



Chapter 4

Tree

Two kinds of data structure

A decorative graphic consisting of a horizontal bar with a color gradient from dark blue on the left to bright yellow on the right. To the right of the bar is a large, stylized arrow pointing to the right, filled with a brown-to-gold gradient.

- Linear: list, stack, queue, string
- Non-linear: tree, graph

4.1 Tree



1. Definition:

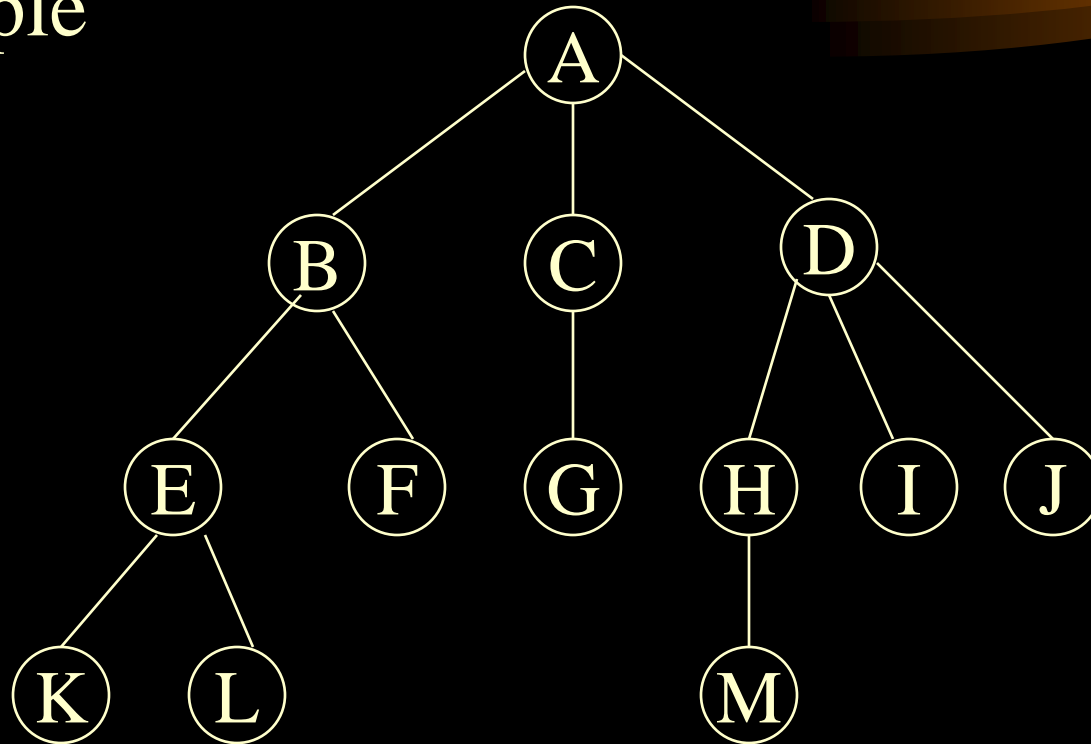
A tree T is a collection of nodes(element).

The collection can be empty;

otherwise, a tree consists of a distinguished node r ,
called the **root**, and zero or more nonempty(sub)trees T_1 ,
 T_2, \dots, T_k

4.1 Tree

example



4.1 Tree



2. Terminology

Degree of an elements(nodes): the number of children it has.

Degree of a tree: the maximum of its element degrees

Leaf: element whose degree is 0

Branch: element whose degree is not 0

4.1 Tree



Level:

the level of root is 0 (1)

the level of an element=

the level of its parent+1

Depth(Height) of a tree:

the maximum level of its elements

4.2 Binary Trees

1. **Definition:** A binary tree t is a finite (possibly empty) collection of elements.

When the binary tree is not empty:

- It has a **root** element
- The remaining elements(if any) are partitioned into two binary trees, which are called the **left** and **right** subtrees of t .

5种基本构型:

4.2 Binary Trees

2. The essential differences between a binary tree and a tree are:

1) Each element **in a binary tree** has exactly two subtrees (one or both of these subtrees may be empty).

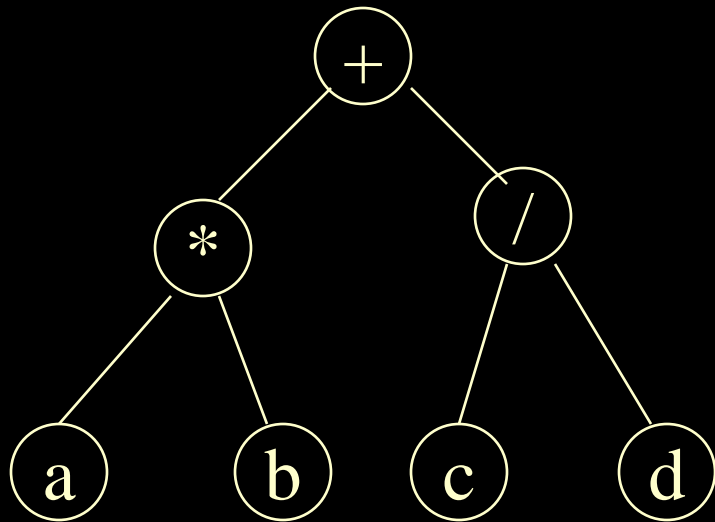
Each element **in a tree** can have any number of subtrees.

2) The subtrees of each element **in a binary tree** are ordered. That is, we distinguish between the left and the right subtrees.

The subtrees **in a tree** are unordered.

4.2 Binary Trees

Example of a binary tree



$(a*b)+(c/d)$

4.3 Properties of binary trees

Property 1. The drawing of every binary tree with n elements ($n > 0$) has exactly $n-1$ edges.

Property 2. The number of elements at level i is at most 2^i ($i \geq 0$).

4.3 Properties of binary trees

Property 3. A binary tree of height h , $h \geq 0$, has at least $h+1$ and at most $2^{h+1} - 1$ elements in it.

proof of property 3:

$$\sum_{i=0}^h 2^i = 2^0 + 2^1 + \dots + 2^h = 1 * (1 - 2^{h+1}) / (1 - 2) = 2^{h+1} - 1$$

4.3 Properties of binary trees

Property 4. If number of leaves is n_0 , and the number of the 2 degree elements is n_2 , then $n_0 = n_2 + 1$.

Proof:

设：度为1的结点数是 n_1 个

$$n = n_0 + n_1 + n_2$$

$$n = B + 1 \quad \text{这里} B \text{为分支数}$$

$$n_0 + n_1 + n_2 = 1 * n_1 + 2 * n_2 + 1$$

$$n_0 = n_2 + 1$$

4.3 Properties of binary trees

Property 5. The height of a binary tree that contains n ($n \geq 0$) element is at most $n-1$ and at least $\lceil \log_2(n+1) \rceil - 1$

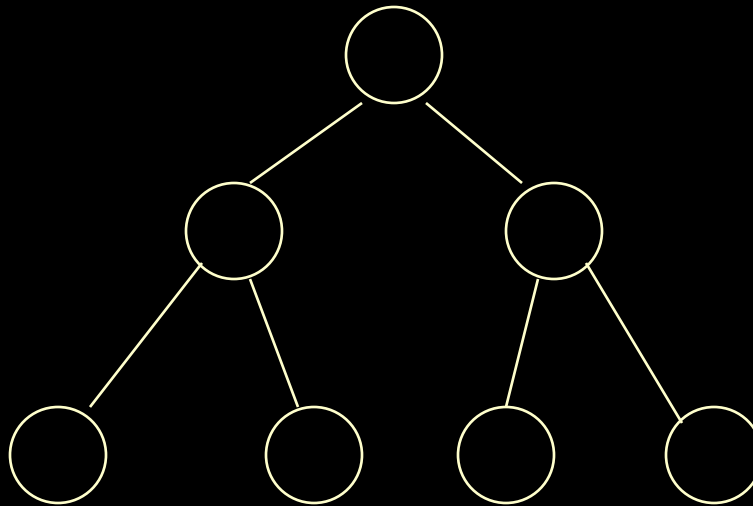
proof: Since there must be at least one element at each level, the height cannot exceed $n-1$.

From property 3, we know $n \leq 2^{h+1} - 1$,
so, $h \geq \log_2(n+1) - 1$, since h is an integer, we get
 $h = \lceil \log_2(n+1) \rceil - 1$

4.3 Properties of binary trees

Definition of a **full binary tree** :

A binary tree of height h that contains exactly $2^{h+1}-1$ elements is called a **full binary tree**.



4.3 Properties of binary trees

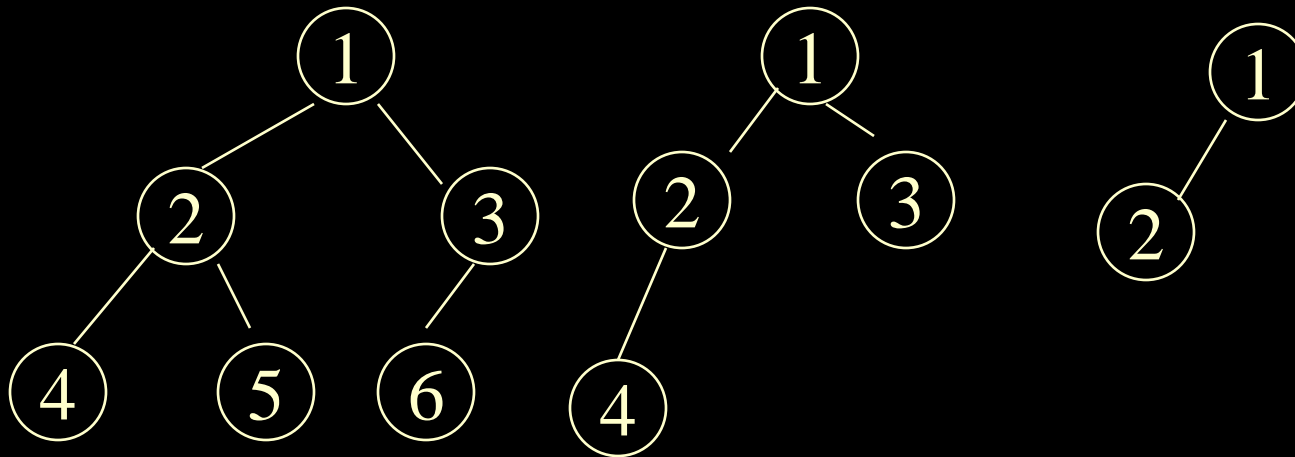
Definition of a complete binary tree:

Suppose we number the elements in a full binary tree of height h using the number 1 through $2^{h+1}-1$. We began at level 0 and go down to level h . Within levels the elements are numbered left to right.

Suppose we delete the k elements numbered $2^{h+1}-i$, $1 \leq i \leq k$, the resulting binary tree is called a complete binary tree.

4.3 Properties of binary trees

Example of complete binary trees



4.3 Properties of binary trees

Property 6. Let i , $0 \leq i \leq n-1$, be the number assigned to an element of a complete binary tree. The following are true.

- 1) if $i=0$, then this element is the root of the binary tree.
if $i>0$, then the parent of this element has been assigned the number $\lfloor (i-1)/2 \rfloor$
- 2) if $2*i+1 \geq n$, then this element has no left child. Otherwise, its left child has been assigned the number $2*i+1$.

4.3 Properties of binary trees

- 3) if $2*i+2 \geq n$, then this element has no right child,
Otherwise its right child has been assigned the number
 $2*i+2$.

4.4 Representation of binary tree

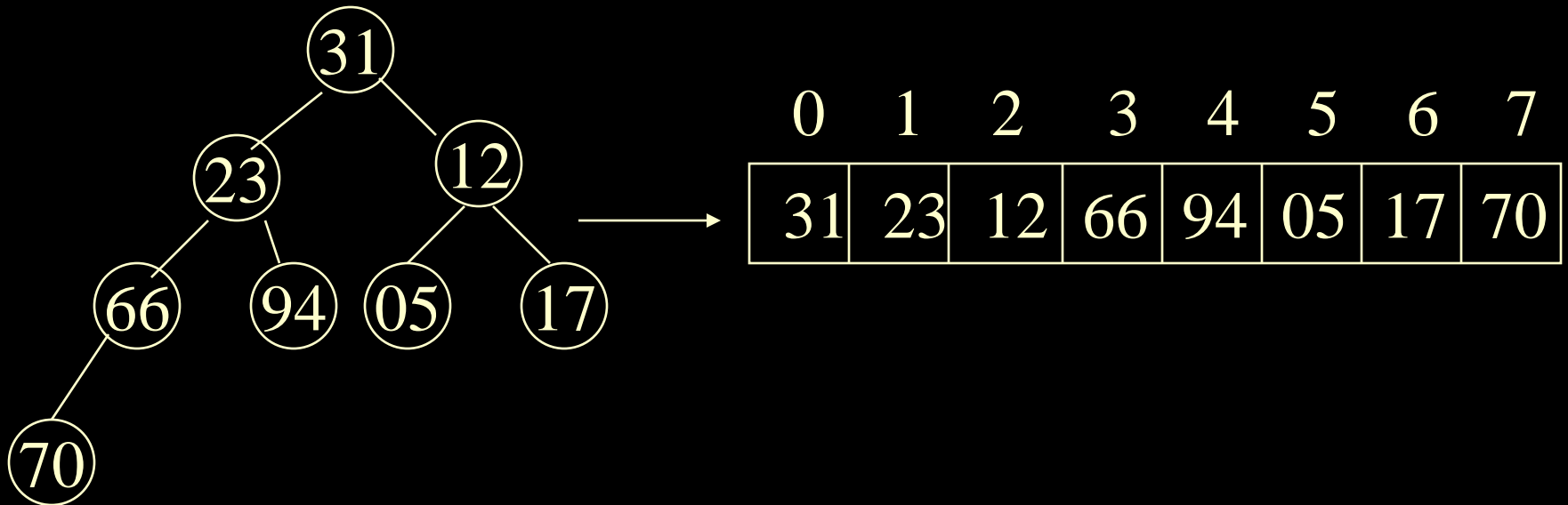


1. Formula-Based Representation (array representation)

The binary tree to be represented is regarded as a complete binary tree with some missing elements.

