1 Assumption and Theory

1.1 Hypothesis

1) The energy emitted by the solar during unit time is constant;
2) The radiant energy absorbed by mercury, Venus, and Earth is negligible;
3) Only considered in the solar system.

1.2 List of Constants

Symbol	Description	Value
$S.$	Solar constant/W·s^{-2}	1367
G	Universal gravitational constant/$m^3kg^{-1}s^{-2}$	6.67259×10^{-11}
R_E	The distance from the sun to the earth/m	1.496×10^{11}
R_M	The distance from the sun to the Mars/m	2.2794×10^{11}
M_S	The masse of the sun/kg	1.9891×10^{30}
M_E	The masse of the earth/kg	5.965×10^{24}
M_M	The masse of the Mars/kg	6.4219×10^{23}
m	Total mass (sail plus payload)/kg	2000
T_E	The period of revolution of earth/s	3.1536×10^7 (365days)
T_M	The period of revolution of Mars/s	5.93568×10^7 (687days)
ω_E	The angular velocity of revolution of earth/rad·s^{-1}	1.9924×10^{-7}
ω_M	The angular velocity of revolution of Mars/rad·s^{-1}	1.0585×10^{-7}

Table 1: List of Constants

1.3 Light pressure received by solar sail

The solar constant (S.) refers to the solar radiation energy that received per second by per unit area of the top of the atmosphere circle perpendicular to the sun light on the average distance from sun to the earth ($D = 1.496 \times 10^8 km$). Similarly we denote the energy flow density at the distance of r from the sun as S(r). According to the conversation of energy under three hypothesis above:

$$S.\frac{4}{3}\pi {R_E}^2 = S(r)\frac{4}{3}\pi r^2$$

$$S.(r) = \frac{S.R_E^2}{r^2} \tag{1.1}$$

The energy density is:

$$\rho_E = \frac{E}{V} = \frac{S.(tAcos\alpha)}{ctAcos\alpha} = \frac{S.}{c} \tag{1.2}$$

The total light pressure is equal to twice its energy density:

$$P(r) = 2\rho_E = \frac{2S(r)}{c} = \frac{2S.{R_E}^2}{cr^2} \tag{1.3}$$

The ideal plane solar sail model assumes that the solar photon impacts on the sail surface as an ideal reflection, and the sail surface is an ideal plane. Therefore, the forces exert by incident photons and reflected photons are equal, shown as follows:

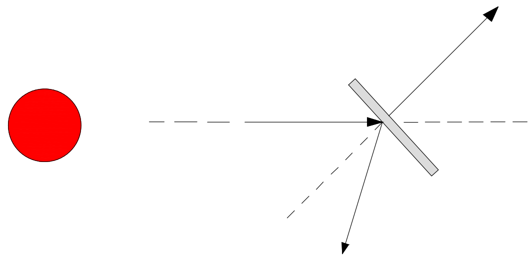

Figure 1: incident and reflected photon

The light pressure produced by the incident photon is:

$$P_i = \frac{P(r)}{2} A_z(cos\alpha\ \vec{n} - sin\alpha\ \vec{t}) \tag{1.4}$$

In this formula, A_z is the effective sail area, α is the angle between the incident ray and the solar sail unit normal direction $\vec{n}$, the solar sail normal $\vec{n}$ is perpendicular to the sail plane and deviates from the sun, and $\vec{t}$ is the tangent unit vector perpendicular to the normal $\vec{n}$, the anticlockwise direction is defined positive. The light pressure produced by the reflected light is as follows:

$$P_r = \frac{P(r)}{2} A_z(cos\alpha\ \vec{n} + sin\alpha\ \vec{t}) \tag{1.5}$$

The effective area refers to the area on which the sail is projected on the plane perpendicular to the sunlight, $A_z = Acos\alpha$. Therefore, the total light pressure acting on the ideal plane solar sail is:

$$\vec{F}_p = \vec{F}_i + \vec{F}_r = P(r)Acos^2\alpha\ \vec{n} = \frac{2S.{R_E}^2 Acos^2\alpha}{cr^2}\ \vec{n} \tag{1.6}$$

Therefore, the force of light pressure F_p is always along the normal direction of solar sail surface, $\vec{f} = \vec{n}$, $\vec{f}$ is the unit vector of thrust, it is always in the direction along the pressure direction and deviates from the sun.

1.4 Force analysis and acceleration

In this problem, the solar sail is mainly affected by the gravity of Sun and the pressure of sunlight, because the magnitude of solar sail due to the gravitational action of Mercury, Venus and Earth is very small which can be neglected in comparing to the relative solar gravitation.

The solar sail is subjected to the small thrust from the continuous light pressure, and thrust itself is inversely proportional to the square of distance from Sun to the solar sail, so it can be realized with spiral trajectory.

Light pressure is always deviated from the sun, by controling the attitude of solar sail, when the solar sail attitude angle α is positive, i.e. $F_p \cdot \alpha > 0$, solar sail get orbital angular momentum, solar sail and spiral outward away from the sun; when the sail angle is negative, i.e. $F_p \cdot \alpha < 0$, solar sail lose orbital angular momentum, and the solar sail moves toward the sun.

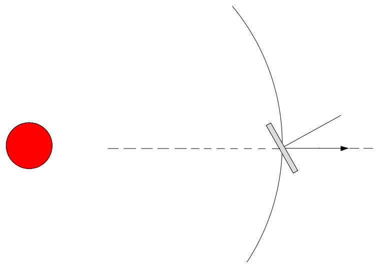

Figure 2: spiralling intward($F_P \cdot \alpha > 0$)

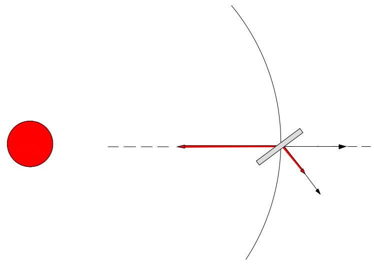

Figure 3: spiralling outward($F_P \cdot \alpha < 0$)

Denoted F_g as the gravity of Sun received by solar sail and $F_g = G\dfrac{M_S m}{r^2}$.
According to the Newton Second Law:

$$\vec{F_g} + \vec{F_p} = m\vec{a} \tag{1.7}$$

projected to the radial direction:

$$G\frac{M_S m}{r^2} + \frac{2S.{R_E}^2 Acos^2\alpha}{cr^2} \cdot cos\alpha = ma \tag{1.8}$$

so we obtain the normal acceleration as:

$$a = \frac{GM_S}{r^2} + \frac{2S.{R_E}^2 Acos^3\alpha}{mcr^2} \tag{1.9}$$

Numerical application:

$$a = \frac{13.27 \times 10^{19}}{r^2} + \frac{1.02 \times 10^{14} Acos^3\alpha}{r^2} \tag{1.10}$$