CSC446 Term Project Report

Project Description

The following statistics will represent various possible queuing systems for a theoretical bank teller system. The systems simulated will be a M/M/1 system, M/G/1 system, M/M/c/N system, and a system comprised of a network of queues.

Problem Mapping

For the M/M/1 simulations customers will arrive with times following an exponential distribution and the teller will serve them with service times also following an exponential distribution.

For the M/G/1 simulations customers will arrive following an exponential distribution and the teller will serve them with service times that follow a uniform distribution.

For the M/M/c/N simulations it is assumed that there are 3 different tellers that will help customers based on availability. It is also assumed that the bank has a maximum capacity of 20 customers and that any customers that arrive when the bank is at capacity will be turned away. Customers will arrive following an exponential distribution and the tellers will all serve them with service times also following an exponential distribution.

For the network of queues there is an initial reception queue (queue 1) where customers will be diverted to one of two queues with different tellers. One of these secondary queues is for customers that are looking for assistance with their investments (queue 2), and the other is for customers that are looking for assistance with their day-to-day banking needs (queue 3). It is assumed that 30% of customers will need help with their investments and 70% will need help with their day-to-day banking. All three queues in this system will follow a M/M/1 model.

Simulation Goals and Parameters

For the analysis of the simulations, the recorded statistics (L, L_Q, W, W_Q) will be plotted against changes in their server utilization to show how they preform under higher customer arrival rates. Each measured value will also contain a vertical bar at each data point that represents the confidence interval. These confidence intervals are often very small, especially at lower utilization, and are not visible due to overlap with theoretical values.

Each simulation was run with 1,000,000 customers and had the following parameters:

M/M/1 – service time mean of 1 with arrival time mean varying from 0.1 to 0.8 in steps of 0.1 in order to obtain utilizations of 0.1 to 0.8

M/G/1 – uniform(0, 2) service time distribution with a mean of 1 and arrival time mean varying from 0.1 to 0.8 in steps of 0.1 in order to obtain utilizations of 0.1 to 0.8

M/M/c/N – 3 servers and 20 total system capacity. Service time mean of 1 with arrival time mean varying from 0.3 to 2.4 in steps of 0.3 in order to obtain utilizations of 0.1 to 0.8

Network of queues – all 3 queues have exponential service distribution and have service time mean of 1, with queue 1 arrival time mean varying from 0.1 to 0.8 in steps of 0.1 in order to obtain queue 1 utilizations of 0.1 to 0.8. Queue 1 also has a routing probability of 0.3 for queue 2 and 0.7 for queue 3.

Methodology

JMT was used to simulate all of the different queueing models. Two different custom python scripts were also made, one to calculate the theoretical values and probability of loss for the M/M/c/N model, and another to calculate the confidence intervals using the confidence provided by JMT, which has a default value of 0.99. The statistics were calculated through JMT's Performance Indices feature and were recorded as follows:

