# 1 Trajectory Optimization Model

## 1.1 Establishment of optimal model

### 1.1.1 Description

In the previous section we have demonstrated the feasibility of launching solar sail spacecraft from earth to Mars orbit. In comparing the previous figures, we can find the attitude angle $\alpha$ and the surface of the solar sail A have certain influence on the trajectory of spacecraft. Thought understanding the differential equation (**??**) (**??**) the transit time t is determined by the attitude angle $\alpha$ and the surface of the solar sail A. So this section is in order to find the optimal solution, to decrease the transit time and insure the payload, to maximize the advantage of solar sails.

### 1.1.2 Conditions of Capture by Mars

According to actual condition, when the spacecraft near Mars, the influence of the Martian gravity is bigger and bigger, so we set up a Martian gravity range. When the spacecraft into the scope, we ignore the role of the sun's gravity, and the Martian gravity plays a leading role. In other words, the spacecraft was captured by Mars. At this time, if the speed constraint is under certain conditions, it can land safely, so we can get the spacecraft to be captured by Mars as one of the constraints.
We suppose that when $\frac{G_M}{G} = \varepsilon$ the spacecraft will be captured by Mars, according to the gravitational equation:

$$\frac{GM_M m}{(r_{range})^2} = \frac{GM_S m}{(R_M - r_{range})^2} \cdot \varepsilon$$

So here we get the distance constrain:

$$\varepsilon \cdot r_{range}^2 + 2 \cdot R_E \cdot \frac{M_M}{M_S} r_{range} - R_E^2 \cdot \frac{M_M}{M_S} = 0$$

E.g. we suppose that $\varepsilon = 10$ after calculating we get the range of gravitational influence of Mars is $2.68 \times 10^7$. So in the constraint condition we set this order of magnitude $(10^7)$ is reasonable.

### 1.1.3 Mathematical formulation

As discussion above, we can take the below mathematical formulation of optimization model

❶ **Objective function:** $\min t = t(\alpha, A)$

❷ **Decision variable:** $\alpha, A$

❸ $t(\alpha, A)$ **satisfy system of differential equations** (**??**) (**??**)

$$\vec{F_x} = \left(-\frac{GMmx}{(\sqrt{x^2+y^2})^3} + \frac{C_1 A cos^2\alpha}{\sqrt{x^2+y^2}}\left(cos\alpha \frac{x}{\sqrt{x^2+y^2}} - sin\alpha \frac{y}{\sqrt{x^2+y^2}}\right)\right)\vec{u_x}$$

$$\vec{F_y} = (-\frac{GMmy}{(\sqrt{x^2+y^2})^3} + \frac{C_1 Acos^2\alpha}{\sqrt{x^2+y^2}}(cos\alpha\frac{y}{\sqrt{x^2+y^2}} + sin\alpha\frac{x}{\sqrt{x^2+y^2}}))\vec{u_y}$$

❹ **Constraint conditions:**

- 4 Initial conditions:

$$\begin{cases} x(t=0) = R_E \\ y(t=0) = 0 \\ v_x(t=0) = 0 \\ v_y(t=0) = V_E(Earth\ revolution\ speed) \end{cases}$$

- 4 End conditions:

$$\begin{cases} x(t=t_{arrive})^2 + y(t=t_{arrive})^2 = R_M^2 \\ v_x(t=t_{arrive})^2 + v_y(t=t_{arrive})^2 \leq 9000m\cdot s^{-1} \\ x(t=t_{arrive}) = R_M\cos\left[\frac{t_{arrive}}{T_M}\cdot 2\pi\right] \\ y(t=t_{arrive}) = R_M\sin\left[\frac{t_{arrive}}{T_M}\cdot 2\pi\right] \end{cases}$$

  Where the last 2 equations mean: when the solar sail get to Mars, time spent is $t_{arrive}$, at the end of this time, the position of Mars are $\left(R_M\cos\left[\frac{t_{arrive}}{T_M}\cdot 2\pi\right], R_M\sin\left[\frac{t_{arrive}}{T_M}\cdot 2\pi\right]\right)$, where $T_M$ is the period of revolution of Mars, the position of solar sail $(x(t = t_{arrive}), y(t = t_{arrive}))$ should meet this point.

