# Chapter 6

# The Buddhabrot and Subsets of the Buddhabrot

## 6.1 The Buddhabrot

If for the Mandelbrot set we plot the paths of points which diverge, rather than how quickly they diverge, we can create another way of displaying the Mandelbrot set [21]. Basically we are overlaying the sequences (3.1) for points in the complex plane but **not** in the Mandelbrot set (if we included only values in the Mandelbrot set we get what is called the Anti-Buddhabrot see figure 6.2). This process produces something like figure 6.1.

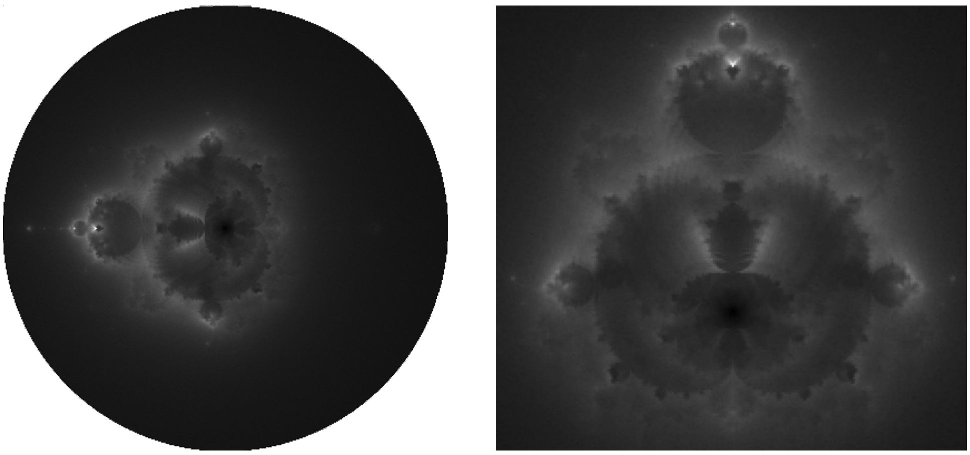

Figure 6.1: The Buddhabrot, the right image shows how it got its name.

You can see how similar figure 6.1 is to the Mandelbrot set, but the inside of the set now shows patterns, which resemble stretched and rotated versions of the Mandelbrot set.

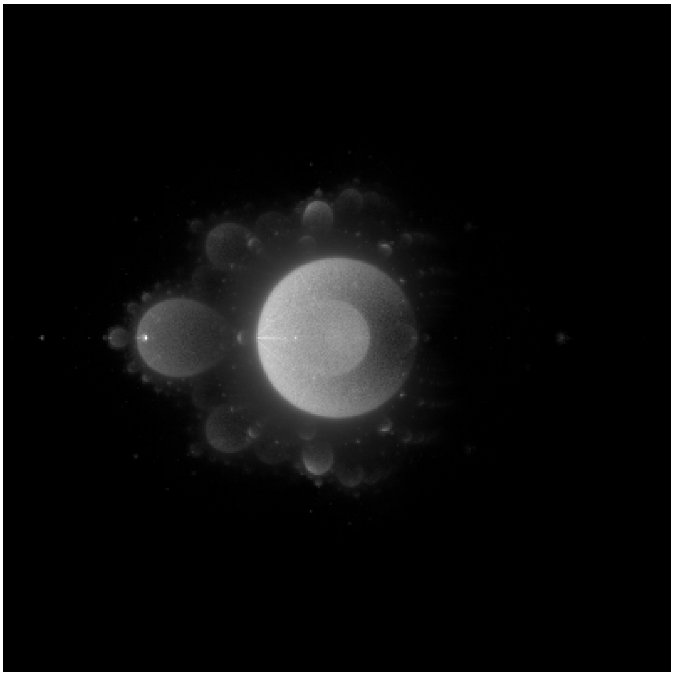

Figure 6.2: The Anti-Buddhabrot.

