Chapter 3

List, Stack, and Queue

- Introduce the concept of Abstract Data Types(ADTS).
- show how to efficiently perform operations on lists.
- Introduce the stack ADT and its use in implementing recursion.
- Introduce the queue ADT and its use in operating systems and algorithm design.

3.1 Abstract Data Types(ADTS)

1. ADT-----a set of objects together with a set of operaions.

   Abstract data types are mathematical abstractions;

   nowhere in an ADT's definition is there any mention of how the set of operations is implemented.

3.1 Abstract Data Types(ADTS)

2. data object---

a set of instances or values

for example:

Boolean={false,true}

Digit={0,1,2,3,4,5,6,7,8,9}

Letter={A,B,……Z,a,b,……z}

NaturalNumber={0,1,2,……}

Integer = {0, +1, -1, +2, -2, +3, -3, …}

String={a,b,……,aa, ab, ac,……}

3.1 Abstract Data Types(ADTS)

3. data structure

is a data object together with the relationships among the instances and among the individual elements that compose an instance

- Data_Structure={D,R}
- D---data object，
- R ---a set of relationships of all the data members in D.

3.2 The List ADT

$L = (e_1, e_2, e_3, \ldots, e_n)$

list size is n

if n=0:empty list

if n(finite)>0:

e_1 is the first element

e_n is the last element

e_i precedes e_{i+1}

3.2 The List ADT

Example:

Students =(Jack, Jill, Abe, Henry, Mary, …, Judy)

Exams =(exam1, exam2, exam3)

Days of Week = (S, M, T, W, Th, F, Sa)

Months = (Jan, Feb, Mar, Apr, …, Nov, Dec)

3.2 The List ADT

Operations:

Create a linear list

determine whether the list is empty

determine the length of the list

find the kth of the element

search for a given element

delete the kth element

insert a new element just after the kth

3.2 The List ADT

ADT specification of a linear list

```
AbstractDateType LinearList
{  instances
      ordered finite collections of zero or more elements
   operations
      Create();          Destroy();
      IsEmpty();         Length();
      Find(k,x);         Search(x);
      Delete(k,x);       Insert(k,x);
      Output(out);
}
```

3.2.1. Simple Array Implementation of Lists

1. Use an array to represent the instance of an object

   $(e_1,e_2,\ldots\ldots\ldots e_n)$

element	0	1	2		n-1		MaxSize-1
	e_1	e_2	e_3		e_n		

length=n

each position of the array is called a cell or a node

mapping formula: location(i)=i-1 O(1)

3.2.1. Simple Array Implementation of Lists

- Search(x)   O(length)

$L = (a,b,d,b,e)$

Search$(d) = 3$

Search$(a) = 1$

Search$(z) = 0$

ACN=(1+2+……+n)/n

=(1+n)*n/(2n)=(n+1)/2

3.2.1. Simple Array Implementation of Lists

- remove(k,x)

  delete the k'th element and return it in x

  L = (a,b,c,d,e)

  delete(2,x) =>

  L=(a,c,d,e), x=b,

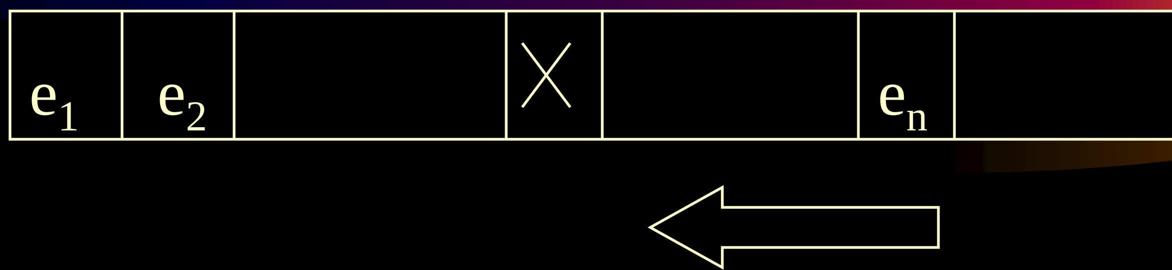
  and index of *c,d,e* decrease by *1*

  delete(0) => error

  delete(20) => error

  O(n)

3.2.1. Simple Array Implementation of Lists



$$AMN=\sum_{i=0}^{n-1}(n-i-1)/n=(n-1+n-2+\ldots\ldots+1+0)/n=(n-1)/2$$

3.2.1. Simple Array Implementation of Lists

- insert (x , i)

insert x after the i'th element

$L = (a,b,c,d,e,f,g)$

insert(0,h) =>

$L = (h,a,b,c,d,e,f,g)$

index of *a,b,c,d,e,f,* and *g* increase by *1*

insert(10,h) => error

insert(-6,h) => error

$O(n)$

3.2.1. Simple Array Implementation of Lists

0	1	2		n-1	n
e_1	e_2			e_n	

$$AMN=\sum_{i=0}^{n}(n-i)/(n+1)=(n+n-1+\ldots\ldots+1+0)/(n+1)=n/2$$

3.2.1. Simple Array Implementation of Lists

2. Use array Implementation

   merit : easy Search.

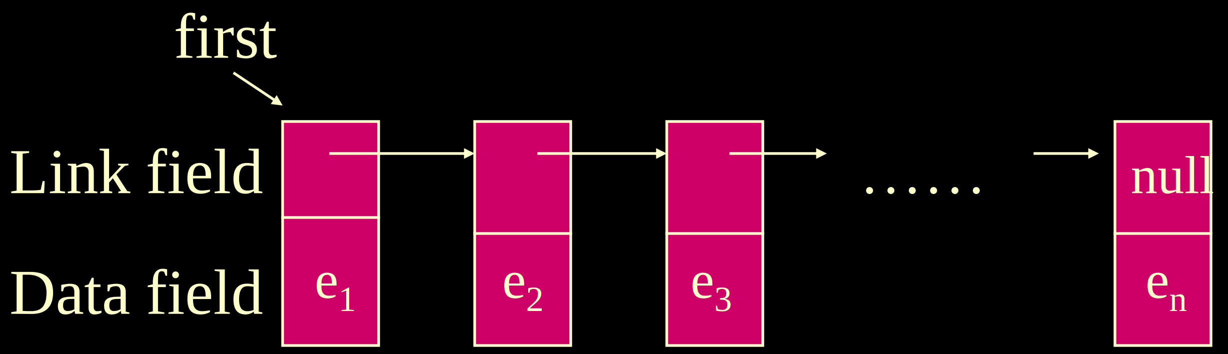
   short coming : Insertion and Removing(Deletion) spend a lot of time.

3.2.2. Linked Lists

In order to avoid the linear cost of insertion and deletion.

1) Each node of a data object keeps a link or a pointer about the location of other relevant nodes

$L=(e_1,e_2,\ldots\ldots\ldots e_n)$



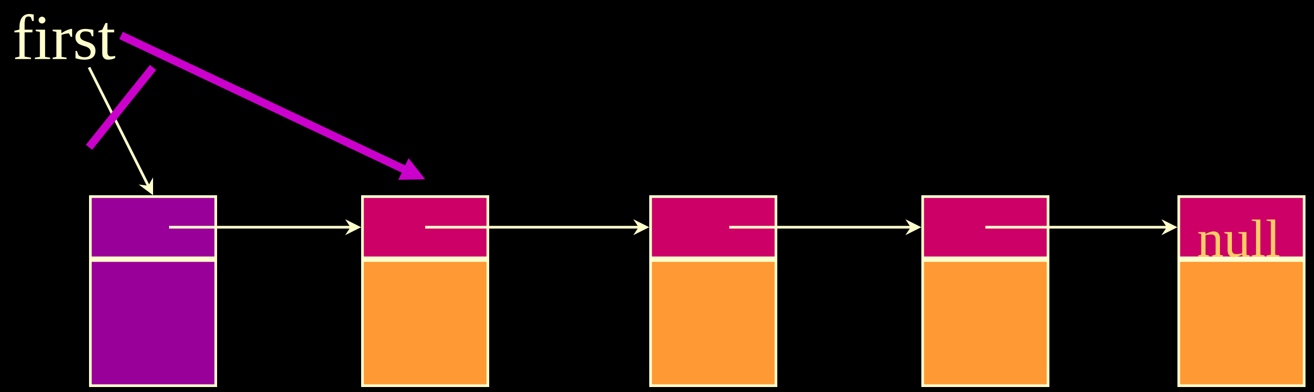
3.2.2. Linked Lists

- The figure above is called a single linked list,and the structure is also called a chain
- A chain is a linked list in which each node represents one element.
- There is a link or pointer from one element to the next.
- The last node has a null pointer.