L = number of customers in system

L_Q = arrival rate * queue time

W = response time of system

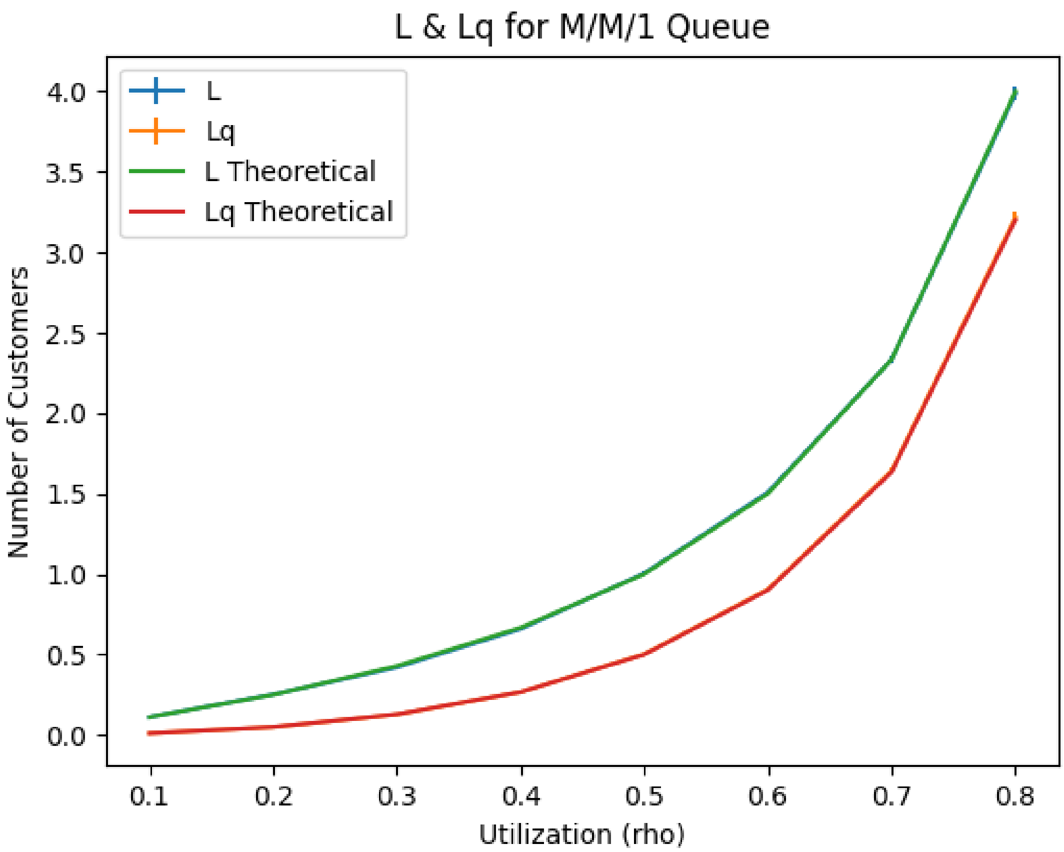
W_Q = queue time

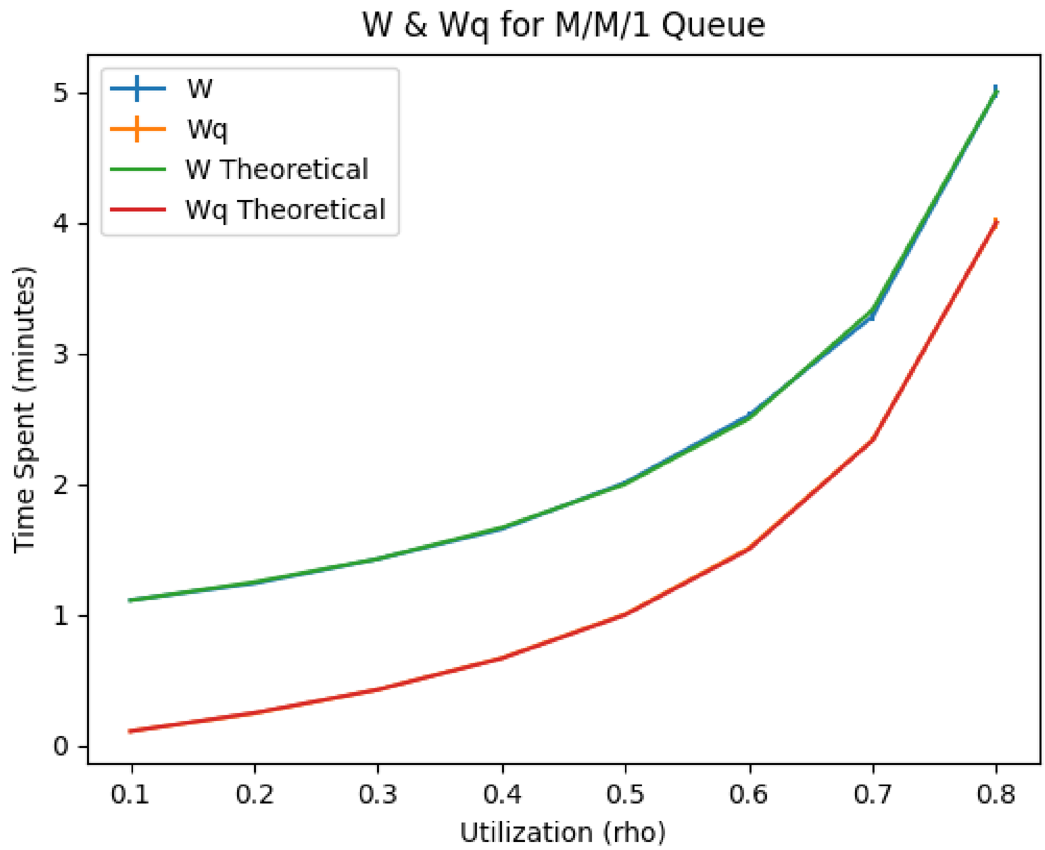
ρ = utilization

Analysis

M/M/1 System

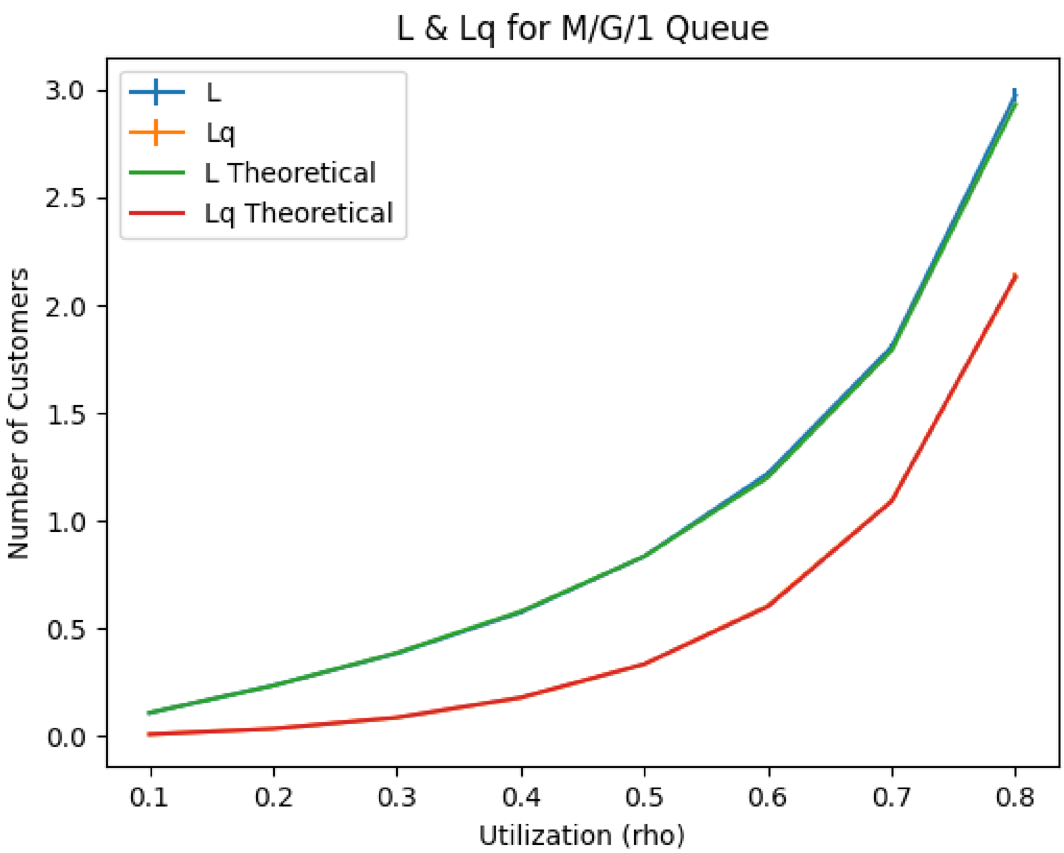
For the M/M/1 model all recorded statistics stay true to their theoretical values, with only the average