## 6.2 Programming the Buddhabrot

As in the other programmes we set up an array acting as the $4 \times 4$ complex plane centred on the origin. Then we pick random points in the complex plane and check if they are in the Mandelbrot set, if they are we discard them otherwise we increment the array representing the plane by one in the grid point they are closest to, and do the same for every point in the sequence (3.1) with an absolute value less than two. It is important for the number of random points we pick to be at least 100 times larger than the number of points in the array, otherwise we will not get a good picture. Note that we start the sequences at $c$ rather than $z_0 = 0$ as otherwise 0 would then be in every random sequence and would out-shine all other pixels. See appendix A.4 for the actual Python code in its entirety.

## 6.3 Subsets of the Buddhabrot

What if instead of plotting random points sampled from all over the $4 \times 4$ square we just sampled from the $\pi$-paths going into pinch points on the Mandelbrot set. For example finding the sequences for $c = -\frac{3}{4} + \varepsilon i$ with $\varepsilon$ a random (non-zero) real number between 0 and 1 say. Well this gives a subset of the Buddhabrot, figure 6.3.

Figure 6.3: A subset of the Buddhabrot for $c = -\frac{3}{4} + \varepsilon i$.

We can do this for other paths into pinch points, such as the line $c = \frac{1}{4} + \varepsilon$ and the parabola $c = -\frac{5}{4} - \varepsilon^2 + \varepsilon i$. The first subset is quite boring and just a line, but the second subset is much more interesting, figure 6.4.

I then thought about what happens for smaller $\varepsilon$ and how if we ordered the random $\varepsilon$s then see how the subset in figure 6.4 forms as we travel away from the pinch point along the path. So I made separate subsets for random $\varepsilon$ between 0 and 0.01, then between 0 and 0.02, and so on. This showed that for smaller $\varepsilon$ more points were added per $\varepsilon$, which makes sense since those points are nearer the Mandelbrot set and therefore diverge slower. Then when picking $0.001 \leq \varepsilon \leq 0.041$ I noticed a familiar shape hiding in the negative space of the image, see figure 6.5. This looks like a Julia set with $c \approx -\frac{5}{4} - 0.021^2 + 0.021i$. Since the Buddhabrot consists of sequences which **do** diverge, every point ($z_n$ for some $n \in \mathbb{N}$) can be thought of as the initial point ($z_0$) in a new Julia sequence (4.1) which will diverge also, therefore the points ($z_n$) will not be in that Julia set. Since in figure 6.5 we used $c$ values close together, the Julia sets for those $c$s look similar, see figure 6.6. Hence the negative space in figure 6.5 is the intersection of the Julia sets for that range of $c$ values $\square$.

Since we used a small 'imax' the points on the path were in the Mandelbrot set more than if we had used a larger 'imax' (see figure 6.7), resulting in the negative image

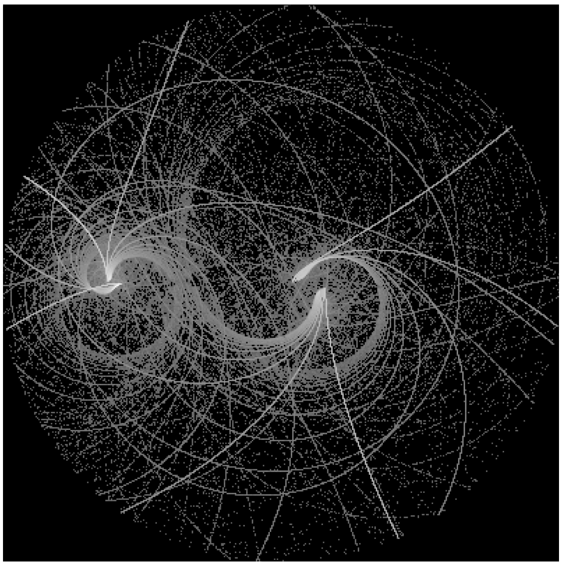

Figure 6.4: A subset of the Buddhabrot for $c = -\frac{5}{4} - \varepsilon^2 + \varepsilon i$.