3.2.2. Linked Lists

- Deletion a element of a chain

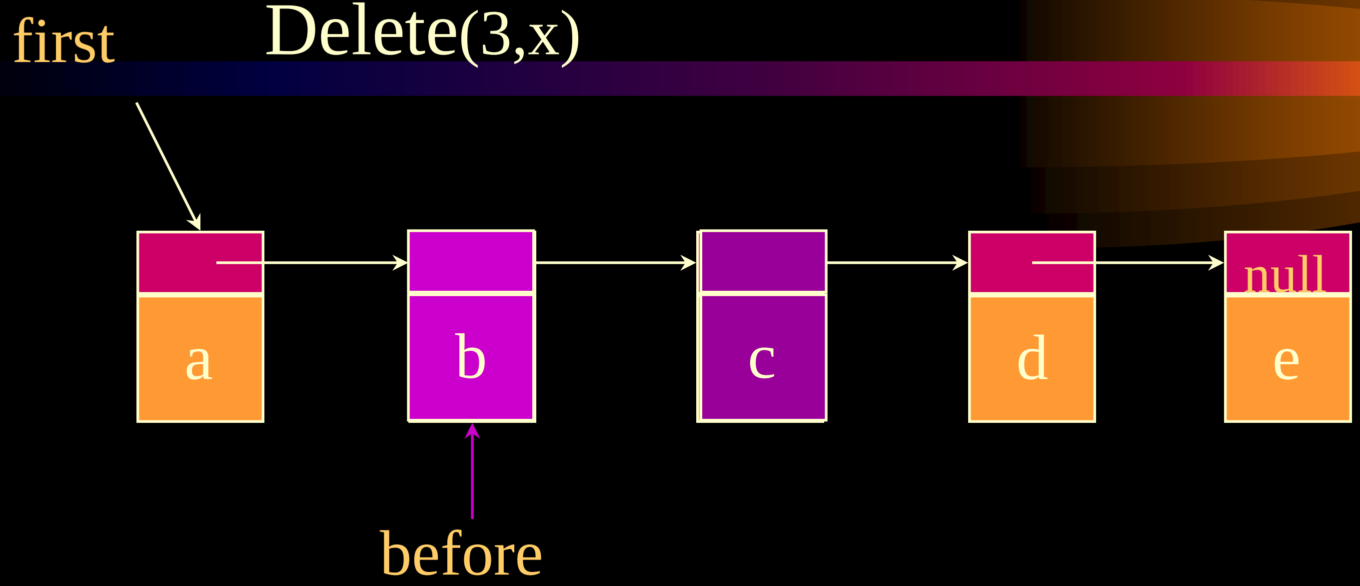
Delete(1,x)



first = first.link;

3.2.2. Linked Lists

Delete(3,x)

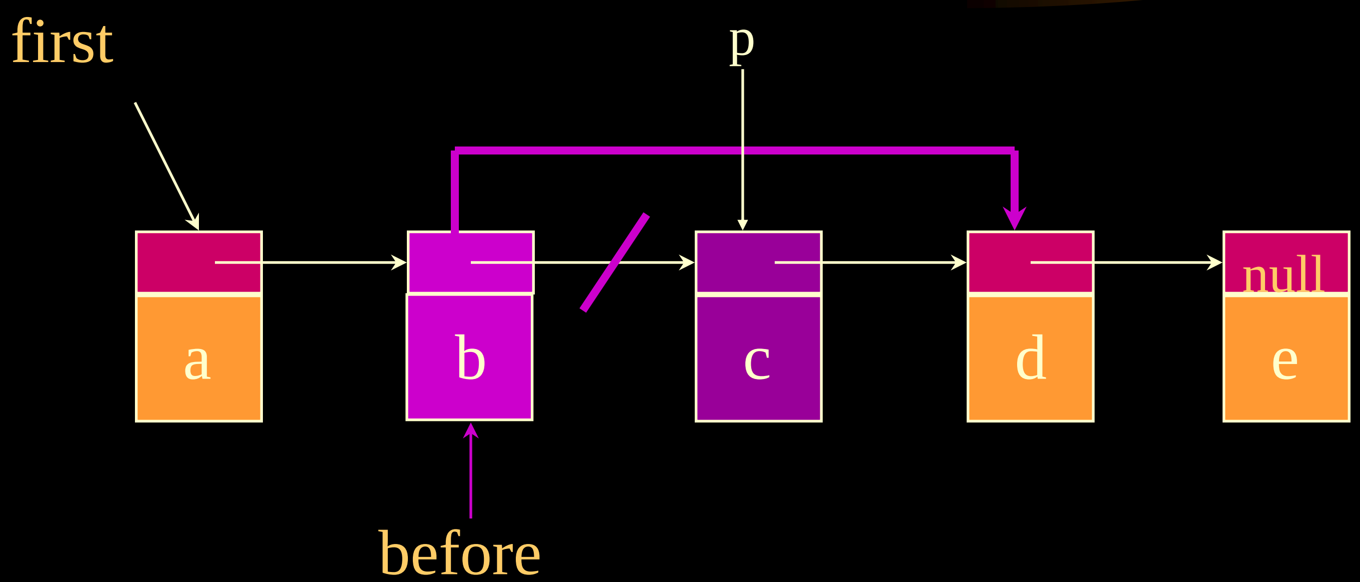


first get to node just before node to be removed

before= first .link;

3.2.2. Linked Lists

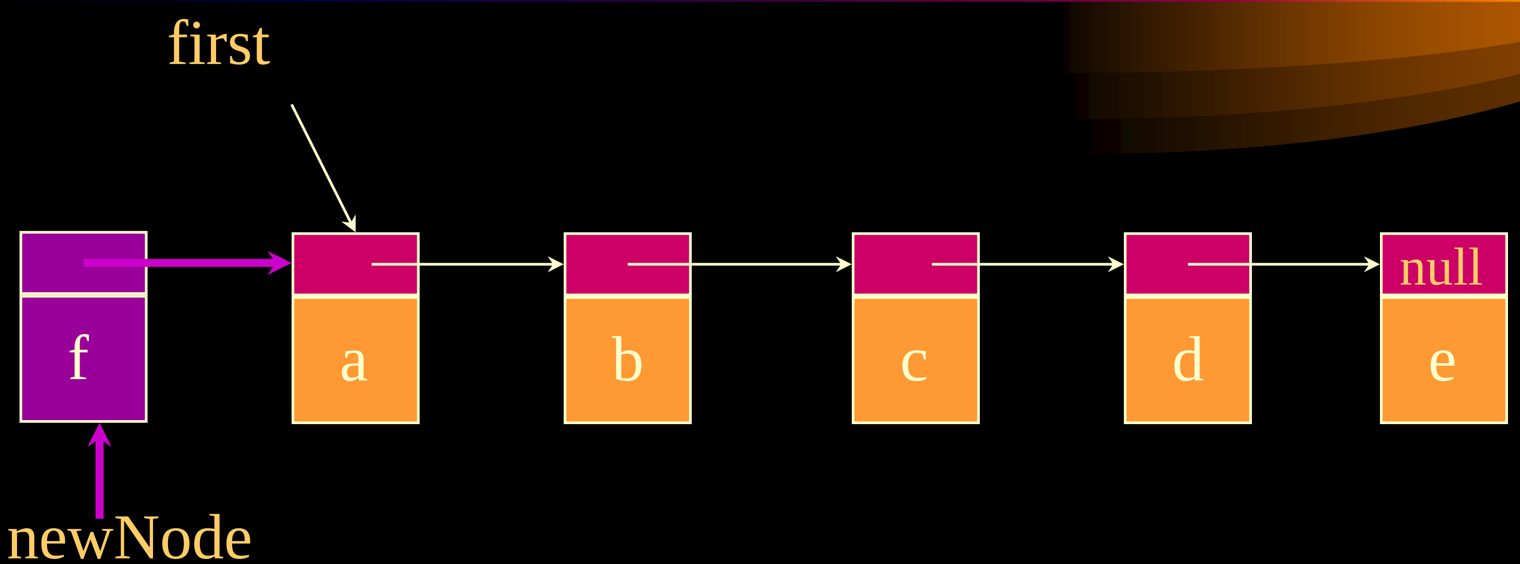
now change pointer in before



before .link = before .link .link;

3.2.2. Linked Lists

- insert operation ----insert(0,’f’)

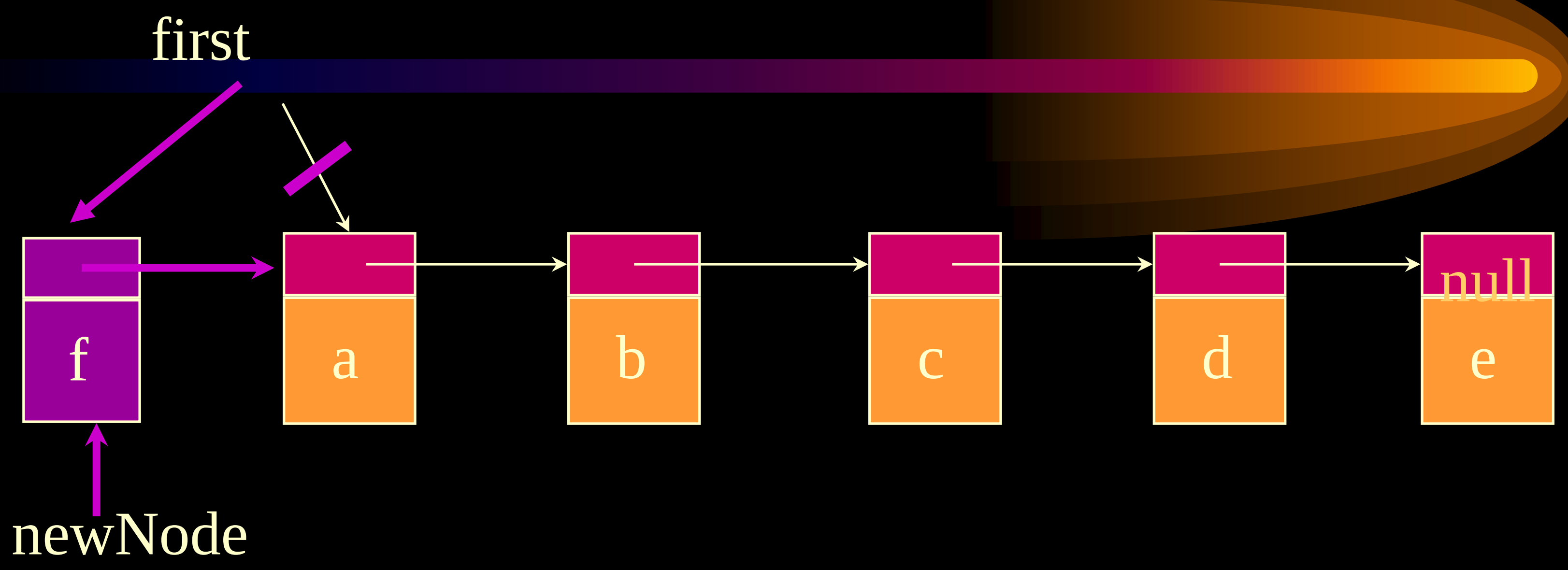


Step 1: get a node, set its data and link fields

ChainNode newNode =

new ChainNode(‘f’, first);

