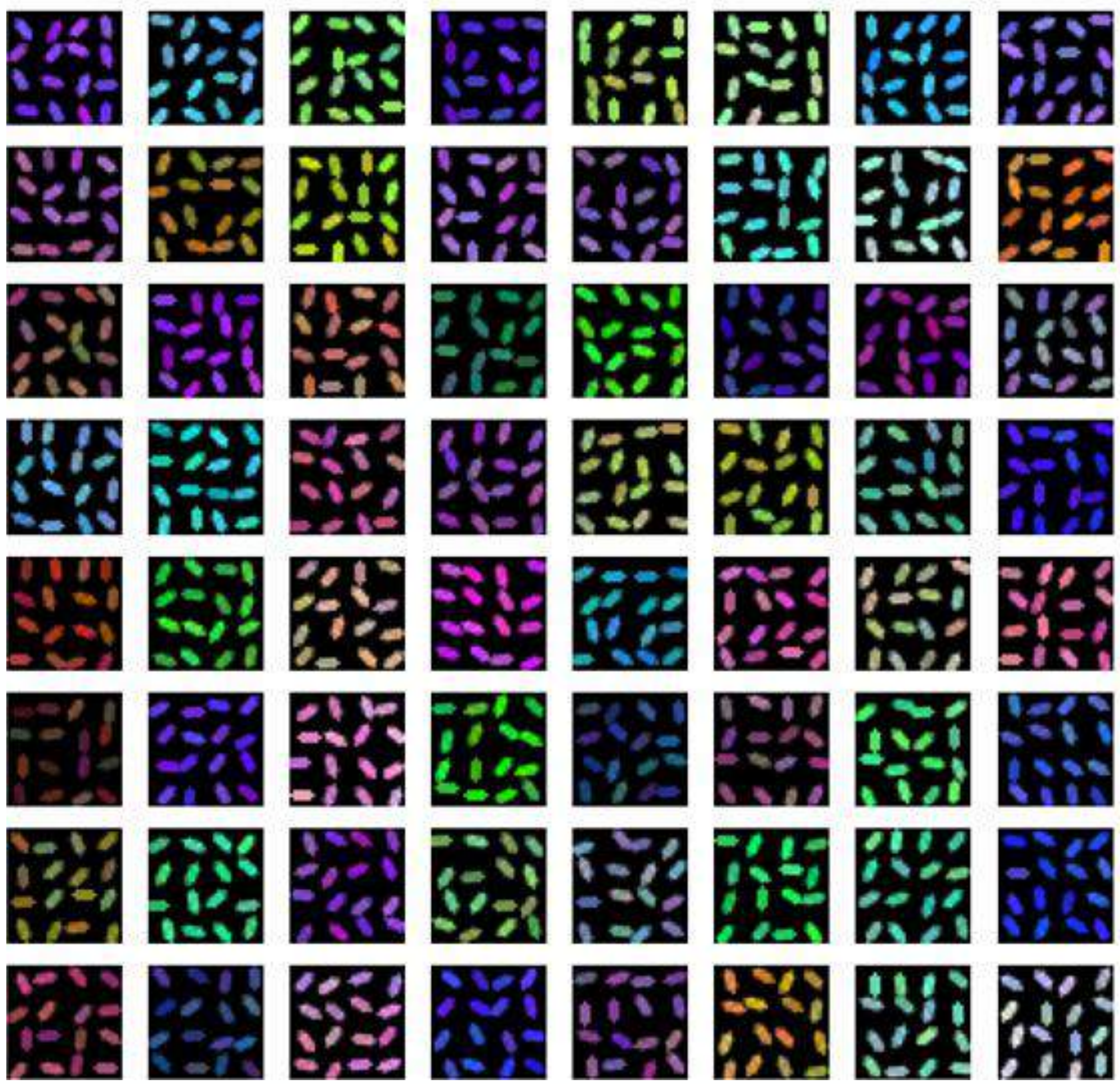



Hong-Ye Hu　Dian Wu　Yubei Chen　Bruno Olshausen

(UCSD)　(UC Berkeley)