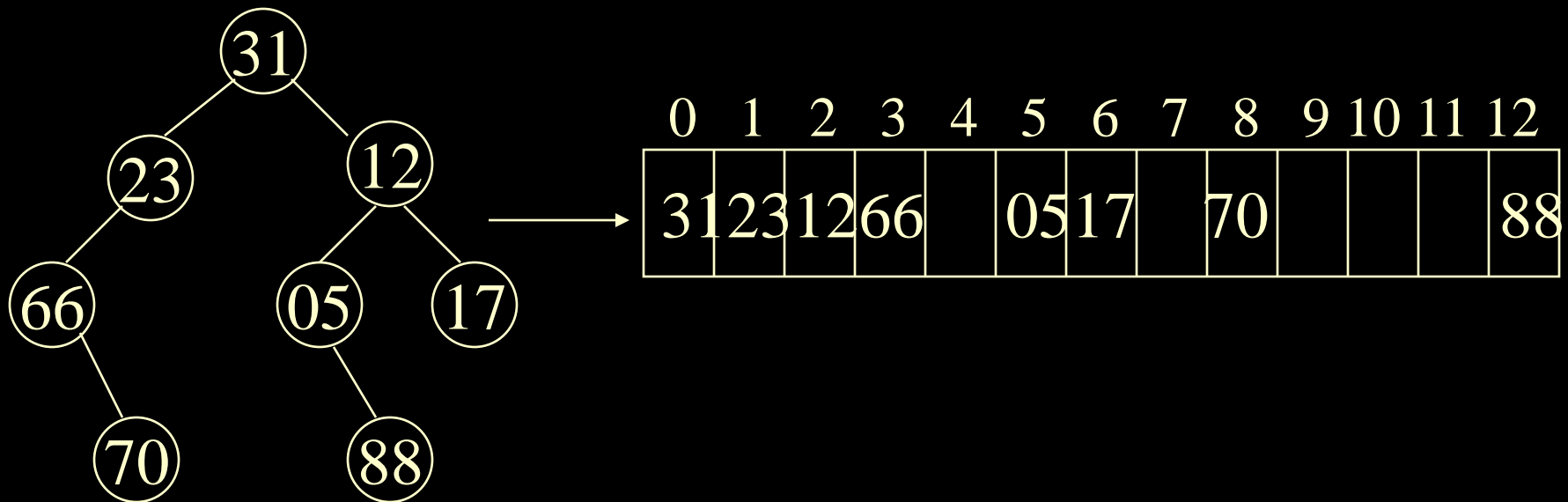
4.4 Representation of binary tree

Example of a complete binary tree (array representation)



4.4 Representation of binary tree

Example of a common binary tree(array representation)



4.4 Representation of binary tree

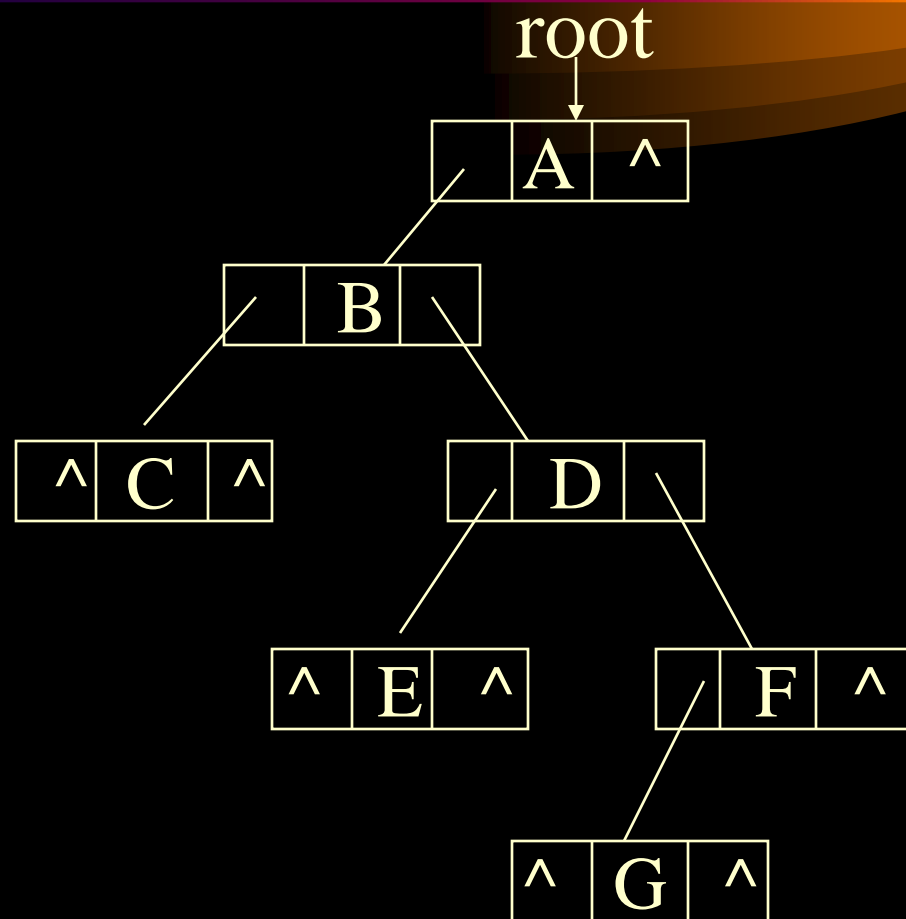
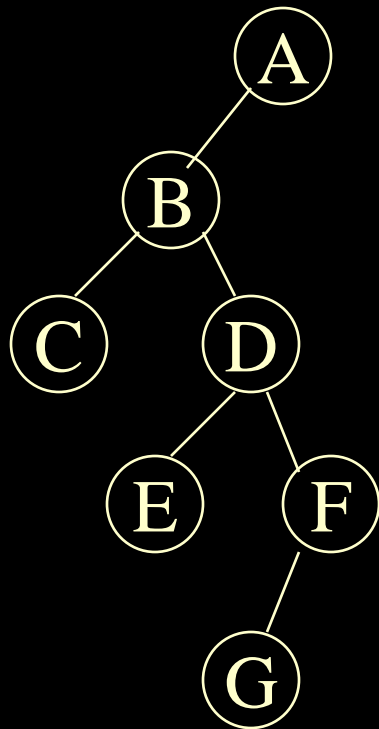
2. Linked representation(also called L-R linked storage)

The node structure:

LeftChild	data	RightChild
-----------	------	------------

4.4 Representation of binary tree

Example



4.4 Representation of binary tree

3. Represented by Cursor

	data	leftchild	rightchild
0	A	1	-1
1	B	2	3
2	C	-1	-1
3	D	4	5
4	E	-1	6
5	F	-1	-1
6	G	-1	-1

4.4 Representation of binary tree

Node class for linked binary trees

```
class BinaryNode
```

```
{
```

```
    BinaryNode(){Left=Right=0;}
```

```
    BinaryNode(Object e)
```

```
        {element=e; Left=Right=0;}
```

```
    BinaryNode(Object e, BinaryNode l, BinaryNode r)
```

```
        {element=e; Left=l; Right=r; }
```

```
    Object element;
```

```
    BinaryNode left; //left subtree
```

```
    BinaryNode right; //right subtree
```

```
};
```

4.5 Common binary tree operations

the abstract data type binary tree

- Create()
- IsEmpty()
- Root(x)
- MakeTree(root, left, right)
- BreakTree(root, left, right)
- PreOrder
- InOrder
- PostOrder
- LevelOrder

4.5 Common binary tree operations

The Class BinaryTree

1. Binary tree class

```
template<class T>class BinaryTree
{ public:
    BinaryTree(){root=0;};
    ~BinaryTree(){ };
    bool IsEmpty()const
        {return ((root)?false:true);}
    bool Root(T& x)const;
    void MakeTree(const T& data,
        BinaryTree<T>& leftch, BinaryTree<T>& rightch);
```

4.5 Common binary tree operations

```
void BreakTree(T& data , BinaryTree<T>& leftch,  
    BinaryTree<T>& rightch);  
void PreOrder(void(*visit)(BinaryNode<T>*u))  
    {PreOrder(visit, root);}   
void InOrder(void(*visit)(BinaryNode<T>*u))  
    {InOrder(visit, root);}   
void PostOrder (void(*visit)(BinaryNode<T>*u))  
    {PostOrder(visit, root);}   
void LevelOrder  
    (void(*visit)(BinaryNode<T> *u));
```

4.5 Common binary tree operations

private:

```
    BinaryNode<T>* root;
```

```
    void PreOrder(void(*visit)(BinaryNode<T> *u),  
                  BinaryNode<T>*t);
```

```
    void InOrder(void(*visit)(BinaryNode<T> *u),  
                 BinaryNode<T>*t);
```

```
    void PostOrder(void(*visit) (BinaryNode<T> *u),  
                   BinaryNode<T>*t);
```

```
};
```

4.5 Common binary tree operations

- In this class ,we employ a linked representation for binary trees.
- The function **visit** is used as parameter to the traversal methods,so that different operations can be implemented easily

4.5 Common binary tree operations

2.Implementation of some member functions

```
Template<class T>
```

```
void BinaryTree<T>::MakeTree(const T& data,  
    BinaryTree<T>& leftch,  BinaryTree<T>& rightch)  
{ root=new  BinaryNode<T>(data, leftch.root,  
                                                                    rightch.root);  
  leftch.root=rightch.root=0;  
}
```

4.5 Common binary tree operations

```
template<class T>
void BinaryTree<T>::BreakTree(T& data,
    BinaryTree<T>& leftch, BinaryTree<T>& rightch)
{ if(!root)throw BadInput(); //tree empty
  data=root.element;
  leftch.root=root.Left;
  rightch.root=root.Right;
  delete root;
  root=0;
}
```


4.5 Common binary tree operations

3. Application of class BinaryTree(Create BinaryTree)

```
#include<iostream.h>
```

```
#include "binary.h"
```

```
int count=0;  BinaryTree<int>a,x,y,z;
```

```
template<class T>
```

```
void ct(BinaryTreeNode<T>*t){ count++;}
```

```
void main(void)
```

```
{  a.MakeTree(1,0,0);
```

```
    z.MakeTree(2,0,0);
```

```
    x.MakeTree(3,a,z);
```

```
    y.MakeTree(4,x,0);
```

```
    y.PreOrder(ct);
```

```
    cout<<count<<endl;
```

```
}
```

4.6 Binary Tree Traversal

Each element is visited exactly once

V-----表示访问一个结点

L-----表示访问V的左子树

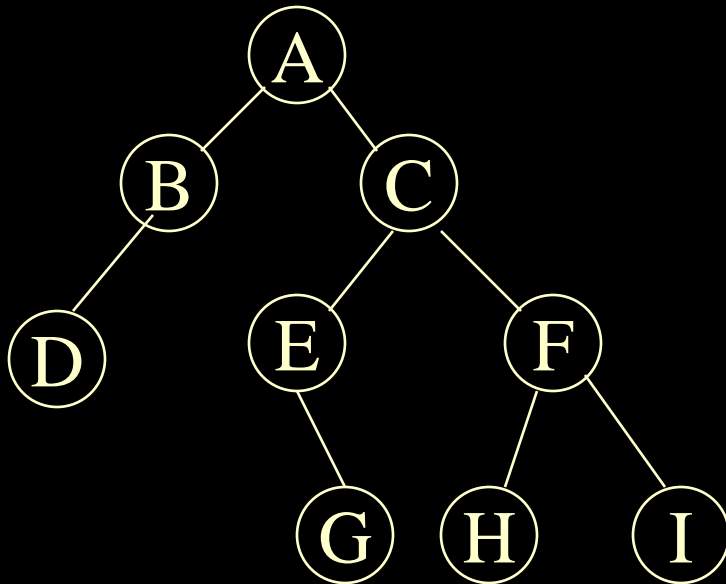
R-----表示访问V的右子树

VLR LVR LRV VRL RVL RLV

- Preorder
- Inorder
- Postorder
- Level order

4.6 Binary Tree Traversal

Example of binary tree traversal



Preorder : ABDCEGFHI

Inorder : DBAEGCHFI

Postorder : DBGEHIFCA

Level order: ABCDEFGHI

4.6 Binary Tree Traversal

Preorder traversal

```
template<class T>
void PreOrder(BinaryNode<T>* t)
{ // preorder traversal of *t.
    if(t){ visit(t);
           PreOrder(t→Left);
           PreOrder(t→Right);
        }
}
```

4.6 Binary Tree Traversal

Inorder traversal

```
template<class T>
void InOrder(BinaryNode<T>* t)
{ if(t){ InOrder(t→Left);
        visit(t);
        InOrder(t→Right);
      }
}
```

4.6 Binary Tree Traversal

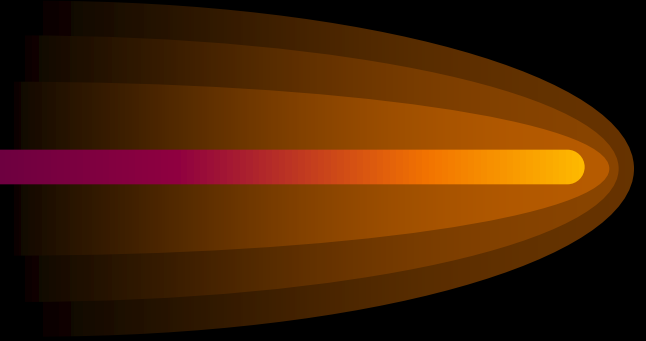
Postorder traversal

```
template<class T>
void PostOrder(BinaryNode<T>* t)
{ if(t){
    PostOrder(t→Left);
    PostOrder(t→Right);
    visit(t);
}
}
```

4.6 Binary Tree Traversal

Example:

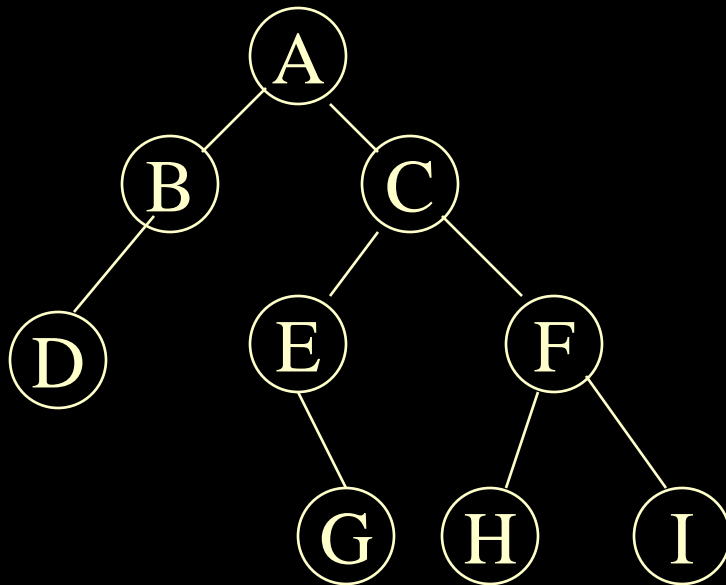
Inorder traversal:



4.6 Binary Tree Traversal

Level order

it is a non-recursive function and a queue is used.



