n阶行列式转置和交换性质 含例1.2.6和例1.2.7的证明

引理 行列式可以按第一列展开,即 $A=\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}=a_{11}A_{11}+a_{21}A_{21}+\cdots+a_{n1}A_{n1}$.

证明思路:

行列式按第1列展开 $a_{11}M_{11}-a_{21}M_{21}+\cdots+(-1)^{i+1}a_{i1}M_{i1}+\cdots+(-1)^{n+1}a_{n1}M_{n1}$

$||$

余子式按第1行展开

$a_{11}M_{11}+$

$$\begin{array}{c|cccccc}
 & a_{12} & a_{13} & \cdots & a_{1k} & \cdots & a_{1n} \\
\hline
a_{21} & (-1)^{2+1+1+2-1}M_{21,12} & (-1)^{2+1+1+3-1}M_{21,13} & \cdots & (-1)^{2+1+1+k-1}M_{21,1k} & \cdots & (-1)^{2+1+1+n-1}M_{21,1n} \\
a_{31} & (-1)^{3+1+1+2-1}M_{31,12} & (-1)^{3+1+1+3-1}M_{31,13} & \cdots & (-1)^{3+1+1+k-1}M_{31,1k} & \cdots & (-1)^{3+1+1+n-1}M_{31,1n} \\
\vdots & \vdots & \vdots & & \vdots & & \vdots \\
a_{i1} & (-1)^{i+1+1+2-1}M_{i1,12} & (-1)^{i+1+1+3-1}M_{i1,13} & \cdots & (-1)^{i+1+1+k-1}M_{i1,1k} & \cdots & (-1)^{i+1+1+n-1}M_{i1,1n} \\
\vdots & \vdots & \vdots & & \vdots & & \vdots \\
a_{n1} & (-1)^{n+1+1+2-1}M_{n1,12} & (-1)^{n+1+1+3-1}M_{n1,13} & \cdots & (-1)^{n+1+1+k-1}M_{n1,1k} & \cdots & (-1)^{n+1+1+n-1}M_{n1,1n}
\end{array}$$

$||$ $\boldsymbol{M_{i1,1k}=M_{1k,i1}}$ 都是去掉i,1行1,k列

余子式按第1列展开

$$a_{11}M_{11}+a_{12}(-1)^{1+2}M_{12}+\cdots+a_{1k}(-1)^{1+k}M_{1k}+\cdots+a_{1n}(-1)^{1+n}M_{1n}$$

$$=a_{11}A_{11}+a_{12}A_{12}+\cdots+a_{1n}A_{1n}=A$$

定理1.2.1. 行列式与它的转置行列式的值相等.

证明思路：数学归纳法，2阶成立，假设阶数<n成立

$$A=\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

利用引理结论，按第一列展开

$$=a_{11}M_{11}-a_{21}M_{21}+\cdots+(-1)^{k+1}a_{k1}M_{k1}+\cdots+(-1)^{n+1}a_{n1}M_{n1}$$

$$\|$$

$$a_{11}N_{11}-a_{21}N_{12}+\cdots+(-1)^{1+k}a_{k1}N_{1k}+\cdots+(-1)^{1+n}a_{n1}N_{1n}$$

$$=\begin{vmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{vmatrix}=A'$$

转置行列式的余子式关系

$$M_{21}:\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$M_{k1}=M_{k1}'$

(归纳假设)

$M_{k1}'=N_{1k}$

$$N_{12}:\begin{vmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{vmatrix}$$

定理1.2.2. 对调两行(列)的位置，行列式的值相差一个负号，即

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = - \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

证明思路：分两步：

第一步：相邻两行交换变号；

第二步：若相邻两行交换变号，则任意两行交换也变号.

先证第二步：利用第一步结论

$$A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = (-1)^{j-i} \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i+1,1} & a_{i+1,2} & \cdots & a_{i+1,n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \end{vmatrix} = (-1)^{j-i}(-1)^{j-(i+1)} \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ a_{i+1,1} & a_{i+1,2} & \cdots & a_{i+1,n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots \end{vmatrix} = -B$$