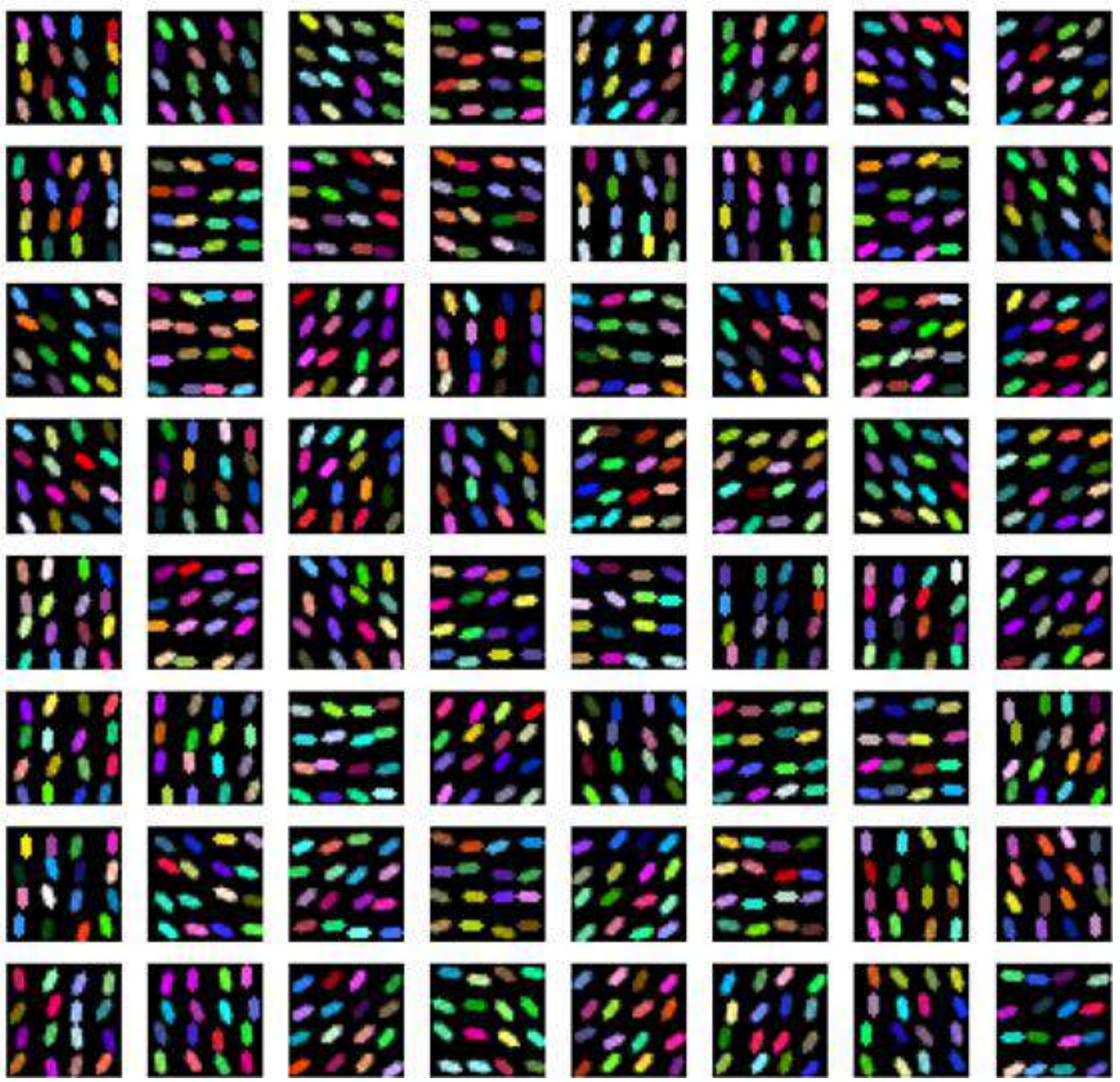
Two Toy Multi-Scale Datasets

MSDS1



High-level: color
Low-level: orientation

MSDS2

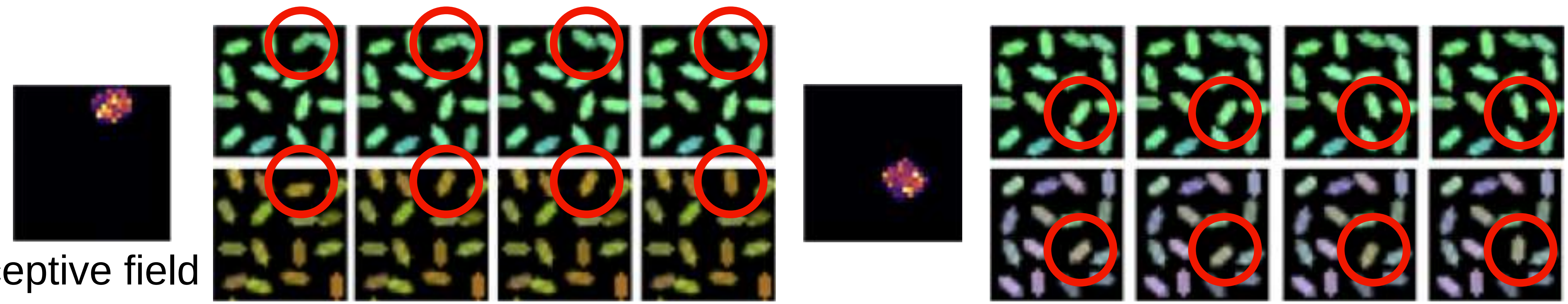


High-level: orientation
Low-level: color

Multi-scale Latent Representation

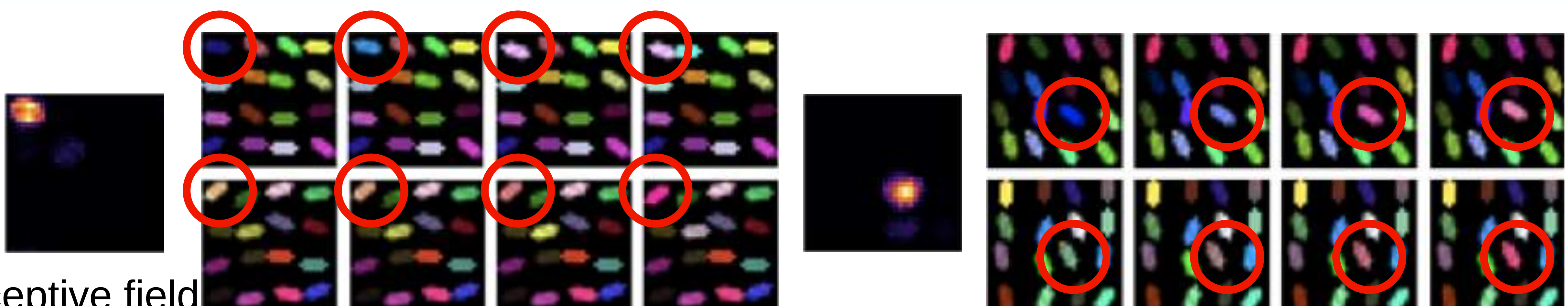
- Tuning a low-level holographic variable

MSDS1 (Low-level: orient.)



Receptive field

MSDS2 (Low-level: color)



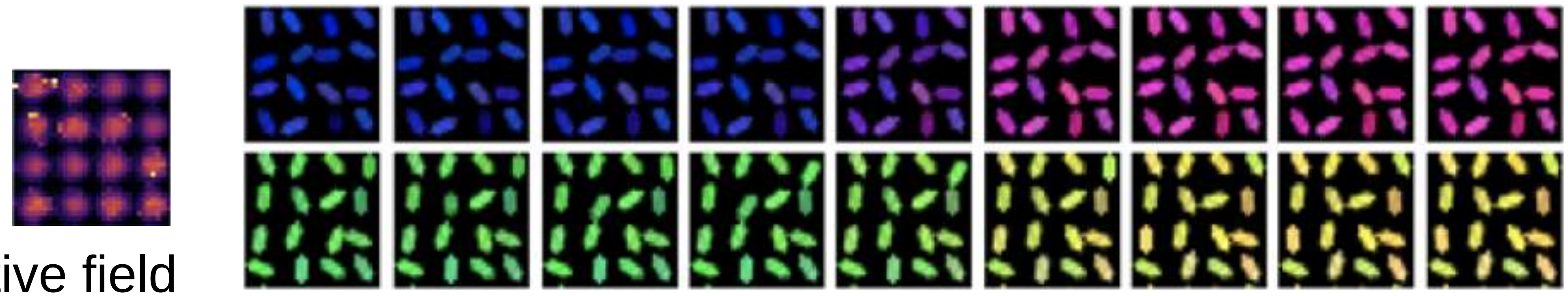
Receptive field

Multi-scale Latent Representation

- Tuning a high-level holographic variable

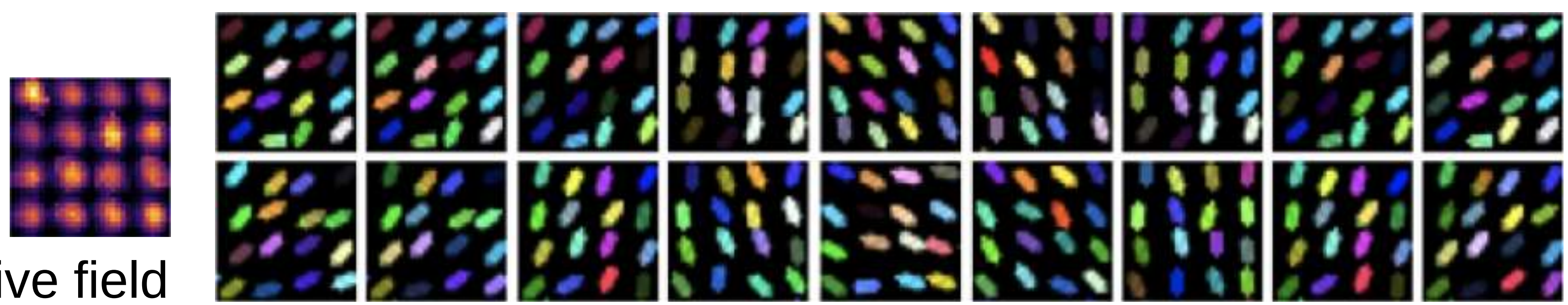
High level

MSDS1 (High-level: color)



Receptive field

MSDS2 (High-level: orient.)



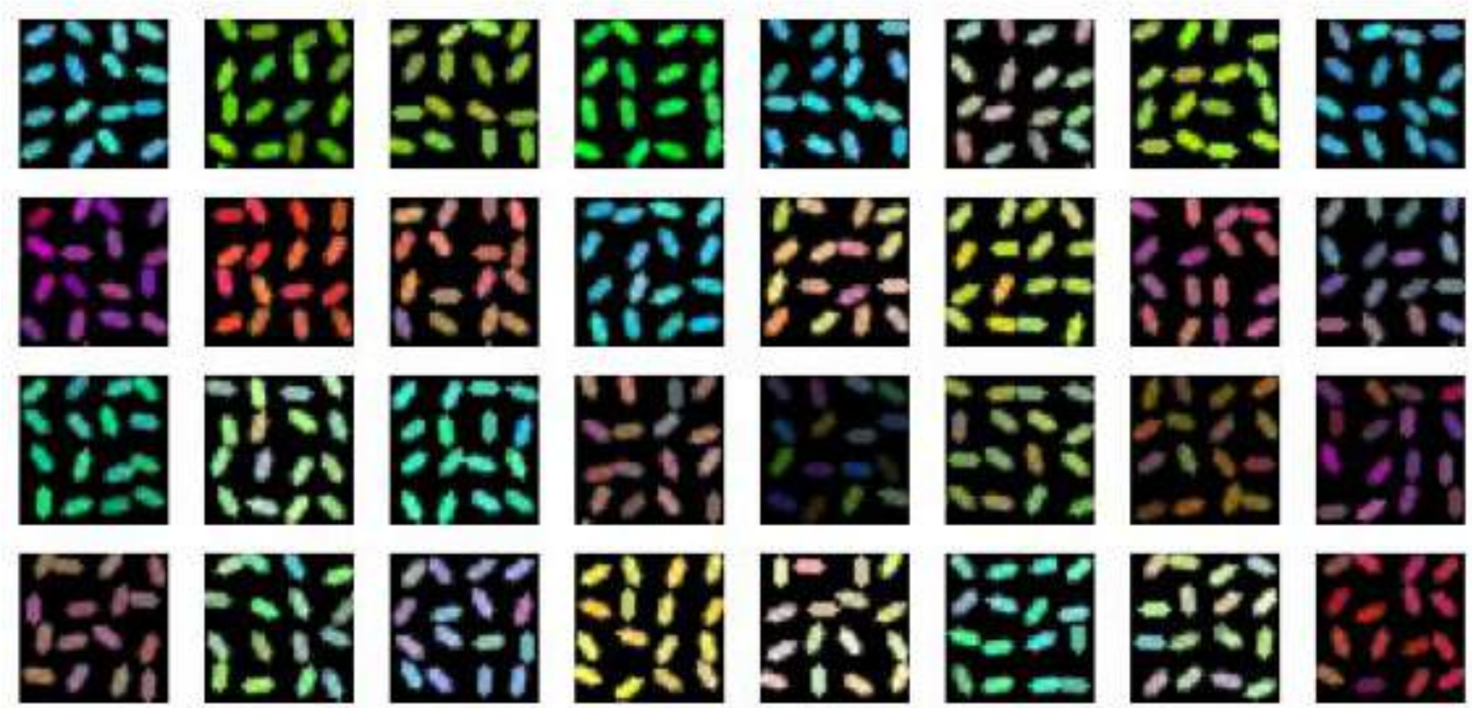
Receptive field

- How is it useful? - Global features are hard to learn.