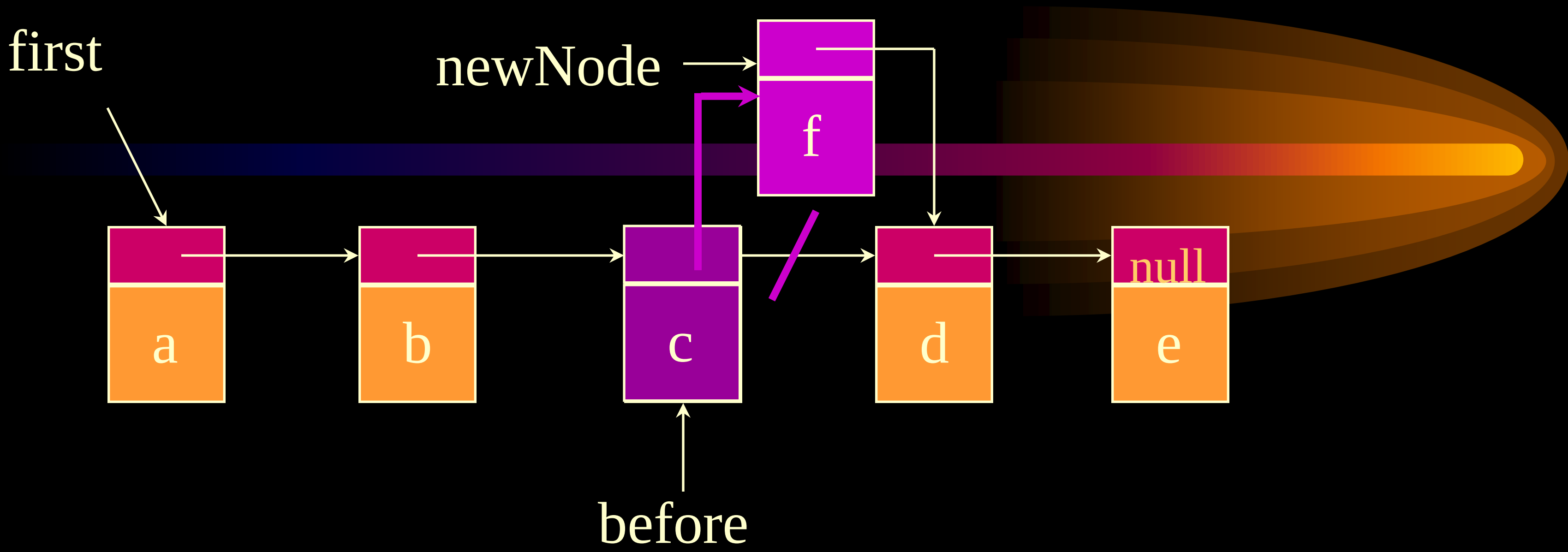
3.2.2. Linked Lists



Step 2: update first

first = newNode;

insert(3,'f')



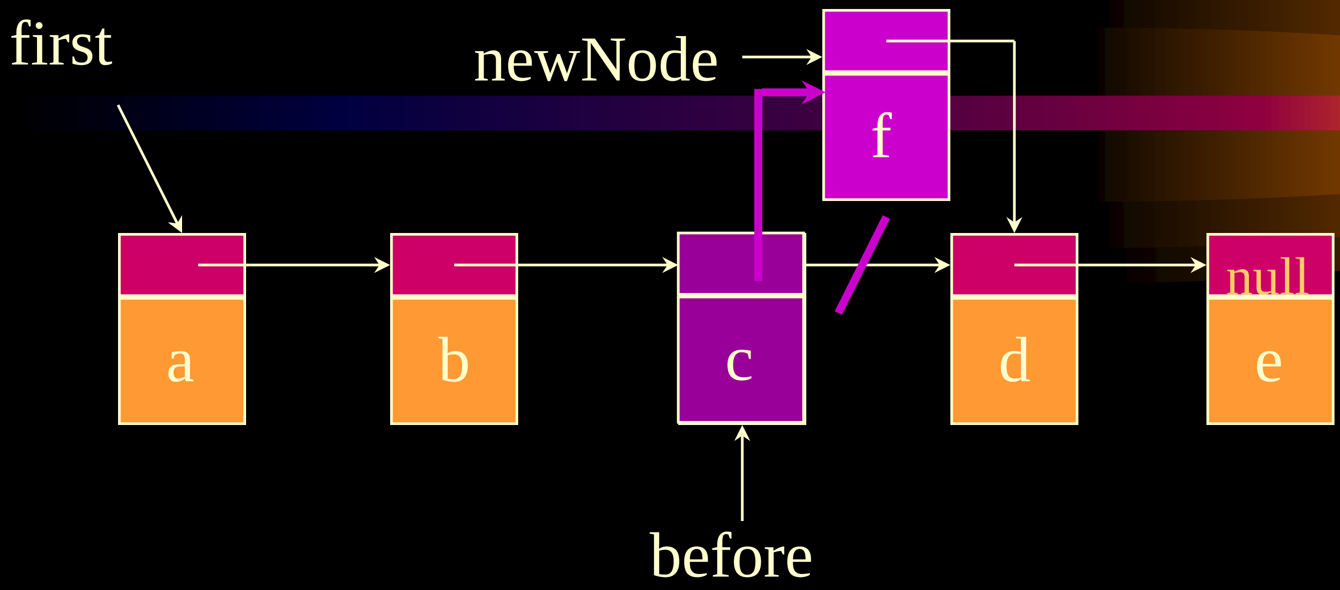
1. first find node whose index is 3
2. next create a node and set its data and link fields

   ChainNode newNode = new ChainNode('f',before .link);

3. finally link before to newNode

   before .link = newNode;

Two-Step insert(3,’f’)



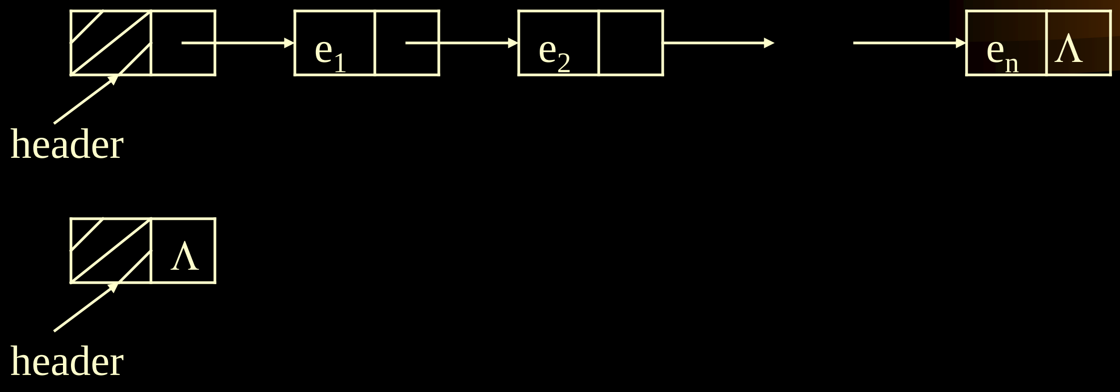
before = first .link .link;

newNode .link = before .link;

before .link = newNode;

3.2.3 Programming Details

1. Header (dummy node)



3.2.2. Linked Lists

2. Class definition

ListNode —— 代表结点的类

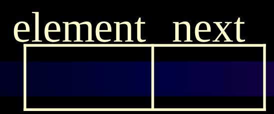
LinkedList —— 代表表本身的类

LinkedListItr —— 代表位置的类

都是包DataStructures的一部分

3.2.2. Linked Lists

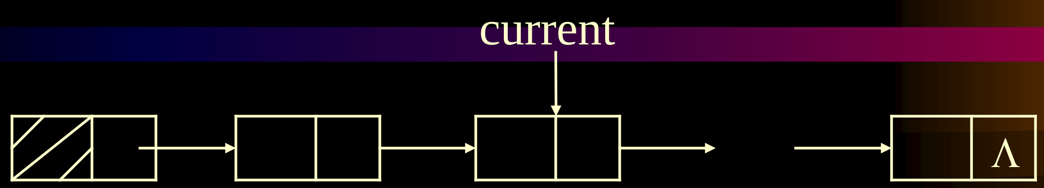
1) ListNode class




```
package DataStructures;
class ListNode
{   ListNode( object theElement)
    {   this( theElement, null);
     }
     ListNode( object theElement, ListNode n)
     {    element = theElement;
          next = n;
      }
     object element;
     ListNode next;
}
```

