Question 2:

a. $T(n) = 3T(\frac{n}{5}) + n^2$ $a=3,\ b=5,\ f(n)=n^2$

$\log_b(a) = \log_5(3)$ $\log_5(3) = \frac{\log(3)}{\log(5)} \approx \frac{.477}{.699} \approx .682$

compare $f(n) \to n^2 > n^{.682}$

By case 3 because $\Omega(n^c)$ where $c > \log_b(a)$ $\to$ $\boxed{T(n) = \Theta(n^2)}$

b. $T(n) = 4T(\frac{n}{3}) + 7n$ $a=4,\ b=3,\ f(n)=7n$

$\log_b(a) = \log_3(4) \to \frac{\log(4)}{\log(3)} \approx \frac{.602}{.477} \approx 1.263$

$n^1 < n^{1.263}$

By case 1 since $7n$ grows slower than $n^{1.263}$, $f(n) = O(n^c)$ where $c < \log_b(a)$

Thus: $\boxed{T(n) = \Theta(n^{1.263})}$

c. $T(n) = 5T(\frac{n}{4}) + 10$ $a=5,\ b=4,\ f(n)=10$

$\log_b(a) = \log_4(5) = \frac{\log(5)}{\log(4)} \approx \frac{.699}{.602} \approx 1.16$

case 1 since $c < \log_b(a)$ because $10 < n^{1.16}$

Thus: $\boxed{T(n) = \Theta(n^{1.16})}$

d. $T(n) = 9T(\frac{n}{3}) + n^4$ $a=9,\ b=3,\ f(n)=n^4$

$\log_b(a) = \log_3(9) = 2$ compare: $n^4 > n^2$

By case 3 because $c > \log_b(a)$ since $n^4 > n^2$

Thus: $\boxed{T(n) = \Theta(n^4)}$

e. $T(n) = 6T(\frac{n}{8}) + n^3$ $a=6,\ b=8,\ f(n)=n^3$

$\log_b(a) = \log_8(6) = \frac{\log(6)}{\log(8)} \approx \frac{.778}{.903} \approx .861$

$n^3 > n^{.861}$ so we use case 3 because $c > \log_b(a)$

Thus: $\boxed{T(n) = \Theta(n^3)}$