Efficient Algorithms for Citation Network Analysis

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Abstract

In the paper very efficient, linear in number of arcs, algorithms for determining Hummon and Doreian's arc weights SPLC and SPNP in citation network are proposed, and some theoretical properties of these weights are presented. The nonacyclicity problem in citation networks is discussed. An approach to identify on the basis of arc weights an important small subnetwork is proposed and illustrated on the citation networks of SOM (self organizing maps) literature and US patents.

Keywords: large network, acyclic, citation network, main path, CPM path, arc weight, algorithm, self organizing maps, patent

1 Introduction

The citation network analysis started with the paper of Garfield et al. (1964) [10] in which the introduction of the notion of citation network is attributed to Gordon Allen. In this paper, on the example of Asimov's history of DNA [1], it was shown that the analysis "demonstrated a high degree of coincidence between an historian's account of events and the citational relationship between these events". An early overview of possible applications of graph theory in citation network analysis was made in 1965 by Garner [13].

The next important step was made by Hummon and Doreian (1989) [14, 15, 16]. They proposed three indices (NPPC, SPLC, SPNP) – weights of arcs that provide us with automatic way to identify the (most) important part of the citation network – the main path analysis.

In this paper we make a step further. We show how to efficiently compute the Hummon and Doreian's weights, so that they can be used also for analysis of very large citation networks with several thousands of vertices. Besides this some theoretical properties of the Hummon and Doreian's weights are presented.

The proposed methods are implemented in **Pajek** – a program, for Windows (32 bit), for analysis of *large networks*. It is freely available, for noncommercial use, at its homepage [4].

For basic notions of graph theory see Wilson and Watkins [18].

	m							
10	60	0	1	35	3	11	7	5
23	60 657	1	5	218	1	16	19	134

network **DNA** Coupling Small world Small & Griffith Cocitation Scientometrics Kroto SOM Zewail Lederberg Desalination

Table 1: Citation network characteristics

Citation Networks

US patents

In a given set of units U (articles, books, works, ...) we introduce a *citing* relation $R \subseteq \mathbf{U} \times \mathbf{U}$

$$uRv \equiv v \text{ cites } u$$

which determines a *citation network* N = (U, R).

In Table 1 some characteristics of real life citation networks are presented. Most of these networks were obtained from the Eugene Garfield's collection of citation data [10, 12] produced using *HistCite* Software (formerly called *HistComp* – *comp*iled *Hist*oriography program) [11]. All of these networks are the result of searches in the Web of Science and are used with the permission of ISI of Philadelphia, www.isinet.com. These networks in Pajek's format are available from Pajek's web site [19].

In Table 1: $n = |\mathbf{U}|$ is the number of vertices; m = |R| is the number of arcs; m_0 is the number of loops; n_0 is the number of isolated vertices; n_C is the size of the largest weakly connected component; k_C is the number of nontrivial weakly connected components; h is the depth of network (minimum number of levels); Δ_{in} is the maximum input degree; and Δ_{out} is the maximum output degree. The last three columns contain the numbers of strongly connected components (cyclic parts) of size 2, 3 and 4.

A citing relation is usually *irreflexive*, $\forall u \in \mathbf{U} : \neg uRu$, and (almost) acyclic – no vertex is reachable from itself by a nontrivial path, or formally $\forall u \in \mathbf{U} \forall k \in \mathbf{N}^+ : \neg u R^k u$. In the following we shall assume that it has this property. We shall postpone the question how to deal with nonacyclic citation networks till the end of the theoretical part of the paper.

For a relation $Q \subseteq \mathbf{U} \times \mathbf{U}$ we denote by Q^{inv} its *inverse* relation, $uQ^{\text{inv}}v \equiv vQu$, and by

$$Q(u) = \{ v \in \mathbf{U} : uQv \}$$

the set of successors of unit $u \in U$. If Q is acyclic then also Q^{inv} is acyclic. This means that the network $N^{\text{inv}} = (U, R^{\text{inv}}), uR^{\text{inv}}v \equiv u$ cites v, is a network of the same type as the original citation network N = (U, R). Therefore it is just a matter of 'taste' which relation to select.

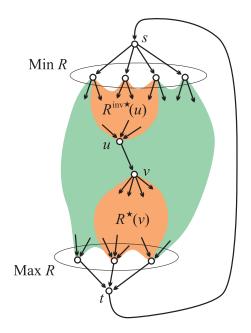


Figure 1: Citation Network in Standard Form

Let $I=\{(u,u):u\in \mathbf{U}\}$ be the *identity* relation on \mathbf{U} and $\overline{Q}=\bigcup_{k\in \mathbf{N}^+}Q^k$ the *transitive* closure of relation Q. Then Q is acyclic iff $\overline{Q}\cap I=\emptyset$. The relation $Q^\star=\overline{Q}\cup I$ is the *transitive* and reflexive closure of relation Q.

Since the set of units U is finite and R is acyclic we know from the theory of relations that:

• The set of units U can be topologically ordered – there exists a surjective mapping (permutation) $i: U \to 1..|U|$ with the property

$$uRv \Rightarrow i(u) < i(v)$$

- Let $\operatorname{Min} R = \{u \in \mathbf{U} : R^{\operatorname{inv}}(u) = \emptyset\}$ be the set of *minimal* elements and $\operatorname{Max} R = \{u \in \mathbf{U} : R(u) = \emptyset\}$ the set of *maximal* elements. Then $\operatorname{Min} R \neq \emptyset$ and $\operatorname{Max} R \neq \emptyset$.
- Every unit $u \in \mathbf{U}$ and every arc $(u, v) \in R$ belong to at least one path from $\operatorname{Min} R$ to $\operatorname{Max} R$:

 $\forall u \in \mathbf{U} : R^{\star}(u) \cap \operatorname{Max} R \neq \emptyset$ $\forall u \in \mathbf{U} : R^{\operatorname{inv}^{\star}}(u) \cap \operatorname{Min} R \neq \emptyset$

To simplify the presentation we transform a citation network $\mathbf{N}=(\mathbf{U},R)$ to its *standard* form $\mathbf{N}'=(\mathbf{U}',R')$ (see Figure 1) by extending the set of units $\mathbf{U}':=\mathbf{U}\cup\{s,t\},\,s,t\notin\mathbf{U}$ with a common *source* (initial unit) s and a common *sink* (terminal unit) t, and by adding the corresponding arcs to relation R

$$R' := R \ \cup \ \{s\} \times \operatorname{Min} R \ \cup \ \operatorname{Max} R \times \{t\} \ \cup \ \{(t,s)\}$$

This eliminates problems with networks with several connected components and/or several initial/terminal units. In the following we shall assume that the citation network $\mathbf{N} = (\mathbf{U}, R)$ is in the standard form. Note that, to make the theory smoother, we added to R' also the 'feedback' arc (t, s), thus destroying its acyclicity.

3 Analysis of Citation Networks

An approach to the analysis of citation network is to determine for each unit / arc its *importance* or *weight*. These values are used afterward to determine the essential substructures in the network. In this paper we shall focus on the methods of assigning weights $w: R \to \mathbb{R}_0^+$ to arcs proposed by Hummon and Doreian [14, 15]:

- node pair projection count (NPPC) method: $w_d(u, v) = |R^{inv^*}(u)| \cdot |R^*(v)|$
- search path link count (SPLC) method: $w_l(u, v)$ equals the number of "all possible search paths through the network emanating from an origin node" through the arc $(u, v) \in R$, [14, p. 50].
- search path node pair (SPNP) method: $w_p(u, v)$ "accounts for all connected vertex pairs along the paths through the arc $(u, v) \in R$ ", [14, p. 51].