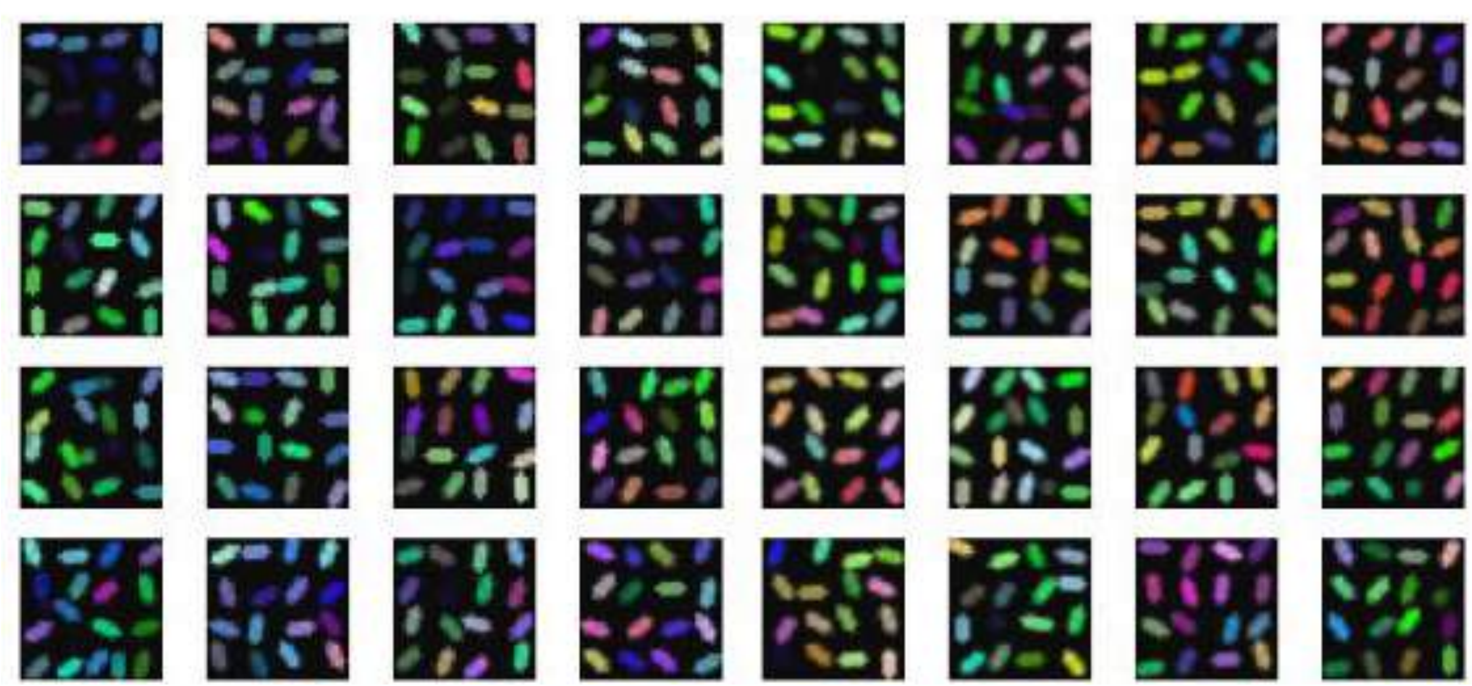
Compare with Real NVP

MSDS1

RG-Flow (Ours)