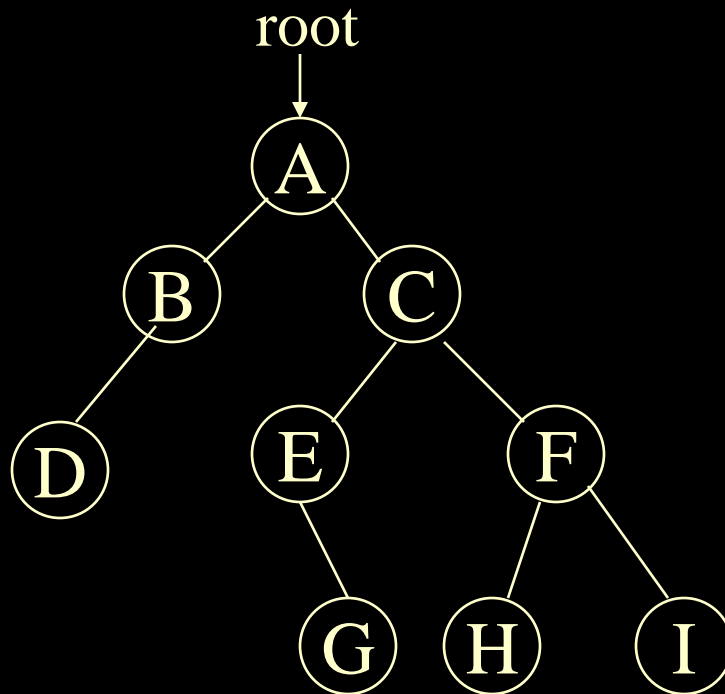
4.6 Binary Tree Traversal

Level order

```
template<class T>
void LevelOrder(BinaryNode<T>* t)
{   LinkedQueue<BinaryNode<T>*> Q;
    while(t){
        visit(t);    //visit t
        if(t→Left) Q.Add(t→Left);
        if(t→Right) Q.Add(t→Right);
        try{ Q.Delete(t);}
        catch(OutOfBounds){return;}
    }
}
```

Inorder, Postorder non-recursive algorithm

- **Inorder** non-recursive algorithm

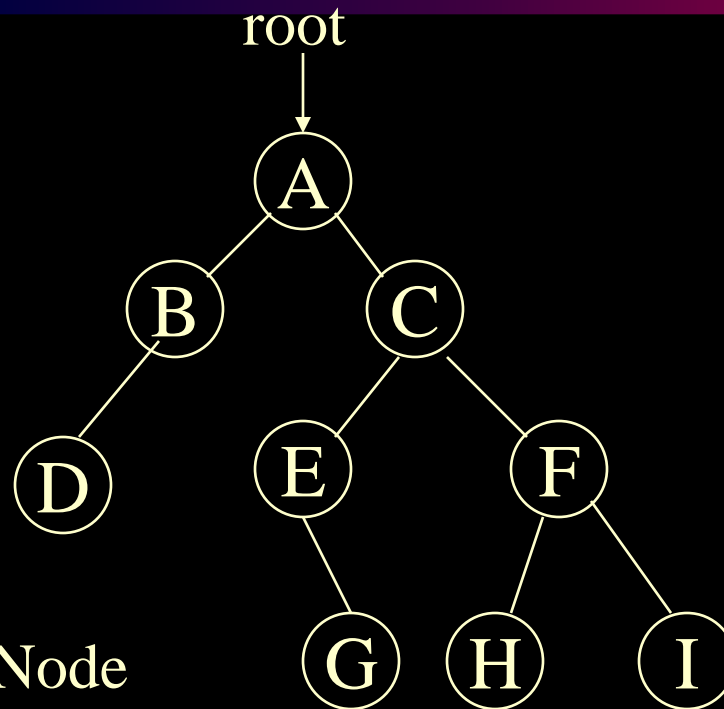


Inorder non-recursive algorithm

```
void Inorder(BinaryNode <T> * t)
{
    Stack<BinaryNode<T>*> s(10);
    BinaryNode<T> * p = t;
    for ( ; ; )
    {
        1) while(p!=NULL)
            { s.push(p); p = p->Left; }
        2) if (!s.IsEmpty( ))
            {
                p = s.pop( );
                cout << p->element;
                p = p->Right;
            }
        else return;
    }
}
```

Inorder, Postorder non-recursive algorithm

- **Postorder** non-recursive algorithm



```
struct StkNode
{
    BinaryNode <T> * ptr;
    int tag;
}
```

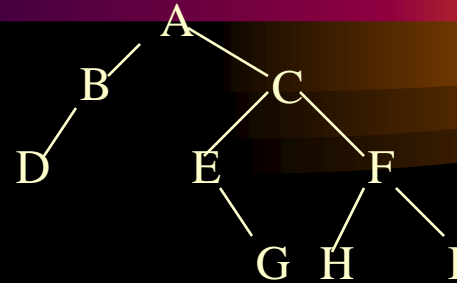
Postorder non-recursive algorithm

```
void Postorder(BinaryNode <T> * t)
{ Stack <StkNode<T>>s(10);
  StkNode<T> Cnode;
  BinaryNode<T> * p = t;
  for( ; ; )
  { 1)while (p!=NULL)
      { Cnode.ptr = p;  Cnode.tag = 0; s.push(Cnode);
        p = p->Left;
      }
    2)Cnode = s.pop( );  p = Cnode.ptr;
    3)while ( Cnode.tag == 1) //从右子树回来
      { cout << p->element;
        if ( !s.IsEmpty( ))
          { Cnode = s.pop( );  p = Cnode.ptr; }
        else return;
      }
    4)Cnode.tag = 1; s.push(Cnode);  p = p->Right;  //从左子树回来
  }//for
}
```

建立一棵二叉树的方法

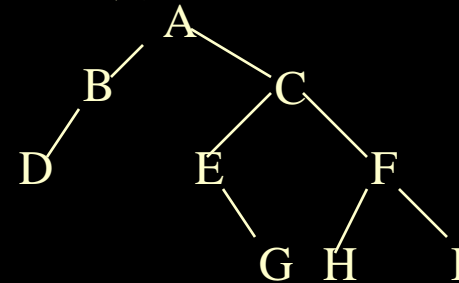
1. 利用MakeTree函数
2. 利用先序、中序唯一的构造一棵二叉树

先序: ABDCEGFHI
中序: DBAEGCHFI } →



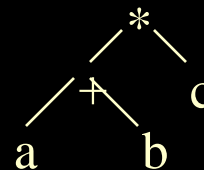
- *3. 利用二叉树的广义表表示来构造一棵二叉树

$A(B(D), C(E(,G), F(H,I))) \longrightarrow$



4. 利用二叉树的后缀表示来构造一棵二叉树

$(a+b)*c \longrightarrow ab+c* \longrightarrow$



建立一棵二叉树的方法

4. 利用二叉树的后缀表示来构造一棵二叉树(习题)

$$(a + b) * c \longrightarrow a b + c *$$

$$(- a + b) * c \longrightarrow a - b + c *$$

$$d + (- a + b) * c \longrightarrow d a - b + c * +$$

2. 利用先序、中序唯一的构造一棵二叉树 string

1. 字符串的有关概念
串的定义、术语、基本操作
2. 字符串的类说明
3. 部分成员函数的实现
4. 利用前序、中序序列建立一棵树

string

1. 字符串（简称串）的定义以及一些术语

*串：是 n ($n \geq 0$) 个字符的一个有限序列，开头结尾用双引号“ ”括起来。

例如： $B = \text{“structure”}$ (B 为串名)

*串的长度：串中所包含的字符个数 n （不包括分界符‘ ’，也不包括串的结束符‘\0’）

*空串：长度为0的串。或者说只包含串结束符‘\0’的串

注意：“\0”不等于“ \0”，空串不等于空白串

*子串：串中任一连续子序列

例子： $B = \text{“peking”}$ 则空串“ ”、“ki”、“peking”都是 B 的子串

但“pk”不是 B 的子串

string

*串的基本操作:

构造一个空串;

求串长;

两个串的连接（并置）;

取子串;

求一个子串在串中第一次出现的位置等。

string

Java与C/C++的不同处:

Java语言的字符串不是字符数组，所以不能以字符数组方式进行一些操作。如， `str[1] = "a"` 是错误的，而只能通过方法（函数）来进行操作。

`int length()`

`boolean equals(Object obj)`

`char charAt(int index)`

`String substring (int beginIndex)`

`String substring (int beginIndex, int endIndex)`

string

2. 字符串的类说明

```
const int maxlen=128;
class String
{ public:
    String(const String & ob);
    String(const char * init);
    String( );
    ~String( ) {delete[ ] ch;}
    int Length( )const {return curlen;}
    String & operator( )(int pos, int len); //取子串
    int operator ==(const String & ob) const
        { return strcmp(ch, ob.ch)==0;} //判别相等否?
    int operator !=(const String &ob) const
        { return strcmp(ch, ob.ch)!=0;}
    int operator ! ( ) const {return curlen==0;}
```

string

String & operator = (const String & ob); //串赋值

String & operator +=(const String & ob); //并置运算

char & operator[](int i);

int Find(String pat) const;

private:

int curLen;

char * ch;

}

string

3. 部分成员函数的实现

Taking Substring

```
String & String::operator()(int pos, int len)
{
    String *temp=new String;

    if (pos<0||pos+len-1>=maxlen ||len<0)
    {
        temp->curLen=0; temp->ch[0]='\0';
    }

    else{ if (pos+len-1>=curLen) len=curLen-pos;

        temp->curLen=len;
        for(int i=0, j=pos; i<len; i++, j++)
            temp->ch[i]=ch[j];
        temp->ch[len]='\0';
    }
    return *temp;
}
```

string

Assigning Operate

```
String & String ::operator=(const String &ob)
```

```
{ if (&ob!=this)
```

```
    {delete [ ] ch;
```

```
      ch=new char[maxLen+1];
```

```
      if(!ch) {cerr<< “Out Of Memory! \n”;exit(1);} 
```

```
      curLen=ob.curLen;
```

```
      strcpy(ch, ob.ch);
```

```
    }
```

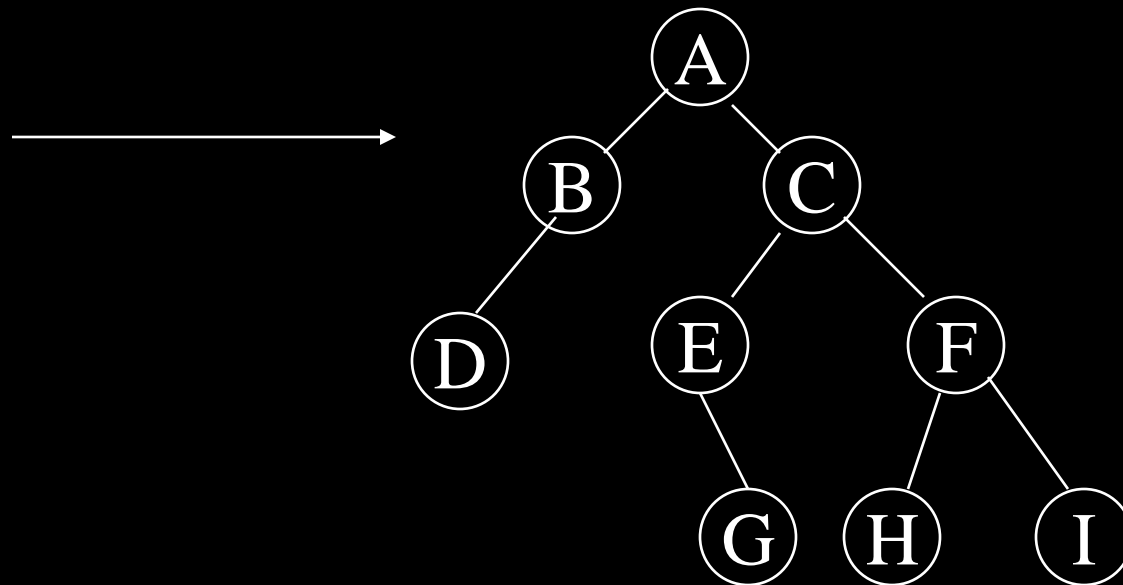
```
    else cout<<“Attempted assignment of a String to itself! \n”;
```

```
    return * this;
```

```
}
```

4. Create BinaryTree recursive algorithm

{ preorder: ABDCEGFHI
inorder: DBAEGCHFI



Create BinaryTree recursive algorithm 1

```
void CreateBT(String pres, ins ; BinaryNode <Type>* & t)
```

```
{  int inpos;
```

```
    String pretemp, instemp ;
```

```
    if (pres.length( )==0) t=NULL;
```

```
    else { t=new BinaryNode;
```

```
          t->element=pres.ch[0];  inpos=0;
```

```
          while (ins.ch[inpos]!=t->element) inpos++;
```

```
          pretemp=pres(1,inpos);
```

```
          instemp=ins(0,inpos-1);
```

```
          CreateBT(pretemp, instemp, t->left);
```

```
          pretemp=pres(inpos+1, pres.length( )-1);
```

```
          instemp=ins(inpos+1, pres.length( )-1);
```

```
          CreateBT(pretemp, instemp, t->right);
```

```
    }
```

```
}
```

Create BinaryTree recursive algorithm 1

public:

```
BinaryTree( string pre, string In )
```

```
{ createBT( pre, In, root ) ;  
}
```

.....

```
main( )
```

```
{ BinaryTree t1( “ABHFDECKG”, “ HBDFAEKCG” ) ;
```

.....

```
}
```

如果已知后序与中序，能否唯一构造一棵二叉树呢？

后序: **DBGHEHIFCA**

中序: **DBAEGCHFI**

如果已知先序与后序呢？

4.7 ADT and Class Extensions

- **PreOutput():**output the data fields in preorder
- **InOutput():**output the data fields in inorder
- **PostOutput():**output the data fields in postorder
- **LevelOutput():**output the data fields in level order
- **Delete():**delete a binary tree,freeing up its nodes
- **Height():**return the tree height
- **Size():**return the number of nodes in the tree

4.7 ADT and Class Extensions

The height of the tree is determined as:

$$\max\{hl, hr\}+1$$

```
template<class T>
```

```
int BinaryTree<T>::Height(BinaryNode<T> *t) const
```

```
{ if(!t) return 0;
```

```
  int hl=Height(t→Left);
```

```
  int hr=Height(t→Right);
```

```
  if(hl>hr) return ++ hl;
```

```
  else return ++hr;
```

```
}
```

4.8 Application

1.Binary-Tree Representation of a Tree

树的存储方式：三种

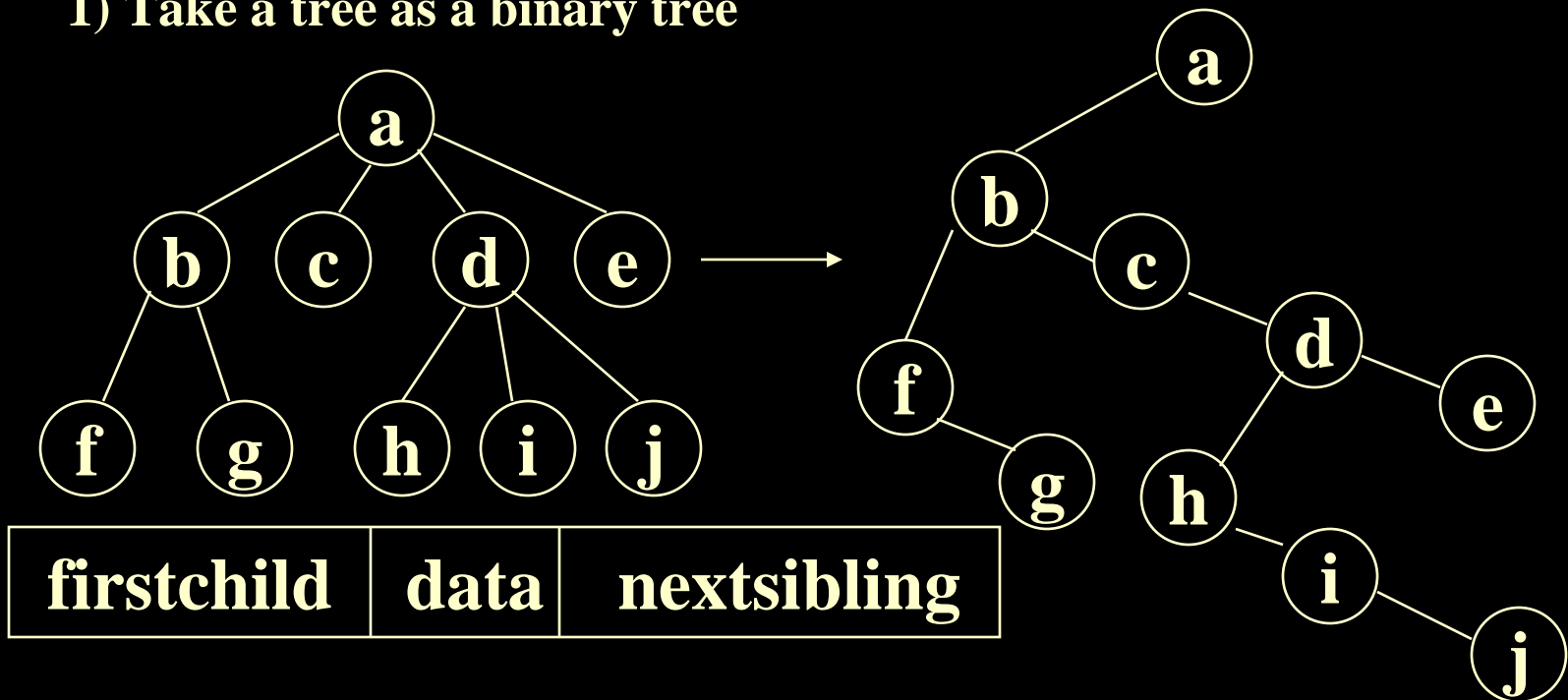
- 广义表表示：a(b(f,g),c,d(h,i,j),e)

