Extra Problems - Root Finding

1 Bracketing Methods

1.1 Question 1 (Chapra, ch5)

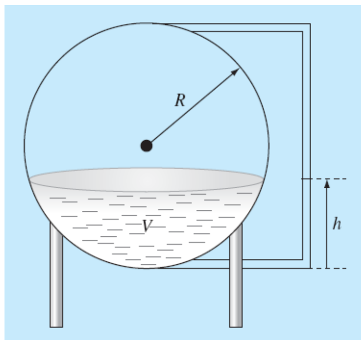
Medical studies have shown that a bungee jumper's chance of sustaining a vertebrae injury increases if the free-fall velocity exceeds 36 m/s after 4 seconds of free-fall. The analytical solution for the fall velocity as a function of time is given by :

$$v(t) = \sqrt{\frac{gm}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}}t\right)$$

We'd like to find the mass (m) at which this criterion is exceeded, given a drag coefficient of $c_d = 0.25$kg/m.

- Cast this as a root finding problem $f(m) = 0$.
- Plot this new function of mass (f(m)) for various mass to see what mass would be the first to reach the critical velocity (g=9.81 m^2/s) (**Answer : about 145kg**)
- Solve using the bisection method with $\epsilon_a = 0.0001\%$. (**Answer : m=142.74 kg after 21 iterations**)

1.2 False position



You are designing a spherical tank to hold water for a small village in a developing country. The volume of liquid it can hold can be computed as

$$V = \pi h^2 \frac{3R - h}{3}$$

where V = volume [m^3], h = depth of water in tank [m], and R = the tank radius[m]. If R = 3 m, to what depth must the tank be filled so that it holds 30 m^3? Use three iterations of the false-position method to determine your answer. Determine the approximate relative error after each iteration. Employ initial guesses of 0 and R.

Compare your answer to the result from a builtin method of root finding.

2 Open Methods

A resistance temperature detector (RTD) is a type of analog temperature sensor. It contains a strip of metal, whose resistance, R, changes with temperature, T, following the equation below.

$$R = R_0\left(1 + AT + BT^2 + CT^4\right)$$

R_0 is the resistance of the metal at 273 K, and A, B, and C are material constants. Consider a copper RTD with $R_0 = 100\,\Omega$, $A = 6.75 \times 10^{-4}\,1/\mathrm{K}$, $B = 1.34 \times 10^{-8}\,1/\mathrm{K}^2$, and $C = 2.77 \times 10^{-13}\,1/\mathrm{K}^4$. Determine the temperature of the RTD if its resistance is $150\,\Omega$.

Use Newton's method and the secant method. Set $N = 10$ and $\epsilon = 0.0001$ and compare the final result and final error of the different methods. Also compare these results to the Newton and Secant methods from scipy, which uses a default error tolerance on the order of 1×10^{-8}. Note that you should plot the function to determine good initial guesses.

Hint: Temperature is given in Kelvin so can't be negative. Solve for the positive root.