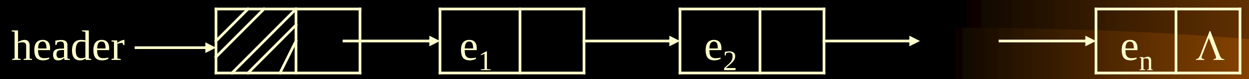
3.2.2. Linked Lists

2) Iterator class for linked lists



```
package DataStructures
public class LinkedListItr
{   LinkedListItr( ListNode  theNode)
    {   current = theNode;
    }
     public boolean isPastEnd( )
     {   return current = = null;
     }
     public object retrieve( )
     {   return isPastEnd( ) ? Null : current.element;
     }
     public void advance( )
     {   if( ! isPastEnd( ) )
             current = current.next;
     }
     ListNode current;
}
```

3) LinkedList class



```
package DataStructures;
public class LinkedList
{   public LinkedList( )
      {   header = new ListNode( null ) ; }
    public boolean isEmpty( )
      {   return header.next = = null ; }
    public void makeEmpty( )
      {   header.next = null; }
    public LinkedListItr zeroth( )
      {  return new LinkedListItr( header ) ; }
    public LinkedListItr first( )
      {  return new LinkedListItr( header.next ) ; }
    public LinkedListItr find( object x )
    public void remove( object x )
    public LinkedListItr findPrevious( object x )
    public void insert( object x, LinkedListItr p )

    private ListNode header;
}
```

Method to print a list

```
public static void printList( LinkedList theList )
{   if ( theList.isEmpty( ) )
        System.out.print( “Empty list” ) ;
    else
    {   LinkedListItr itr = theList.first( );
        for(  ; ! Itr.isPastEnd( ); itr. Advance( ) )
            System.out.print( itr.retrieve( ) + “ “ ) ;
    }

    System.out.println( );
}
```

Operation:

- Constructors
- isEmpty
- makeEmpty
- Zeroth and first return iterators corresponding to the header and first element.
- Find(x)

```
public  LinkedListItr  find (object x)
{    ListNode itr = header.next;
     while ( itr != null && !itr.element.equals( x ) )
             itr = itr.next;
      return new LinkedListItr( itr );
}
```

O(N)

- Remove(x)

```
public void remove( object x )
{   LinkedListItr p = findprevious( x );
    if( p.current.next != null )
        p.current.next = p.current.next.next;
}
```

O(1)

- Findprevious (x)

```
public  LinkedListItr findPrevious( object x )
{   ListNode itr = header;
    while( itr.next !=null && !itr.next.element.equals( x ))
        itr = itr.next;
     return new LinkedListItr( itr );
}
```

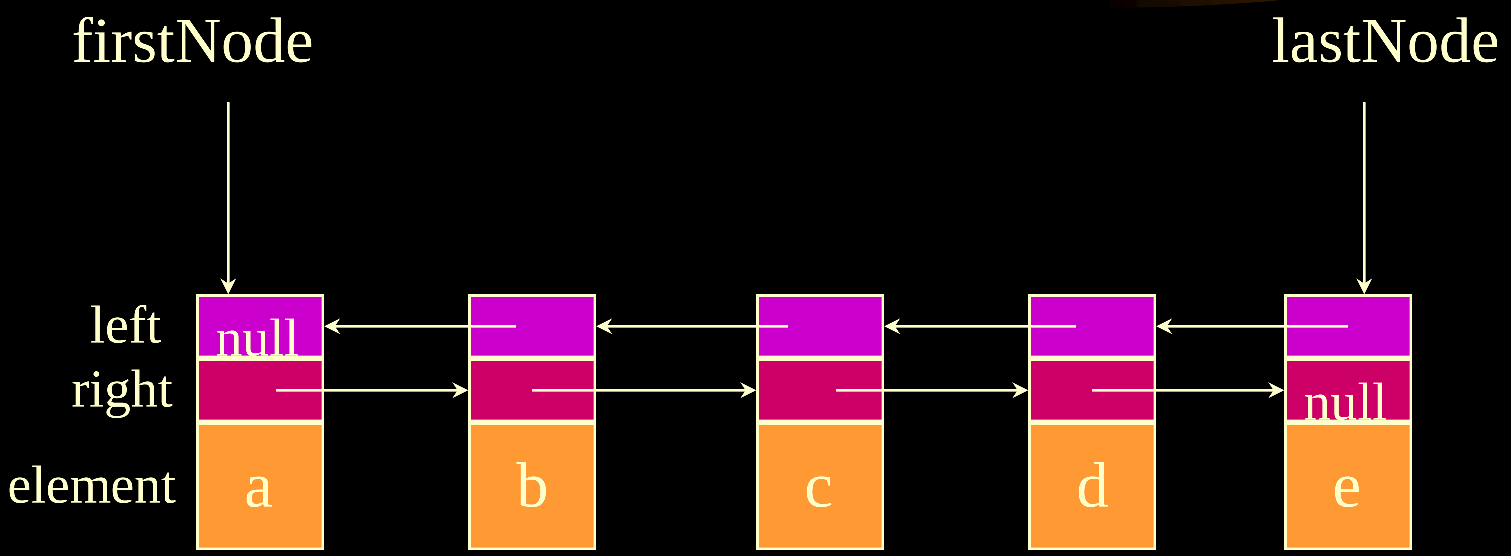
O(N)

- Insert(x, p)

```
public  void insert( object x, LinkedListItr p)
{   if( p!=null && p.current != null )
        p.current.next = new ListNode( x, p.current.next );
}
```

O(1)