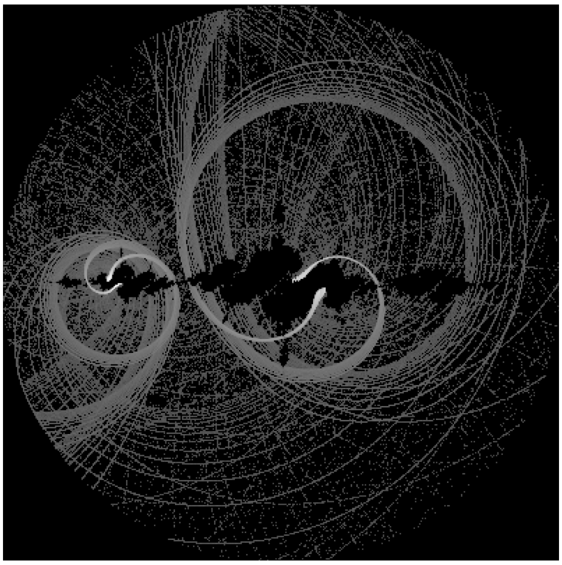

Figure 6.5: A subset of the Buddhabrot for $c = -\frac{5}{4} - \varepsilon^2 + \varepsilon i$, with small $\varepsilon$.

looking like the intersection of mostly Julia sets rather than Fatou sets.

Another result from this effect is shown by marking the last few points in the end of each sequence before it grows beyond 2. If we mark the last term in each sequence then

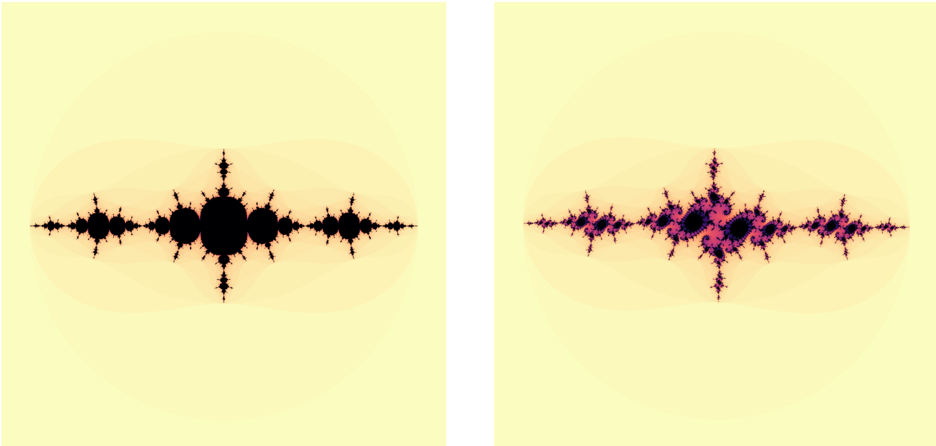

Figure 6.6: The Julia sets with $c = -1.250001 + 0.001i$ (left) and $c = -1.251681 + 0.041i$ (right), both with imax = 100.

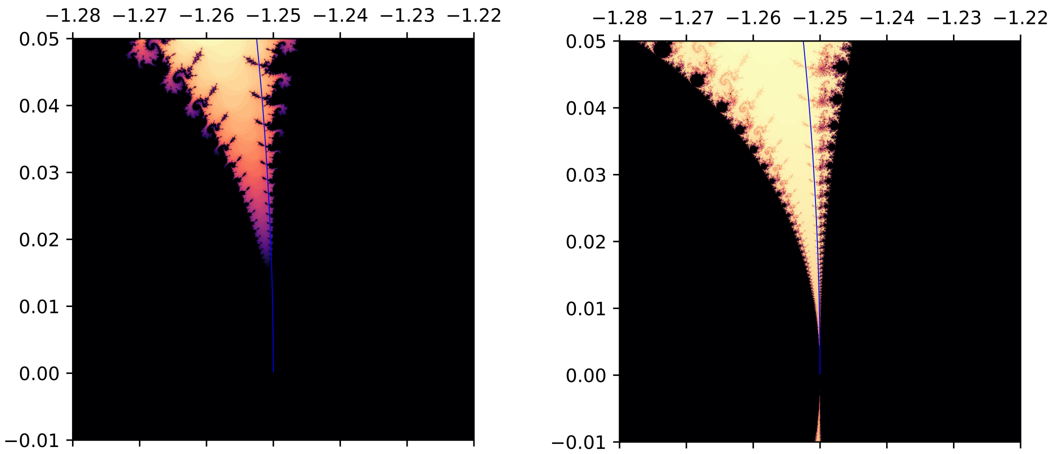


Figure 6.7: The point-bridge at -1.25 in the Mandelbrot set, with the blue line being the parabola into the point, the left image has imax = 100 and the right imax = 1000.