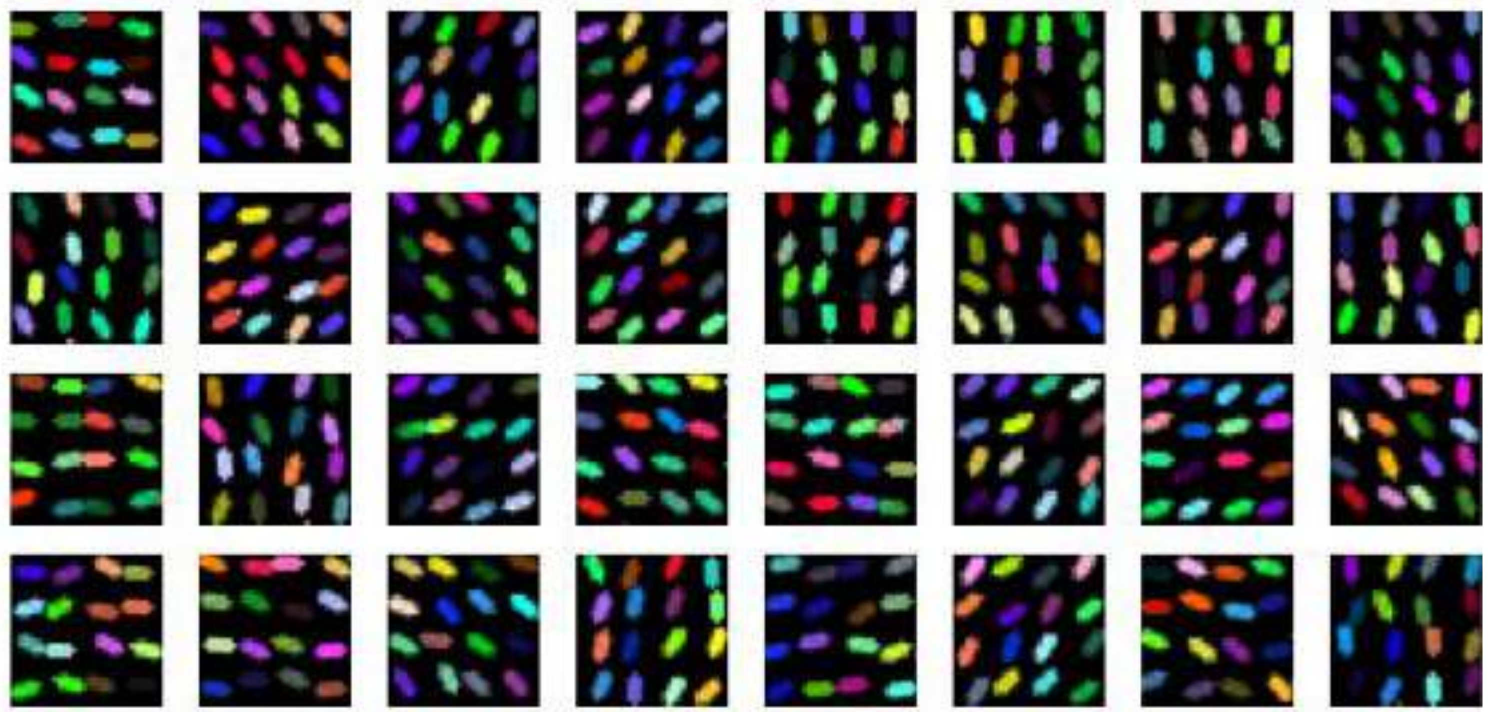
Real NVP



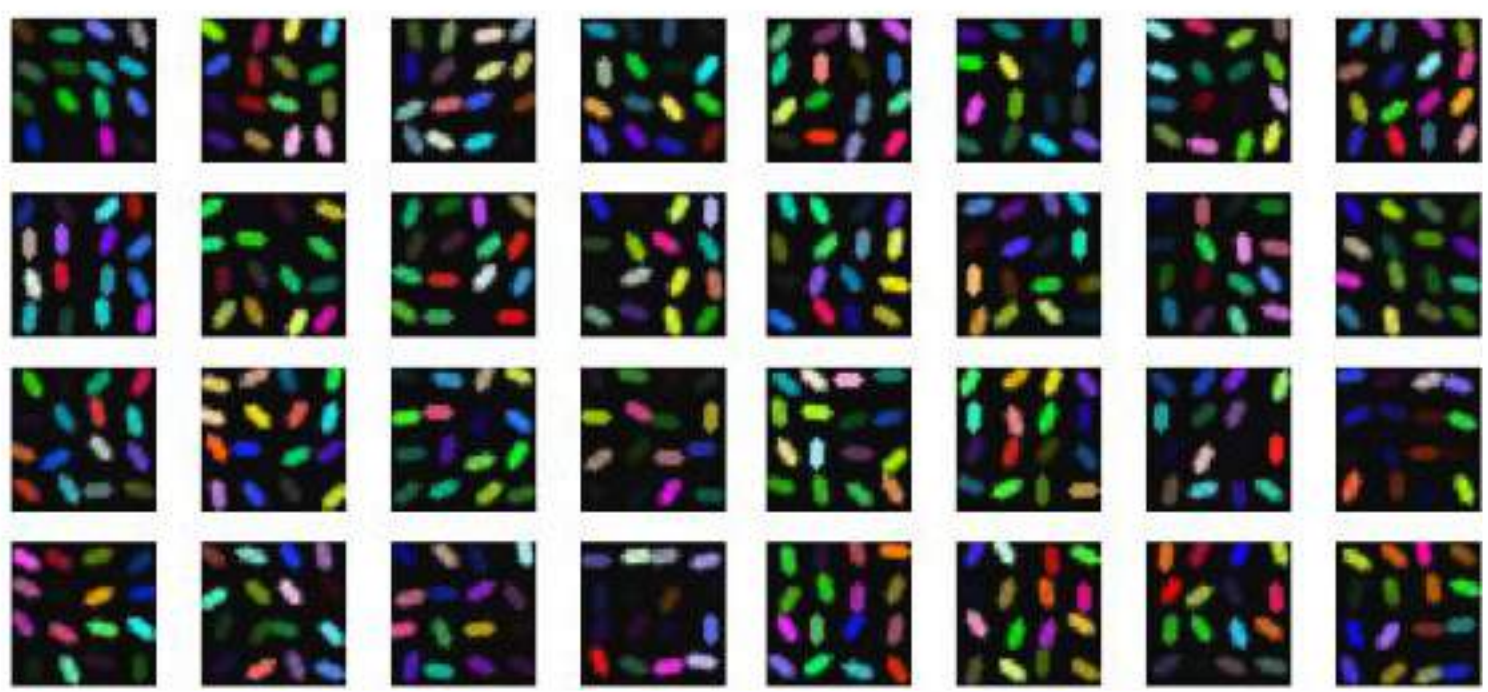
Compare with Real NVP

MSDS2

RG-Flow (Ours)

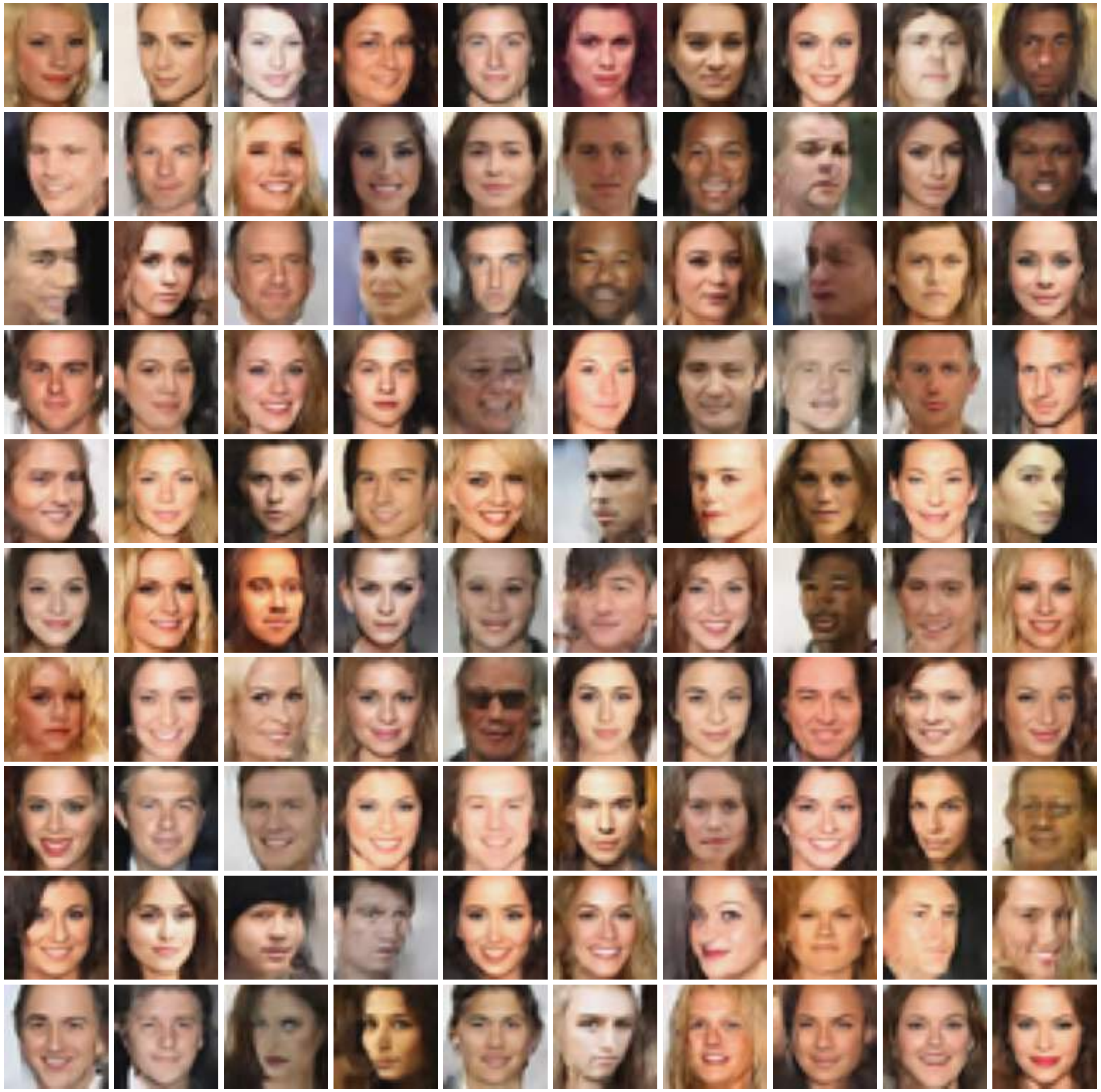


Real NVP



Human Face Dataset (CelebA)

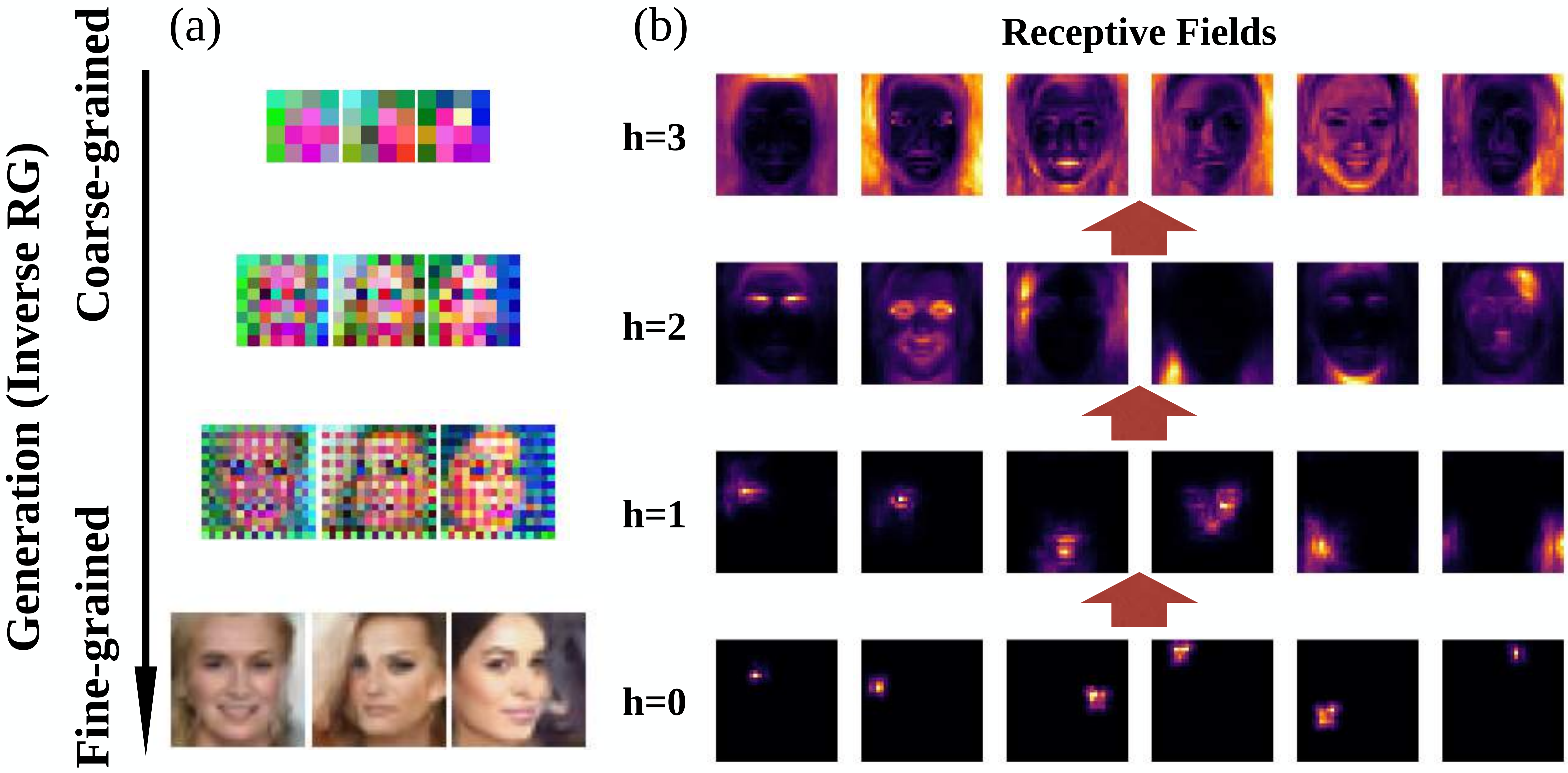
- Samples generated by RG-Flow:



Emergent Hierarchical Representations

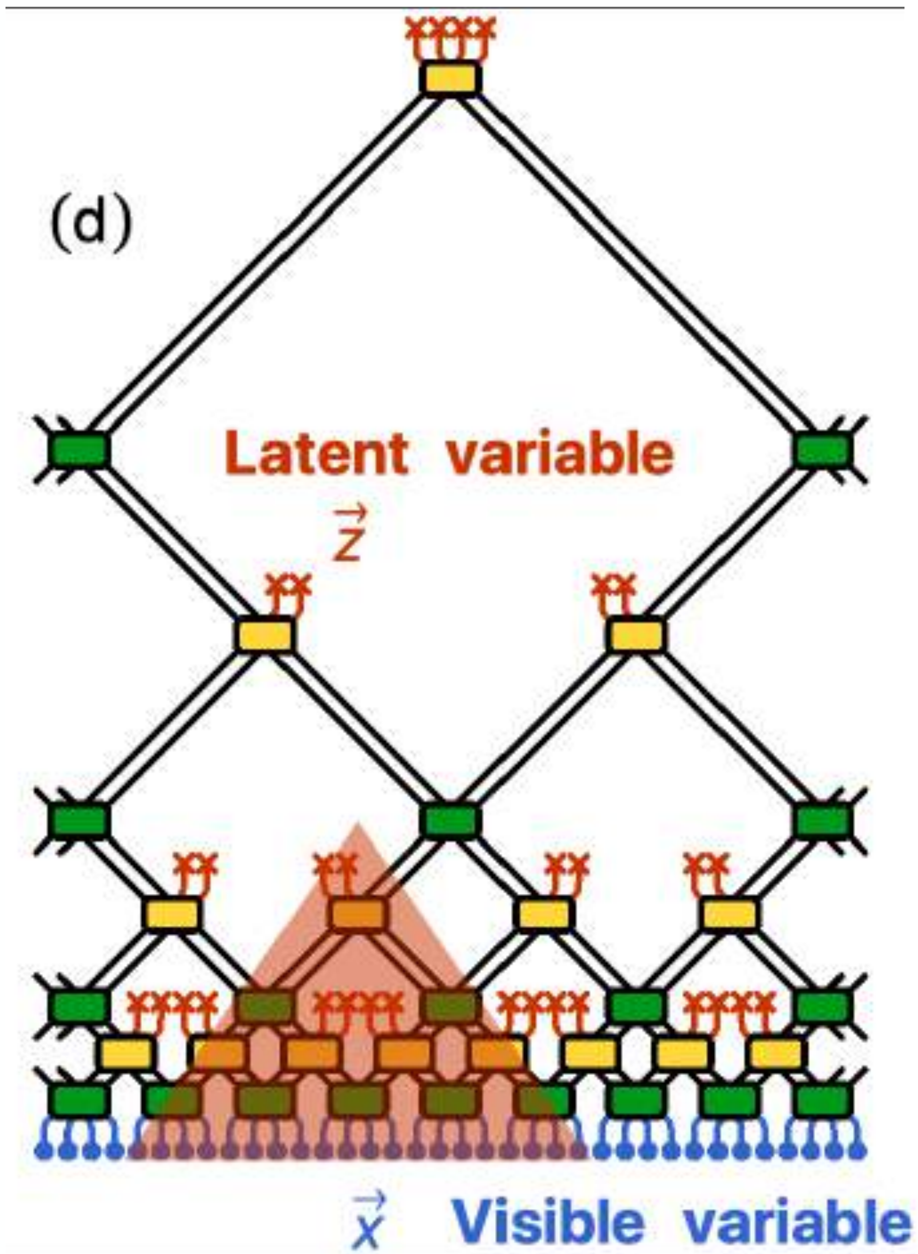
- After training, probe how visible variables respond to perturbations of holographic variables.

Emergent features (unsupervised)