3.2.4. Doubly Linked Lists

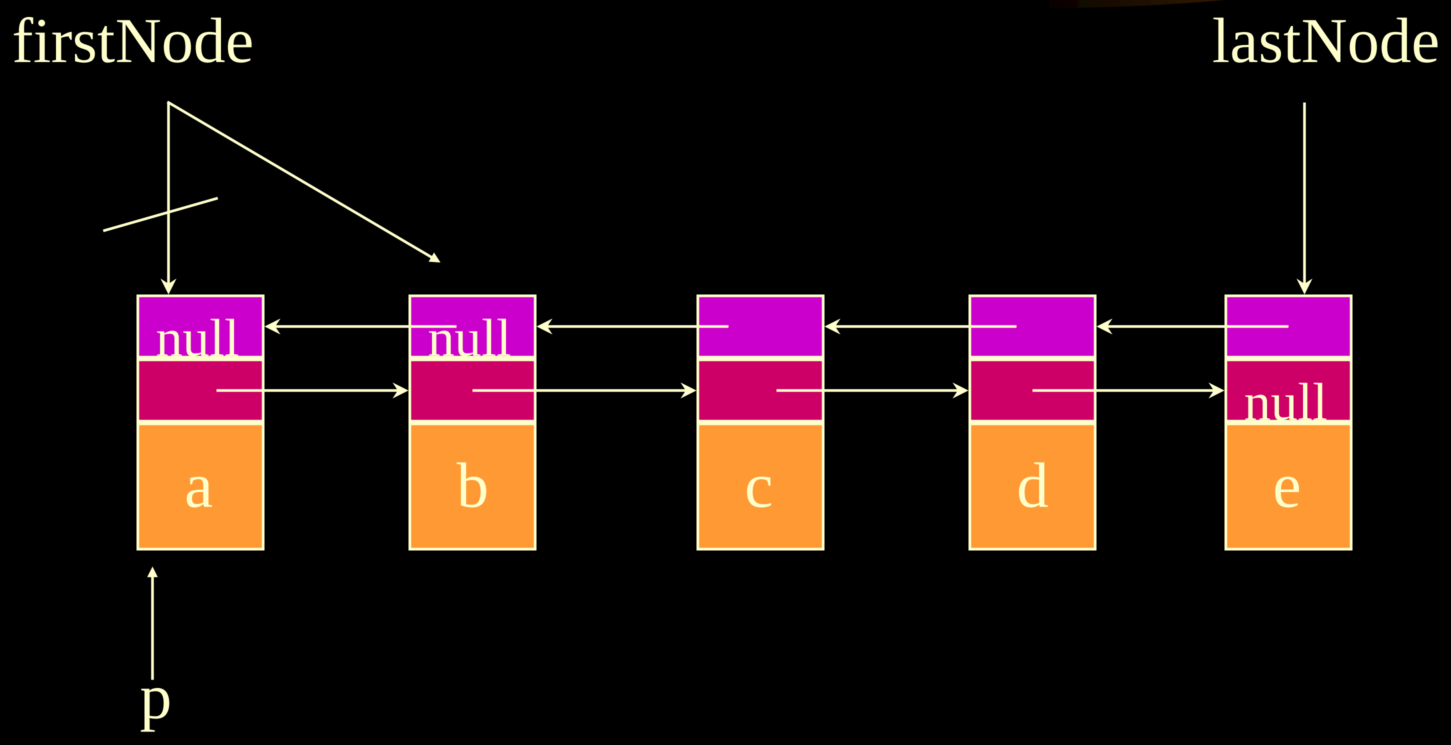


3.2.4. Doubly Linked Lists

operations: insert delete

3.2.4. Doubly Linked Lists

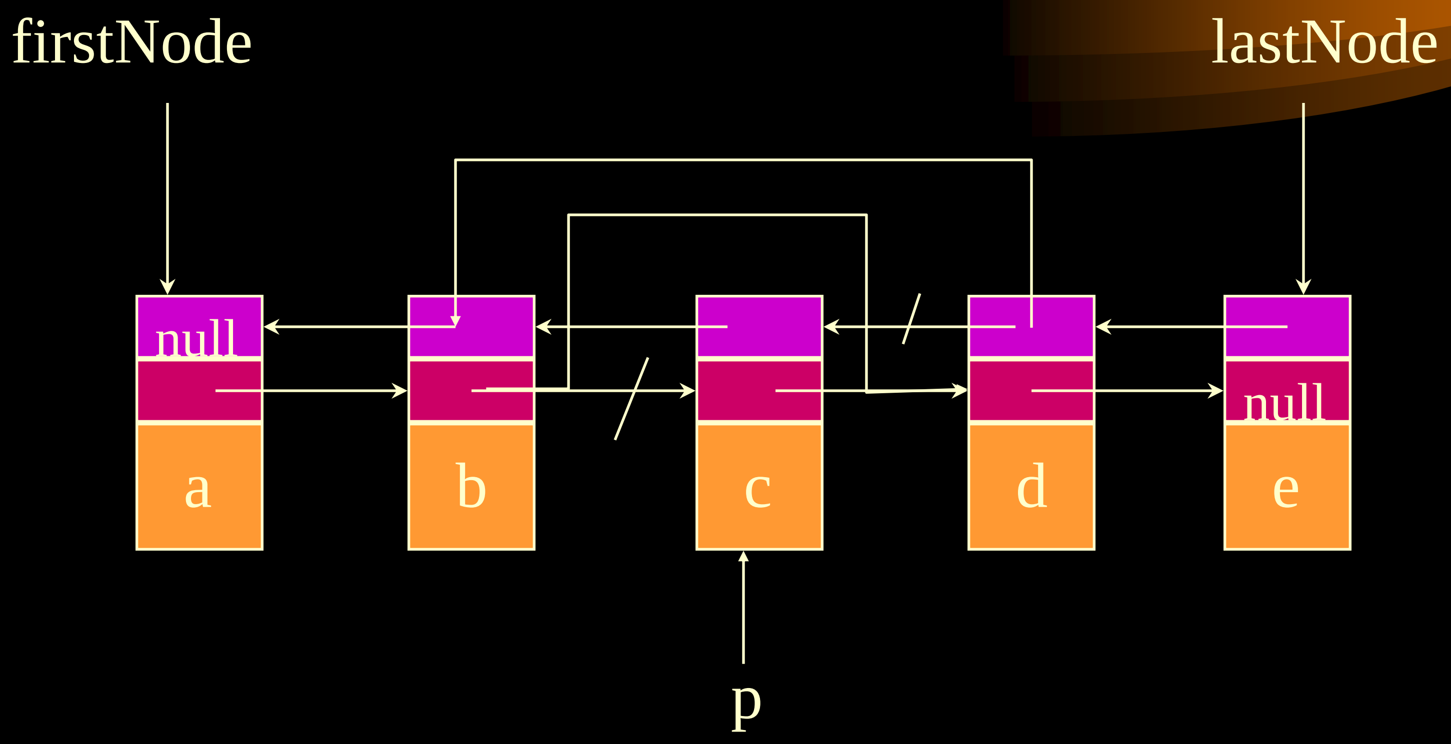
Delete(1)



P=firstNode; firstNode=p .right; firstNode .left=null;

3.2.4. Doubly Linked Lists

Delete(3)

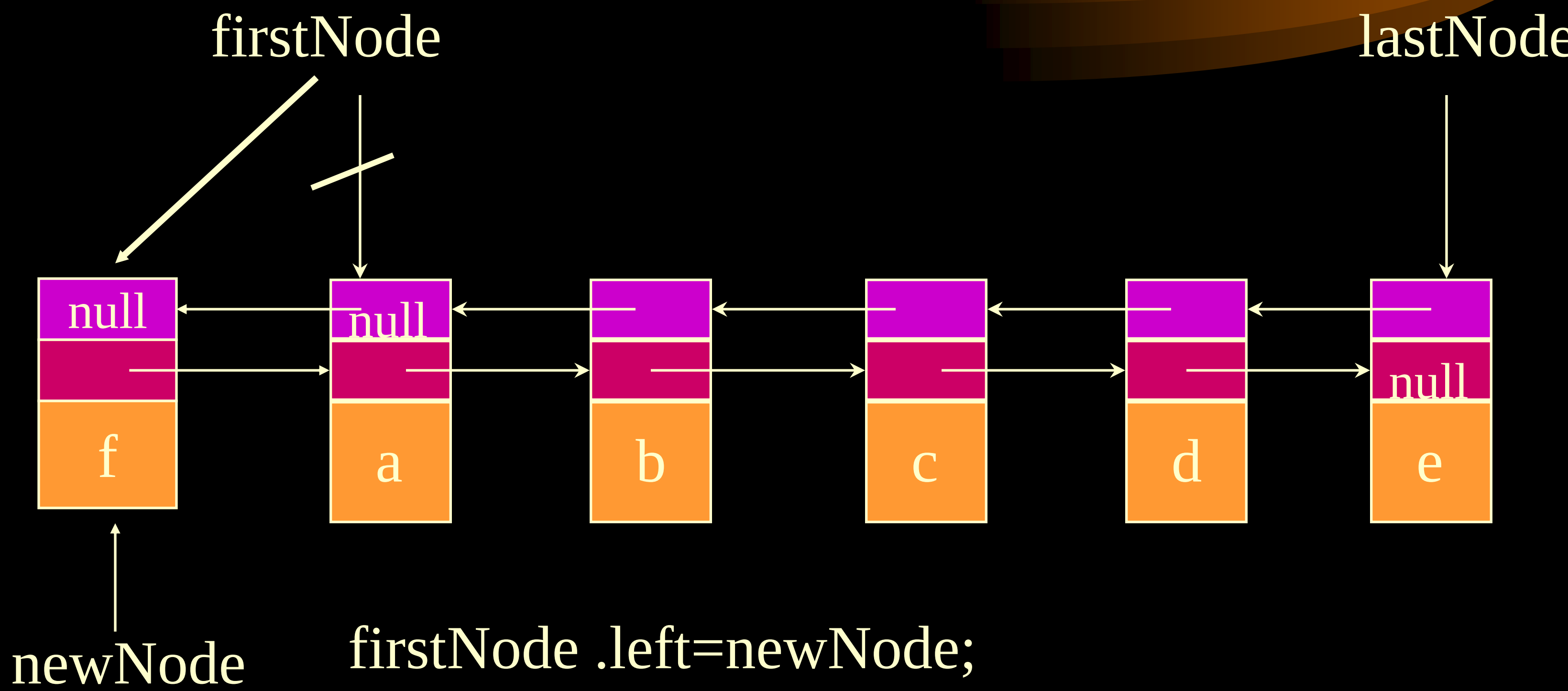


P .left .right=p .right;

P .right .left=p .left;

3.2.4. Doubly Linked Lists

insert(0,'f')

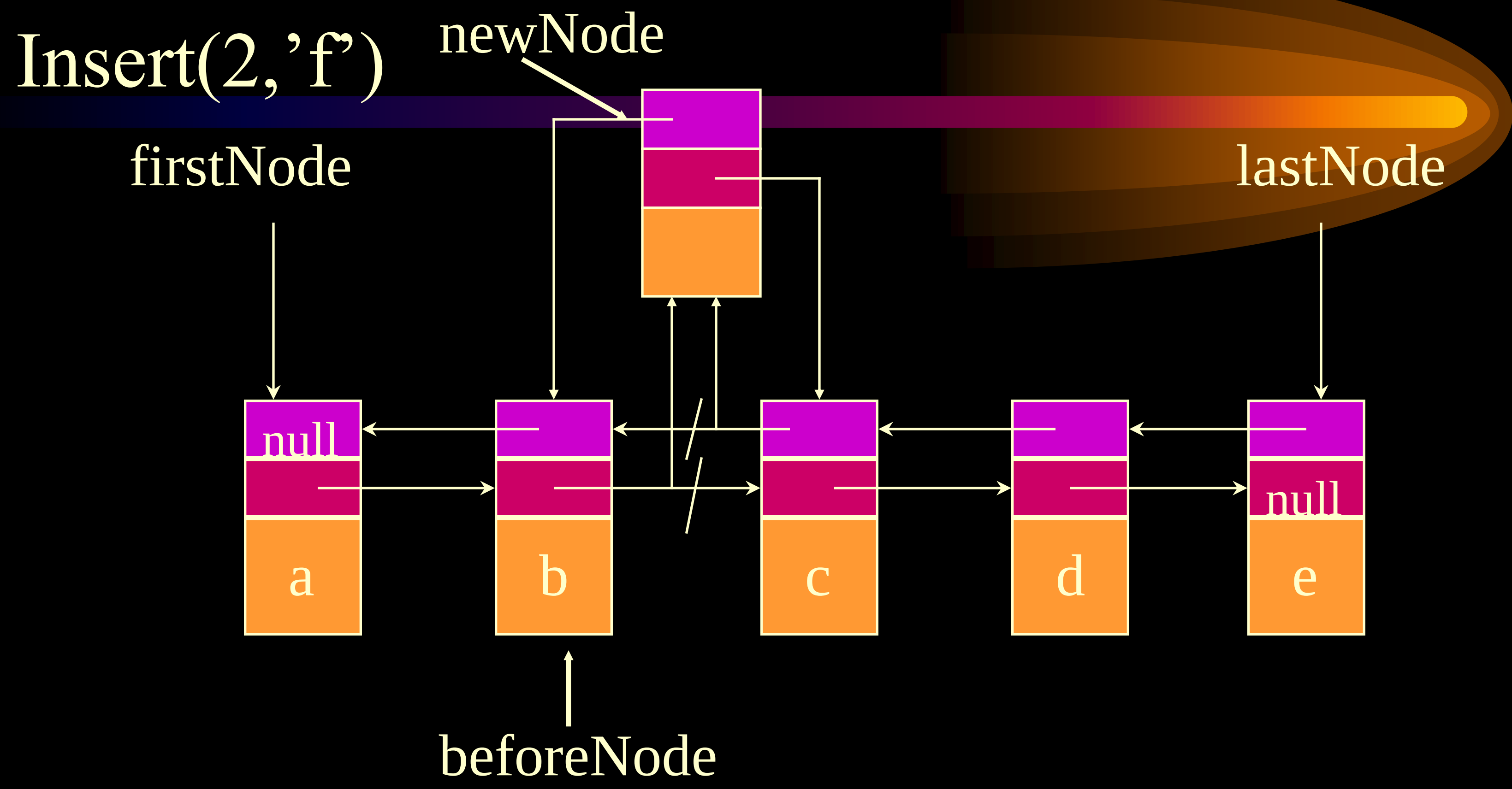


firstNode .left=newNode;