- 双亲表示法

- 左子女—右兄弟表示法

	a	b	f	g	
	0	1	2	2		

1) Take a tree as a binary tree



4.8 Application

```
class TreeNode:
```

```
    T data;
```

```
    TreeNode *firstchild, *nextsibling;
```

```
class Tree:
```

```
    TreeNode * root, *current;
```

4.8 Application

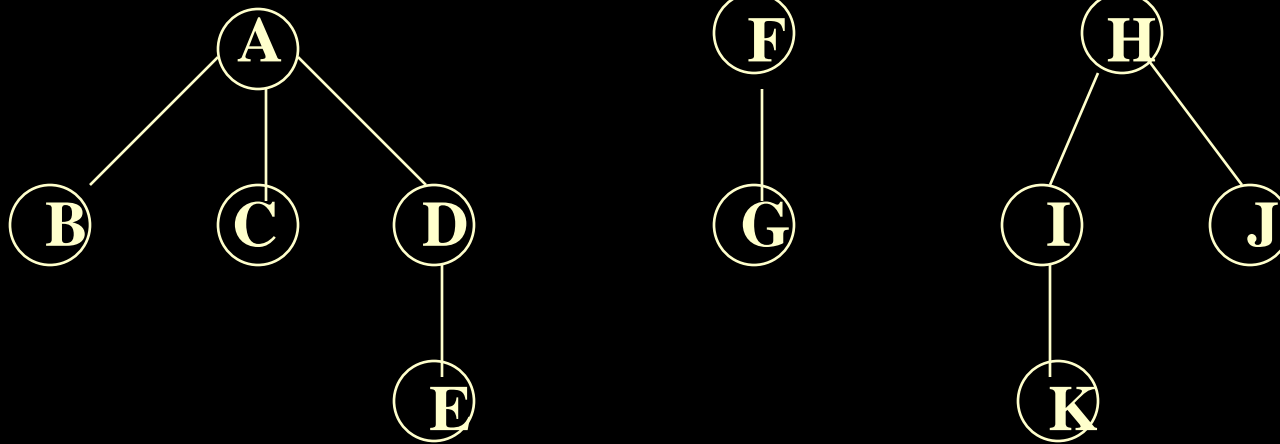
insert new node in tree

```
template <class T> void Tree <T>::Insertchild(T value)
{  TreeNode<T>*newnode = new TreeNode<T>(value);
    if(current->firstchild == NULL)
        current->firstchild = newnode;
    else
    {  TreeNode<T>*p = current->firstchild;
        while ( p->nextsibling!=NULL) p = p->nextsibling;
        p->nextsibling = newnode;
    }
}
```

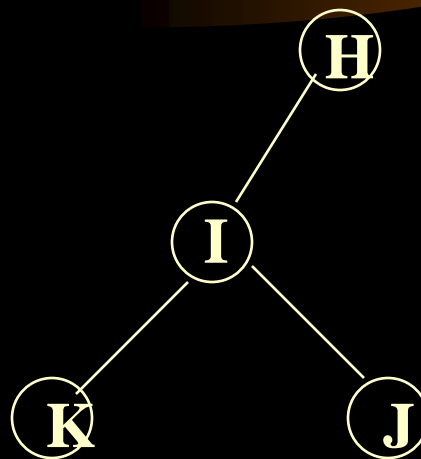
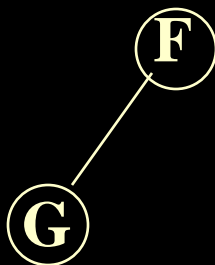
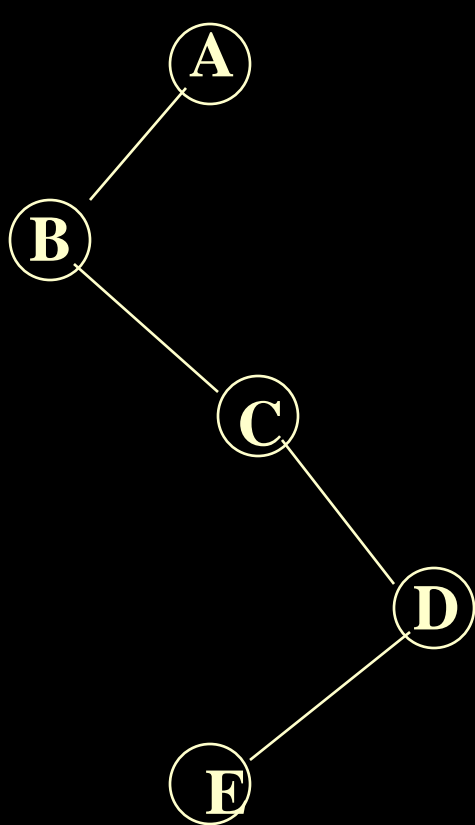
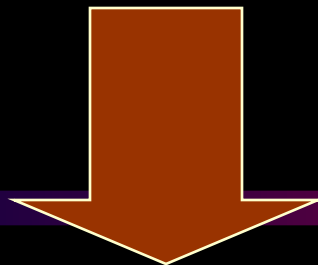