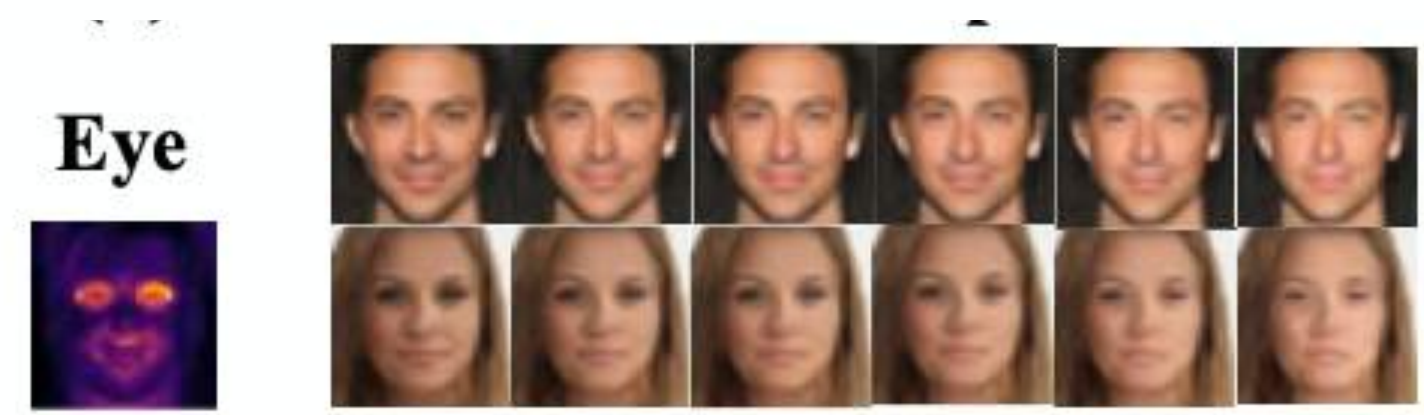
Emergent Hierarchical Representations

- Features at different scales



High-level features

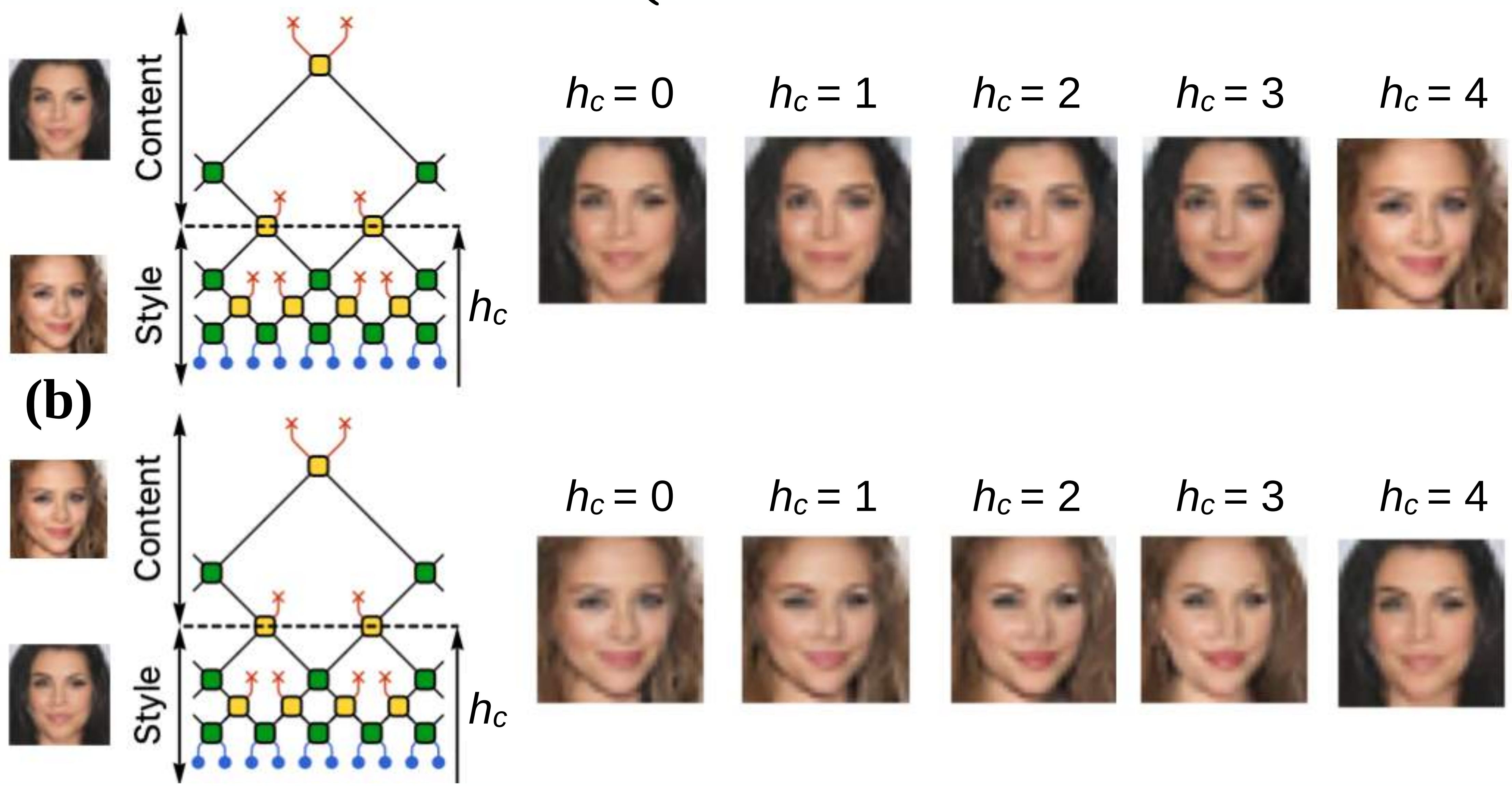
Mid-level features



Multi-Scale Feature Mixing

- Mixing high-level features of A with low-level features of B

$$\boldsymbol{z}^{(h)} = \begin{cases} \boldsymbol{z}_A^{(h)} & \text{for } h \geq h_c, \\ \boldsymbol{z}_B^{(h)} & \text{for } h < h_c \end{cases}$$

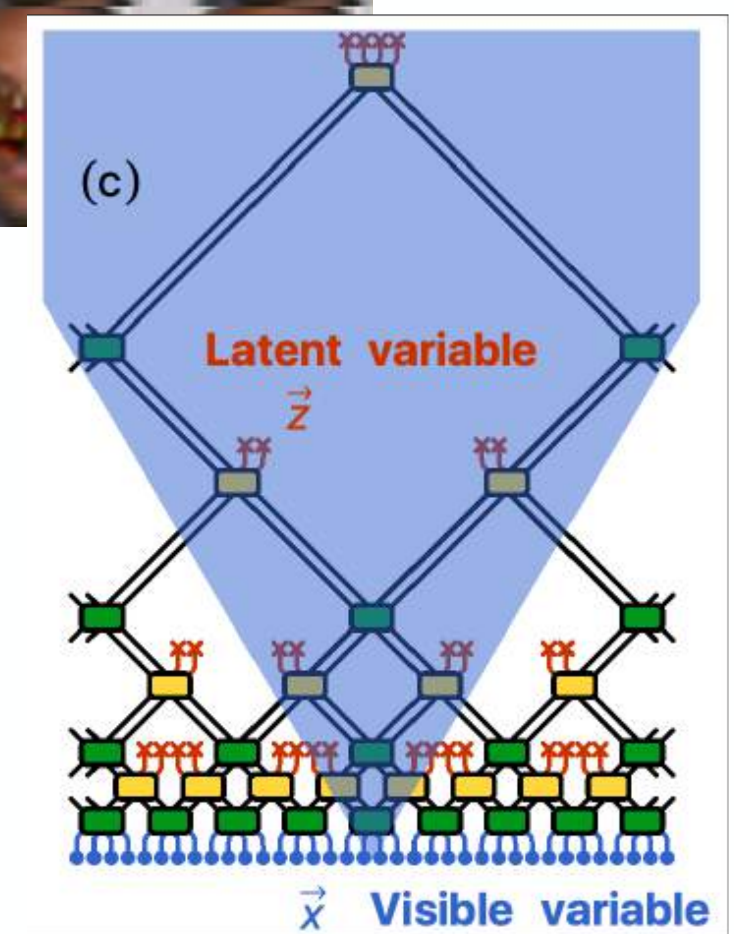


Efficient Error Correction

- Error correction: restore locally corrupted images

RG-Flow

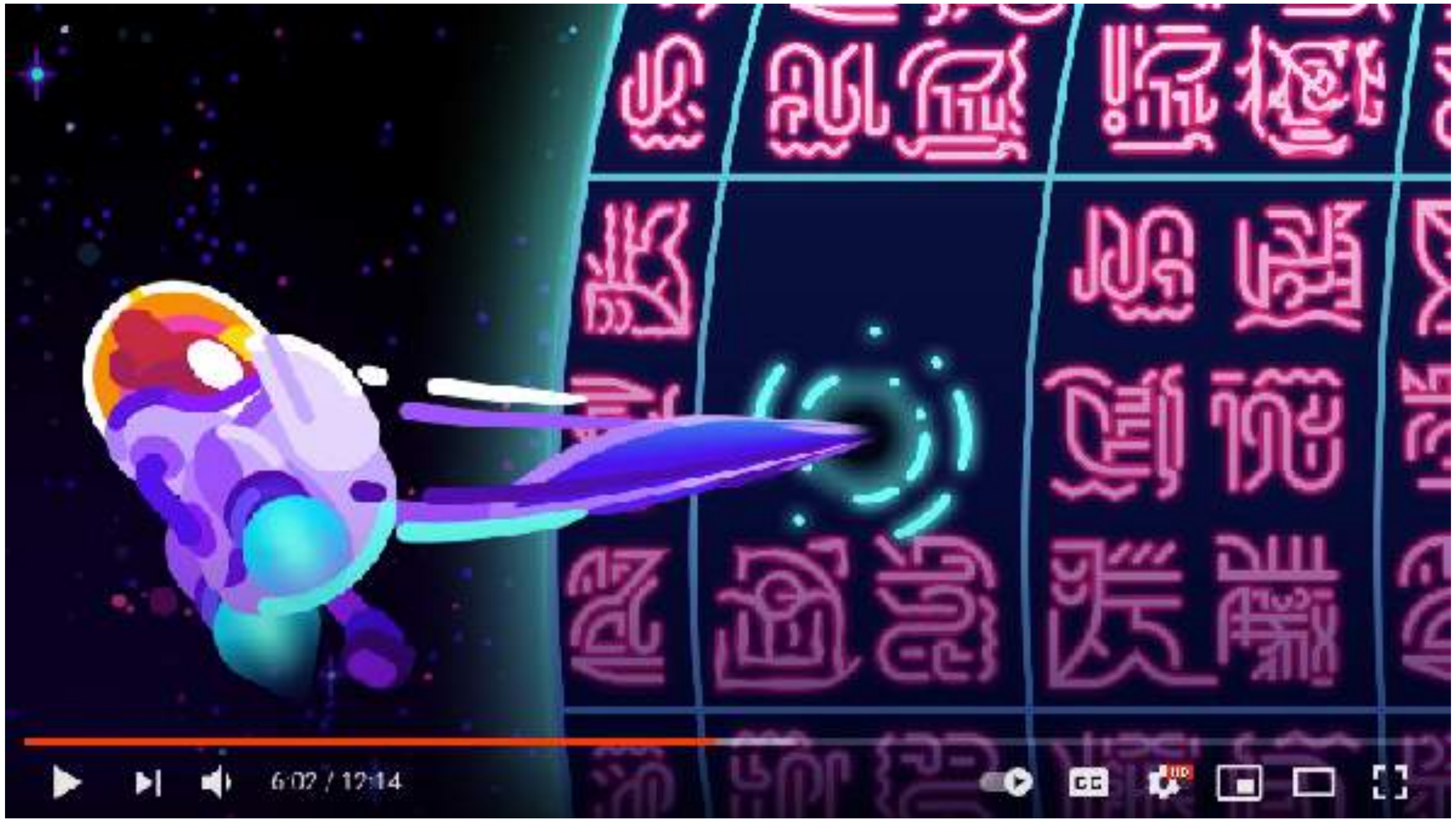
Constrained Real NVP



  - Past light-cone only covers ~ O(log N) volume in the hyperbolic space
  - Only need to resample ~ O(log N) holographic variables to restore the corrupted image