newNode .left=null;   newNode .right=firstNode;

firstNode=newNode

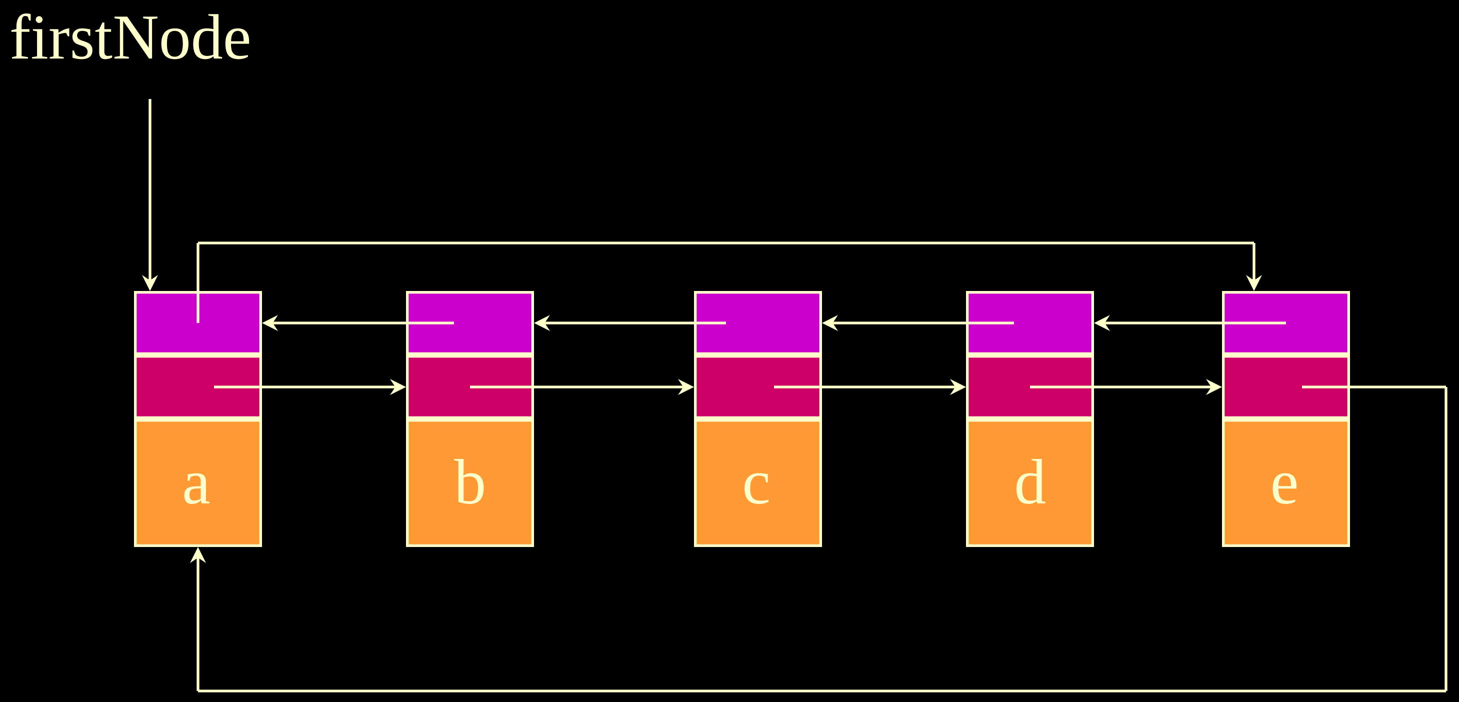
3.2.4. Doubly Linked Lists



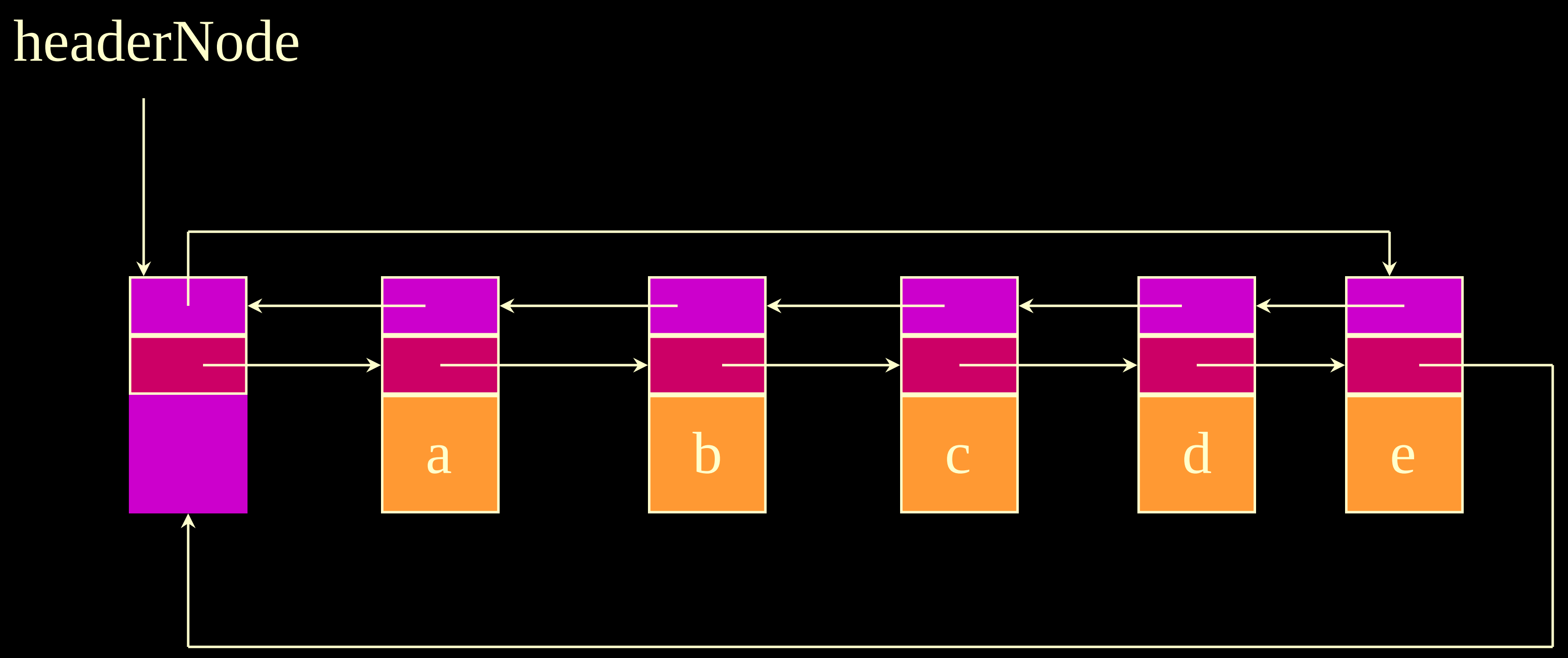
newNode .left=beforeNode; newNode .right=beforeNode .right;

beforeNode .right .left=newNode; beforeNode .right=newNode;