3.1 Computing NPPC weights

To compute w_d for sets of units of moderate size (up to some thousands of units) the matrix representation of R can be used and its transitive closure computed by Roy-Warshall's algorithm [9]. The quantities $|R^*(v)|$ and $|R^{\text{inv}^*}(u)|$ can be obtained from closure matrix as row/column sums. An O(nm) algorithm for computing w_d can be constructed using Breath First Search from each $u \in U$ to determine $|R^{\text{inv}^*}(u)|$ and $|R^*(v)|$. Since it is of order at least $O(n^2)$ this algorithm is not suitable for larger networks (several ten thousands of vertices).

3.2 Search path count method

To compute the SPLC and SPNP weights we introduce a related *search path count* (SPC) method for which the weights N(u, v), uRv count the number of different paths from s to t (or from $\operatorname{Min} R$ to $\operatorname{Max} R$) through the arc (u, v).

To compute N(u, v) we introduce two auxiliary quantities: let $N^-(v)$ denotes the number of different s-v paths, and $N^+(v)$ denotes the number of different v-t paths.

Every s-t path π containing the arc $(u, v) \in R$ can be uniquely expressed in the form

$$\pi = \sigma \circ (u,v) \circ \tau$$

where σ is a s-u path and τ is a v-t path. Since every pair (σ, τ) of s-u / v-t paths gives a corresponding s-t path it follows:

$$N(u, v) = N^{-}(u) \cdot N^{+}(v), \qquad (u, v) \in R$$

where

$$N^{-}(u) = \begin{cases} 1 & u = s \\ \sum_{v:vRu} N^{-}(v) & \text{otherwise} \end{cases}$$

and

$$N^{+}(u) = \begin{cases} 1 & u = t \\ \sum_{v:uRv} N^{+}(v) & \text{otherwise} \end{cases}$$

This is the basis of an efficient algorithm for computing the weights N(u, v) – after the topological sort of the network [9] we can compute, using the above relations in topological order, the weights in time of order O(m). The topological order ensures that all the quantities in the right side expressions of the above equalities are already computed when needed. The counters N(u, v) are used as SPC weights $w_c(u, v) = N(u, v)$.

3.3 Computing SPLC and SPNP weights

The description of SPLC method in [14] is not very precise. Analyzing the table of SPLC weights from [14, p. 50] we see that we have to consider **each** vertex as an origin of search paths. This is equivalent to apply the SPC method on the extended network $N_l = (U', R_l)$

$$R_l := R' \cup \{s\} \times (\mathbf{U} \setminus \cup R(s))$$

It seems that there are some errors in the table of SPNP weights in [14, p. 51]. Using the definition of the SPNP weights we can again reduce their computation to SPC method applied on the extended network $\mathbf{N}_p = (\mathbf{U}', R_p)$

$$R_p := R \cup \{s\} \times \mathbf{U} \cup \mathbf{U} \times \{t\} \cup \{(t,s)\}$$

in which every unit $u \in U$ is additionally linked from the source s and to the sink t.

3.4 Computing the numbers of paths of length k

We could use also a direct approach to determine the weights w_p . Let $L^-(u)$ be the number of different paths terminating in u and $L^+(u)$ the number of different paths originating in u. Then for uRv it holds $w_p(u,v) = L^-(u) \cdot L^+(v)$.

The procedure to determine $L^-(u)$ and $L^+(u)$ can be compactly described using two families of polynomial generating functions

$$P^{-}(u;x) = \sum_{k=0}^{h(u)} p^{-}(u,k)x^{k}$$
 and $P^{+}(u;x) = \sum_{k=0}^{h^{-}(u)} p^{+}(u,k)x^{k}$, $u \in \mathbf{U}$

where h(u) is the depth of vertex u in network (\mathbf{U}, R) , and $h^-(u)$ is the depth of vertex u in network $(\mathbf{U}, R^{\text{inv}})$, The coefficient $p^-(u, k)$ counts the number of paths of length k to u, and $p^+(u, k)$ counts the number of paths of length k from u.

Again, by the basic principles of combinatorics

$$P^{-}(u;x) = \begin{cases} 0 & u = s \\ 1 + x \cdot \sum_{v:vRu} P^{-}(v;x) & \text{otherwise} \end{cases}$$

and

$$P^{+}(u;x) = \begin{cases} 0 & u = t \\ 1 + x \cdot \sum_{v:uRv} P^{+}(v;x) & \text{otherwise} \end{cases}$$

and both families can be determined using the definitions and computing the polynomials in the (reverse for P^+) topological ordering of U. The complexity of this procedure is at most O(hm). Finally

$$L^{-}(u) = P^{-}(u; 1)$$
 and $L^{+}(v) = P^{+}(v; 1)$

In real life citation networks the depth h is relatively small as can be seen from the Table 1.

The complexity of this approach is higher than the complexity of the method proposed in subsection 3.3 – but we get more detailed information about paths. May be it would make sense to consider 'aging' of references by $L^-(u) = P^-(u; \alpha)$, for selected α , $0 < \alpha \le 1$.

3.5 Vertex weights

The quantities used to compute the arc weights w can be used also to define the corresponding vertex weights t

$$t_d(u) = |R^{\text{inv}^*}(u)| \cdot |R^*(u)|$$

 $t_c(u) = N^-(u) \cdot N^+(u)$
 $t_l(u) = N'^-(u) \cdot N'^+(u)$
 $t_p(u) = L^-(u) \cdot L^+(u)$

They are counting the number of paths of selected type through the vertex u.

3.6 Implementation details

In our first implementation of the SPNP method the values of $L^-(u)$ and $L^+(u)$ for some large networks (Zewail and Lederberg) exceeded the range of Delphi's LargeInt (20 decimal places). We decided to use the Extended real numbers (range $= 3.6 \times 10^{-4951} \dots 1.1 \times 10^{4932}$, 19-20 significant digits) for counters. This range is safe also for very large citation networks.

To see this, let us denote $N^*(k) = \max_{u:h(u)=k} N^-(u)$. Note that h(s) = 0 and $uRv \Rightarrow h(u) < h(v)$. Let $u^* \in \mathbf{U}$ be a unit on which the maximum is attained $N^*(k) = N^-(u^*)$. Then

$$N^{*}(k) = \sum_{v:vRu^{*}} N^{-}(v) \le \sum_{v:vRu^{*}} N^{*}(h(v)) \le \sum_{v:vRu^{*}} N^{*}(k-1) =$$

$$= \deg_{in}(u^{*}) \cdot N^{*}(k-1) \le \Delta_{in}(k) \cdot N^{*}(k-1)$$

where $\Delta_{in}(k)$ is the maximal input degree at depth k. Therefore $N^*(h) \leq \prod_{k=1}^h \Delta_{in}(k) \leq \Delta_{in}^h$. A similar inequality holds also for $N^+(u)$. From both it follows

$$N(u, v) \leq \Delta_{in}^{h(u)} \cdot \Delta_{out}^{h^-(v)} \leq \Delta^{H-1}$$

where H=h(t) and $\Delta=\max(\Delta_{in},\Delta_{out})$. Therefore for $H\leq 1000$ and $\Delta\leq 10000$ we get $N(u,v)\leq \Delta^{H-1}\leq 10^{4000}$ which is still in the range of Extended reals. Note also that in the derivation of this inequality we were very generous – in real-life networks N(u,v) will be much smaller than Δ^{H-1} .