- 2 conditions of range of decision conditions:

$$0 < A < \frac{m}{\sigma} = 285714m^2$$

$$0 < \alpha < \frac{\pi}{2}$$

## 1.2 Realization by program

**STEP0** : Start;

**STEP1** : Input kinematical differential equations (**??**) (**??**) and initial conditions;

**STEP2** : Give the range of the surface A and the attitude angle $\alpha$;

**STEP3** : Set the step length of A and $\alpha$, start to loop;

**STEP4** : Judge the solution : the relative speed between Mars and the spacecraft is less than $9km/s$, the distance between Mars and the spacecraft is less than $r_{range}$;

**STEP5** : Output the (A,$\alpha$) that meet the judgement;

The detailed program code will be pasted to Appendix.

## 1.3 Result

After the iterative process by Matlab , luckily we seek 7 optimal values that satisfy all constraint conditions. By the program, we also obtain the transit time(time when the solar sailing arrive the destination) $t_{arrive}$ and the effective load(the total mass removes the quality of the solar sail) $m_e$, and

$$m_e = m - \sigma \cdot A$$

where $A$ is the area of solar sail, $\sigma = 7g \cdot m^{-2}$ is the density of solar sail and $m = 2000kg$ is the total mass of solar sail and effective load $m_e$.

Next is the table of 7 optimal values found by program and we list the corrosponding values of $A$, $t_{arrive}$ and $m_e$.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $\alpha(rad)$ | 0.21195 | 0.2198 | 0.14915 | 0.1099 | 0.1256 | 0.0658 | 0.0942 |
| $\sqrt{A}(m)$ | 420 | 420 | 440 | 460 | 460 | 480 | 480 |
| $A(m^2)$ | 176400 | 176400 | 193600 | 211600 | 211600 | 230400 | 230400 |
| $t_{arrive}(day)$ | 765 | 770 | 548 | 525 | 564 | 483 | 840 |
| $m_e(kg)$ | 765.2 | 765.2 | 644.8 | 518.8 | 518.8 | 387.2 | 387.2 |

Table 1: List of 7 optimal values

By this table, we can find the minimum transit time $t_{arrive}$ is 483 days, the corrosponding effective load is 387.2 kg account for 19.36% of the total mass. If we want to get the maximum effective load $m_e$, we need about 770 days.

Next are specific trajectories of the 7 cases above.