3.2.4. Doubly Linked Circular Lists

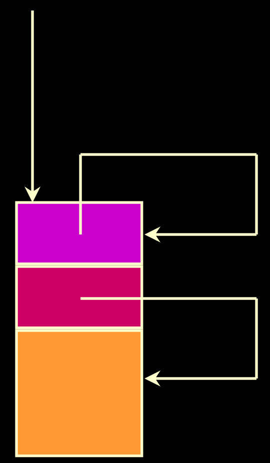


Doubly Linked Circular List With Header Node

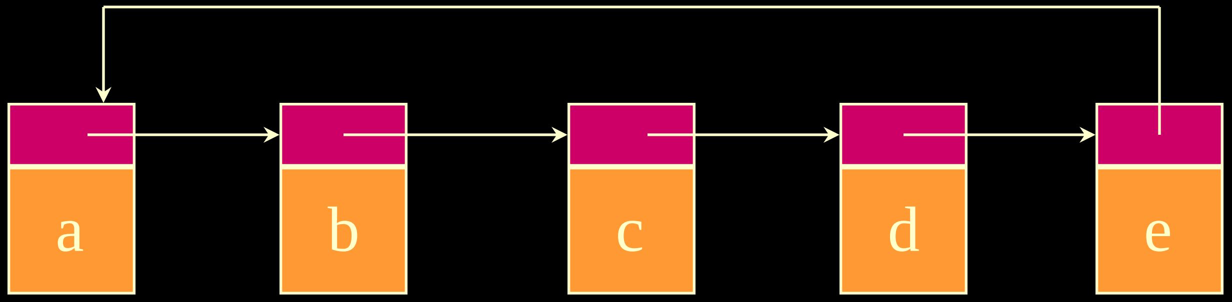


Empty Doubly Linked Circular List With Header Node

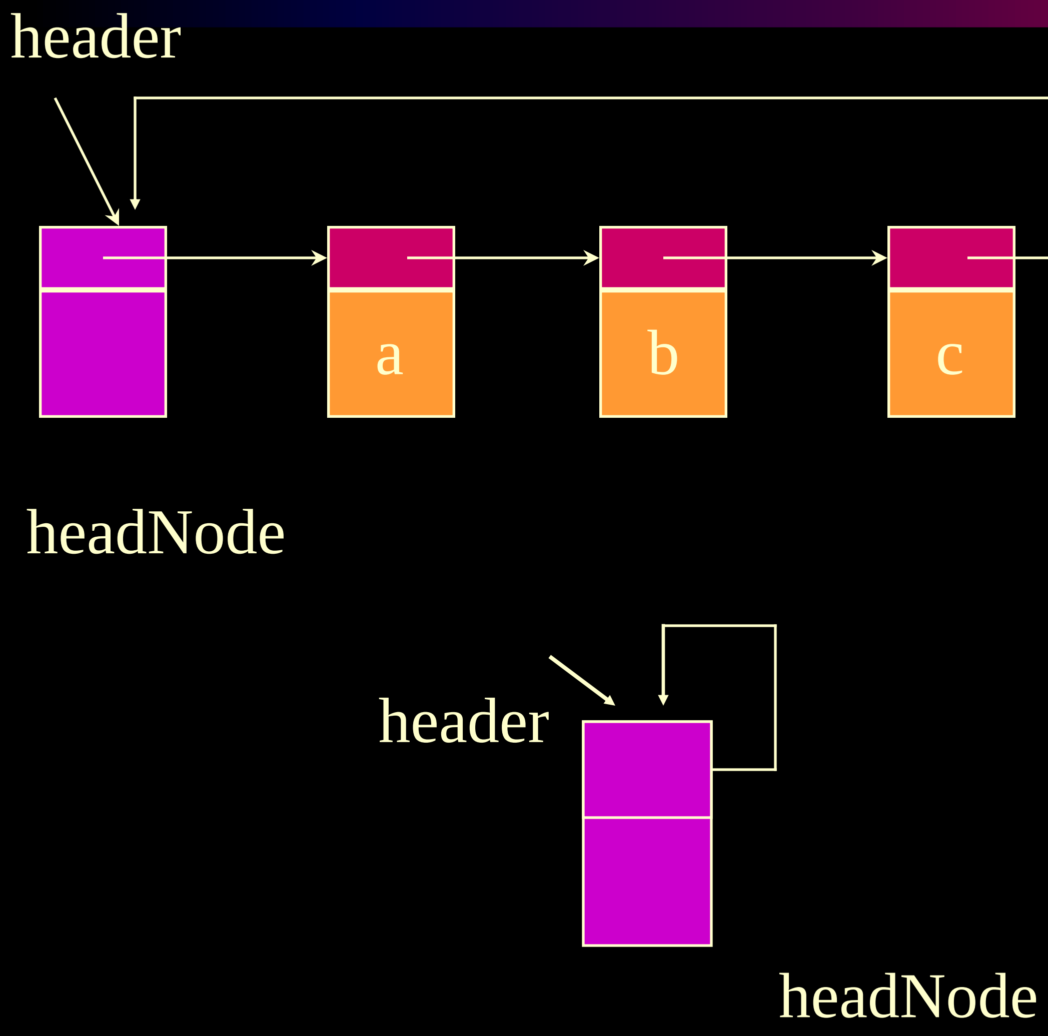
headerNode



3.2.5. Circular Linked Lists



3.2.5. Circular Linked Lists

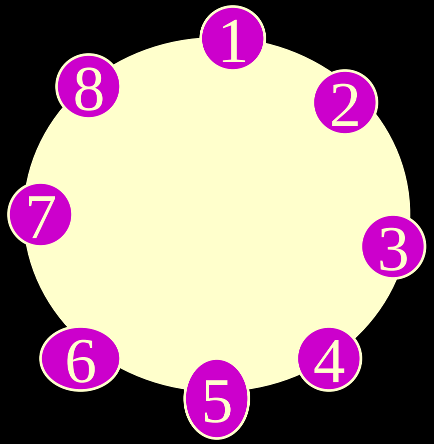


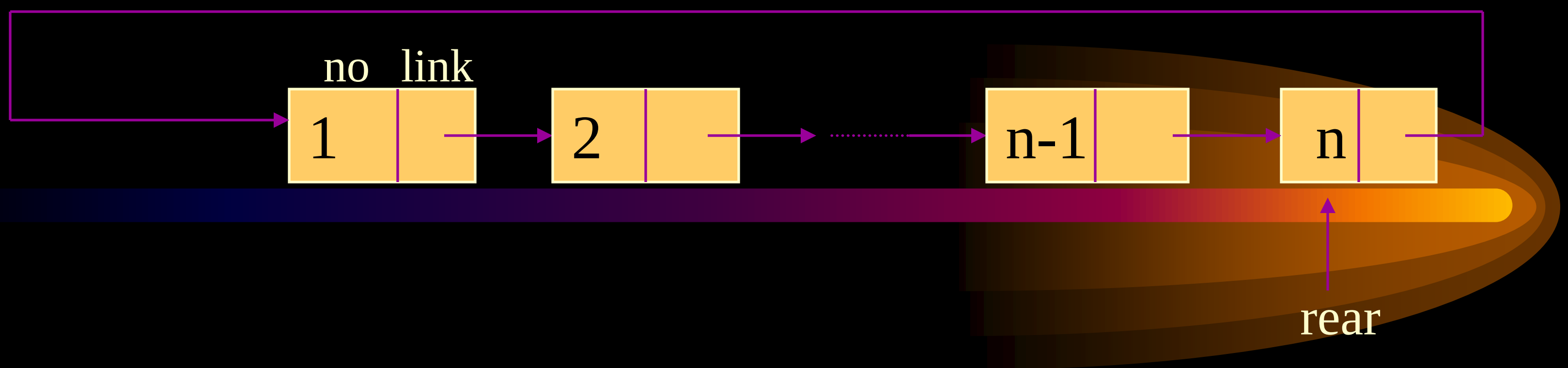
例子:

用循环链表求解约瑟夫(josephus)问题

约瑟夫问题：实际上是一个游戏。书中以旅行社从n个旅客中选出一名旅客，为他提供免费环球旅行服务。

例，n=8, m=3（报数）从1号开始报数出列顺序为：3, 6, 1, 5, 2, 8, 4。最后一个编号7的旅客将赢得环球旅游。





rear: 每次指向要出队列的前一个结点

出队列的人也用链表来表示:

head: 指向出队列结点链表的开头结点

p: 指向出队列结点链表的尾结点

以上rear, head, p都是ListNode的一个对象引用。

1. w = m;
2. for(int i = 1; i<= n-1; i++)

```
{   1) for (int j =1; j<=w-1; j++)  rear = rear.link;
    2) if (i = = 1)
        {  head = rear.link ; p = head; }
       else
          {  p.link = rear.link; p = rear.link; }
    3) rear.link = p.link;
 }
```

3. P.link = rear;

   rear.link = null;

3.2.6. Examples

1. Polynomial ADT

$p_n(x) = a_0x^{e0}+a_1x^{e1}+a_2x^{e2}+\ldots\ldots.+a_nx^{en}$

- Array implementation

example: $p(x) = 3x^4-5x^3+8x^2+2x-1$

coeffArray	3	-5	8	2	-1

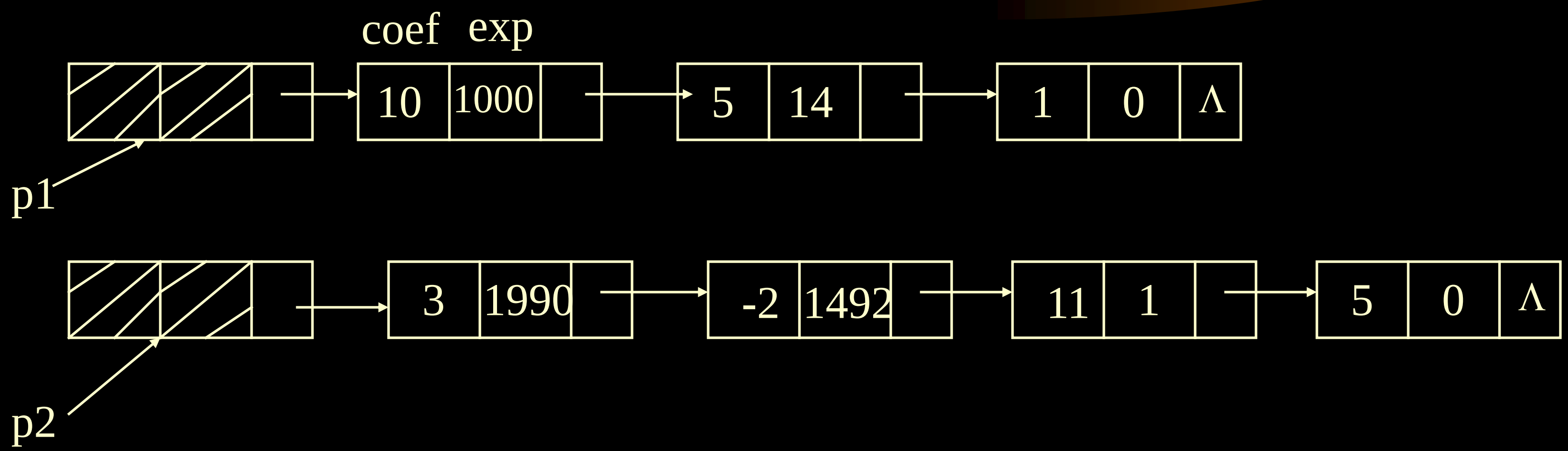
$p(x) = 2x^{1000}+8x^{50}-2$

3.2.6. Examples

- Linked list representations

$p_1(x) = 10x^{1000}+5x^{14}+1$

$p_2(x) = 3x^{1990}-2x^{1492}+11x+5$



polynomial operations: addition , multiplication, and so on.