time spent in the bank per customer (W) varying slightly from its theoretical with utilizations in the range of 0.6 – 0.8. All values fall within the given 0.99 confidence interval.

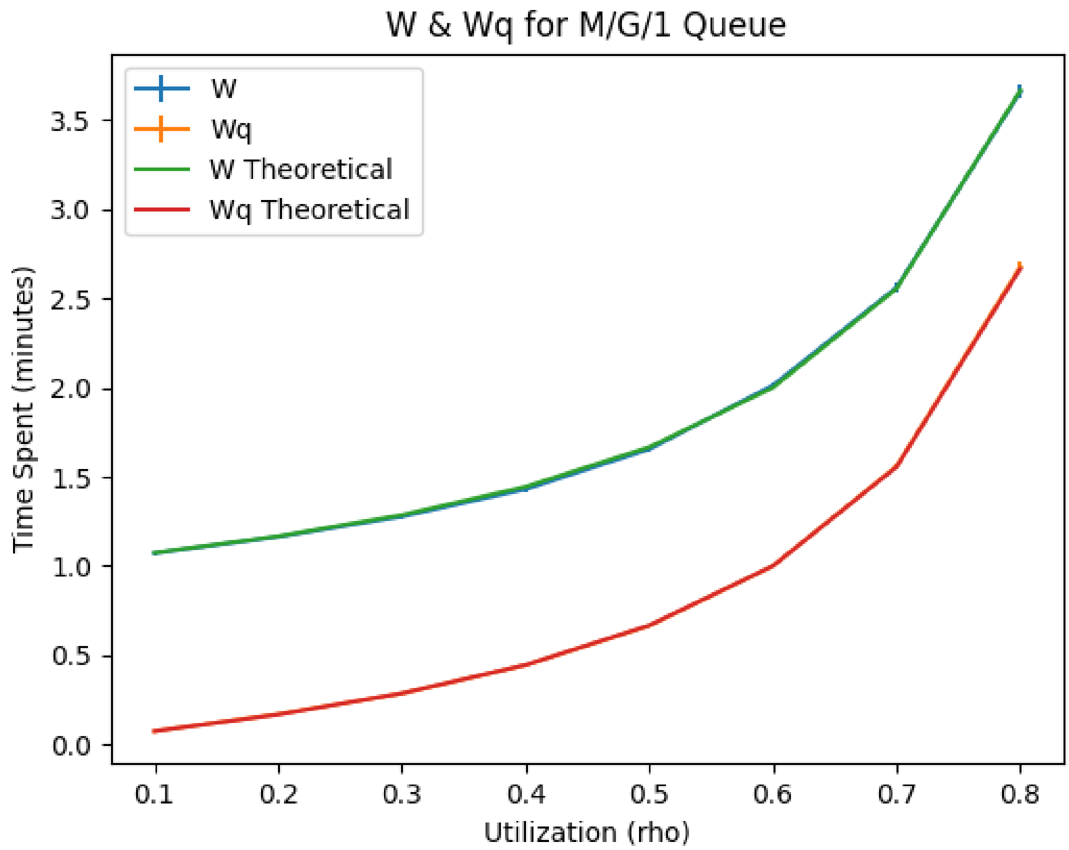



W & Wq for M/M/1 Queue
W
Wq
W Theoretical
Wq Theoretical
Time Spent (minutes)
Utilization (rho)
0
1
2
3
4
5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