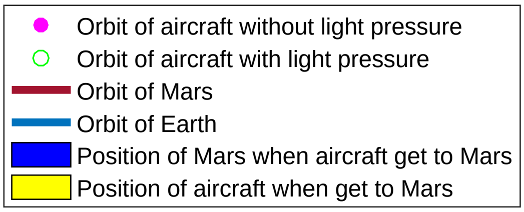


Figure 1: Legend of lines and points in figures below

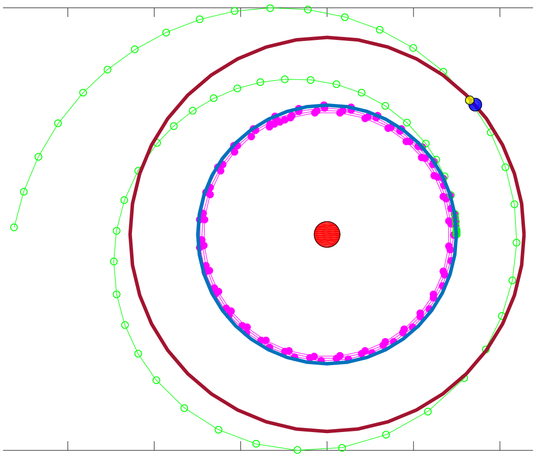

Figure 2: $alpha = 0.21195, A = 176400$

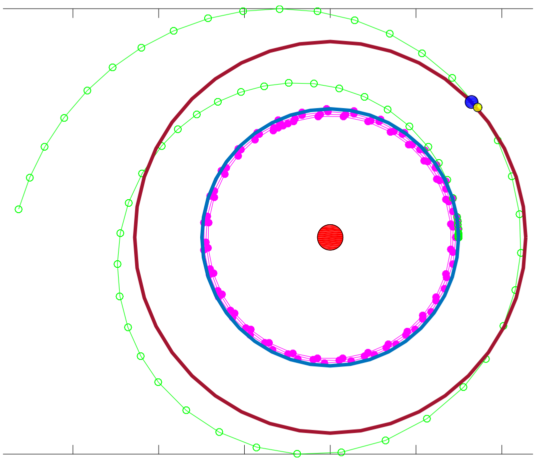

Figure 3: $alpha = 0.2198, A = 176400$

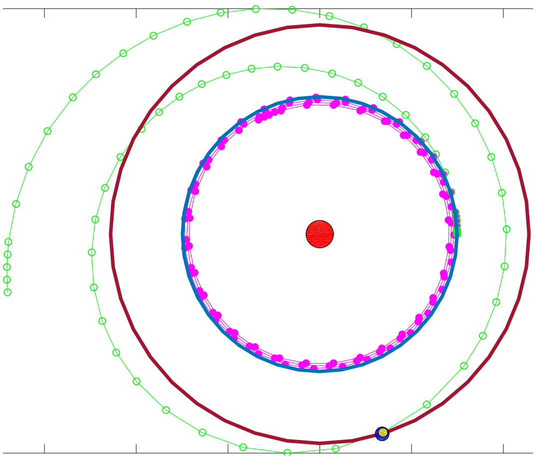

Figure 4: $alpha = 0.14915, A = 193600$

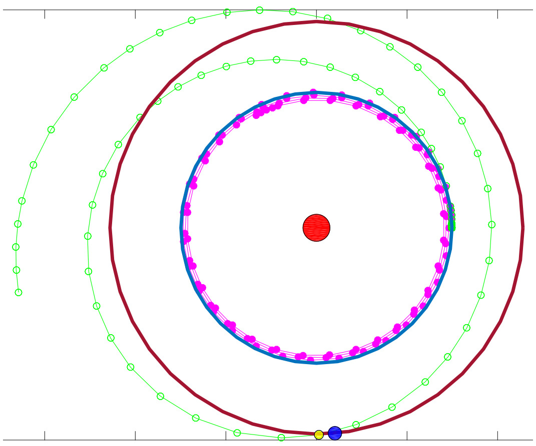

Figure 5: $alpha = 0.1099, A = 211600$

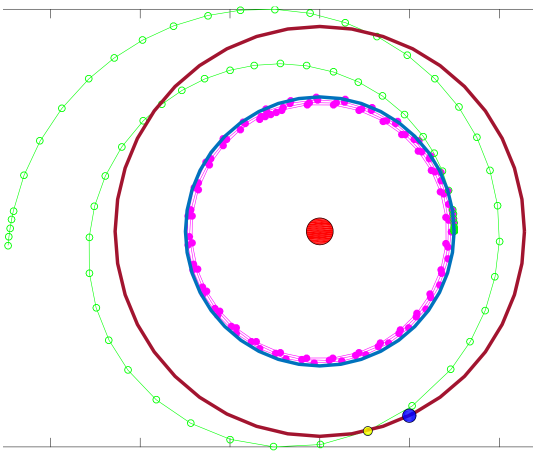

Figure 6: $alpha = 0.1256, A = 211600$

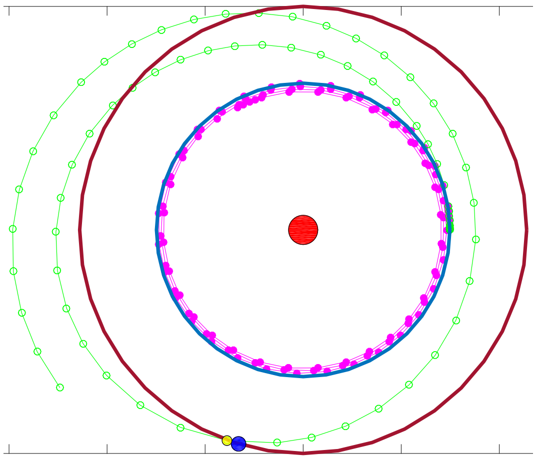

Figure 7: $alpha = 0.0658, A = 230400$

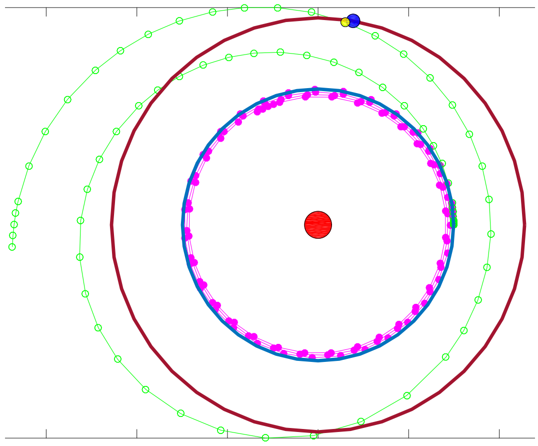

Figure 8: $alpha = 0.0942, A = 230400$