第一步：相邻两行交换变号

数学归纳法：2阶成立，假设阶数<n成立，n阶时按第一列展开

$$A=\begin{vmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{k1} & \cdots & a_{kj} & \cdots & a_{kn} \\ \vdots & & \vdots & & \vdots \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{in} \\ a_{i+1,1} & \cdots & a_{i+1,j} & \cdots & a_{i+1,n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nj} & \cdots & a_{nn} \end{vmatrix}$$

$$= a_{11}A_{11}+\cdots+a_{k1}A_{k1}+\cdots+a_{i1}A_{i1}+a_{i+1,1}A_{i+1,1}+\cdots+a_{n1}A_{n1}$$

$$= a_{11}A_{11}+\cdots+a_{i-1,1}A_{i-1,1}+a_{i+2,1}A_{i+2,1}+\cdots+a_{n1}A_{n1}$$
$$+a_{i1}A_{i1}+a_{i+1,1}A_{i+1,1}$$

$$B=\begin{vmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{vmatrix}=\begin{vmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{k1} & \cdots & a_{kj} & \cdots & a_{kn} \\ \vdots & & \vdots & & \vdots \\ a_{i+1,1} & \cdots & a_{i+1,j} & \cdots & a_{i+1,n} \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nj} & \cdots & a_{nn} \end{vmatrix}$$

$$= b_{11}B_{11}+\cdots+b_{k1}B_{k1}+\cdots+b_{i1}B_{i1}+b_{i+1,1}B_{i+1,1}+\cdots+b_{n1}B_{n1}$$

$$= b_{11}B_{11}+\cdots+b_{i-1,1}B_{i-1,1}+b_{i+2,1}B_{i+2,1}+\cdots+b_{n1}B_{n1}$$
$$+b_{i1}B_{i1}+b_{i+1,1}B_{i+1,1}$$

当$k\neq i, i+1$时，比较$(k,1)$元素的代数余子式 $A_{k1}=(-1)^{k+1}M_{k1}$，$B_{k1}=(-1)^{k+1}N_{k1}$，如图

$$\begin{vmatrix} a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ a_{i+1,2} & \cdots & a_{i+1,j} & \cdots & a_{i+1,n} \\ \vdots & & \vdots & & \vdots \\ a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{vmatrix} = M_{k1} \qquad \begin{vmatrix} a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ a_{i+1,2} & \cdots & a_{i+1,j} & \cdots & a_{i+1,n} \\ a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & & \vdots & & \vdots \\ a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{vmatrix} = N_{k1}$$

利用归纳法有
$M_{k1}=-N_{k1}$，
故 $a_{k1}A_{k1}=-b_{k1}B_{k1}$

当 $k=i$, $i+1$时，$a_{i1}A_{i1}=(-1)^{i+1}a_{i1}M_{i1}$，　$a_{i+1,1}A_{i+1,1}=(-1)^{i+2}a_{i+1,1}M_{i+1,1}$，
$b_{i1}B_{i1}=(-1)^{i+1}a_{i+1,1}N_{i1}$，　$b_{i+1,1}B_{i+1,1}=(-1)^{i+2}a_{i1}N_{i+1,1}$，　如图

$$\begin{vmatrix} a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{i-1,2} & \cdots & a_{i-1,j} & \cdots & a_{i-1,n} \\ a_{i+1,2} & \cdots & a_{i+1,j} & \cdots & a_{i+1,n} \\ a_{i+2,2} & \cdots & a_{i+2,j} & \cdots & a_{i+2,n} \\ \vdots & & \vdots & & \vdots \\ a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{vmatrix} = M_{i1} \qquad \begin{vmatrix} a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{i-1,2} & \cdots & a_{i-1,j} & \cdots & a_{i-1,n} \\ a_{i+1,2} & \cdots & a_{i+1,j} & \cdots & a_{i+1,n} \\ a_{i+2,2} & \cdots & a_{i+2,j} & \cdots & a_{i+2,n} \\ \vdots & & \vdots & & \vdots \\ a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{vmatrix} = N_{i+1,1}$$

交叉比较有
$M_{i1}=N_{i+1,1}$, $M_{i+1,1}=N_{i1}$
故 $a_{i1}A_{i1}=-b_{i+1,1}B_{i+1,1}$,
$a_{i+1,1}A_{i+1,1}=-b_{i1}B_{i1}$

$$A=a_{11}A_{11}+\ldots+a_{i1}A_{i1}+a_{i+1,1}A_{i+1,1}+\ldots+a_{n1}A_{n1}=-(b_{11}B_{11}+\ldots+b_{i1}B_{i1}+b_{i+1,1}B_{i+1,1}+\ldots+b_{n1}B_{n1})=-B$$

下述定理及推论在之前介绍性质时已说明了证明思路：

推论1.2.3. 两行(列)相等的行列式的值为0.

推论1.2.4. 行列式可以按任一行(列)展开.

定理1.2.5. 行列式的任一行(列)元素的公因子可以提到行列式外面.

推论1.2.6. 若行列式某两行(列)对应元素成比例，则行列式的值为零.

定理1.2.7. 行列式的第i行(列) 的每一个元素都可以表示为两数的和，则该行列式可以表示为两个行列式之和.

定理1.2.8. 将行列式的任意一行(列) 乘以数k加到另一行(列)上去，行列式的值不变.