Very large/small numbers that result as weights in large networks are not easy to use. One possibility to overcome this problem is to use the logarithms of the obtained weights – logarithmic transformation is monotone and therefore preserve the ordering of weights (importance of vertices and arcs). The transformed values are also more convenient for visualization with line thickness of arcs.

4 Properties of weights

4.1 General properties of weights

Directly from the definitions of weights we get

$$w_k(u, v; R) = w_k(v, u; R^{\text{inv}}), \qquad k = d, c, p$$

and

$$w_c(u,v) \le w_l(u,v) \le w_p(u,v)$$

Let $\mathbf{N}_A = (\mathbf{U}_A, R_A)$ and $\mathbf{N}_B = (\mathbf{U}_B, R_B)$, $\mathbf{U}_A \cap \mathbf{U}_B = \emptyset$ be two citation networks, and $\mathbf{N}_1 = (\mathbf{U}_A', R_A')$ and $\mathbf{N}_2 = ((\mathbf{U}_A \cup \mathbf{U}_B)', (R_A \cup R_B)')$ the corresponding standardized networks of the first network and of the union of both networks. Then it holds for all $u, v \in \mathbf{U}_A$ and for all $p, q \in R_A$

$$\frac{t_k^{(1)}(u)}{t_k^{(1)}(v)} = \frac{t_k^{(2)}(u)}{t_k^{(2)}(v)}, \quad \text{and} \quad \frac{w_k^{(1)}(p)}{w_k^{(1)}(q)} = \frac{w_k^{(2)}(p)}{w_k^{(2)}(q)}, \quad k = d, c, l, p$$

where $t^{(1)}$ and $w^{(1)}$ is a weight on network N_1 , and $t^{(2)}$ and $w^{(2)}$ is a weight on network N_2 . This means that adding or removing components in a network do not change the ratios (ordering) of the weights inside components.

Let $N_1 = (U, R_1)$ and $N_2 = (U, R_2)$ be two citation networks over the same set of units U and $R_1 \subseteq R_2$ then

$$w_k(u, v; R_1) \le w_k(u, v; R_2), \qquad k = d, c, p$$

4.2 NPPC weights

In an acyclic network for every arc $(u, v) \in R$ hold

$$R^{\operatorname{inv}^{\star}}(u) \cap R^{\star}(v) = \emptyset$$
 and $R^{\operatorname{inv}^{\star}}(u) \cup R^{\star}(v) \subseteq \mathbf{U}$

therefore $|R^{\text{inv}^*}(u)| + |R^*(v)| \le n$ and, using the inequality $\sqrt{ab} \le \frac{1}{2}(a+b)$, also

$$w_d(u, v) = |R^{\text{inv}^*}(u)| \cdot |R^*(v)| \le \frac{1}{4}n^2$$

Close to the source or sink the weights w_d are small, since the sets $R^*(u)$ (and $R^{\text{inv}^*}(u)$) are monotonic along the paths in a sense

$$u\overline{R}v \Rightarrow R^{\star}(u) \subset R^{\star}(v)$$

The weights w_d are larger in the 'middle' of the network.

A more uniform (but less sensitive) weight would be $w_s(u,v) = |R^{\text{inv}^*}(u)| + |R^*(v)|$ or in the normalized form $w'_s(u,v) = \frac{1}{n}w_s(u,v)$.

4.3 SPC weights

For the flow N(u, v) the Kirchoff's node law holds:

For every node v in a citation network in standard form it holds

incoming flow = outgoing flow =
$$t_c(v)$$

Proof:

$$\sum_{x:xRv} N(x,v) = \sum_{x:xRv} N^{-}(x) \cdot N^{+}(v) = (\sum_{x:xRv} N^{-}(x)) \cdot N^{+}(v) = N^{-}(v) \cdot N^{+}(v)$$

$$\sum_{y:vRy} N(v,y) = \sum_{y:vRy} N^-(v) \cdot N^+(y) = N^-(v) \cdot \sum_{y:vRy} N^+(y) = N^-(v) \cdot N^+(v)$$

From the Kirchoff's node law it follows that the *total flow* through the citation network equals N(t,s). This gives us a natural way to normalize the weights

$$w(u,v) = \frac{N(u,v)}{N(t,s)} \quad \Rightarrow \quad 0 \le w(u,v) \le 1$$

If C is a minimal arc-cut-set

$$\sum_{(u,v)\in C} w(u,v) = 1$$

Let $\overrightarrow{\mathbf{K}}_n = \{(u,v) : u,v \in 1..n \land u < v\}$ be the complete acyclic directed graph on n vertices then the value of $N(u,v;\overrightarrow{\mathbf{K}}_n)$ is maximum over all citation networks on n units. It is easy to verify that

$$N(1, n; \overrightarrow{\mathbf{K}}_n) = 2^{n-2}$$

and in general

$$N(i, j; \overrightarrow{\mathbf{K}}_n) = 2^{j-i-1}, i < j$$

From this result we see that the exhaustive search algorithm proposed in Hummon and Doreian [14, 15] can require exponential time to compute the arc weights w.

5 Nonacyclic citation networks

The problem with cycles is that if there is a cycle in a network then there is also an infinite number of trails between some units. There are some standard approaches to overcome the problem:

- to introduce some 'aging' factor which makes the total weight of all trails converge to some finite value;
- to restrict the definition of a weight to some finite subset of trails for example paths or geodesics.

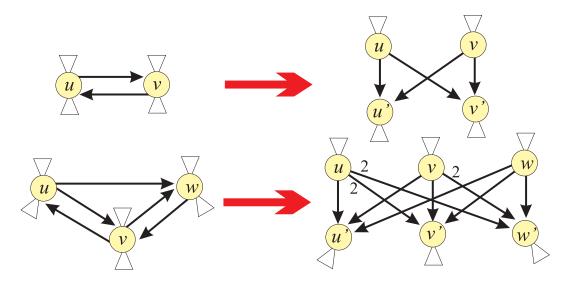


Figure 2: Preprint transformation

But, new problems arise: What is the right value of the 'aging' factor? Is there an efficient algorithm to count the restricted trails?

The other possibility, since a citation network is usually almost acyclic, is to transform it into an acyclic network

- by identification (shrinking) of cyclic groups (nontrivial strong components), or
- by deleting some arcs, or
- by transformations such as the 'preprint' transformation (see Figure 2) which is based on the following idea: Each paper from a strong component is duplicated with its 'preprint' version. The papers inside strong component cite preprints.

Large strong components in citation network are unlikely – their presence usually indicates an error in the data. An exception from this rule is the citation network of High Energy Particle Physics literature [20] from **arXiv**. In it different versions of the same paper are treated as a unit. This leads to large strongly connected components. The idea of preprint transformation can be used also in this case to eliminate cycles.

6 First Example: SOM citation network

The purpose of this example is not the analysis of the selected citation network on SOM (self-organizing maps) literature [12, 24, 23], but to present typical steps and results in citation network analysis. We made our analysis using program **Pajek**.