M/G/1 System

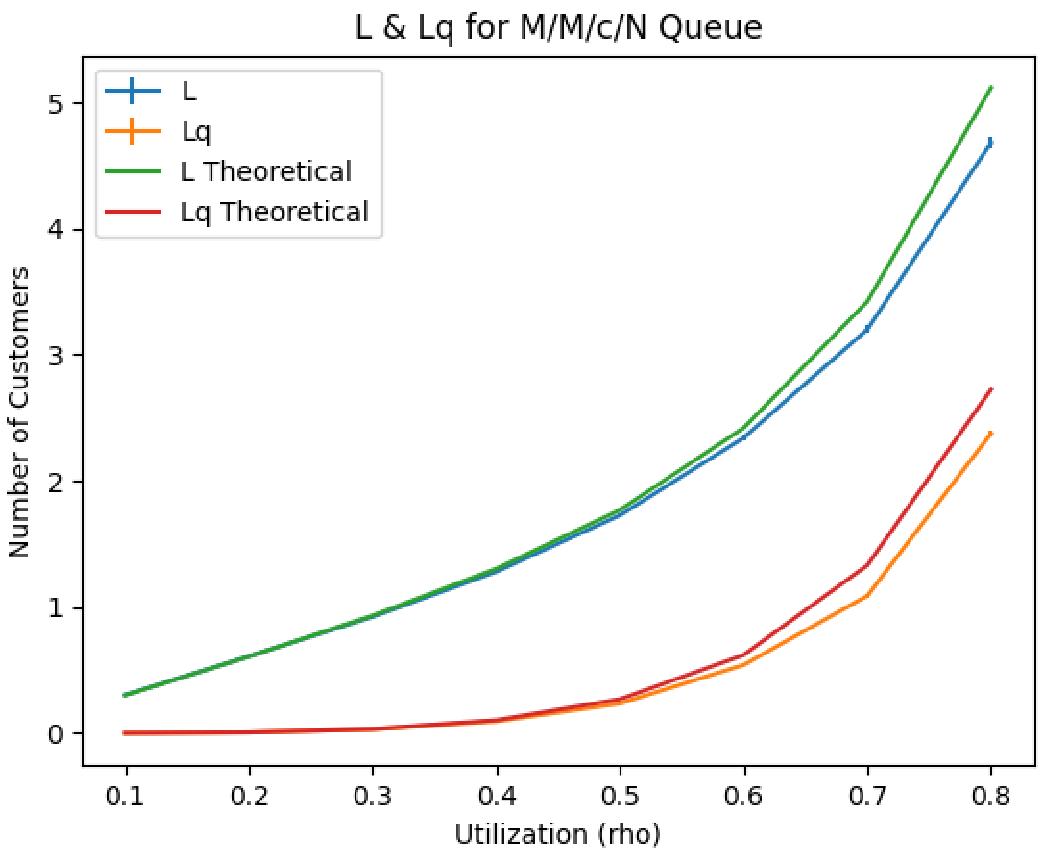
For the M/G/1 model all recorded statistics stay true to their theoretical values, with only the average number of customers in the bank (L) varying slightly from its theoretical with utilizations in the range of 0.6 - 0.8. All values fall within the given 0.99 confidence interval.