定理1.2.9. 行列式任一行(列) 的元素与另一行(列)元素的代数余子式对应乘积之和为零. 即，若设 $A=\left|a_{ij}\right|_{n\times n}$ 则有

$$\sum_{k=1}^{n} a_{ik}A_{jk}=a_{i1}A_{j1}+a_{i2}A_{j2}+\cdots+a_{in}A_{jn}=\begin{cases} A, & i=j, \\ 0, & i\neq j, \end{cases}$$

$$\left(\sum_{k=1}^{n} a_{ki}A_{kj}=a_{1i}A_{1j}+a_{2i}A_{2j}+\cdots+a_{ni}A_{nj}=\begin{cases} A, & i=j, \\ 0, & i\neq j. \end{cases}\right)$$

例1.2.6 证明n阶范德蒙德(Vandermonde)行列式($n\geq 2$)

$$D_n(x_1,x_2,\cdots,x_n)=\begin{vmatrix}1&1&\cdots&1\\x_1&x_2&\cdots&x_n\\\vdots&\vdots&&\vdots\\x_1^{n-2}&x_2^{n-2}&\cdots&x_n^{n-2}\\x_1^{n-1}&x_2^{n-1}&\cdots&x_n^{n-1}\end{vmatrix}=\prod_{1\leq i<j\leq n}(x_j-x_i).$$

证明 将第n-1行($-x_1$)倍加到第n行，第n-2行($-x_1$)倍加到第n-1行，这样依次下去，最后将第1行($-x_1$)倍加到第2行，得

$$D_n(x_1,x_2,\cdots,x_n)=\begin{vmatrix}1&1&\cdots&1\\0&x_2-x_1&\cdots&x_n-x_1\\0&x_2(x_2-x_1)&\cdots&x_n(x_n-x_1)\\\vdots&\vdots&&\vdots\\0&x_2^{n-2}(x_2-x_1)&\cdots&x_n^{n-2}(x_n-x_1)\end{vmatrix}=\begin{vmatrix}x_2-x_1&x_3-x_1&\cdots&x_n-x_1\\x_2(x_2-x_1)&x_3(x_3-x_1)&\cdots&x_n(x_n-x_1)\\\vdots&\vdots&&\vdots\\x_2^{n-2}(x_2-x_1)&x_3^{n-2}(x_3-x_1)&\cdots&x_n^{n-2}(x_n-x_1)\end{vmatrix}$$

$$=(x_2-x_1)(x_3-x_1)\cdots(x_n-x_1)\begin{vmatrix}1&1&\cdots&1\\x_2&x_3&\cdots&x_n\\\vdots&\vdots&&\vdots\\x_2^{n-2}&x_3^{n-2}&\cdots&x_n^{n-2}\end{vmatrix}=(x_2-x_1)(x_3-x_1)\cdots(x_n-x_1)D_{n-1}(x_2,x_3,\cdots,x_n)$$

$$=(x_2-x_1)(x_3-x_1)\cdots(x_n-x_1)(x_3-x_2)\cdots(x_n-x_2)D_{n-2}(x_3,x_4,\cdots,x_n)=\cdots$$

$$=(x_2-x_1)\cdots(x_n-x_1)(x_3-x_2)\cdots(x_n-x_2)\cdots(x_{n-1}-x_{n-2})(x_n-x_{n-2})(x_n-x_{n-1})=\prod_{1\leq i<j\leq n}(x_j-x_i).$$

证法二

设 $f(y)=\begin{vmatrix} 1 & 1 & \cdots & 1 & 1 \\ x_1 & x_2 & \cdots & x_{n-1} & y \\ \vdots & \vdots & & \vdots & \vdots \\ x_1^{n-2} & x_2^{n-2} & \cdots & x_{n-1}^{n-2} & y^{n-2} \\ x_1^{n-1} & x_2^{n-1} & \cdots & x_{n-1}^{n-1} & y^{n-1} \end{vmatrix} = a_{n-1}y^{n-1}+a_{n-2}y^{n-2}+\cdots+a_1y+a_0,$

显然 $f(x_i)=\begin{vmatrix} 1 & \cdots & 1 & \cdots & 1 \\ x_1 & \cdots & x_i & \cdots & x_i \\ \vdots & & \vdots & & \vdots \\ x_1^{n-2} & \cdots & x_i^{n-2} & \cdots & x_i^{n-2} \\ x_1^{n-1} & \cdots & x_i^{n-1} & \cdots & x_i^{n-1} \end{vmatrix}=0, i=1,2,\cdots,n-1.$ 即 $x_1,x_2,,x_{n-1}$是$f(y)=0$ 的解.