First we test the network for acyclicity. Since in the SOM network there are 11 nontrivial strong components of size 2, see Table 1, we have to transform the network into acyclic one. We decided to do this by shrinking each component into a single vertex. This operation produces some loops that should be removed.



Figure 3: Main path and CPM path in SOM network with SPC weights

Now, we can compute the citation weights. We selected the SPC (search path count) method. It returns the following results: the network with citation weights on arcs, the main path network and the vector with vertex weights.

In a citation network, a *main path* (sub)network is constructed starting from the source vertex and selecting at each step in the end vertex/vertices the arc(s) with the highest weight, until a sink vertex is reached.

Another possibility is to apply on the network $\mathbf{N} = (\mathbf{U}, R, w)$ the critical path method (CPM) from operations research.

First we draw the main path network. The arc weights are represented by the thickness of arcs. To produce a nice picture of it we apply the Pajek's macro Layers which contains a sequence of operations for determining a layered layout of an acyclic network (used also in analysis of genealogies represented by p-graphs). Some experiments with settings of different options are needed to obtain a right picture, see left part of Figure 3. In its right part the CPM

Table 2: 15 Hubs and Authorities

Rank	h	Hub Id	a	Authority Id
1	0.06442	CLARK-JW-1991-V36-P1259	0.85214	HOPFIELD-JJ-1982-V79-P2554
2	0.06366	#GARDNER-E-1988-V21-P257	0.33427	KOHONEN-T-1982-V43-P59
3	0.05794	HUANG-SH-1994-V17-P212	0.14531	KOHONEN-T-1990-V78-P1464
4	0.05721	GULATI-S-1991-V33-P173	0.12398	CARPENTER-GA-1987-V37-P54
5	0.05513	SHUBNIKOV-EI-1997-V64-P989	0.10376	#GARDNER-E-1988-V21-P257
6	0.05496	MARSHALL-JA-1995-V8-P335	0.09353	HOPFIELD-JJ-1986-V233-P625
7	0.05488	VEMURI-V-1993-V36-P203	0.07882	MCELIECE-RJ-1987-V33-P461
8	0.05409	CHENG-B-1994-V9-P2	0.07656	KOHONEN-T-1988-V1-P3
9	0.05360	BUSCEMA-M-1998-V33-P17	0.07372	RUMELHART-DE-1985-V9-P75
10	0.05258	XU-L-1993-V6-P627	0.07271	KOSKO-B-1988-V18-P49
11	0.05249	WELLS-DM-1998-V41-P173	0.07246	ANDERSON-JA-1977-V84-P413
12	0.05233	SCHYNS-PG-1991-V15-P461	0.07033	AMARI-SI-1977-V26-P175
13	0.05173	SMITH-KA-1999-V11-P15	0.06709	KOSKO-B-1987-V26-P4947
14	0.05149	BONABEAU-E-1998-V9-P1107	0.05802	PERSONNAZ-L-1985-V46-PL359
15	0.05126	KOHONEN-T-1990-V78-P1464	0.05702	GROSSBERG-S-1987-V11-P23

path is presented.

We see that the upper parts of both paths are identical, but they differ in the continuation. The arcs in the CPM path are thicker.

We could display also the complete SOM network using essentially the same procedure as for the displaying of main path. But the obtained picture would be too complicated (too many vertices and arcs). We have to identify some simpler and important subnetworks inside it.

Inspecting the distribution of values of weights on arcs (lines) we select a threshold 0.007 and determine the corresponding arc-cut – delete all arcs with weights lower than selected threshold and afterwards delete also all isolated vertices (degree = 0).

Now, we are ready to draw the reduced network. We first produce an automatic layout. We notice some small unimportant components. We preserve only the large main component, draw it and improve the obtained layout manually. To preserve the level structure we use the option that allows only the horizontal movement of vertices.

Finally we label the 'most important vertices' with their labels. A vertex is considered important if it is an endpoint of an arc with the weight above the selected threshold (in our case 0.05).

The obtained picture of SOM 'main subnetwork' is presented in Figure 4. We see that the SOM field evolved in two main branches. From CARPENTER-1987 the strongest (main path) arc is leading to the right branch that after some steps disappears. The left, more vital branch is detected by the CPM path. Further investigation of this is left to the readers with additional knowledge about the SOM field.

As a complementary information we can determine Kleinberg's hubs and authorities vertex weights [17]. Papers that are cited by many other papers are called authorities; papers that cite many other documents are called hubs. Good authorities are those that are cited by good hubs

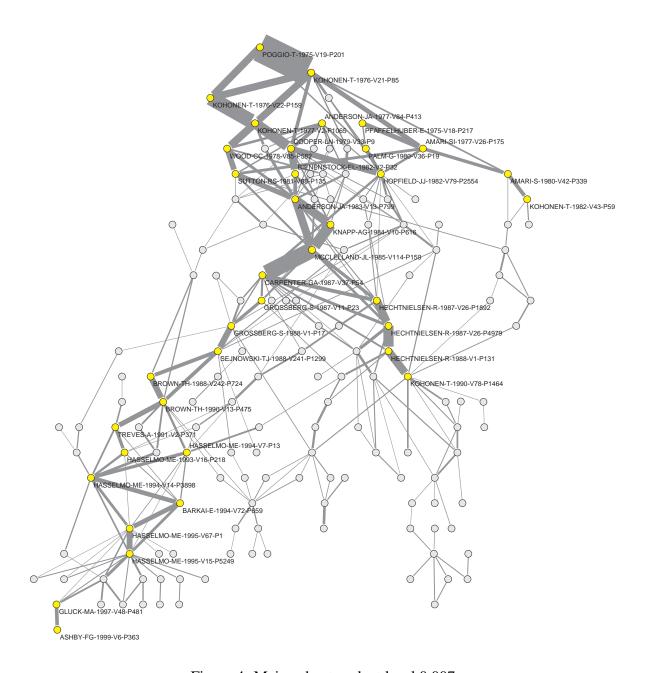


Figure 4: Main subnetwork at level 0.007

and good hubs cite good authorities. The 15 highest ranked hubs and authorities are presented in Table 2. We see that the main authorities are located in eighties and the main hubs in nineties. Note that, since we are using the relation $uRv \equiv u$ is cited by v, we have to interchange the roles of hubs and authorities produced by **Pajek**.

An elaboration of the hubs and authorities approach to the analysis of citation networks complemented with visualization can be found in Brandes and Willhalm (2002) [8].

7 Second Example: US patents

The network of US patents from 1963 to 1999 [21] is an example of very large network (3774768 vertices and 16522438 arcs) that, using some special options in **Pajek**, can still be analyzed on PC with at least 1 G memory. The SPC weights are determined in a range of 1 minute. This shows that the proposed approach can be used also for very large networks.

The obtained main path and CPM path are presented in Figure 5. Collecting from the *United States Patent and Trademark Office* [22] the basic data about the patents from both paths, see Table 3-6, we see that they deal with 'liquid crystal displays'.

But, in this network there should be thousands of 'themes'. How to identify them? Using the arc weights we can define a *theme* as a connected small subnetwork of size in the interval k .. K (for example, between $k=\frac{1}{3}h$ and K=3h) with stronger internal cohesion relatively to its neighborhood.