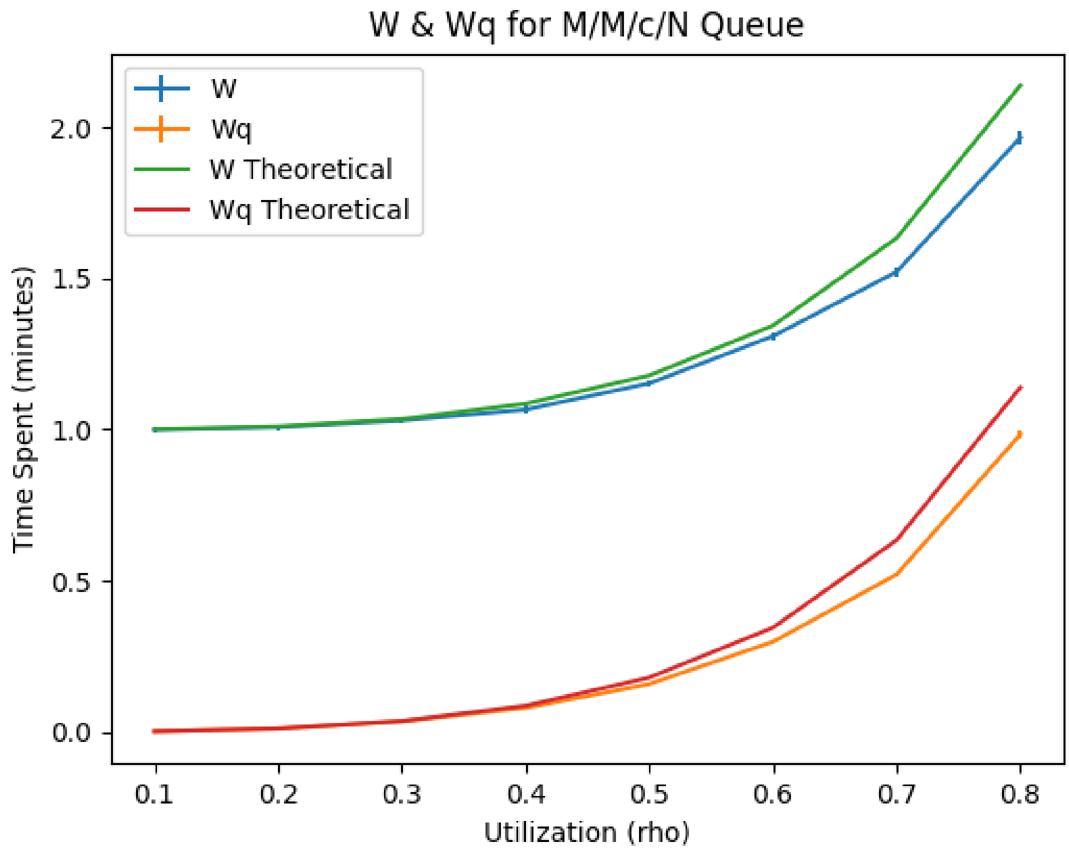



W & Wq for M/G/1 Queue
W
Wq
W Theoretical
Wq Theoretical
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Time Spent (minutes)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Utilization (rho)

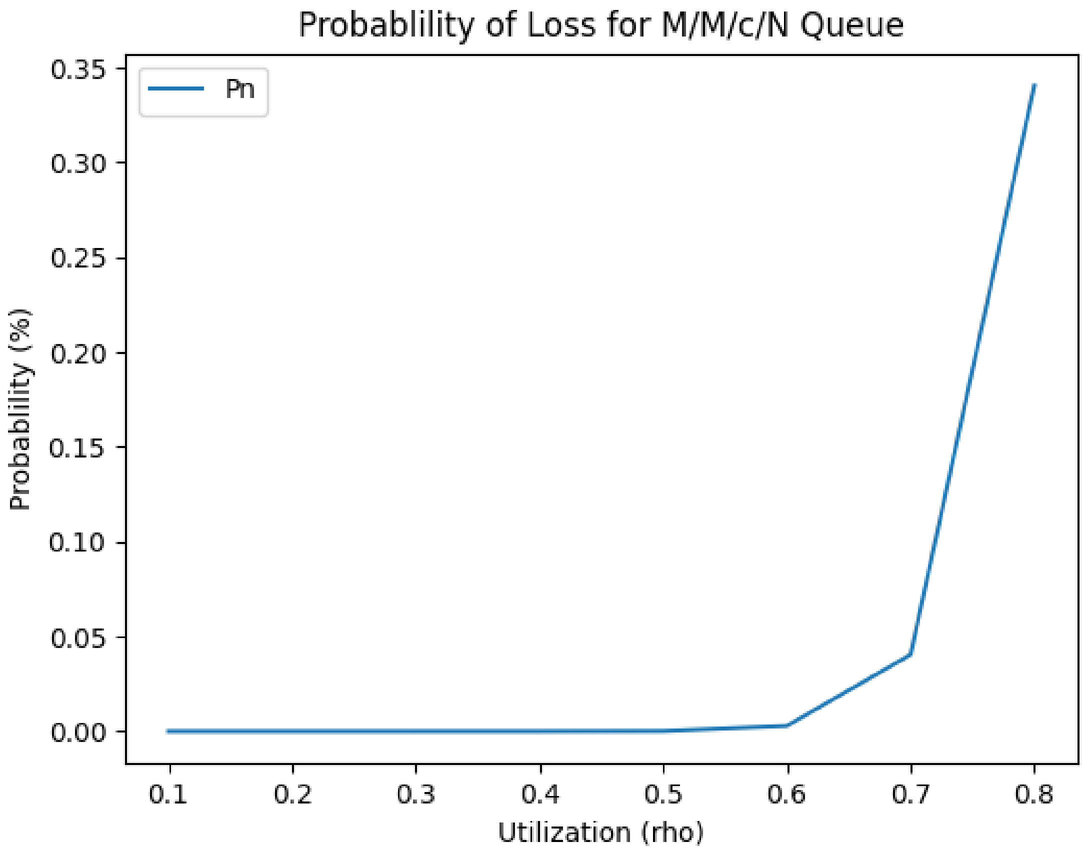
M/M/c/N System

For the M/M/c/N model all recorded statistics stay true to their theoretical values in the utilization range of 0.1 - 0.5, Beyond that range, all simulation statistics fall below the theoretical values with increasing magnitude as utilization levels increase. For the M/M/c/N system the probability of customer loss (Pn) was also studied. Pn stays negligible and close to 0 until a utilization level of 0.7 where it rises to approximately 0.04%, and then spikes to approximately 0.3% by a utilization of 0.8. All values still fall within the given 0.99 confidence interval.