全息对偶与量子引力

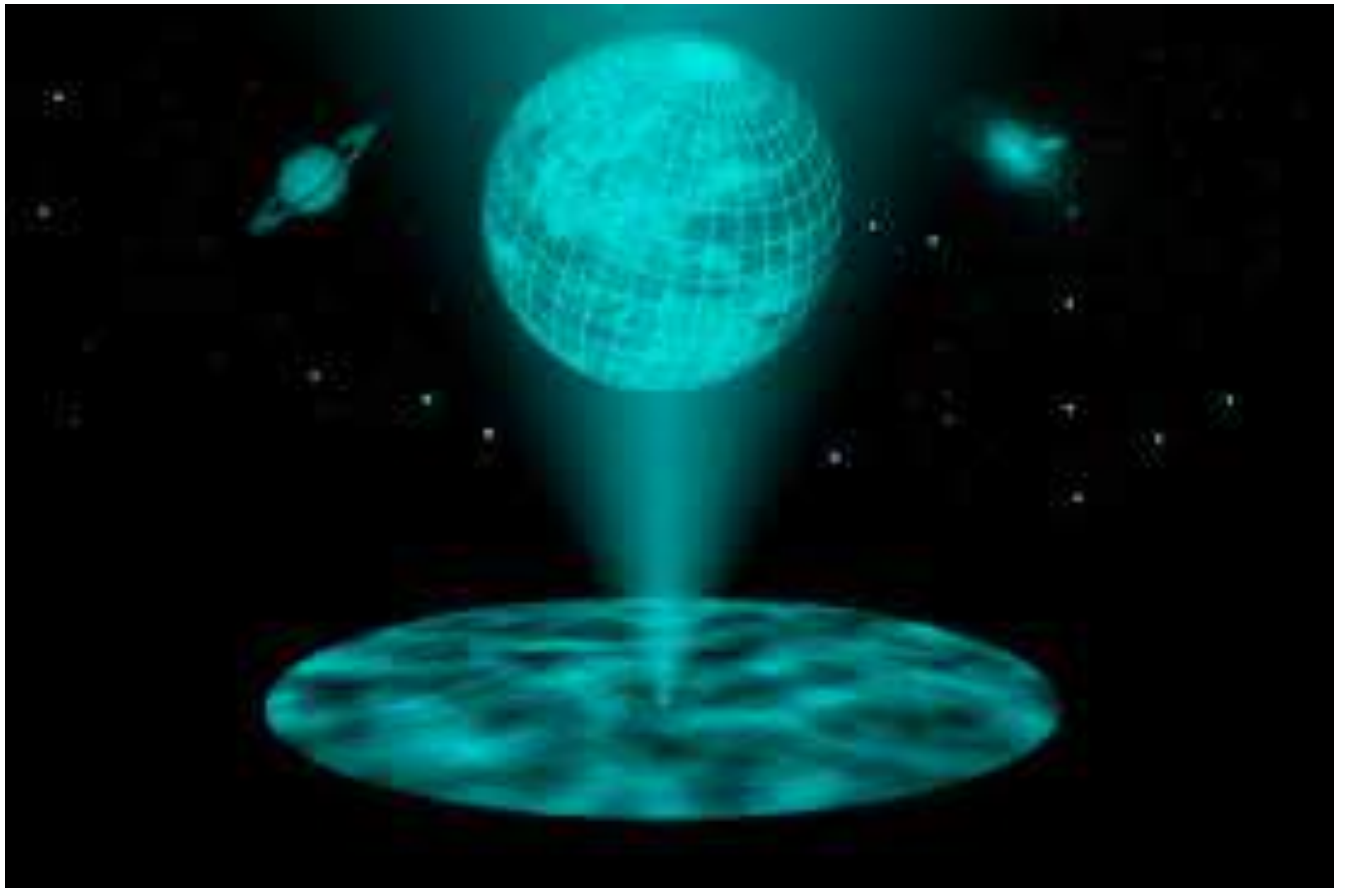
- 黑洞信息悖论：落入黑洞的信息消失了，违反量子力学要求的信息守恒（信息不生不灭，只能改变）
  - 主流解释：三维物体落入黑洞后，其信息以量子编码形式呈现于黑洞的二维表面(视界)，这些信息会随黑洞蒸发而复现