To find such subnetworks we use again the arc-cuts. We select a treshold t and delete all arcs with weight lower than t. In the so reduced network we determine (weakly) connected components. The components of size in range k..K, we call them (k,K)-islands, represent the themes since:

- they are connected and of selected size,
- all arcs linking them to their outside neighbors have weight lower than t, and
- each vertex of an island is linked with some other vertex in the same island with an arc with a weight at least t.

We discard components of size smaller than k as 'noninteresting'.

The components of size larger then K are too large. They contain several themes. To identify them we repeat the procedure on the network of these components with a higher threshold value t'. Recently we developed an algorithm, named Islands [7], that by 'continuosly' changing the threshold identifies all maximal (k, K)-islands.

We determined for SPC weights all (2,90)-islands in the US Patents network. The reduced network of islands has 470137 vertices, 307472 arcs and for different k: $C_2 = 187610$, $C_5 = 8859$, $C_{30} = 101$, $C_{50} = 30$ islands. The detailed island size frequency distribution is given in Table 7 and presented in a log-log scale in Figure 6 that shows that it obeys the power law.

The main island has 90 vertices and contains middle parts of the main path and the CPM path. They also have a short common part. Again, the greedy strategy of the main path leads to a less vital branch. Considering the basic data about the patents from Table 3-5, we see that also the main island deals with 'liquid crystal displays'.

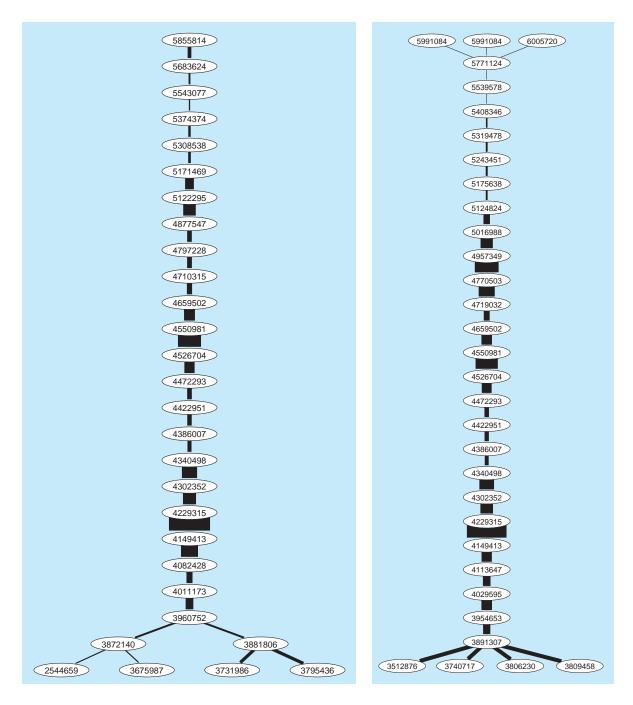


Figure 5: Main path and CPM path subnetwork of Patents

Table 3: Patents on the liquid-crystal display

patent	date	author(s) and title	
2544659	Mar 13, 1951	Dreyer. Dichroic light-polarizing sheet and the like and the	
		formation and use thereof	
2682562	Jun 29, 1954	Wender, et al. Reduction of aromatic carbinols	
3322485	May 30, 1967	Williams. Electro-optical elements utilazing an organic	
3322 103	1 via y 50, 1507	nematic compound	
3512876	May 19, 1970	Marks. Dipolar electro-optic structures	
3636168	Jan 18, 1972	Josephson. Preparation of polynuclear aromatic compounds	
3666948	May 30, 1972	Mechlowitz, et al. Liquid crystal termal imaging system	
2000710	1,14, 50, 15, 2	having an undisturbed image on a disturbed background	
3675987	Jul 11, 1972	Rafuse. Liquid crystal compositions and devices	
3691755	Sep 19, 1972	Girard. Clock with digital display	
3697150	Oct 10, 1972	Wysochi. Electro-optic systems in which an electrophoretic-	
005,100	00010,1572	like or dipolar material is dispersed throughout a liquid	
		crystal to reduce the turn-off time	
3731986	May 8, 1973	Fergason. Display devices utilizing liquid crystal light	
0,01300	1.141) 0, 15 / 6	modulation	
3740717	Jun 19, 1973	Huener, et al. Liquid crystal display	
3767289	Oct 23, 1973	Aviram, et al. Class of stable trans-stilbene compounds,	
	, , , , , , , , , , , , , , , , , , , ,	some displaying nematic mesophases at or near room	
		temperature and others in a range up to 100°C	
3773747	Nov 20, 1973	Steinstrasser. Substituted azoxy benzene compounds	
3795436	Mar 5, 1974	Boller, et al. Nematogenic material which exhibit the Kerr	
	,	effect at isotropic temperatures	
3796479	Mar 12, 1974	Helfrich, et al. Electro-optical light-modulation cell	
		utilizing a nematogenic material which exhibits the Kerr	
		effect at isotropic temperatures	
3806230	Apr 23, 1974	Haas. Liquid crystal imaging system having optical storage	
		capabilities	
3809458	May 7, 1974	Huener, et al. Liquid crystal display	
3872140	Mar 18, 1975	Klanderman, et al. Liquid crystalline compositions and	
		method	
3876286	Apr 8, 1975	Deutscher, et al. Use of nematic liquid crystalline substances	
3881806	May 6, 1975	Suzuki. Electro-optical display device	
3891307	Jun 24, 1975	Tsukamoto, et al. Phase control of the voltages applied to	
		opposite electrodes for a cholesteric to nematic phase	
20.45255	M 20 1075	transition display	
3947375	Mar 30, 1976	Gray, et al. Liquid crystal materials and devices	
3954653	May 4, 1976	Yamazaki. Liquid crystal composition having high dielectric	
2060752	Jun 1 1076	anisotropy and display device incorporating same	
3960752	Jun 1, 1976	Klanderman, et al. Liquid crystal compositions	
3975286	Aug 17, 1976	Oh. Low voltage actuated field effect liquid crystals	
4000094	Dec 29 1076	compositions and method of synthesis	
4000084	Dec 28, 1976	Hsieh, et al. Liquid crystal mixtures for electro-optical	
4011173	Mar 8, 1977	display devices Steinstrasser. Modified nematic mixtures with	
+0111/3	1 v1 a1 0, 17//	positive dielectric anisotropy	
4013582	Mar 22, 1977	Gavrilovic. Liquid crystal compounds and electro-optic	
7013362	1v1ai 44, 19//	devices incorporating them	
4017416	Apr 12, 1977	Inukai, et al. P-cyanophenyl 4-alkyl-4'-biphenylcarboxylate,	
701/410	Apr 12, 17//	method for preparing same and liquid crystal compositions	
		using same	
		dome baine	