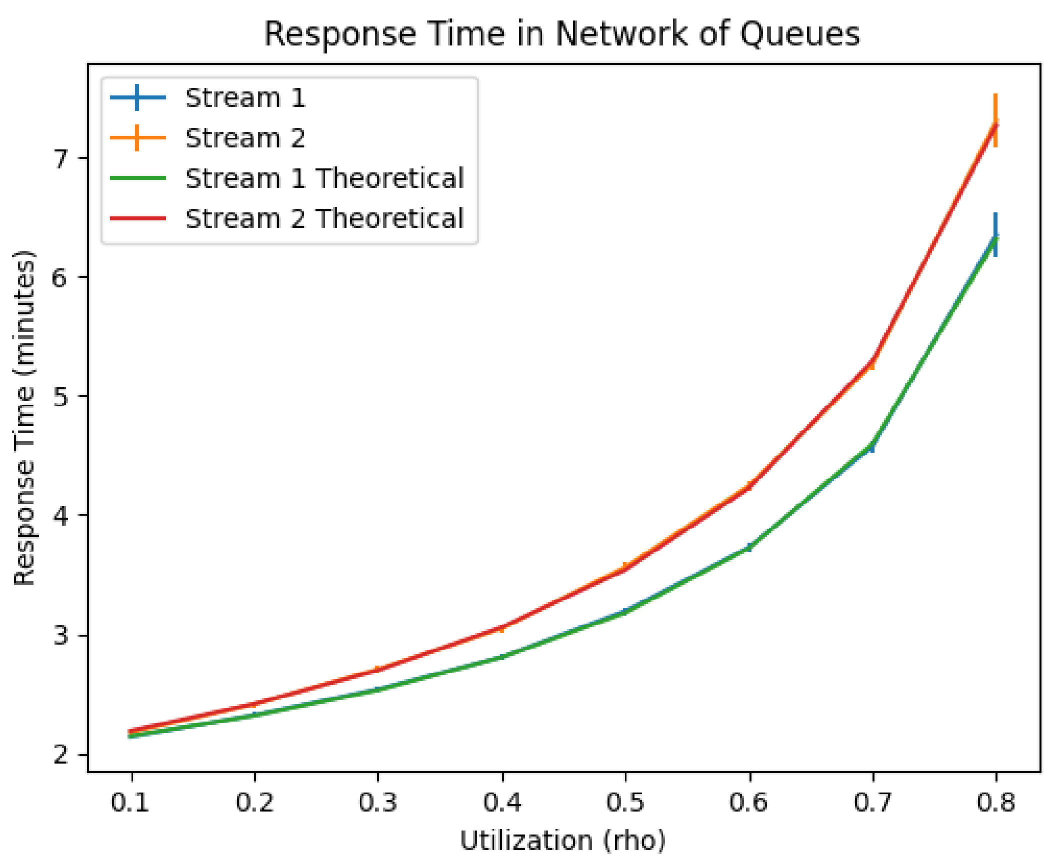
W & Wq for M/M/c/N Queue
W
Wq
W Theoretical
Wq Theoretical
Time Spent (minutes)
2.0
1.5
1.0
0.5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Utilization (rho)



Probablility of Loss for M/M/c/N Queue
Pn
Probablility (%)
Utilization (rho)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Network of Queues

For the network of queues only the time spent in the bank (W) was recorded but separately for customers looking for assistance with their investments (represented by stream 1) and for customers that are looking for assistance with their day-to-day banking (represented by stream 2). In both streams W stayed consistent with its theoretical values for all utilizations. All values fall within the given 0.99 confidence interval up until a utilization of 0.8 where the initial reception queue (queue 1) failed to converge within its 0.99 confidence interval and had to be reduced to 0.97.



Conclusion

The simulations of all possible queuing systems for the bank stayed relatively true to their theoretical values, only ever varying slightly at higher utilizations. The one exception is the M/M/c/N queue statistics. This is possibly due to volatility of the probability of loss, which the other systems did not have. All values also stayed within JMT's 0.99 confidence except for the average time spent in the bank (W) at 0.8 utilization in the network of queues system which had to be changed to 0.97 in order for the system to converge. This is most likely due to the randomness of the routing between streams.

When comparing the M/M/1 system to the M/G/1 system, the M/G/1 system had lower average number of customers in the bank and in the queue (L and Lq) and lower maximum average time spent in the bank and in the queue (W and Wq) than the M/M/1 system. This is most likely because the M/G/1 queue had the faster service rate due to the uniformly distributed service time.

The M/M/c/N system had the lowest maximum average time spent in the bank and in the queue (W and Wq) of all the systems. It is unlikely to be due to the probability of loss since the maximum probability of loss is less than 0.35% at utilizations as high as 0.8. Therefore, the lower time spent in the bank is most due to the total of 3 tellers.