Kurzgesagt – The Information Paradox
YouTube

全息对偶与量子引力

- 三维物体是如何被呈现于二维表面的? (二向箔?)
- 全息对偶：边界量子系统可以与体内的引力系统对应

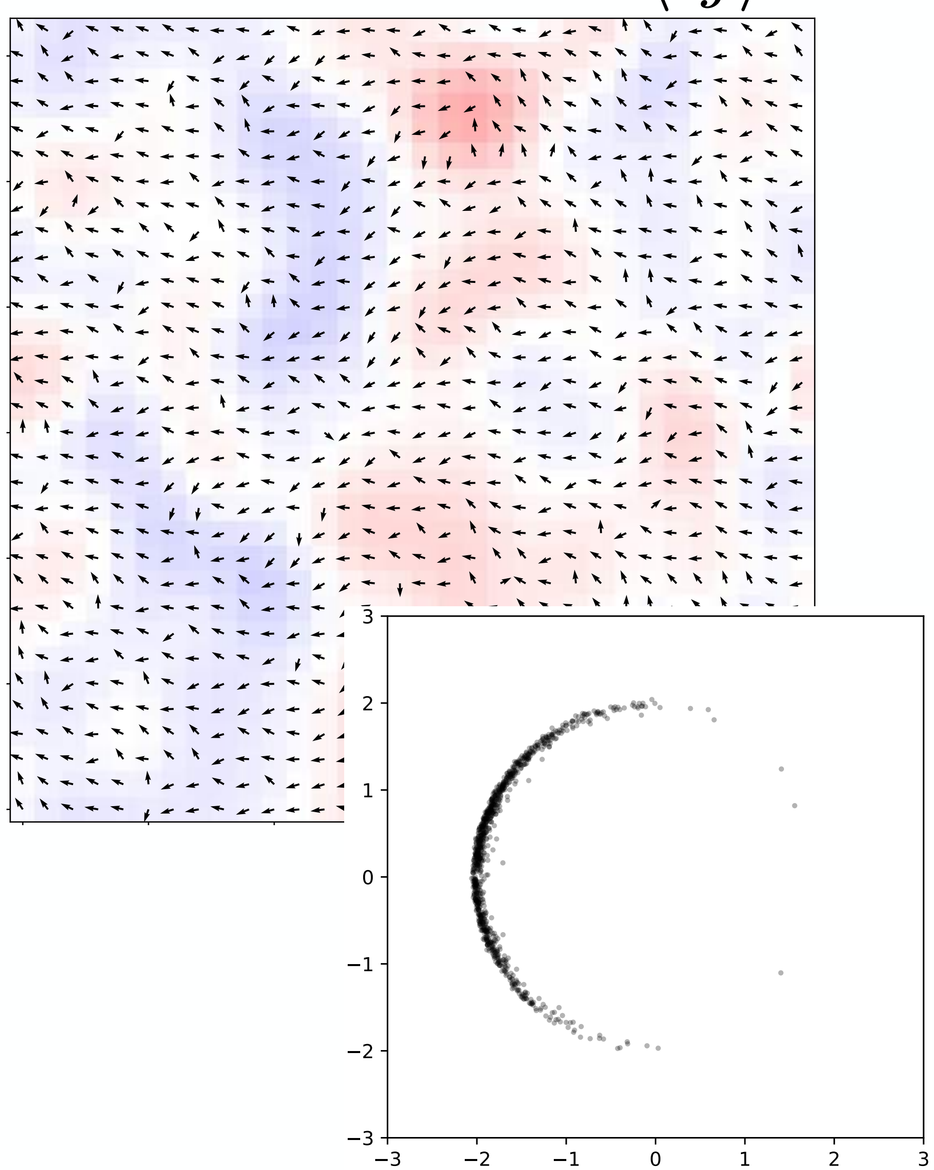


- 如何解码隐藏于量子纠缠中的时空几何?
  - 对量子场论进行全息重整化 → 全息对偶的时空结构

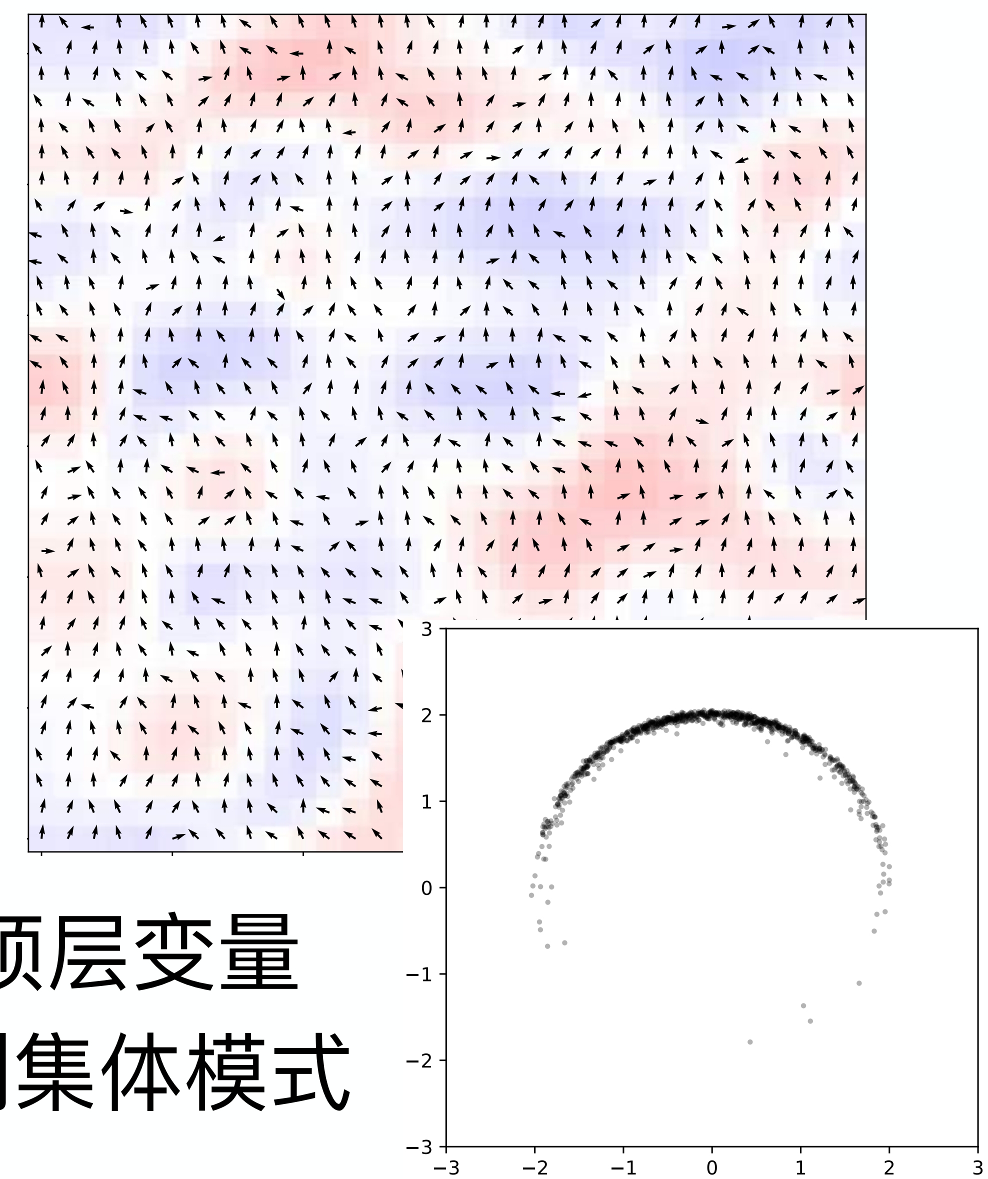
全息对偶与量子引力

- 2D复标量场的共形场论 (conformal field theory)

$$S[\phi] = -t\sum_{\langle ij\rangle}\phi_i^*\phi_j + \sum_i(\mu|\phi_i|^2 + \lambda|\phi_i|^4)$$

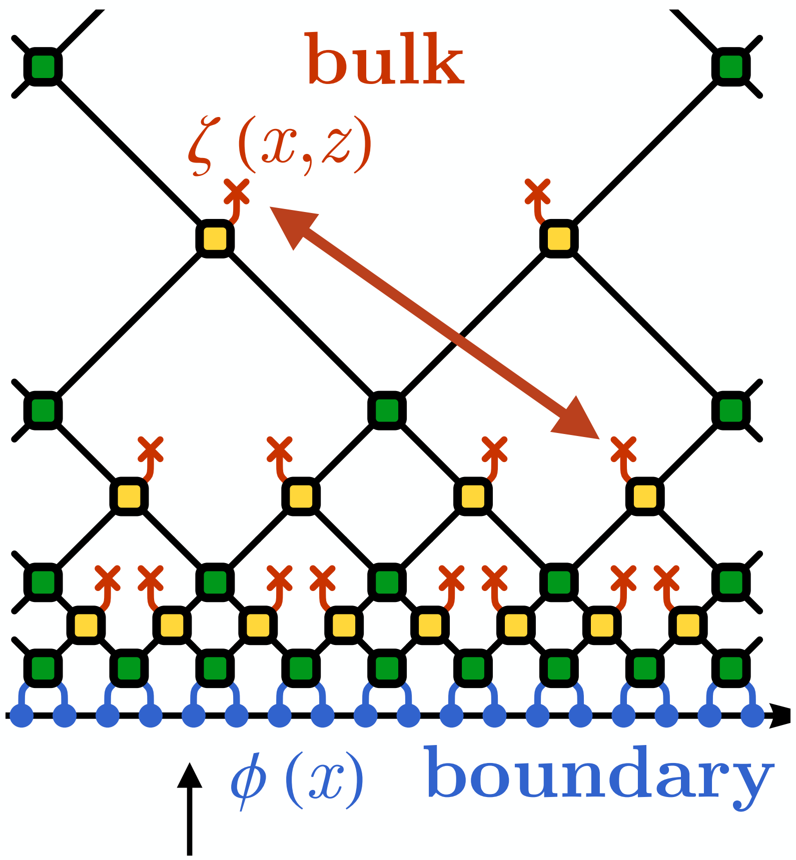


调整顶层变量
- 控制集体模式



全息对偶与量子引力

- 进一步研究全息变量之间的残余互信息



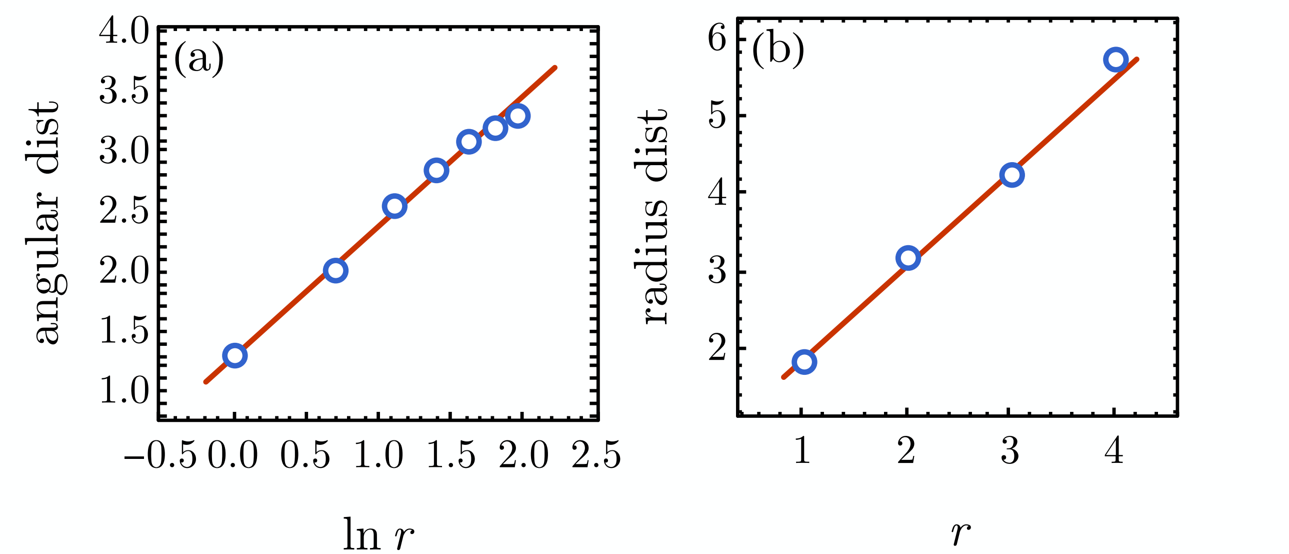
将量子场论的概率模型送回经过训练的网络

$$P_{\text{post}}[\xi] = P_{\text{QFT}}[\phi]\,\det\left(\frac{\partial R[\phi]}{\partial \phi}\right)^{-1}$$

- 时空几何从互信息中涌现

$$d(a,b) = -\xi \ln \frac{I(\zeta(x_a,z_a):\zeta(x_b,z_b))}{I_0}$$

$d(x,y,z|x+r,y,z) \sim \ln r$ $\quad$ $d(x,y,z|x,y,z+r) \sim r$



- 结果与双曲几何吻合

$$\mathrm{d}s^2 = \frac{1}{z^2}(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$$