Table 4: Patents on the liquid-crystal display

patent	date	author(s) and title	
4029595	Jun 14, 1977	Ross, et al. Novel liquid crystal compounds and electro-optic	
		devices incorporating them	
4032470	Jun 28, 1977	Bloom, et al. Electro-optic device	
4077260	Mar 7, 1978	Gray, et al. Optically active cyano-biphenyl compounds and	
	·	liquid crystal materials containing them	
4082428	Apr 4, 1978	Hsu. Liquid crystal composition and method	
4083797	Apr 11, 1978	Oh. Nematic liquid crystal compositions	
4113647	Sep 12, 1978	Coates, et al. Liquid crystalline materials	
4118335	Oct 3, 1978	Krause, et al. Liquid crystalline materials of reduced viscosity	
4130502	Dec 19, 1978	Eidenschink, et al. Liquid crystalline cyclohexane derivatives	
4149413	Apr 17, 1979	Gray, et al. Optically active liquid crystal mixtures and	
	1 ,	liquid crystal devices containing them	
4154697	May 15, 1979	Eidenschink, et al. Liquid crystalline hexahydroterphenyl	
	,	derivatives	
4195916	Apr 1, 1980	Coates, et al. Liquid crystal compounds	
4198130	Apr 15, 1980	Boller, et al. Liquid crystal mixtures	
4202791	May 13, 1980	Sato, et al. Nematic liquid crystalline materials	
4229315	Oct 21, 1980	Krause, et al. Liquid crystalline cyclohexane derivatives	
4261652	Apr 14, 1981	Gray, et al. Liquid crystal compounds and materials and	
	•	devices containing them	
4290905	Sep 22, 1981	Kanbe. Ester compound	
4293434	Oct 6, 1981	Deutscher, et al. Liquid crystal compounds	
4302352	Nov 24, 1981	Eidenschink, et al. Fluorophenylcyclohexanes, the preparation	
	·	thereof and their use as components of liquid crystal dielectrics	
4330426	May 18, 1982	Eidenschink, et al. Cyclohexylbiphenyls, their preparation and	
		use in dielectrics and electrooptical display elements	
4340498	Jul 20, 1982	Sugimori. Halogenated ester derivatives	
4349452	Sep 14, 1982	Osman, et al. Cyclohexylcyclohexanoates	
4357078	Nov 2, 1982	Carr, et al. Liquid crystal compounds containing an alicyclic	
		ring and exhibiting a low dielectric anisotropy and liquid	
		crystal materials and devices incorporating such compounds	
4361494	Nov 30, 1982	Osman, et al. Anisotropic cyclohexyl cyclohexylmethyl ethers	
4368135	Jan 11, 1983	Osman. Anisotropic compounds with negative or positive	
		DC-anisotropy and low optical anisotropy	
4386007	May 31, 1983	Krause, et al. Liquid crystalline naphthalene derivatives	
4387038	Jun 7, 1983	Fukui, et al. 4-(Trans-4'-alkylcyclohexyl) benzoic acid	
		4"'-cyano-4"-biphenylyl esters	
4387039	Jun 7, 1983	Sugimori, et al. Trans-4-(trans-4'-alkylcyclohexyl)-cyclohexane	
44005		carboxylic acid 4"'-cyanobiphenyl ester	
4400293	Aug 23, 1983	Romer, et al. Liquid crystalline cyclohexylphenyl derivatives	
4415470	Nov 15, 1983	Eidenschink, et al. Liquid crystalline fluorine-containing	
		cyclohexylbiphenyls and dielectrics and electro-optical display	
4410262	D < 1002	elements based thereon	
4419263	Dec 6, 1983	Praefcke, et al. Liquid crystalline cyclohexylcarbonitrile	
4.40005.5	D 07 1000	derivatives	
4422951	Dec 27, 1983	Sugimori, et al. Liquid crystal benzene derivatives	
4455443	Jun 19, 1984	Takatsu, et al. Nematic halogen Compound	
4456712	Jun 26, 1984	Christie, et al. Bismaleimide triazine composition	
4460770	Jul 17, 1984	Petrzilka, et al. Liquid crystal mixture	
4472293	Sep 18, 1984	Sugimori, et al. High temperature liquid crystal substances of	
		four rings and liquid crystal compositions containing the same	

Table 5: Patents on the liquid-crystal display

patent date author(s) and title 4472592 Sep 18, 1984 Takatsu, et al. Nematic liquid crystalline compounds 4480117 Oct 30, 1984 Takatsu, et al. Nematic liquid crystalline compounds 4502974 Mar 5, 1985 Sugimori, et al. High temperature liquid-crystalline est compounds 4510069 Apr 9, 1985 Eidenschink, et al. Cyclohexane derivatives	
4480117 Oct 30, 1984 Takatsu, et al. Nematic liquid crystalline compounds Sugimori, et al. High temperature liquid-crystalline est compounds	
4502974 Mar 5, 1985 Sugimori, et al. High temperature liquid-crystalline est compounds	
compounds	
	er
4510069 Apr 9, 1985 Eidenschink, et al. Cyclohexane derivatives	
4514044 Apr 30, 1985 Gunjima, et al. 1-(Trans-4-alkylcyclohexyl)-2-(trans-4'	
stituted phenyl) cyclohexyl)ethane and liquid crystal m	ixture
4526704 Jul 2, 1985 Petrzilka, et al. Multiring liquid crystal esters	
4550981 Nov 5, 1985 Petrzilka, et al. Liquid crystalline esters and mixtures	
4558151 Dec 10, 1985 Takatsu, et al. Nematic liquid crystalline compounds	
4583826 Apr 22, 1986 Petrzilka, et al. Phenylethanes	
4621901 Nov 11, 1986 Petrzilka, et al. Novel liquid crystal mixtures	
4630896 Dec 23, 1986 Petrzilka, et al. Benzonitriles	
4657695 Apr 14, 1987 Saito, et al. Substituted pyridazines	
4659502 Apr 21, 1987 Fearon, et al. Ethane derivatives	
4695131 Sep 22, 1987 Balkwill, et al. Disubstituted ethanes and their use in li	quid
crystal materials and devices	1
4704227 Nov 3, 1987 Krause, et al. Liquid crystal compounds	
4709030 Nov 24, 1987 Petrzilka, et al. Novel liquid crystal mixtures	
4710315 Dec 1, 1987 Schad, et al. Anisotropic compounds and liquid crystal	
mixtures therewith	
4713197 Dec 15, 1987 Eidenschink, et al. Nitrogen-containing heterocyclic co	ompounds
4719032 Jan 12, 1988 Wachtler, et al. Cyclohexane derivatives	omp o unus
4721367 Jan 26, 1988 Yoshinaga, et al. Liquid crystal device	
4752414 Jun 21, 1988 Eidenschink, et al. Nitrogen-containing heterocyclic co	omnounds
4770503 Sep 13, 1988 Buchecker, et al. Liquid crystalline compounds	mpounds
4795579 Jan 3, 1989 Vauchier, et al. 2,2'-difluoro-4-alkoxy-4'-hydroxydipho	envls and
their derivatives, their production process and	city is and
their use in liquid crystal display devices	
4797228 Jan 10, 1989 Goto, et al. Cyclohexane derivative and liquid crystal	
composition containing same	
4820839 Apr 11, 1989 Krause, et al. Nitrogen-containing heterocyclic esters	
4832462 May 23, 1989 Clark, et al. Liquid crystal devices	
	of
television pictures, control system and process for prod	lucing
said screen	- 4
5016988 May 21, 1991 Iimura. Liquid crystal display device with a birefringer	ıt
compensator	1
5016989 May 21, 1991 Okada. Liquid crystal element with improved contrast a	ana
brightness	
5122295 Jun 16, 1992 Weber, et al. Matrix liquid crystal display	
5124824 Jun 23, 1992 Kozaki, et al. Liquid crystal display device comprising	
retardation compensation layer having a maximum prin	ncıpal
refractive index in the thickness direction	
5171469 Dec 15, 1992 Hittich, et al. Liquid-crystal matrix display	
5175638 Dec 29, 1992 Kanemoto, et al. ECB type liquid crystal display device	
birefringent layer with equal refractive indexes in the th	hickness
and plane directions	