4.8 Application

2) Forest \longleftrightarrow Binary tree

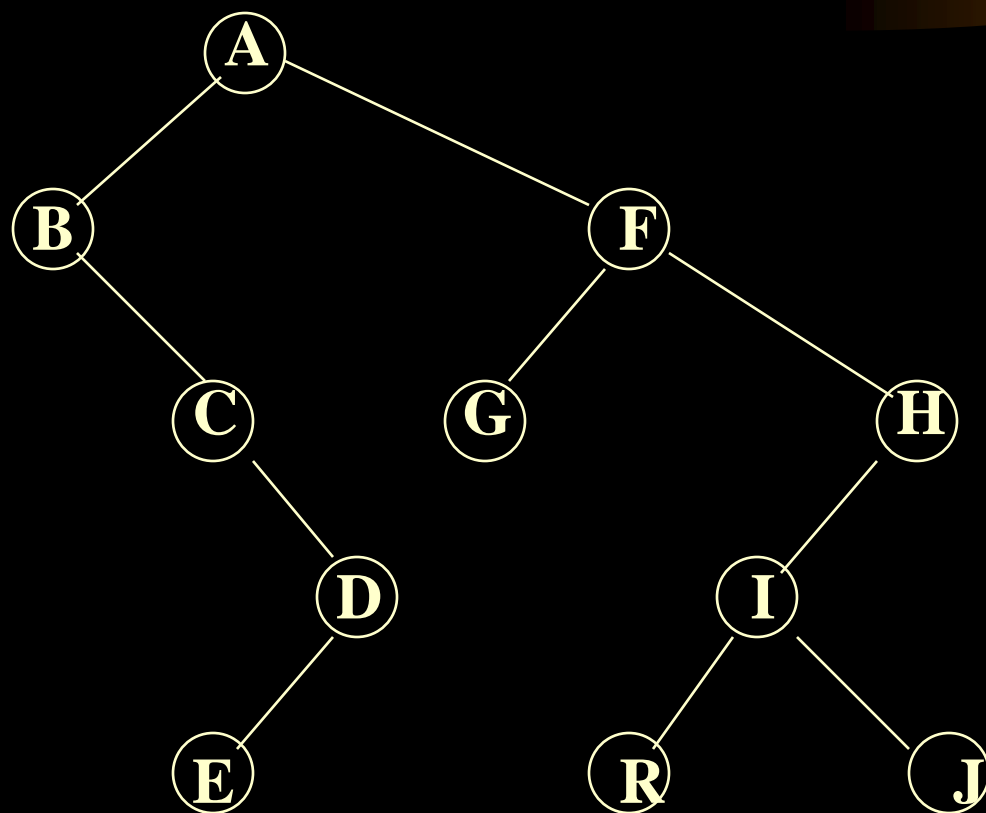
- Forest \longrightarrow Binary tree



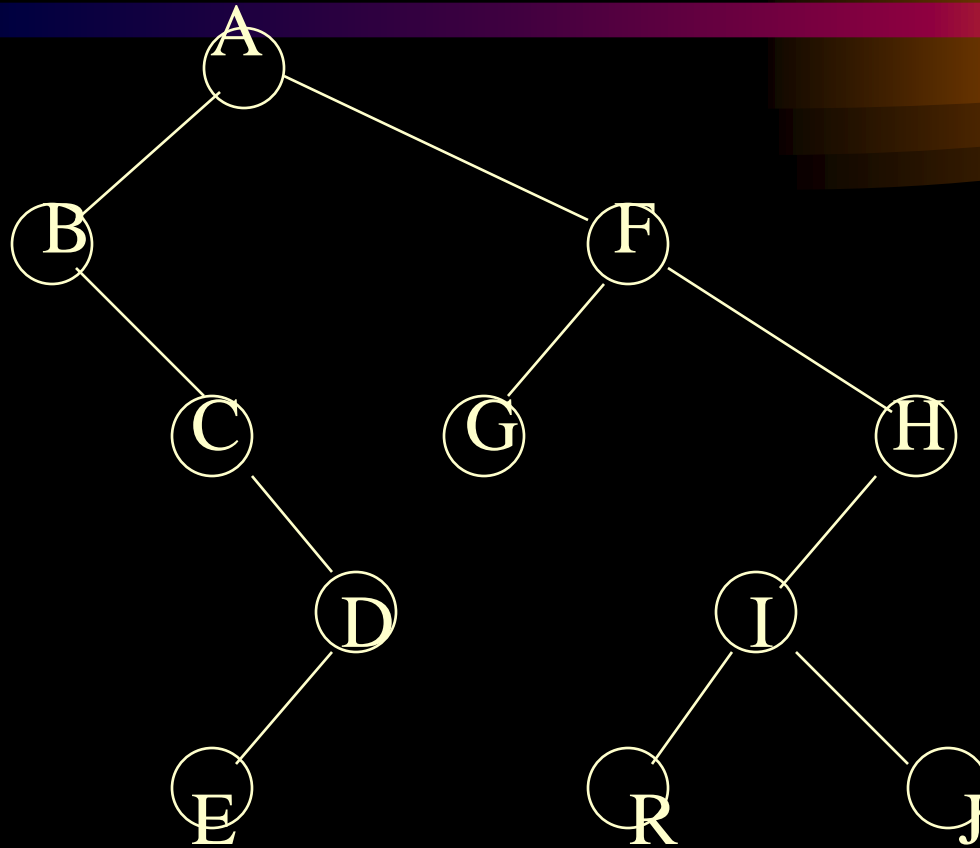
每棵树转为二叉树



把每棵二叉树根用右链相连



- **Binary tree** \longrightarrow **Forest**



4.8 Application

3) 树的遍历：深度优先遍历，广度优先遍历

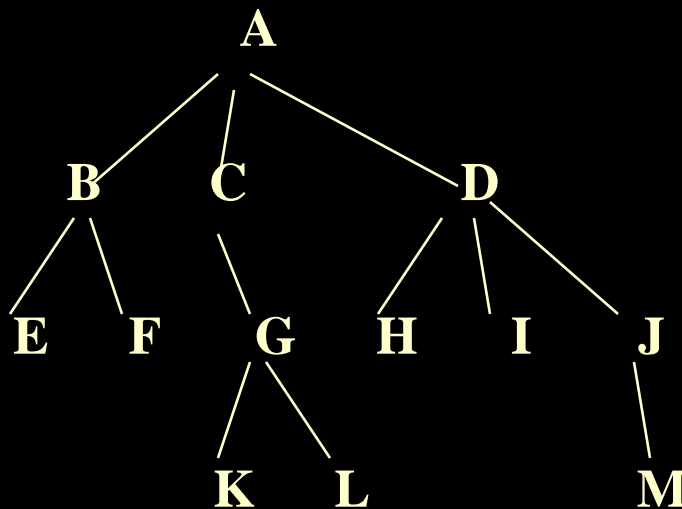
- 深度优先遍历

先序次序遍历（先序）

访问树的根 ——> 按先序遍历根的第一棵子树，第二棵子树，.....等。

后序次序遍历（后序）

按后序遍历根的第一棵子树，第二棵子树，.....等 ——> 访问树的根。

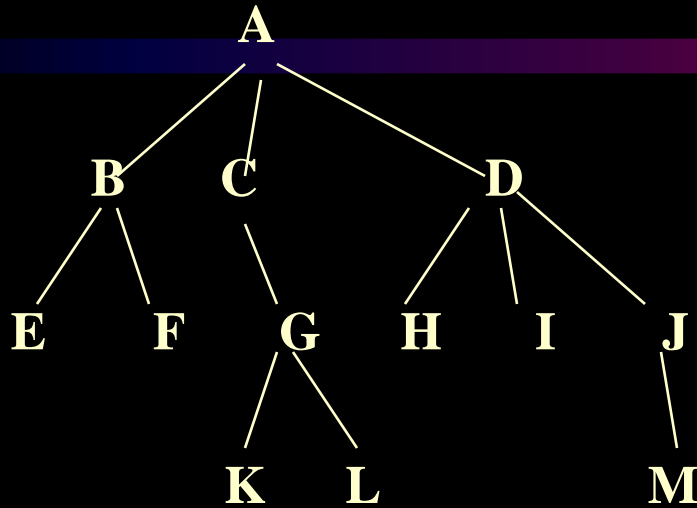


先根：ABEFCGKLDHIJM与
对应的二叉树的先序一致

后根：EFBKL GCHIMJDA与
对应的二叉树的~~中序~~一致

4.8 Application

- 广度优先遍历



分层访问: ABCDEFGHIJKLM

4) 森林的遍历

深度优先遍历

* 先根次序遍历

访问F的第一棵树的根

按先根遍历第一棵树的子树森林

按先根遍历其它树组成的森林

二叉树的先序

* 中根次序遍历

按中根遍历第一棵树的子树森林

访问F的第一棵树的根

按中根遍历其它树组成的森林

二叉树的中序

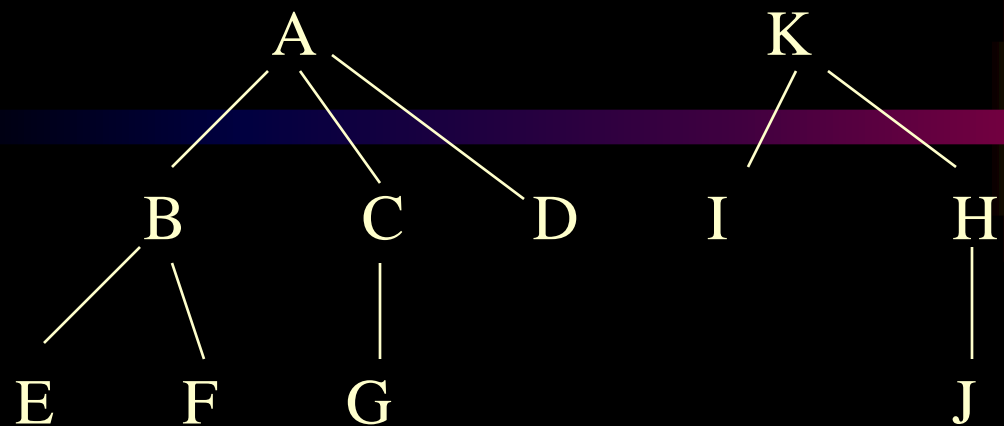
* 后根次序遍历

按后根遍历第一棵树的子树森林

按后根遍历其它树组成的森林

访问F的第一棵树的根

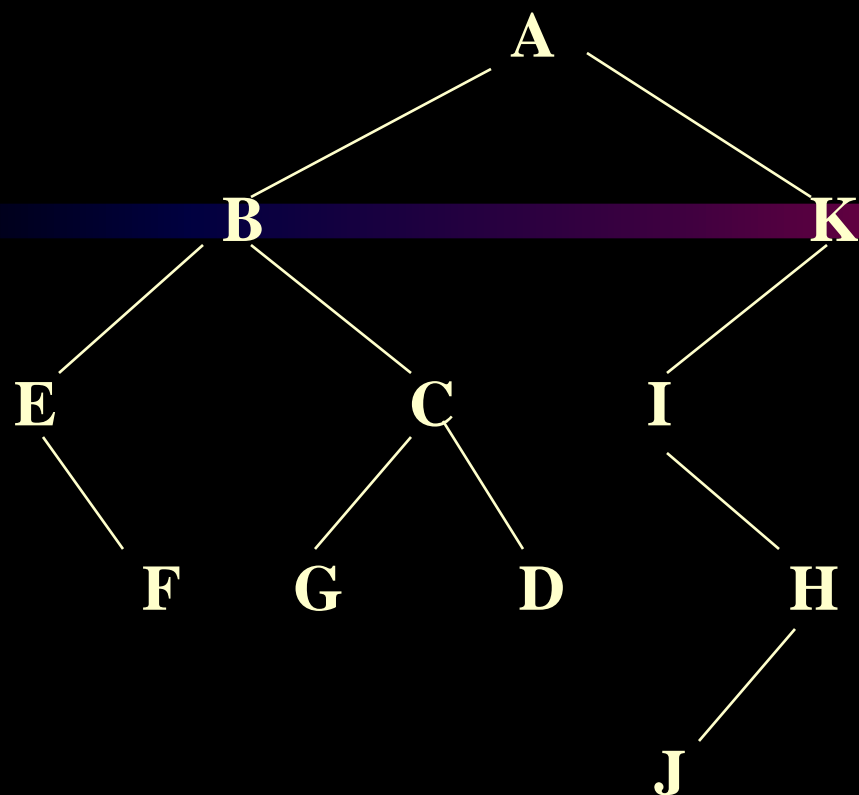
二叉树的后序



先根: **ABEFCGDKIHJ**

中根: **EFBGCDALJHK**

后根: **FEGDCBJHIKA**



后序: **FEGDCBJHIKA**

广度优先遍历 (层次遍历)

AKBCDIHEFGJ

Thread Tree

1.Purpose:

2. Thread Tree Representation

left Thread Tree and right Thread Tree

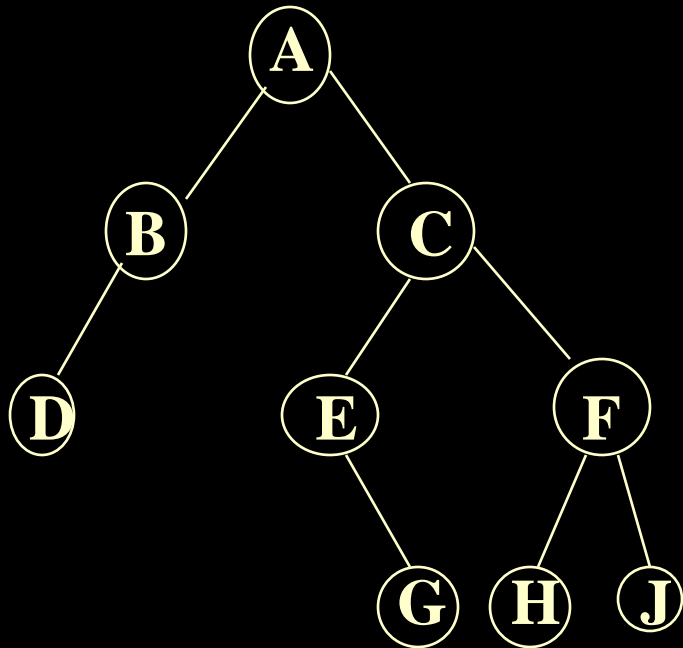
3.Thread Tree class

1.Purpose:

n 个结点的二叉树有 $2n$ 个链域，
其中真正有用的是 $n - 1$ 个，其它 $n + 1$ 个都是空域。
为了充分利用结点中的空域，使得对某些运算更快，如
前驱或后继等运算。

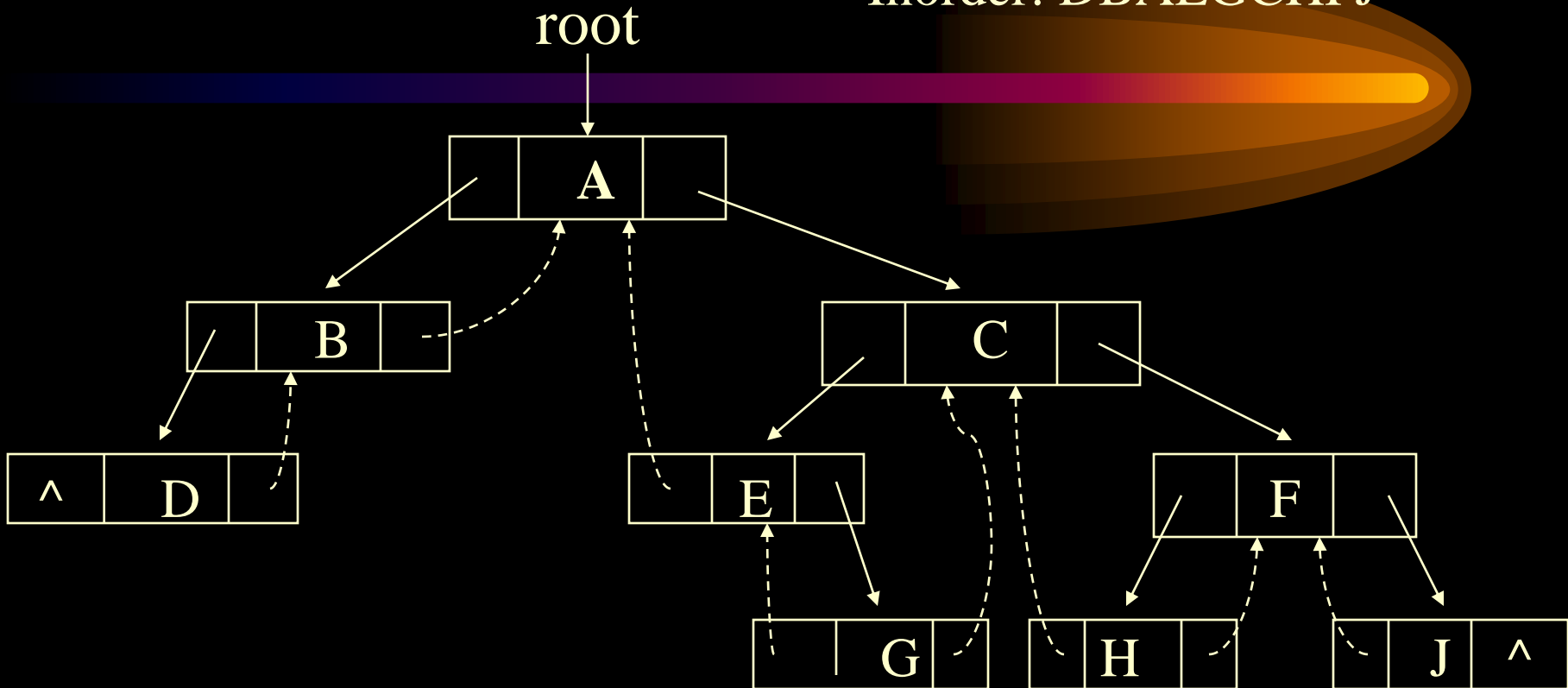
Thread Tree

Example:



Thread Tree

Inorder: DBAEGCHFJ



2. 机内如何存储

一个结点增加两个标记域：

leftchild	leftthread	data	rightthread	rightchild
-----------	------------	------	-------------	------------

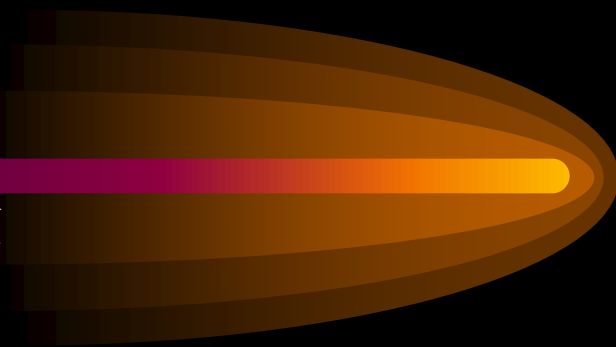
leftThread == $\begin{cases} 0 & \text{leftchild 指向左子女} \\ 1 & \text{leftchild 指向前驱 (某线性序列)} \end{cases}$

rightThread == $\begin{cases} 0 & \text{rightchild 指向右子女} \\ 1 & \text{rightchild 指向后继} \end{cases}$

3. 线索化二叉树的类声明。

```
template< class Type> class ThreadNode
{
    friend class ThreadTree;
private:
    int leftThread, rightThread;
    ThreadNode<Type>* leftchild, *rightchild;
    Type data;
public:
    ThreadNode(const Type item): data(item), leftchild(0),
        rightchild(0), leftThread(0), rightThread(0) { }
};
```

```
template< class Type> class ThreadTree
{
    public:
        // 线索二叉树的公共操作
    private:
        ThreadNode<Type> * root;
        ThreadNode<Type> *current
};
```



讨论线索化二叉树的几个算法

1) 按中序遍历中序线索树

遍历算法（以中序为例）：

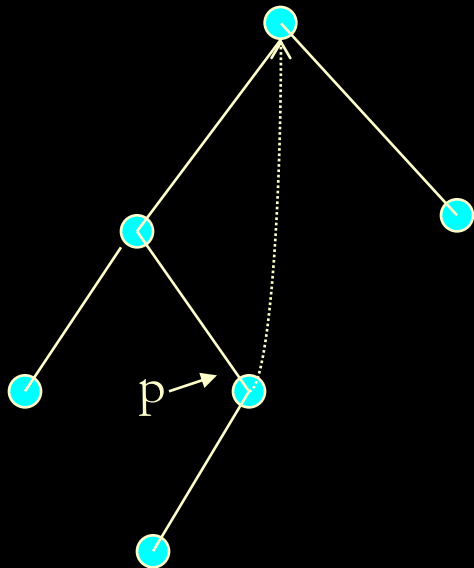
递归， 非递归（需用工作栈）。

这里前提是中序线索树， 所以既不要递归， 也不要栈。

遍历算法： * 找到中序下的第一个结点（first）

* 不断找后继（Next）

如何找后继？

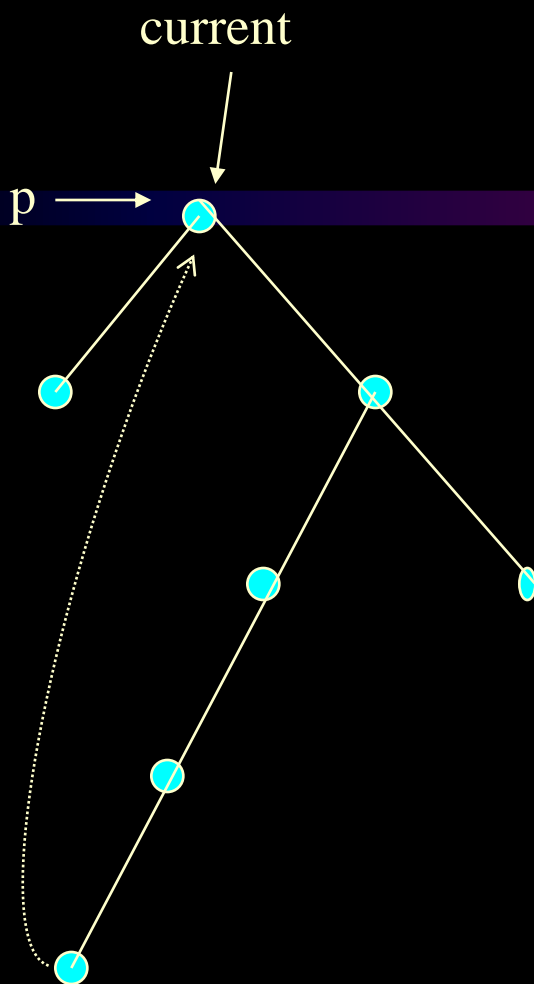


p指结点没有右子树

$(p \rightarrow \text{rightthread} = 1)$

则 $p = p \rightarrow \text{rightchild}$

（右链就是后继）



p有右子树

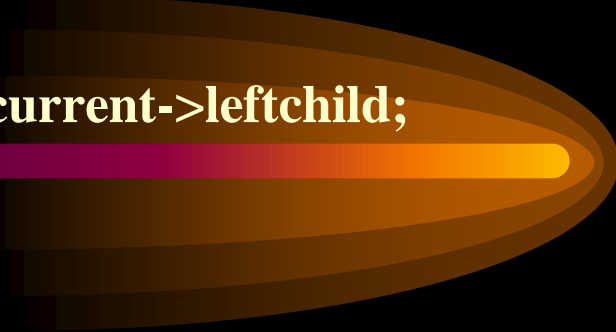
(p->rightThread == 0)

则 (1) p=p->rightchild

(2) while(p->leftThread == 0)

p=p->leftchild;

```
template<class Type>
ThreadNode<Type>* ThreadInorderIterator<Type>::First( )
{
    while (current->leftThread==0) current=current->leftchild;
    return current;
}
```



```
template<class Type>
ThreadNode<Type>* ThreadInorderIterator<Type>::Next( )
{
    ThreadNode<Type>* p=current->rightchild;
    if(current->rightThread==0)
        while(p->leftThread==0) p=p->leftchild;
    current=p;
}
```

