Question1: Repeated substitution

expand $T(\frac{n}{4})$

$T(n) = 3T(\frac{n}{4}) + 4n \rightarrow T(\frac{n}{4}) = 3T(\frac{n}{16}) + 4 \times \frac{n}{4} \rightarrow 3T(\frac{n}{16}) + n$

Substitute into original: $T(n) = 3(3T(\frac{n}{16}) + n) + 4n = 9T(\frac{n}{16}) + 3n + 4$

$\rightarrow T(n) = 9T(\frac{n}{16}) 7n$

Second expansion: $T(\frac{n}{16}) = 3T(\frac{n}{64}) + 4 \cdot \frac{n}{16} = 3T(\frac{n}{64}) + \frac{n}{4}$

$\rightarrow T(n) = 27T(\frac{n}{64}) + \frac{37n}{4}$

Pattern: $T(n) = 3^k T(\frac{n}{4^k}) + (4n + 3n + \frac{9n}{4} \ldots)$

Base Case when $n/4^k = 1$: $T(n) = 3^{\log_4(n)} T(1) + \sum_{i=0}^{\log_4(n)-1} 4n \cdot 3^i \rightarrow \sum_{i=0}^{\log_4(n)-1} 4n \cdot 3^i$

$\rightarrow$ Geometric Series: $4n \cdot \frac{1 - 3^{\log_4(n)}}{1 - 3} = \frac{3^{\log_4(n)} - 1}{2}$

$\rightarrow O(n^{\log_4(3)})$ $\quad T(n) = \Theta(n^{\log_4(3)}) \rightarrow T(n) = \boxed{\Theta(n)}$

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master theorem: $a = 3, b = 4, f(n) = 4n$

$\log_b(a) = \log_4(3) = \frac{\log(3)}{\log(4)}$ by change of base $\rightarrow \frac{.477}{.602} \approx .793 = n^{.793}$

$f(n) = 4n = n^1$, $n^1 > n^{.793}$

. If $f(n) = \Omega(n^c)$ where $c > \log_b(a)$, then solution is $T(n) = \Theta(f(n))$

Since $1 > .793$ we know that $T(n) = \boxed{\Theta(n)}$