Table 6: Patents on the liquid-crystal display

	patent	date	author(s) and title		
Ļ					
)	243451	Sep 7, 1993	Kanemoto, et al. DAP type liquid crystal device with cholesteric		
			liquid crystal birefringent layer		
5	283677	Feb 1, 1994	Sagawa, et al. Liquid crystal display with ground regions		
		ŕ	between terminal groups		
5	308538	May 3, 1994	Weber, et al. Supertwist liquid-crystal display		
5	319478	June 7, 1994	Funfschilling, et al. Light control systems with a circular polarizer		
			and a twisted nematic liquid crystal having a minimum path		
			difference of .lambda./2		
5	374374	Dec 20, 1994	Weber, et al. Supertwist liquid-crystal display		
5	408346	Apr 18, 1995	Trissel, et al. Optical collimating device employing cholesteric		
		-	liquid crystal and a non-transmissive reflector		
5	539578	Jul 23, 1996	Togino, et al. Image display apparatus		
5	543077	Aug 6, 1996	Rieger, et al. Nematic liquid-crystal composition		
5	555116	Sep 10, 1996	Ishikawa, et al. Liquid crystal display having adjacent		
		1 ,			
5	683624	Nov 4, 1997	Sekiguchi, et al. Liquid crystal composition		
5	771124	Jun 23, 1998	Kintz, et al. Compact display system with two stage magnification		
		ŕ	and immersed beam splitter		
5	855814	Jan 5, 1999	Matsui, et al. Liquid crystal compositions and liquid crystal		
		ĺ			
5	991084	Nov 23, 1999			
		, , , , , , , , , , , , , , , , , , , ,			
6	005720	Dec 21, 1999			
5 5 5	6683624 771124	Nov 4, 1997 Jun 23, 1998	electrode terminals set equal in length Sekiguchi, et al. Liquid crystal composition Kintz, et al. Compact display system with two stage magnificatio and immersed beam splitter		

Table 7: Island size frequency distribution

[1]	0	139793	29670	9288	3966	1827	997	578	362	250
[11]	190	125	104	71	47	37	36	33	21	23
[21]	17	16	8	7	13	10	10	5	5	5
[31]	12	3	7	3	3	3	2	6	6	2
Ī 41 Ī	1	3	4	1	5	2	1	1	1	1
Ī 51 Ī	2	3	3	2	0	0	0	0	0	1
[61]	0	0	0	0	1	0	0	2	0	0
[71]	0	0	1	1	0	0	0	1	0	0
[81]	2	0	0	0	0	1	2	0	0	7

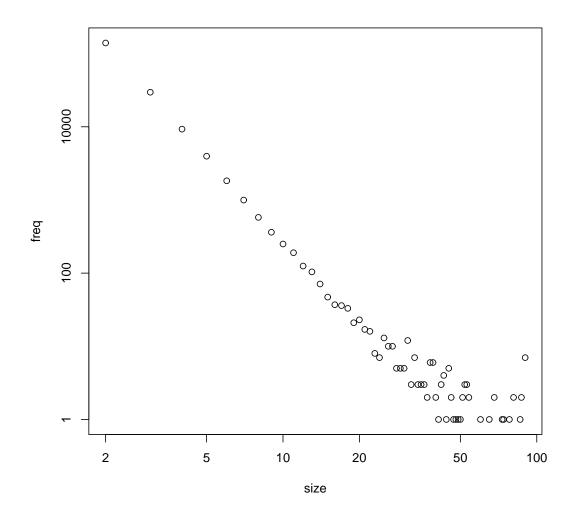


Figure 6: Island size frequency distribution

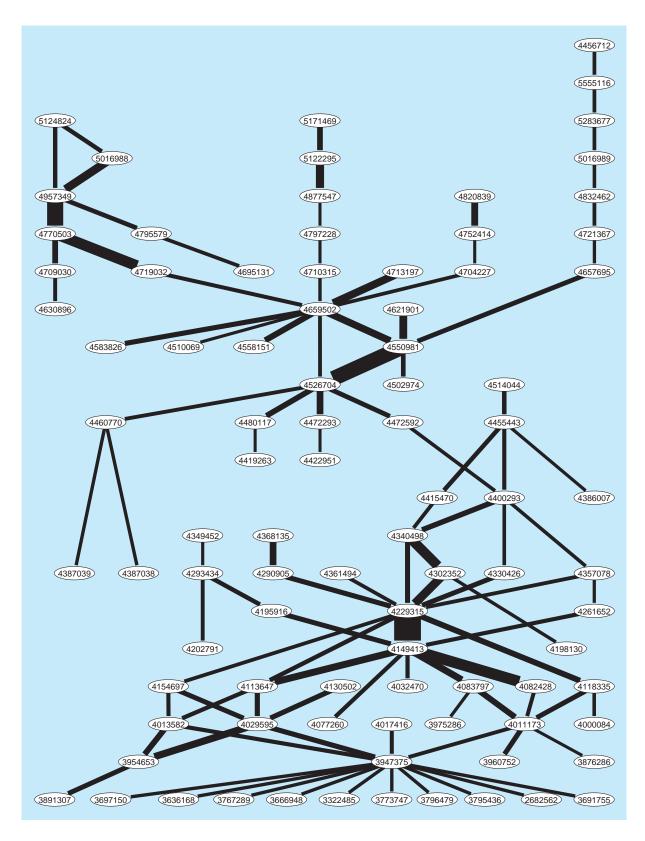


Figure 7: Main island 'liquid-crystal display'

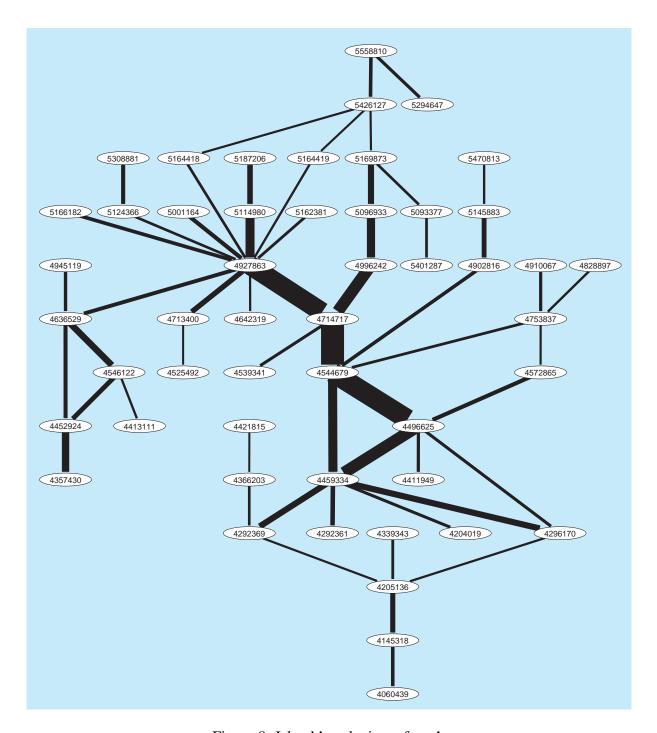


Figure 8: Island 'producing a foam'