the resulting shape will be the part of the "average" Julia set where the sequence (4.1) with those $z_0$s will be beyond 2 after only 1 iteration. If we mark the last two terms this will be the area where the sequence goes past 2 after more than 2 iterations, and so on, see figure 6.8. In fact plotting the last "imax" points will give the outline of the Julia set computed for that same "imax". Going back to section 4.1, since for these values of $c$ we would have a Fatou set and thus would appear empty not leaving the negative image we see. But since the "imax" used was not sufficiently large these points would be in the Mandelbrot set for that same "imax" and thus the negative image in figure 6.5 looks more like a continuous Julia set than a Fatou set.

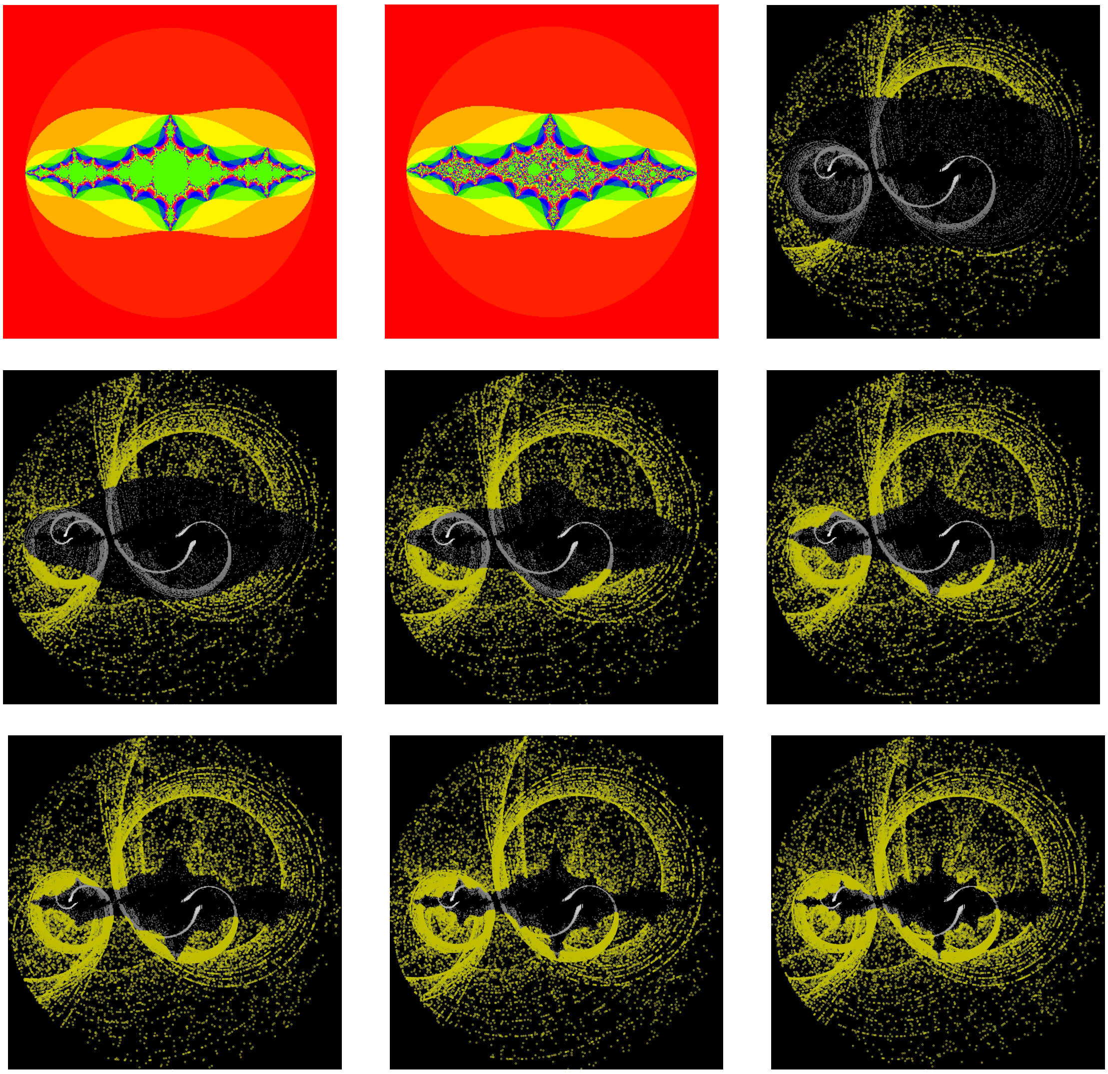

Figure 6.8: From left to right and top to bottom: The Julia set with $c = -1.250001 + 0.001i$, the Julia set with $c = -1.251681+0.041i$ both in a colour scheme to emphasise the different number of iterations, a subset of the Buddhabrot for the path $c = -1.25-\varepsilon^2+\varepsilon i$ with the last terms highlighted, a subset with the last two terms highlighted, etc.