故 $f(y)=a_{n-1}(y-x_1)\cdots(y-x_{n-1})$, 其中 $a_{n-1}=\begin{vmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_{n-1} \\ \vdots & \vdots & & \vdots \\ x_1^{n-2} & x_2^{n-2} & \cdots & x_{n-1}^{n-2} \end{vmatrix}=D_{n-1}(x_1,\cdots,x_{n-1}).$

于是 $D_n(x_1,x_2,\cdots,x_n)=f(x_n)=D_{n-1}(x_1,\cdots,x_{n-1})(x_n-x_1)\cdots(x_n-x_{n-1})$

$=D_{n-2}(x_1,\cdots,x_{n-2})(x_{n-1}-x_1)\cdots(x_{n-1}-x_{n-2})(x_n-x_1)\cdots(x_n-x_{n-1})=\cdots$

$=(x_2-x_1)(x_3-x_1)(x_3-x_2)\cdots(x_n-x_1)\cdots(x_n-x_{n-1})=\prod\limits_{1\le i<j\le n}(x_j-x_i).$

例1.2.7 证明$n+m$阶行列式(块三角行列式)

$$D_n=\begin{vmatrix} a_{11} & \cdots & a_{1n} & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} & 0 & \cdots & 0 \\ c_{11} & \cdots & c_{1n} & b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots & \vdots & & \vdots \\ c_{m1} & \cdots & c_{mn} & b_{m1} & \cdots & b_{mm} \end{vmatrix}=\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}\begin{vmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots \\ b_{m1} & \cdots & b_{mm} \end{vmatrix}. \qquad (1.15)$$

证明　记

$$A=\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}=a_{11}M_{11}-a_{12}M_{12}+\cdots+(-1)^{1+n}a_{1n}M_{1n}\ ,\quad B=\begin{vmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots \\ b_{m1} & \cdots & b_{mm} \end{vmatrix}.$$

利用数学归纳法证明.

当$n=1$时，将D_n按第一行展开D_1，得

$$D_1=\begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ c_{11} & b_{11} & \cdots & b_{1m} \\ \vdots & \vdots & & \vdots \\ c_{m1} & b_{m1} & \cdots & b_{mm} \end{vmatrix}=a_{11}\begin{vmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots \\ b_{m1} & \cdots & b_{mm} \end{vmatrix}=a_{11}B=A\cdot B,$$ **结论成立.**

设A为$(n\text{-}1)$阶行列式时结论成立，当A为n阶时，按第一行展开D_n得

$$D_n=\begin{vmatrix} a_{11} & \cdots & a_{1n} & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} & 0 & \cdots & 0 \\ c_{11} & \cdots & c_{1n} & b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots & \vdots & & \vdots \\ c_{m1} & \cdots & c_{mn} & b_{m1} & \cdots & b_{mm} \end{vmatrix}=a_{11}\begin{vmatrix} a_{22} & \cdots & a_{2n} & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ a_{n2} & \cdots & a_{nn} & 0 & \cdots & 0 \\ c_{12} & \cdots & c_{1n} & b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots & \vdots & & \vdots \\ c_{m2} & \cdots & c_{mn} & b_{m1} & \cdots & b_{mm} \end{vmatrix}$$

$$-a_{12}\begin{vmatrix} a_{21} & \cdots & a_{2n} & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} & 0 & \cdots & 0 \\ c_{11} & \cdots & c_{1n} & b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots & \vdots & & \vdots \\ c_{m1} & \cdots & c_{mn} & b_{m1} & \cdots & b_{mm} \end{vmatrix}+\cdots+(-1)^{1+n}a_{1n}\begin{vmatrix} a_{21} & \cdots & a_{2,n-1} & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ a_{n1} & \cdots & a_{n,n-1} & 0 & \cdots & 0 \\ c_{11} & \cdots & c_{1,n-1} & b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots & \vdots & & \vdots \\ c_{m1} & \cdots & c_{m,n-1} & b_{m1} & \cdots & b_{mm} \end{vmatrix}$$

按归纳假设

$$=a_{11}M_{11}B-a_{12}M_{12}B+\cdots+(-1)^{1+n}a_{1n}M_{1n}B$$

$$=(a_{11}M_{11}-a_{12}M_{12}+\cdots+(-1)^{1+n}a_{1n}M_{1n})B=AB.$$

最常用的行列式性质

行列式转置，值不变.

交换行列式两行(列)，值反号.

k乘以行列式的任一行(列) 等于k乘以行列式.

行列式的任一行(列) 的k倍加到另一行(列)上去，值不变.

行列式可以按任一行(列)展开.

$$\begin{vmatrix} A & O \\ * & B \end{vmatrix} = |A||B|, \quad \begin{vmatrix} A & * \\ O & B \end{vmatrix} = |A||B|.$$