*NOTE – If interested, here is a GitHub link to the JMT models used along with the python scripts created that contain all of the raw data and deal with the M/M/c/N and confidence interval calculations and plotting https://github.com/EthanWelburn/CSC446.git

Raw Data

M/M/1 Queue

For $\rho = 0.1$

Theoretical values:

$L = 0.1111$, $L_Q = 0.01111$, $W = 1.1111$, $W_Q = 0.1111$, $\rho = 0.1$

Collected values:

$L = 0.1115$, $L_Q = 0.01119$, $W = 1.1125$, $W_Q = 0.1119$, $\rho = 0.0999$

For $\rho = 0.2$

Theoretical values:

$L = 0.25$, $L_Q = 0.05$, $W = 1.25$, $W_Q = 0.25$, $\rho = 0.2$

Collected values:

$L = 0.2518$, $L_Q = 0.0492$, $W = 1.2408$, $W_Q = 0.2467$, $\rho = 0.2$

For $\rho = 0.3$

Theoretical values:

$L = 0.42857$, $L_Q = 0.12857$, $W = 1.42857$, $W_Q = 0.42857$, $\rho = 0.3$

Collected values:

$L = 0.4206$, $L_Q = 0.1278$, $W = 1.4274$, $W_Q = 0.4289$, $\rho = 0.2979$

For $\rho = 0.4$

Theoretical values:

$L = 0.6667$, $L_Q = 0.2667$, $W = 1.6667$, $W_Q = 0.6667$, $\rho = 0.4$

Collected values:

$L = 0.6585$, $L_Q = 0.2668$, $W = 1.6554$, $W_Q = 0.6699$, $\rho = 0.3995$

For $\rho = 0.5$

Theoretical values:

$L = 1$, $L_Q = 0.5$, $W = 2$, $W_Q = 1$, $\rho = 0.5$

Collected values:

$L = 1.0045$, $L_Q = 0.5026$, $W = 2.0107$, $W_Q = 1.0049$, $\rho = 0.5041$

For $\rho = 0.6$

Theoretical values:

$L = 1.5$, $L_Q = 0.9$, $W = 2.5$, $W_Q = 1.5$, $\rho = 0.6$

Collected values:

$L = 1.5090$, $L_Q = 0.9054$, $W = 2.5258$, $W_Q = 1.5105$, $\rho = 0.5938$

For $\rho = 0.7$

Theoretical values:

$L = 2.3333$, $L_Q = 1.6333$, $W = 3.3333$, $W_Q = 2.3333$, $\rho = 0.7$

Collected values:

$L = 2.3333$, $L_Q = 1.6422$, $W = 3.2831$, $W_Q = 2.3376$, $\rho = 0.7087$

For $\rho = 0.8$

Theoretical values:

$L = 4$, $L_Q = 3.2$, $W = 5$, $W_Q = 4$, $\rho = 0.8$

Collected values:

$L = 3.9848$, $L_Q = 3.2161$, $W = 4.9976$, $W_Q = 3.9976$, $\rho = 0.8011$

M/G/1 Queue (using uniform(0, 2) mean = 1)

For $\rho = 0.1$

Theoretical values:

$L = 0.1074$, $L_Q = 0.0074$, $W = 1.0741$, $W_Q = 0.0741$, $\rho = 0.1$

Collected values:

$L = 0.1068$, $L_Q = 0.0074$, $W = 1.0722$, $W_Q = 0.0741$, $\rho = 0.0996$

For $\rho = 0.2$

Theoretical values:

$L = 0.2333$, $L_Q = 0.03333$, $W = 1.1667$, $W_Q = 0.1667$, $\rho = 0.2$

Collected values:

$L = 0.2338$, $L_Q = 0.0336$, $W = 1.1636$, $W_Q = 0.1685$, $\rho = 0.2000$

For $\rho = 0.3$

Theoretical values:

$L = 0.3857$, $L_Q = 0.0857$, $W = 1.2857$, $W_Q = 0.2857$, $\rho = 0.3$

Collected values:

$L = 0.3830$, $L_Q = 0.0845$, $W = 1.2772$, $W_Q = 0.2849$, $\rho = 0.2982$

For $\rho = 0.4$

Theoretical values:

$L = 0.5778$, $L_Q = 0.1778$, $W = 1.4444$, $W_Q = 0.4444$, $\rho = 0.4$

Collected values:

$L = 0.5728$, $L_Q = 0.1770$, $W = 1.4304$, $W_Q = 0.4440$, $\rho = 0.3985$

For $\rho = 0.5$

Theoretical values:

$L = 0.8333$, $L_Q = 0.3333$, $W = 1.6667$, $W_Q = 0.6667$, $\rho = 0.5$

Collected values:

$L = 0.8334$, $L_Q = 0.3322$, $W = 1.6545$, $W_Q = 0.6647$, $\rho = 0.4979$

For $\rho = 0.6$

Theoretical values:

$L = 1.2$, $L_Q = 0.6$, $W = 2$, $W_Q = 1$, $\rho = 0.6$

Collected values:

$L = 1.2191$, $L_Q = 0.6051$, $W = 2.0132$, $W_Q = 1.0003$, $\rho = 0.6047$

For $\rho = 0.7$

Theoretical values:

$L = 1.7889$, $L_Q = 1.0889$, $W = 2.5556$, $W_Q = 1.5556$, $\rho = 0.7$

Collected values:

$L = 1.8069$, $L_Q = 1.0911$, $W = 2.5621$, $W_Q = 1.5592$, $\rho = 0.6922$

For $\rho = 0.8$

Theoretical values:

$L = 2.9333$, $L_Q = 2.1333$, $W = 3.6667$, $W_Q = 2.6667$, $\rho = 0.8$

Collected values:

$L = 2.9749$, $L_Q = 2.1272$, $W = 3.6582$, $W_Q = 2.6781$, $\rho = 0.7999$

M/M/c/N Queue ($c = 3$, $N = 20$)

For $\rho = 0.1$

Theoretical values:

$L = 0.3004$, $L_Q = 0.0004$, $W = 1.0014$, $W_Q = 0.00138$, $\rho = 0.1$

Collected values:

$L = 0.3025$, $L_Q = 0.000426$, $W = 0.9995$, $W_Q = 0.00142$, $\rho = 0.1007$

For $\rho = 0.2$

Theoretical values:

$L = 0.6063$, $L_Q = 0.0063$, $W = 1.0105$, $W_Q = 0.0105$, $\rho = 0.2$

Collected values:

$L = 0.6087$, $L_Q = 0.00608$, $W = 1.0079$, $W_Q = 0.0101$, $\rho = 0.2012$

For $\rho = 0.3$

Theoretical values:

$L = 0.9316$, $L_Q = 0.0316$, $W = 1.0351$, $W_Q = 0.0350$, $\rho = 0.3$

Collected values:

$L = 0.9215$, $L_Q = 0.03009$, $W = 1.0305$, $W_Q = 0.0339$, $\rho = 0.2963$

For $\rho = 0.4$

Theoretical values:

$L = 1.3028$, $L_Q = 0.1028$, $W = 1.0857$, $W_Q = 0.0857$, $\rho = 0.4$

Collected values:

$L = 1.2792$, $L_Q = 0.0935$, $W = 1.0660$, $W_Q = 0.0785$, $\rho = 0.3970$

For $\rho = 0.5$

Theoretical values:

$L = 1.7686$, $L_Q = 0.2686$, $W = 1.1791$, $W_Q = 0.1790$, $\rho = 0.5$

Collected values:

$L = 1.7268$, $L_Q = 0.2350$, $W = 1.1529$, $W_Q = 0.1575$, $\rho = 0.4970$

For $\rho = 0.6$

Theoretical values:

$L = 2.4192$, $L_Q = 0.6193$, $W = 1.3441$, $W_Q = 0.3441$, $\rho = 0.6$

Collected values:

$L = 2.3409$, $L_Q = 0.5407$, $W = 1.3091$, $W_Q = 0.2974$, $\rho = 0.6058$

For $\rho = 0.7$

Theoretical values:

$L = 3.4210$, $L_Q = 1.3306$, $W = 1.6339$, $W_Q = 0.6339$, $\rho = 0.7$

Collected values:

$L = 3.1997$, $L_Q = 1.0884$, $W = 1.5222$, $W_Q = 0.5201$, $\rho = 0.6955$

For $\rho = 0.8$

Theoretical values:

$L = 5.1165$, $L_Q = 2.7247$, $W = 2.1392$, $W_Q = 1.1392$, $\rho = 0.8$

Collected values:

$L = 4.6799$, $L_Q = 2.3752$, $W = 1.9668$, $W_Q = 0.9838$, $\rho = 0.7973$

Probability of loss

For $\rho = 0.1$

3.3444816053511686e-18

For $\rho = 0.2$

2.637558412521046e-12

For $\rho = 0.3$

6.656790123454948e-09

For $\rho = 0.4$

1.5899108149426105e-06

For $\rho = 0.5$

0.00010248461421261183

For $\rho = 0.6$

0.0027992283689701304

For $\rho = 0.7$

0.04036921023468757

For $\rho = 0.8$

0.3405057079364419

Network of queues (stream 1 has probability of 0.3, stream 2 has a probability of 0.7)

For $\rho = 0.1$

Stream 1 response time = 2.1364

Stream 2 response time = 2.1741

For $\rho = 0.2$

Stream 1 response time = 2.3181

Stream 2 response time = 2.4094

For $\rho = 0.3$

Stream 1 response time = 2.5347

Stream 2 response time = 2.7026

For $\rho = 0.4$

Stream 1 response time = 2.8045

Stream 2 response time = 3.0453

For $\rho = 0.5$

Stream 1 response time = 3.1916

Stream 2 response time = 3.5612

For $\rho = 0.6$

Stream 1 response time = 3.7270

Stream 2 response time = 4.2448

For $\rho = 0.7$

Stream 1 response time = 4.5770

Stream 2 response time = 5.2676

For $\rho = 0.8$

Stream 1 response time = 6.3543

Stream 2 response time = 7.3127