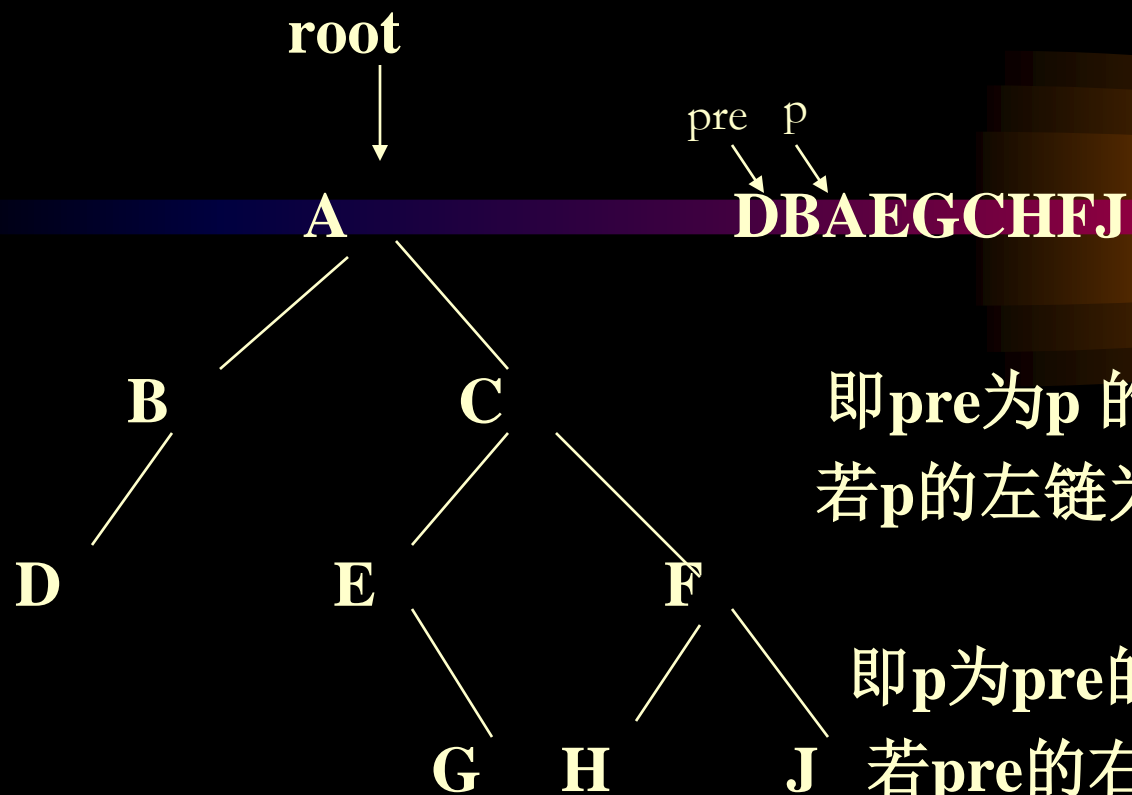
```
template<class Type>
void ThreadInorderIterator<Type>:: Inorder( )
{
    ThreadNode<Type> *p;
    for ( p=First( ); p!=NULL; p=Next( ))
        cout<< p->data<<endl;
}
```

2) 构造中序线索树

对已存在的一棵二叉树建立中序线索树

分析： 与中序遍历算法差不多，但是要填左空域右空域的前驱、后继指针。所以除了流动指针 p 外，还要加一个 pre 指针，它总是指向遍历指针 p 的中序下的前驱结点。

例如：



即pre为p 的中序下的前驱，
若p的左链为空，则可填pre

即p为pre的中序下的后继
若pre的右链为空，则可填p

Thread Tree

Create inorder threadTree:

```
Void Inthread(threadNode<T> * T)
```

```
{ stack <threadNode <T> *> s (10)
```

```
  ThreadNode <T> * p = T ; ThreadNode <T> * pre = NULL;
```

```
  for ( ; ; )
```

```
    { 1.while (p!=NULL)
```

```
      { s.push(p); p = p ->leftchild; }
```

```
    2.if (!s.IsEmpty( ))
```

```
      { 1) p = s.pop;
```

```
        2) if (pre != NULL)
```

```
          { if (pre ->rightchild == NULL)
```

```
            { pre ->rightchild = p; pre ->rightthread = 1;}
```

```
          if ( p -> leftchild == NULL)
```

```
            { p -> leftchild = pre ; p ->leftthread = 1; }
```

```
          }
```

```
        3) pre = p ; p = p -> rightchild ;
```

```
      }
```

```
    else return;
```

```
  }//for
```

```
}
```

4.8 Application

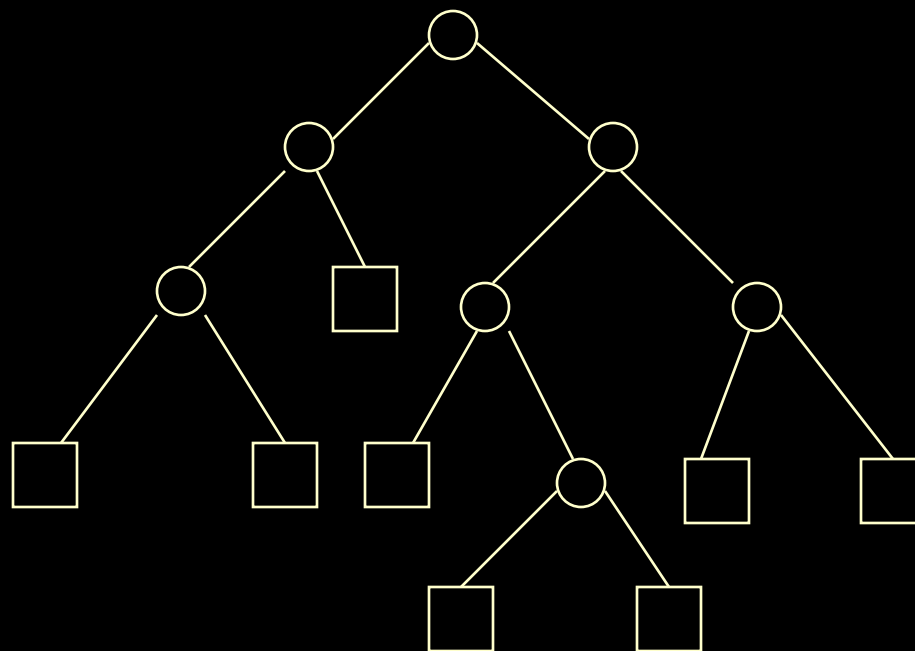
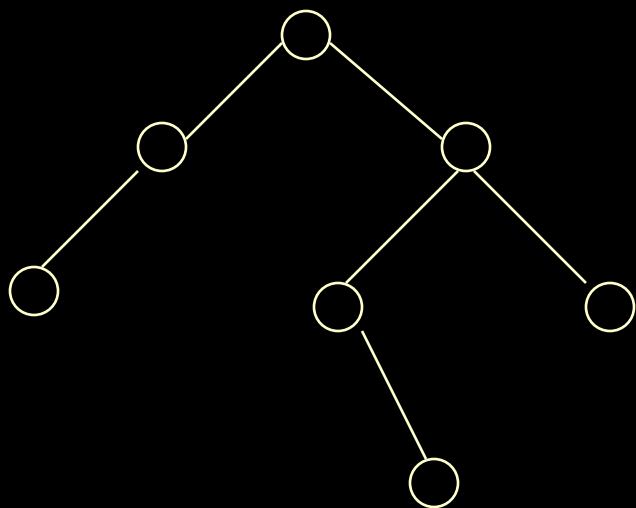
2. 霍夫曼树(Huffman Tree)

● 增长树的概念

1) 增长树

对原二叉树中度为1的结点，增加一个空树叶

对原二叉树中的树叶， 增加两个空树叶



2) 外通路长度（外路径）E：根到每个外结点（增长树的叶子）的路径长度的总和（边数）

$$E=3+3+2+3+4+4+3+3=25$$

3) 内通路长度（内路径）I：根到每个内结点（非叶子）的路径长度的总和（边数）。

$$I=2+1+0+3+2+2+1=11$$

4) 结点的带权路径长度：一个结点的权值与结点的路径长度的乘积。

5) 带权的外路径长度：各叶结点的带权路径长度之和。

6) 带权的内路径长度：各非叶结点的带权路径长度之和。

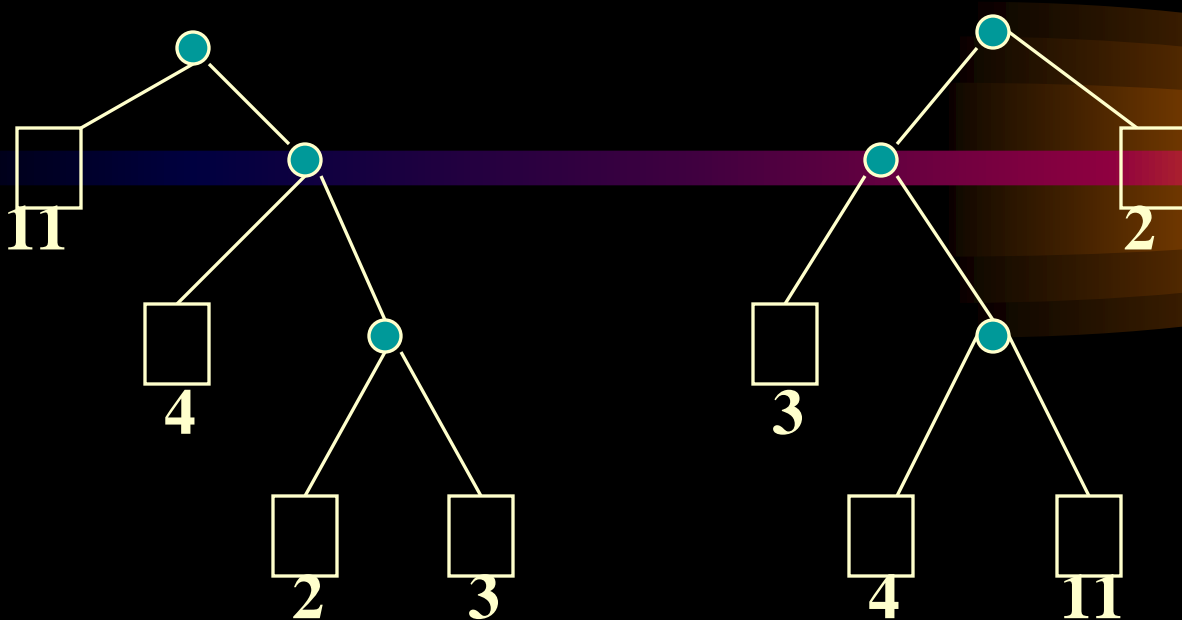
- 霍夫曼树

1) 给出m个实数 W_1, W_2, \dots, W_m ($m \geq 2$) 作为m个外结点的权构造一棵增长树，使得带权外路径长度

$$\sum_{i=1}^m W_i l_i \text{ 最小。}$$

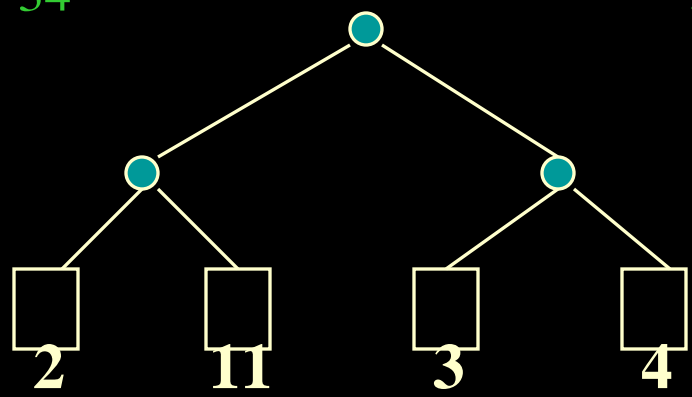
其中 l_i 为从根结点出发到具有权为 w_i 的外结点的通路长。

2) 例子：外结点权为 2, 3, 4, 11 则可构造



34

53

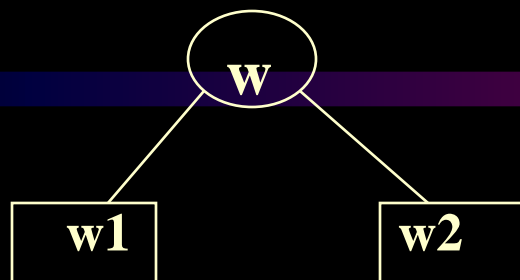


40

3) Huffman 算法

思想： 权大的外结点靠近根， 权小的远离根。

算法： 从 m 个权值中找出两个最小值 w_1 ， w_2 构成



$w = w_1 + w_2$ 表通过该结点的频度。

然后对 $m-1$ 个权值 w ， w_3 ， w_4 ， ... w_m 经由小到大排序， 求解这个问题。

例子： 2， 3， 5， 7， 11， 13， 17， 19， 23， 29， 31， 37， 41

注意：当内结点的权值与外结点的权值相等的情况下， 内结点应排在外结点之后。除了保证 $\sum w_i l_i$ 最小外， 还保证 $\max l_j$ ， $\sum l_j$ 也有最小值

例如： 7, 8, 9, 15

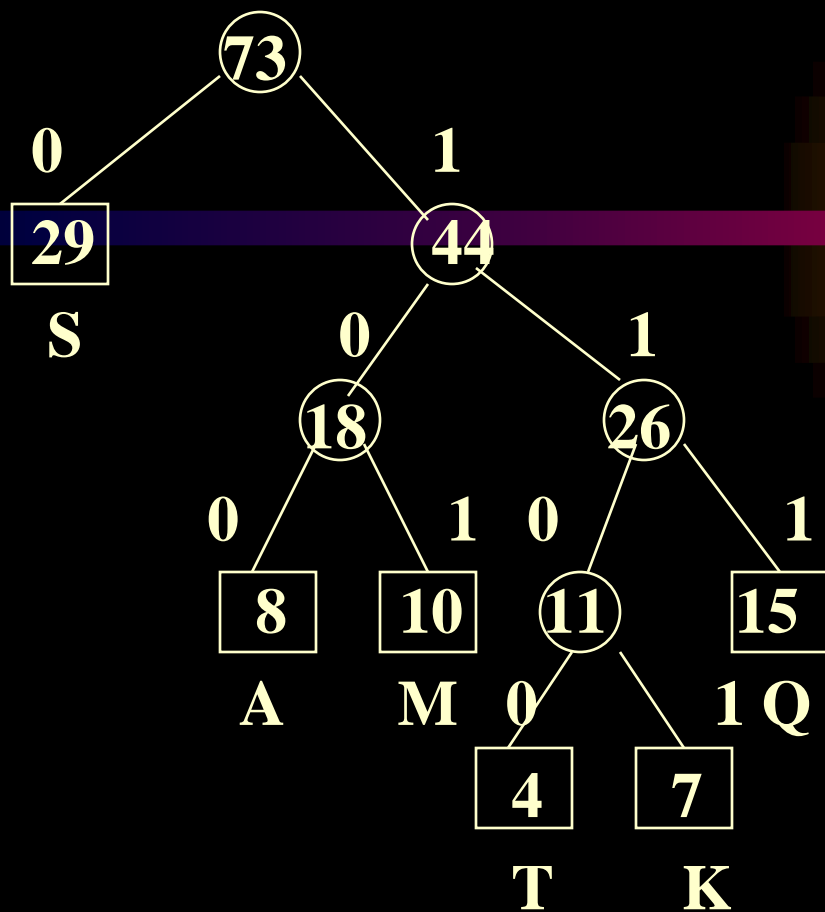
- 霍夫曼编码

是霍夫曼树在数据编码中一种应用。具体的讲用于通信的二进制编码中。设一电文出现的字符为 $D=\{M, S, T, A, Q, K\}$ ，每个字符出现的频率为 $W=\{10, 29, 4, 8, 15, 7\}$ ，如何对上面的诸字符进行二进制编码，使得