You can see in figure 6.3 and 6.4 that the points are along curves emanating from 2 and 4 small regions respectively. This is because of the behaviour of the Mandelbrot sequence for $c = -0.75$ and $c = -1.25$, more specifically from the periodicity of bulbs of the Mandelbrot set. See chapter 5.

The curves we see in figure 6.4 are transformations of the parabola $c = -1.25 - \varepsilon^2 + \varepsilon i$

through the function $f(z) = z^2 + c$. The function has a cyclical nature for this $c$ and the curve will emanate from the same point as it initially did after four iterations. We can see this by plotting a set of points on the parabola and iterating each, see figure 6.9 where the colours match modulo 4, for the complete code see appendix A.5. So basically figure 6.5 is figure 6.9 with points over the Julia set removed.

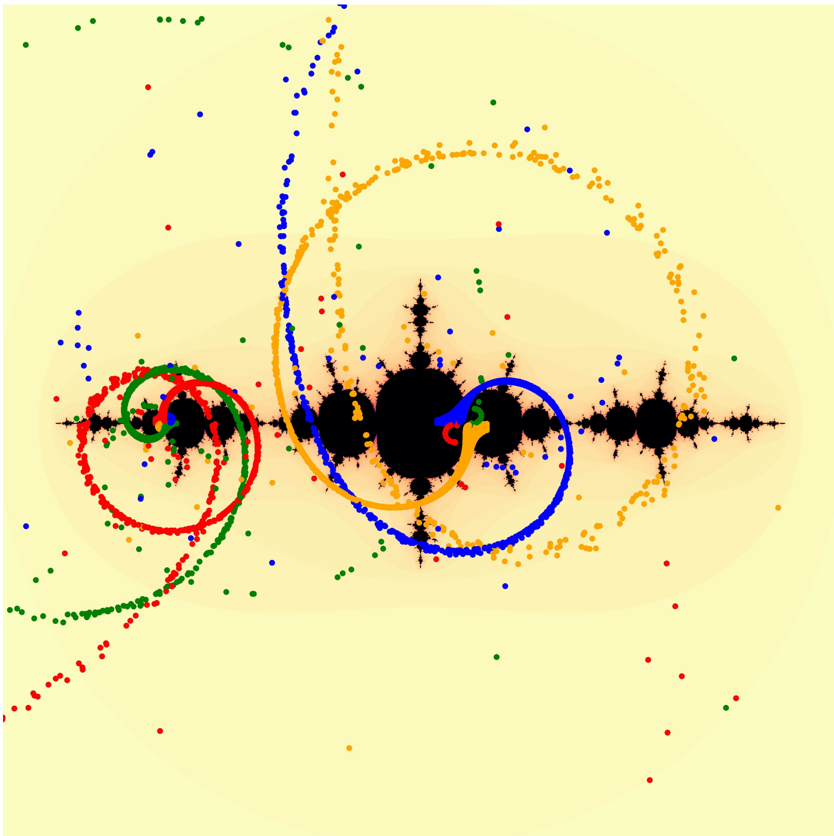

Figure 6.9: Points on the small parabola iterated with the Julia set of $c = -1.250001 + 0.001i$ as the background.