我们来看一些行列式：

$$A=\begin{vmatrix} 12345 & 77684 \\ 24690 & 255368 \end{vmatrix}$$

$$B=\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{vmatrix}.$$

$$C=\begin{vmatrix} 2 & 1 & 6 & -2 & 5 & 2 \\ 3 & 2 & 8 & 11 & -9 & -23 \\ 0 & 0 & 3 & -5 & 7 & 1 \\ 0 & 0 & -2 & 4 & -2 & 7 \\ 0 & 0 & 0 & 0 & 7 & 2 \\ 0 & 0 & 0 & 0 & 9 & 3 \end{vmatrix}$$

直接计算计算量将会很大，若能利用行列式的一些性质进行化简，就能很简单地算出结果.

1.2.2 n阶行列式的性质

定理1.2.1. 行列式与它的转置行列式的值相等.

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{vmatrix}$$

定理的意义：

*** 行列式如果有某种行(列)的性质，则列(行)也有同样性质**

实例验证：

$$\begin{vmatrix} 2 & -4 & 1 \\ 1 & -5 & 3 \\ 1 & -1 & 1 \end{vmatrix} = -8 \qquad \begin{vmatrix} 2 & 1 & 1 \\ -4 & -5 & -1 \\ 1 & 3 & 1 \end{vmatrix}$$

定理1.2.2. 对调两行(列)的位置，行列式的值相差一个负号，即

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = - \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

实例验证：

$$\begin{vmatrix} 2 & -4 & 1 \\ 1 & -5 & 3 \\ 1 & -1 & 1 \end{vmatrix} = -8$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 1 & -5 & 3 \\ 2 & -4 & 1 \end{vmatrix}$$

推论1.2.3. 两行(列)相等的行列式的值为0.

$$\begin{array}{l} \\ \\ i\text{行} \\ \\ j\text{行} \\ \\ \\ \end{array}\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = 0$$

说明:

$$\begin{array}{l} \\ \\ i\text{行} \\ \\ j\text{行} \\ \\ \\ \end{array} A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \overset{\text{交换}i,\,j\text{行}}{=} (-1)\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = -A$$

应用:

$$\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0$$

推论1.2.4.行列式可以按任一行(列)展开.

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{i1}A_{i1} + \cdots + a_{ik}A_{ik} + \cdots + a_{in}A_{in}$$

对照行列式定义

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12} + \cdots + a_{1n}A_{1n}$$

$$= a_{11}M_{11} - \cdots + (-1)^{1+k}a_{1k}M_{1k} + \cdots + (-1)^{1+n}a_{1n}M_{1n}$$

说明：

$$A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{i-1,1} & a_{i-1,2} & \cdots & a_{i-1,n} \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ a_{i+1,1} & a_{i+1,2} & \cdots & a_{i+1,n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = (-1)^{i-1} \begin{vmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \\ a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i-1,1} & a_{i-1,2} & \cdots & a_{i-1,n} \\ a_{i+1,1} & a_{i+1,2} & \cdots & a_{i+1,n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= (-1)^{i-1}\left((-1)^{1+1}a_{i1}M_{i1} - \cdots + (-1)^{1+k}a_{ik}M_{ik} + \cdots + (-1)^{1+n}a_{in}M_{in}\right)$$

$$= (-1)^{i+1}a_{i1}M_{i1} + \cdots + (-1)^{i+k}a_{ik}M_{ik} + \cdots + (-1)^{i+n}a_{in}M_{in}$$

$$= a_{i1}A_{i1} + \cdots + a_{ik}A_{ik} + \cdots + a_{in}A_{in}$$

代数余子式符号：

$$\begin{vmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{vmatrix} \qquad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

按第二行展开

$$\begin{vmatrix} 2 & -4 & 1 \\ 3 & 2 & -4 \\ 1 & -1 & 1 \end{vmatrix} = -3\begin{vmatrix} -4 & 1 \\ -1 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - (-4)\begin{vmatrix} 2 & -4 \\ 1 & -1 \end{vmatrix} = 19$$

应用：

$$\begin{vmatrix} 5 & 0 & 4 & 2 \\ 2 & 0 & 3 & 2 \\ 3 & 0 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 1\times\begin{vmatrix} 5 & 4 & 2 \\ 2 & 3 & 2 \\ 3 & 1 & -1 \end{vmatrix} = -15+24+4-18-10+8=-7$$

定理1.2.5. 行列式的任一行(列)元素的公因子可以提到行列式外面.