- 1) 该电文的总长度最短。

- 2) 为了译码，任一字符的编码不应是另一字符的编码的前缀

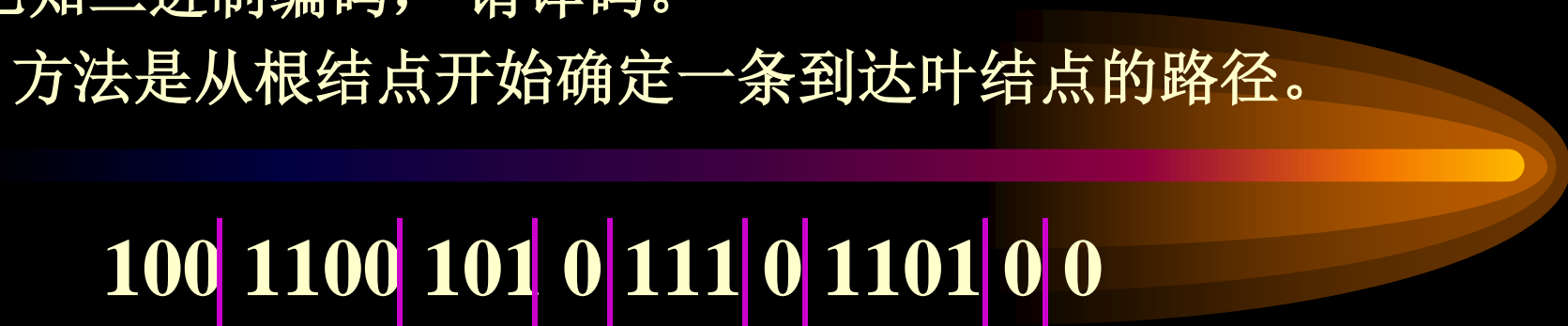
算法：利用Huffman算法，把 $\{10, 29, 4, 8, 15, 7\}$ 作为外部结点的权，构造具有最小带权外路径长度的扩充二叉树，把每个结点的左子女的边标上0，右子女标上1。这样从根到每个叶子的路径上的号码连接起来，就是外结点的字符编码。



编码: S: 0 A: 100 M: 101 Q: 111
T: 1100 K: 1101

已知二进制编码， 请译码。

方法是从根结点开始确定一条到达叶结点的路径。



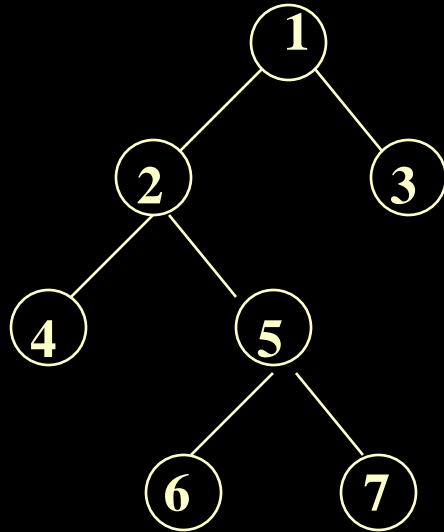
100	1100	101	0	111	0	1101	0	0
A	T	M	S	Q	S	K	S	S

Chapter 4

2009年统考题(单项选择):

3. 给定二叉树如下图所示. 设N 代表二叉树的根, L 代表二叉树的左子树, R 代表根结点的右子树. 若遍历后的结点序列为 3, 1, 7, 5, 6, 2, 4, 则其遍历方式是

A. LRN B. NRL C. RLN D. RNL



Chapter 4

2009年统考题(单项选择):

4. 已知一棵完全二叉树的第6层(设根为第1层)有8个叶结点, 则该完全二叉树的结点个数最多是

A. 39 B. 52 C. 111 D. 119

5. 将森林转换为对应的二叉树, 若在二叉树中, 结点 u 是结点 v 的父结点的父结点, 则在原来的森林中, u 和 v 可能具有的关系是

1) 父子关系 2) 兄弟关系 3) u 的父结点与 v 的父结点是兄弟关系

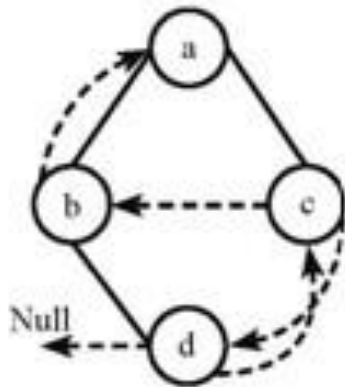
A. 只有2) B. 1)和2) C. 1)和3) D. 1), 2)和3)

Chapter 4

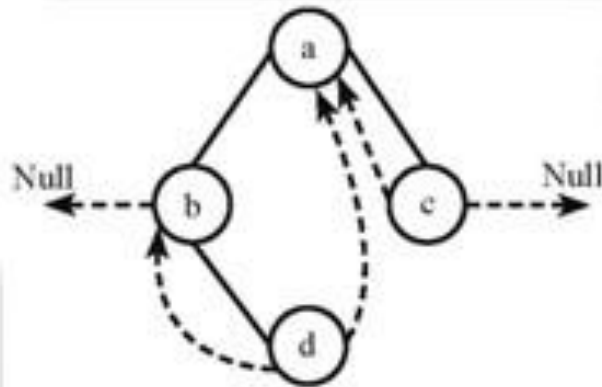
2010年全国考研题

3、下列线索二叉树中（用虚线表示线索），符合后序线索树定义的是（ ）

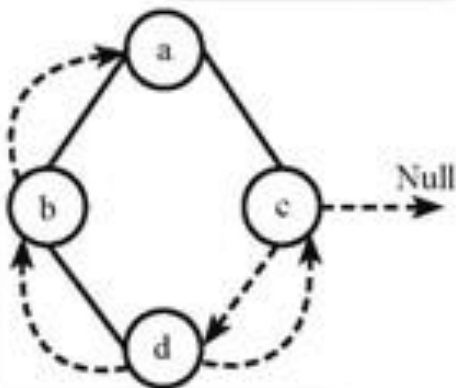
A:



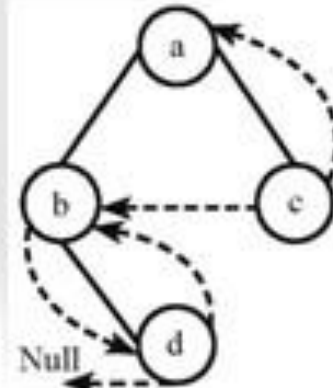
B:



C:



D:



Chapter 4

5、在一棵度为4的树T中，若有20个度为4的结点，10个度为3的结点，1个度为2的结点，10个度为1的结点，则树T的叶节点个数是（）

A: 41 B: 82 C: 113 D: 122

6、对 n (n 大于等于2)个权值均不相同的字符构成哈夫曼树，关于该树的叙述中，错误的是（）

A: 该树一定是一棵完全二叉树

B: 树中一定没有度为1的结点

C: 树中两个权值最小的结点一定是兄弟结点

D: 树中任一非叶结点的权值一定不小于下一任一结点的权值

Chapter 4

1. 给出如下各表达式的二叉树：

1) $(a+b)/(c-d*e)+e+g*h/a$

2) $-x-y*z+(a+b+c/d*e)$

3) $((a+b)>(c-d))\parallel a<f \ \&\&(x<y \parallel y>z)$

2. 如果一棵树有 n_1 个度为1的结点，有 n_2 个度为2的结点，.....， n_m 个度为 m 的结点，试问有多少个度为0的结点？写出推导过程。

3. 分别找出满足以下条件的所有二叉树：

1) 二叉树的前序序列与中序序列相同

2) 二叉树的中序序列与后序序列相同

3) 二叉树的前序序列与后序序列相同

4. 若用二叉链表作为二叉树的存储表示，试对以下问题编写递归算法。

1) 统计二叉树中叶结点的个数。

2) 以二叉树为参数，交换每个结点的左子女和右子女

Chapter 4

5. 已知一棵二叉树的先序遍历结果是 **ABECDFGHIJ**,
中序遍历结果是 **EBCDAFHIGJ**,
试画出这棵二叉树。
6. 编写一个Java函数, 输入后缀表达式, 构造其二叉树表示。设每个操作符有一个或两个操作数。
7. 给定权值{ 15, 03, 14, 02, 06, 09, 16, 17 }, 构造相应的霍夫曼树, 并计算它的带权外路径长度。
8. c1, c2, c3, c4, c5, c6, c7, c8这八个字母的出现频率分别{ 5,25,3,6,10,11,36,4,} 为这八个字母设计不等长的Huffman编码, 并给出该电文的总码数。

实习题:

6. 建立一棵二叉树, 并输出前序、中序、后序遍历结果。

** General Lists

1. 广义表的概念
2. 广义表的性质
3. 广义表的操作
4. 广义表的存储结构
5. 广义表的类声明
6. 输入二叉树的广义表表示来建立一棵树

** General Lists

1. 广义表的概念(LS)

*定义为 $n(n \geq 0)$ 个表元素 $a_0, a_1, a_2, \dots, a_{n-1}$ 组成的有限序列, 记作:

$$LS = (a_0, a_1, a_2, \dots, a_{n-1})$$

其中每个表元素 a_i 可以是原子,也可以是子表.

原子: 某种类型的对象,在结构上不可分(用小写字母表示).

子表: 有结构的.(用大写字母表示)

example:

$$L = (3, (), ('b', 'c'), (((('d')))))$$

*广义表的长度:表中元素的个数

** General Lists

*广义表的表头(head),表尾(tail)

head= a_0 ;

tail=(a_1, a_2, \dots, a_{n-1})

*广义表的深度: 表中所含括号的最大层数

1)A=();

2)B=(6, 2)

3)C=('a', (5, 3, 'x')) 表头为 'a',表尾为((5, 3, 'x'))

4)D=(B, C, A)

5)E=(B, D)

6)F=(4, F) 递归的表

2. 广义表的性质(特点)

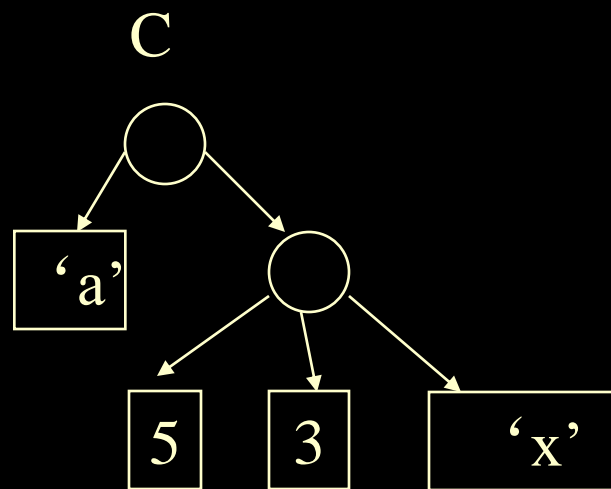
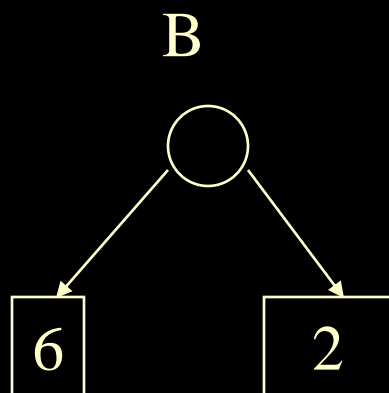
1)有序性

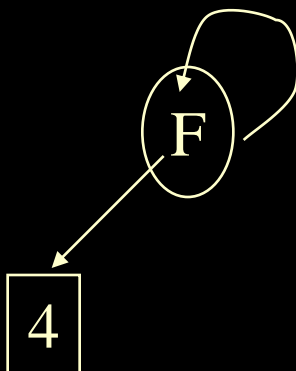
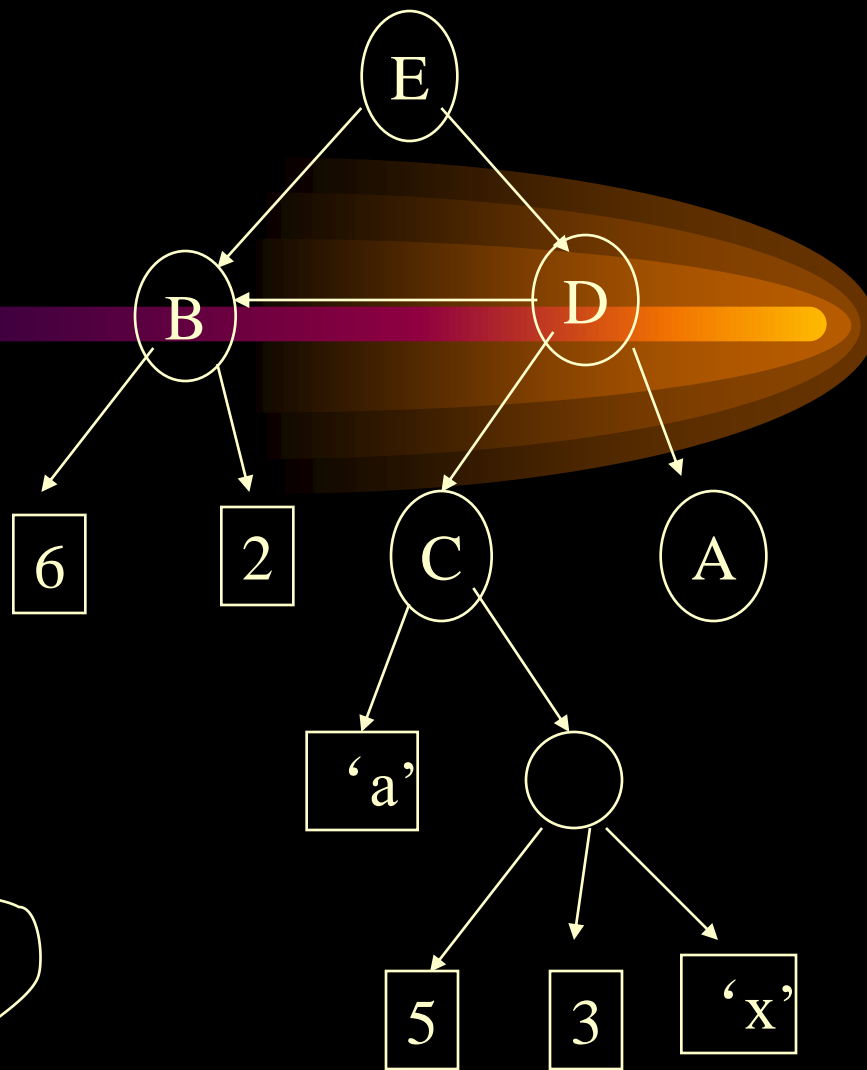
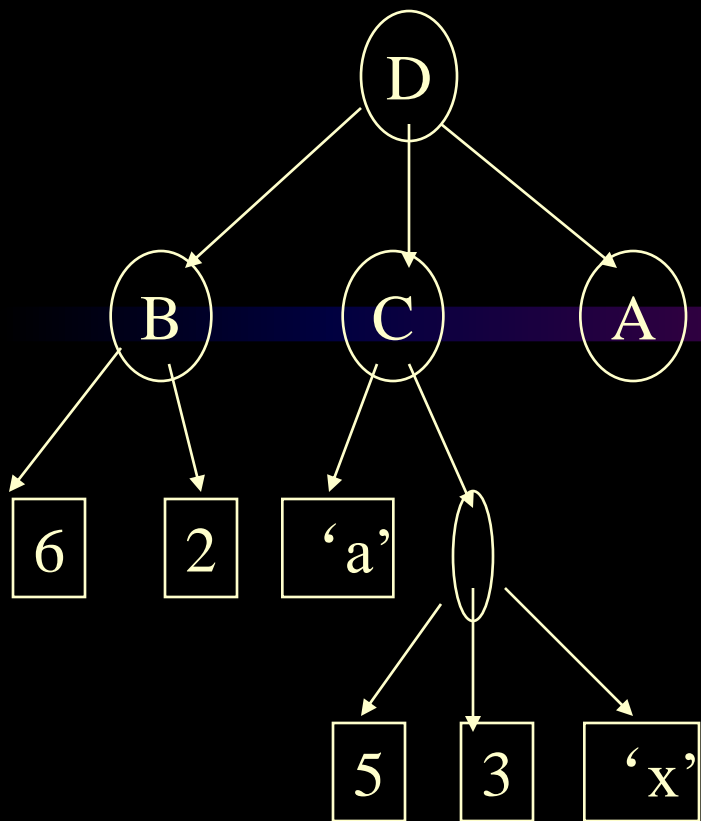
2)有长度,有深度