Table 8: Some patents from the 'foam' island

patent	date	author(s) and title
4060439	Nov 29, 1977	Rosemund, et al. Polyurethane foam composition and method of
		making same
4292369	Sep 29, 1981	Ohashi, et al. Fireproof laminates
4357430	Nov 2, 1982	VanCleve. Polymer/polyols, methods for making same and
		polyurethanes based thereon
4459334	Jul 10, 1984	Blanpied, et al. Composite building panel
4496625	Jan 29, 1985	Snider, et al. Alkoxylated aromatic amine-aromatic polyester
		polyol blend and polyisocyanurate foam therefrom
4544679	Oct 1, 1985	Tideswell, et al. Polyol blend and polyisocyanurate foam
		produced therefrom
4714717	Dec 22, 1987	Londrigan, et al. Polyester polyols modified by low molecular
		weight glycols and cellular foams therefrom
4927863	May 22, 1990	Bartlett, et al. Process for producing closed-cell polyurethane
		foam compositions expanded with mixtures of blowing agents
4996242	Feb 26, 1991	Lin. Polyurethane foams manufactured with mixed
		gas/liquid blowing agents
5169873	Dec 8, 1992	Behme, et al. Process for the manufacture of foams with the aid
		of blowing agents containing fluoroalkanes and fluorinated
		ethers, and foams obtained by this process
5187206	Feb 16, 1993	Volkert, et al. Production of cellular plastics by the
		polyisocyanate polyaddition process, and low-boiling,
		fluorinated or perfluorinated, tertiary alkylamines
		as blowing agent-containing emulsions for this purpose
5308881	May 3, 1994	Londrigan, et al. Surfactant for polyisocyanurate foams
		made with alternative blowing agents
5558810	Sep 24, 1996	Minor, et al. Pentafluoropropane compositions

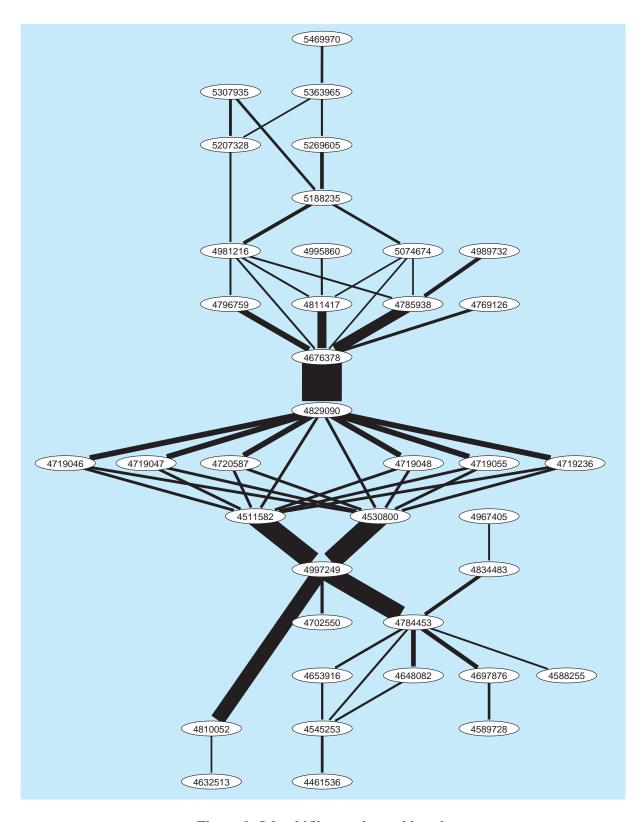


Figure 9: Island 'fiber optics and bags'

Table 9: Some patents from 'fiber optics and bags' island

patent	date	author(s) and title	
4461536	Jul 24, 1984	Shaw, et al. Fiber coupler displacement transducer	
4511582	Apr 16, 1985	Bair. Phenanthrene derivatives	
4530800	Jul 23, 1985	Bair. Perylene derivatives	
4589728	May 20, 1986	Dyott, et al. Optical fiber polarizer	
4676378	Jun 30, 1987	Baxley, et al. Bag pack	
4719047	Jan 12, 1988	Bair. Anthracene derivatives	
4784453	Nov 15, 1988	Shaw, et al. Backward-flow ladder architecture and method	
4785938	Nov 22, 1988	Benoit, Jr., et al. Thermoplastic bag pack	
4810052	Mar 7, 1989	7, 1989 Fling. Fiber optic bidirectional data bus tap	
4811417	Mar 7, 1989	Prince, et al. Handled bag with supporting slits in handle	
4829090	May 9, 1989	Bair. Chrysene derivatives	
4981216	Jan 1, 1991	Wilfong, Jr. Easy opening bag pack and supporting rack	
		system and fabricating method	
4997249	Mar 5, 1991	ar 5, 1991 Berry, et al. Variable weight fiber optic transversal filter	
5188235	Feb 23, 1993	Pierce, et al. Bag pack	
5307935	May 3, 1994	Kemanjian. Packs of self opening plastic bags and method of	
		fabricating the same	
5363965	Nov 15, 1994	Nguyen. Self-opening thermoplastic bag system	

For additional illustration of results obtained by Islands algorithm we selected two smaller islands at lower levels – see Figure 8 (50 vertices) and Figure 9 (38 vertices). Retreiving the basic data about some patents in these islands from *United States Patent and Trademark Office*, see Table 8 and Table 9, we can label the corresponding theme of the first island as 'producing a foam'. The theme of the second island deals initially with 'fiber optics', but in the upper part it switches to 'bag pack system'.

8 Conclusions

In the paper we proposed an approach to the analysis of citation networks that can be used also for very large networks with millions of vertices and arcs.

On test cases, the methods SPC, SPLC, NPPC produced almost the same results. Since the method SPC has additional 'nice' properties it could be considered as a 'first choice' – but, to make a grounded recommendation, additional experiences should be gained from the analyses of real-life large citation networks.

The granularity of the results strongly depends on the range for 'interesting themes' k .. K – varying these two parameters we get larger or smaller sets of themes.

Instead of arc-cuts we could consider also vertex-cuts with respect to p-cores on SPC weights [6] with a p-function

$$p(v, W) = \max(\sum_{u \in W: uRv} w(u, v), \sum_{u \in W: vRu} w(v, u))$$

The subnetworks approach only filters out the structurally important subnetworks thus providing a researcher with a smaller manageable structures which can be further analyzed using more sophisticated and/or substantial methods.

9 Acknowledgments

The search path count algorithm was developed during my visit in Pittsburgh in 1991 and presented at the Network seminar [2]. It was presented to the broader audience at EASST'94 in Budapest [3]. In 1997 it was included in program Pajek [4]. The 'preprint' transformation was developed as a part of the contribution for the Graph drawing contest 2001 [5]. The algorithm for the path length counts was developed in August 2002 and the Islands algorithm in August 2003.

The author would like to thank Patrick Doreian and Norm Hummon for introducing him into the field of citation network analysis, Eugene Garfield for making available the data on real-life networks and providing some relevant references, and Andrej Mrvar and Matjaž Zaveršnik for implementing the algorithms in **Pajek**.

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