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ ka_{i1} & ka_{i2} & \cdots & ka_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = k \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

k可以为0

说明:

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ ka_{i1} & ka_{i2} & \cdots & ka_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = ka_{i1}A_{i1} + \cdots + ka_{ik}A_{ik} + \cdots + ka_{in}A_{in}$$

$$\| $$

$$k(a_{i1}A_{i1} + \cdots + a_{ik}A_{ik} + \cdots + a_{in}A_{in}) = k\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

应用:

$$\begin{vmatrix} 20 & -40 & 10 \\ 1 & -5 & 3 \\ 1 & -1 & 1 \end{vmatrix} = 10\begin{vmatrix} 2 & -4 & 1 \\ 1 & -5 & 3 \\ 1 & -1 & 1 \end{vmatrix} = -80.$$

推论1.2.6. 若行列式某两行(列)对应元素成比例，则行列式的值为零.

包括k为0

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ ka_{j1} & ka_{j2} & \cdots & ka_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = 0$$

说明：

$$A=\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ ka_{j1} & ka_{j2} & \cdots & ka_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = k\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = k\times 0 = 0$$

应用：

$$\begin{vmatrix} 2 & -4 & 1 \\ 1 & -5 & 3 \\ 20 & -40 & 10 \end{vmatrix} = 0.$$

定理1.2.7. 行列式的第i行(列) 的每一个元素都可以表示为两数的和，则该行列式可以表示为两个行列式之和.

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1}+b_{i1} & a_{i2}+b_{i2} & \cdots & a_{in}+b_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

说明：

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1}+b_{i1} & a_{i2}+b_{i2} & \cdots & a_{in}+b_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = (a_{i1}+b_{i1})A_{i1}+\cdots+(a_{ik}+b_{ik})A_{ik}+\cdots+(a_{in}+b_{in})A_{in}$$

$$\|$$

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \qquad \begin{aligned} &= a_{i1}A_{i1}+\cdots+a_{ik}A_{ik}+\cdots+a_{in}A_{in} \\ &+ b_{i1}A_{i1}+\cdots+b_{ik}A_{ik}+\cdots+b_{in}A_{in} \end{aligned}$$

应用：

$$\begin{vmatrix} 2 & -4 & 0 \\ 1 & -5 & 0 \\ 1 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 \\ 1 & -3 & 0 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & -4 & 0 \\ 1 & -2 & 0 \\ 1 & -2 & 1 \end{vmatrix} = -6+0=-6$$

定理1.2.8. 将行列式的任意一行(列) 乘以数k加到另一行(列)上去，行列式的值不变.

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1}+ka_{j1} & a_{i2}+ka_{j2} & \cdots & a_{in}+ka_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

说明：

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1}+ka_{j1} & a_{i2}+ka_{j2} & \cdots & a_{in}+ka_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ ka_{j1} & ka_{j2} & \cdots & ka_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

应用：

$$A = \begin{vmatrix} 12345 & 77684 \\ 24690 & 255368 \end{vmatrix} = \begin{vmatrix} 12345 & 77684 \\ 0 & 100000 \end{vmatrix} = 1234500000.$$

$$B = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0.$$

定理1.2.9. 行列式任一行(列) 的元素与另一行(列)元素的代数余子式对应乘积之和为零. 即，若设 $A=\left|a_{ij}\right|_{n\times n}$ 则有

$$\sum_{k=1}^{n} a_{ik}A_{jk}=a_{i1}A_{j1}+a_{i2}A_{j2}+\cdots+a_{in}A_{jn}=\begin{cases}A, & i=j,\\ 0, & i\neq j,\end{cases}$$

$$\left(\sum_{k=1}^{n} a_{ki}A_{kj}=a_{1i}A_{1j}+a_{2i}A_{2j}+\cdots+a_{ni}A_{nj}=\begin{cases}A, & i=j,\\ 0, & i\neq j.\end{cases}\right)$$

说明：

$$\sum_{k=1}^{n} a_{jk}A_{jk}=a_{j1}A_{j1}+a_{j2}A_{j2}+\cdots+a_{jn}A_{jn}=\begin{array}{c|cccc|l} & a_{11} & a_{12} & \cdots & a_{1n} & \\ & \vdots & \vdots & \cdots & \vdots & \\ & a_{j1} & a_{j2} & \cdots & a_{jn} & j\text{行} \\ & \vdots & \vdots & \cdots & \vdots & \\ & a_{n1} & a_{n2} & \cdots & a_{nn} & \end{array}=A$$

显然也有关系：

$$\sum_{k=1}^{n} s_{k}A_{jk}=s_{1}A_{j1}+s_{2}A_{j2}+\cdots+s_{n}A_{jn}=\begin{array}{c|cccc|l} & a_{11} & a_{12} & \cdots & a_{1n} & \\ & \vdots & \vdots & \cdots & \vdots & \\ & s_{1} & s_{2} & \cdots & s_{n} & j\text{行} \\ & \vdots & \vdots & \cdots & \vdots & \\ & a_{n1} & a_{n2} & \cdots & a_{nn} & \end{array}$$