3)可递归,如上面例6

4)可共享,如E中B为E, D所共享

5)各种广义表都可用一种示意图来表示,用 ○ 表示表元素,用 □ 表示原子





** General Lists

3. General Lists operate:

- 1) head (list)
- 2) tail (list)
- 3) first (list)
- 4) info (elem)
- 5) next (elem)
- 6) nodetype (elem)
- 7) push (list,x)
- 8) addon (list,x)
- 9) setinfo (elem,x)
- 10) sethead (list,x)
- 11) setnext (elem1,elem2)
- 12) settail (list1,list2)

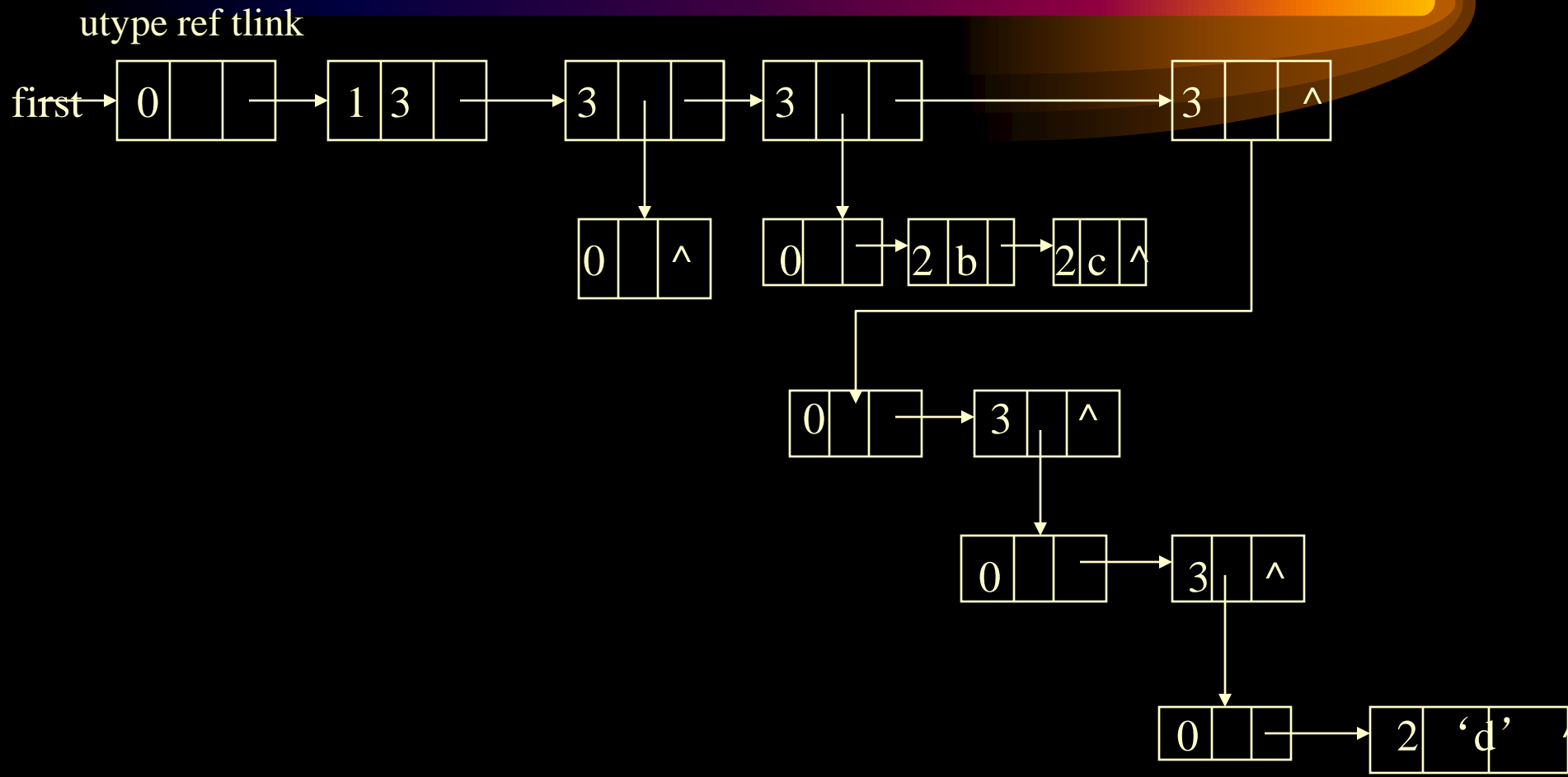
** General Lists

4. General Lists representation:

utype	value	tlink
-------	-------	-------

utype=0	ref
utype=1	intgrinfo
utype=2	charinfo
utype=3	hlink

example: L=(3,(),('b' , 'c'),((('d'))))



** General Lists

5. 广义表的类声明

```
#define HEAD 0
#define INTGR 1
#define CH 2
#define LST 3
class GenList;
class GenListNode
{ friend class GenList;
private:
    int utype;
    GenListNode * tlink;
    union { int ref;
            int intgrinfo;
            char charinfo;
            GenListNode * hlink;
        } value;
```

**** General Lists**

public:

GenListNode & info (GenListNode * elem);

int nodetype (GenListNode * elem) {**return** elem->utype;}

void setinfo (GenListNode * elem, GenListNode & x);

};

class GenList

{ **private:**

GenListNode * first;

GenListNode * Copy (GenListNode * ls);

int depth (GenListnode * ls);

int equal (GenlistNode * s, Genlistnode * t);

void Remove (GenlistNode * ls);

public:

GenList ();

**** General Lists**

```
~GenList ( );  
GenListNode & Head ( );  
GenListNode * Tail ( );  
GenListNode * First ( );  
GenListNode * Next (GenListNode * elem);  
void Push (GenListNode & x);  
GenList & Addon ( GenList & list, GenListNode & x);  
void setHead (GenListNode & x);  
void setNext (GenListNode * elem1, GenListNode * elem2);  
void setTail(GenList & list);  
void Copy (const GenList & l);  
int depth ( );  
int Createlist (GenListNode * ls, char * s);  
}
```

6. 广义表的递归算法

递归算法:1)递归函数的外部调用-----公有函数 界面

2)递归函数的内部调用-----私有函数 真正实现部分

1)求广义表的深度

广义表的深度为广义表中最大括号的重数

广义表 $LS = (a_0, a_1, a_2, \dots, a_{n-1})$, 其中 $a_i (0 \leq i \leq n-1)$ 或者是原子或者是子表.

$$\text{Depth}(LS) = \begin{cases} 0 & \text{当} LS \text{ 为原子时} \\ 1 & \text{当} LS \text{ 为空表时} \\ 1 + \max\{\text{depth}(a_0), \text{depth}(a_1), \dots, \text{depth}(a_{n-1})\} & \end{cases} \quad \left. \begin{matrix} \text{ } \\ \text{ } \\ \text{ } \end{matrix} \right\} \text{递归结束条件}$$

公共函数:

```
int GenList::depth( )  
{ return depth(first);  
}
```

私有函数:

```
int GenList::depth(GenListNode*ls)  
{ if( ls-->tlink==NULL) return 1;  
  GenListNode*temp=ls-->tlink; int m=0;  
  while( temp!=NULL)  
  { if( temp-->utype==LST)  
    { int n=depth(temp-->value.hlink);  
      if(m<n)m=n;  
    }  
    temp=temp-->tlink;  
  }  
  return m+1; }
```


2) 判断两个广义表相等否

相等的条件: 具有相同的结构

对应的数据元素具有相等的值

if(两个广义表都为空表)return相等

else if(都为原子^值相等)递归比较同一层的后面的表元素

else return 不相等.

```
int operator==(const GenList&l,const GenList&m)//假设是友元
```

```
{ return equal(l.first, m.first);
```

```
}
```

```
int equal(GenListNode*s, GenListNode*t)//假设是友元
```

```
{  int x;
```

```
  if(s-->tlink==NULL&&t-->tlink==NULL)return 1;
```

```
  if((s-->tlink!=NULL&&t-->tlink!=NULL&&s-->tlink-->utype==t-->tlink-->utype)
```

```
    {  if(s-->tlink-->utype==INTGR)
```

```
        if(s-->tlink-->value.intgrinfo== t-->tlink-->value.intgrinfo) x=1;
```

```
        else x=0;
```

```
    else if(s-->tlink-->utype==CH)
```

```
        if(s-->tlink-->value.charinfo==t-->tlink-->value.charinfo)x=1;
```

```
        else x=0;
```

```
        else x=equal(s-->tlink-->value.hlink, t-->tlink-->value.hlink);
```

```
    if(x)return equal(s-->tlink, t-->tlink);
```

```
  }
```

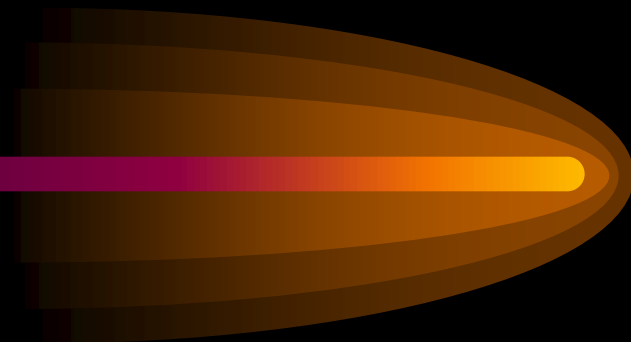
```
  return 0;
```

```
}
```

3) 广义表的复制算法

分别复制表头,表尾,然后合成

前提是广义表不可以是共享表或递归表



公共函数:

```
void GenList::copy(const GenList&l)
{ first=copy(l.first);
}
```

私有函数:

```
GenListNode*GenList::copy(GenListNode*ls)
{
    GenListNode*q=NULL;
    if(ls!=NULL)
        {q=new GenListNode;
          q-->utype=ls-->utype;
```

```
Switch(ls-->utype)
```

```
{ case HEAD: q-->value.ref=ls-->value.ref; break; //表头结点
```

```
case INTGR: q-->value.intgrinfo=ls-->value.intgrinfo; break;
```

```
case CH: q-->value.charinfo=ls-->value.charinfo; break;
```

```
case LST: q-->value.hlink=ls-->value.hlink; break;
```

```
}
```

```
q-->tlink=copy(ls-->tlink);
```

```
}
```

```
return q;
```

```
}
```

4) 广义表的析构造函数----- ~GenList()

公共函数:

GenList::~~GenList()

```
{ remove(first); }
```

私有函数:

void GenList::remove(GenListNode*ls)

```
{ ls→value.ref--;
```

```
  if (!ls→value.ref)
```

```
    { 1) GenListNode*y=ls;
```

```
      2) while(y-->tlink!=NULL)
```

```
        { y=y-->tlink;
```

```
          if(y-->utype==LST)remove(y-->value.hlink);
```

```
        }
```

```
      3) y-->tlink=av; av=ls;      //回收顶点到可利用空间表中
```

```
    }
```

```
}
```

2005. 六. 1 广义表

对广义表的回收算法如下, 请回答在回收广义表ls后, 在可利用空间表av中原广义表中的结点是按何种次序链接起来的(请用原广义表结点上方的字母组成一个序列来表示链接的顺序: 序列的头部是新的av所指向的地方, 序列的尾部连接原来av的空间).

```
GenList :: ~GenList()
```

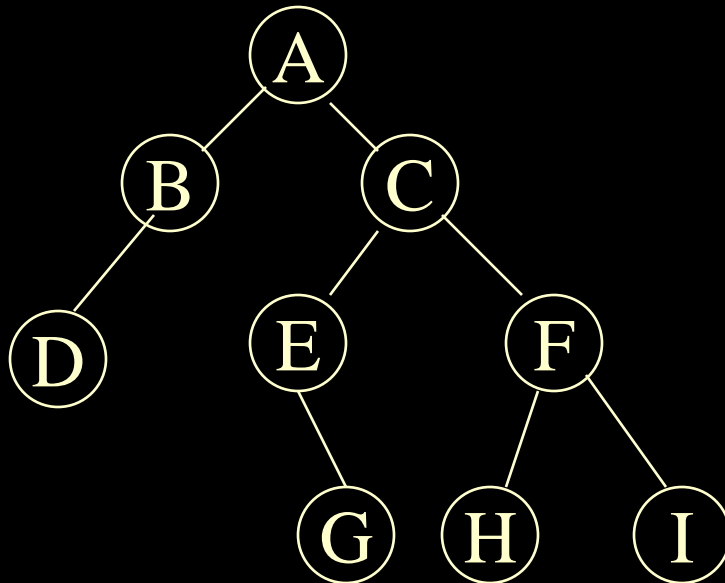
```
{ Remove(first);  
}
```

```
Void GenList :: Remove(GenListNode *ls)
```

```
{ ls->value.ref--;  
  if ( !ls -> value.ref)  
    { GenListNode * y = ls;  
      while ( y ->tlink != NULL)  
        { y = y ->tlink;  
          if ( y -> utype == LST) Remove( y ->value.hlink);  
        }  
      y -> tlink = av; av = ls;  
    }  
}
```

7. Create BinaryTree another method:

A(B(D), C(E(,G), F(H, I)))@



Create BinaryTree another method:

```
void CreateBTree(BTreeNode * & BT, char *a)
{ BTreeNode * s[10];
  int top = -1;
  BT = NULL;
  BTreeNode * p;
  int k;
  istream ins (a);
  char ch;
  ins >> ch;
```

Create BinaryTree another method:

```
while( ch != '@')
{
    switch (ch)
    {
        case '(': top++; s[top] = p; k = 1; break;
        case ')': top--; break;
        case ',': k = 2; break;
        default:
            p = new BTreeNode;
            p->data = ch; p->left = p->right = NULL;
            if(BT == NULL) BT = p ;
            else { if (k == 1) s[top]->left = p;
                    else s[top]->right = p;
                }
            }
    } //switch end
    ins >> ch;
} //while end
}
```