故有关系：

$$\sum_{k=1}^{n} a_{ik}A_{jk}=a_{i1}A_{j1}+a_{i2}A_{j2}+\cdots+a_{in}A_{jn}=\begin{array}{c|cccc|l} & a_{11} & a_{12} & \cdots & a_{1n} & \\ & \vdots & \vdots & \cdots & \vdots & \\ & a_{i1} & a_{i2} & \cdots & a_{in} & i\text{行} \\ & \vdots & \vdots & \cdots & \vdots & \\ & a_{i1} & a_{i2} & \cdots & a_{in} & j\text{行} \\ & \vdots & \vdots & \cdots & \vdots & \\ & a_{n1} & a_{n2} & \cdots & a_{nn} & \end{array}=0$$

应用： 设行列式 $D=\begin{vmatrix}3 & 0 & 4 & 0\\ 2 & 2 & 2 & 2\\ 0 & -7 & 0 & 0\\ 5 & 3 & -2 & 2\end{vmatrix}$.

计算 (1) $2M_{41}-2M_{42}+2M_{43}-2M_{44}$； (2) $M_{41}+M_{42}+M_{43}+M_{44}$.

解 (1) $2M_{41}-2M_{42}+2M_{43}-2M_{44} = -2A_{41}-2A_{42}-2A_{43}-2A_{44}$

$= -(2A_{41}+2A_{42}+2A_{43}+2A_{44})=0$.

(2) $M_{41}+M_{42}+M_{43}+M_{44} = -A_{41}+A_{42}-A_{43}+A_{44}$

例1.2.6 证明n阶范德蒙德(Vandermonde)行列式($n\geq 2$)

结论可作为公式使用

$$D_n(x_1,x_2,\cdots,x_n)=\begin{vmatrix}1 & 1 & \cdots & 1\\ x_1 & x_2 & \cdots & x_n\\ x_1^2 & x_2^2 & \cdots & x_n^2\\ \vdots & \vdots & & \vdots\\ x_1^{n-1} & x_2^{n-1} & \cdots & x_n^{n-1}\end{vmatrix}=\prod_{1\leq i<j\leq n}(x_j-x_i).$$

应用：

$$A=\begin{vmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 2 & -2\\ 1 & 1 & 4 & 4\\ 1 & -1 & 8 & -8\end{vmatrix}$$

$$=(-1-1)(2-1)(-2-1)(2-(-1))(-2-(-1))(-2-2)$$

$$=(-2)\times 1\times(-3)\times 3\times(-1)\times(-4)$$

$$=72.$$

例1.2.7 证明$n+m$阶行列式 (块三角行列式)

结论可作为公式使用

$$D_n=\begin{vmatrix} a_{11} & \cdots & a_{1n} & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} & 0 & \cdots & 0 \\ c_{11} & \cdots & c_{1n} & b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots & \vdots & & \vdots \\ c_{m1} & \cdots & c_{mn} & b_{m1} & \cdots & b_{mm} \end{vmatrix}=\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}\begin{vmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots \\ b_{m1} & \cdots & b_{mm} \end{vmatrix}. \qquad (1.15)$$

式 (1.15) 常简记为： $D_n=\begin{vmatrix} A & O \\ * & B \end{vmatrix}=|A||B|$,类似可得$D_n=\begin{vmatrix} A & * \\ O & B \end{vmatrix}=|A||B|$.

另外要注意：$D_n=\begin{vmatrix} O & A \\ B & * \end{vmatrix}\neq|A||B|,\begin{vmatrix} O & A \\ B & * \end{vmatrix}=(-1)^{nm}\begin{vmatrix} A & O \\ * & B \end{vmatrix}=(-1)^{nm}|A||B|$.

应用：

$$C=\left|\begin{array}{cc:cccc} 2 & 1 & 6 & -2 & 5 & 2 \\ 3 & 2 & 8 & 11 & -9 & -23 \\ \hdashline 0 & 0 & 3 & -5 & 7 & 1 \\ 0 & 0 & -2 & 4 & -2 & 7 \\ 0 & 0 & 0 & 0 & 7 & 2 \\ 0 & 0 & 0 & 0 & 9 & 3 \end{array}\right|=\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}\times\left|\begin{array}{cc:cc} 3 & -5 & 7 & 1 \\ -2 & 4 & -2 & 7 \\ \hdashline 0 & 0 & 7 & 2 \\ 0 & 0 & 9 & 3 \end{array}\right|=1\times\begin{vmatrix} 3 & -5 \\ -2 & 4 \end{vmatrix}\times\begin{vmatrix} 7 & 2 \\ 9 & 3 \end{vmatrix}=1\times2\times3=6.$$