1) Array implementation of the polynomial ADT


```
public class Polynomial
{   public Polynomial( ) { zeroPolynomial( ); }
    public void insertTerm( int coef, int exp )
    public void zeroPolynomial( )
    public Polynomial add( Polynomial rhs )
    public Polynomial multiply( Polynomial rhs ) throws Overflow
    public void print( )

    public static final int MAX-DEGREE = 100;
    private int coeffArray[ ] = new int [MAX-DEGREE + 1];
    private int highPower = 0;
}
```

```
public void zeroPolynomial( )
{    for( int i = 0; i<=MAX-DEGREE; i++)
            coeffArray[i] = 0;
     highPower = 0;
}

public Polynomial add( Polynomial rhs )
{   Polynomial sum = new Polynomial( );
    sum.highPower = max( highPower, rhs.highPower );
    for( int i = sum.highPower;  i>=0;  i--)
       sum.coeffArray[i] = coeffArray[i] + rhs.coeffArray[i] ;
    return sum;
}
```

Example:

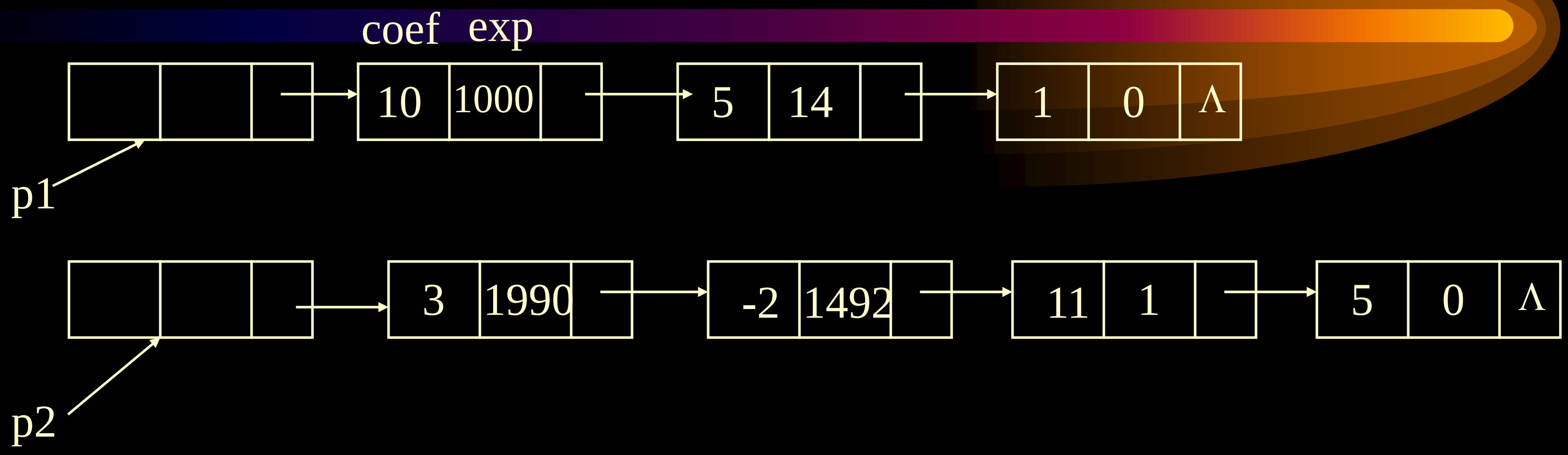
$p_1(x) = 3x^8 – 5x^3 + 3x – 1$

$p_2(x) = 4x^6 + 2x^2 + 2$

0	1	2	3	4	5	6	7	8
-1	3	0	–5	0	0	0	0	3
2	0	2	0	0	0	4	0	0

```
Public Polynomial multiply( Polynomial rhs ) throws overflow
{   Polynomial product = new Polynomial( );
    product.highPower = highPower + rhs.highPower;
    if( product.highPower > MAX-DEGREE )
          throw new overflow( );
    for( int i = 0; i<= highPower; i++ )
      for( int j = 0; j<= rhs.highPower; j++ )
          product.coeffArray[ i + j ] +=  coeffArray[i] * rhs.coeffArray[j];
     return product;
}
```

2) Class skeletons for linked list implemetation of the Polynomial ADT



```
public class Literal
{   //Vavious constractors(not shown)
     int coefficient;
     int exponent;
}
public class Polynomial
{   public Polynomial( ) { /* Exercise */ }
    public void insertTerm( int coef, int exp ) { /* Exercise */ }
    public void zeroPolynomial( ) { /* Exercise */ }
    public Polynomial add( Polynomial rhs ) { /* Exercise */ }
    public Polynomial multiply( Polynomial rhs ) { /* Exercise */ }
    public void print( ) { /* Exercise */ }

    private List  terms;  /*  A List of Literals, sorted by exponent */
}
```

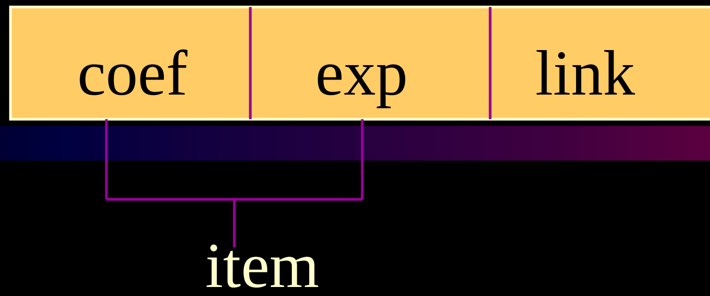
多项式相加：

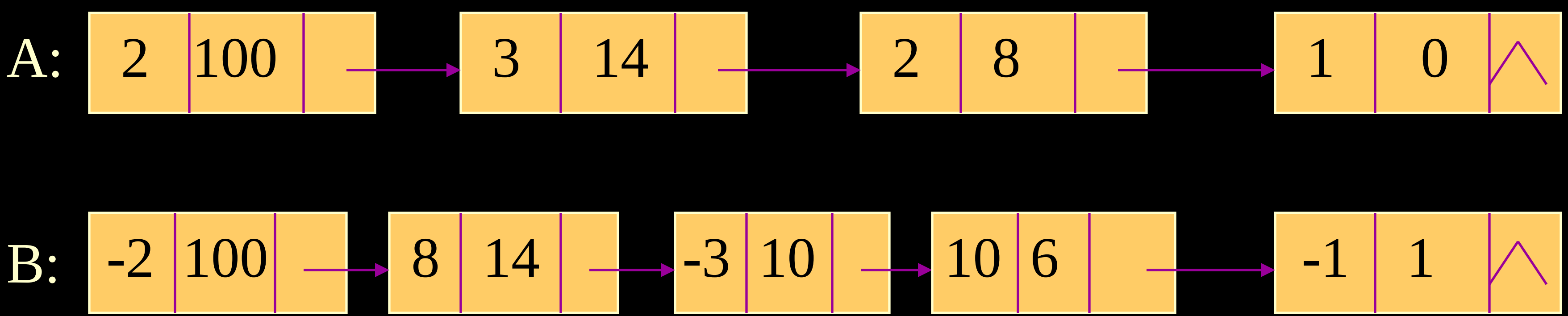
*问题：$A(X)=2X^{100} +3X^{14} +2X^{8} +1$

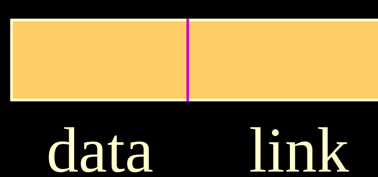
$B(X)= -2X^{100} +8X^{14} -3X^{10} +10X^{6} -X$

$A(X) + B(X) = 11X^{14} – 3X^{10} + 2X^{8} + 10X^{6} – X + 1$

存放非零指数的系数与指数，因此每个结点有三个域组成。







A(X)+B(X)==》A(X)

具体实现时，并不要再重新申请结点，完全利用原来两个链表的结点。

方法：设4个引用变量：

pa, pb, pc, p(c++需要）

1)初始化：pc , pa , pb ;

2)当pa和pb都有项时

pc永远指向相加时结果链表的最后一个结点。

a)指数相等(pa. exp= =pb. exp)

对应系数相加：pa. coef=pa. coef + pb. coef ;

p= pb(c++需要） ; pb前进 ;

if (系数相加结果为0){ p=pa; pa前进; }

else { pc. link=pa; pc=pa; pa前进}

b)指数不等 pa. exp< pb.exp //pb要插入结果链表

{pc. link=pb ; pc=pb ; pb前进}

c)指数不等 pa. exp> pb. exp //pa要插入结果链表

{pc. link=pa ; pc=pa ; pa前进}

3)当两链表中有一链表为空，则将另一链表链入结果链表就可以

if (pb空了){ pc. link=pa;}

else pc. link=pb;

算法分析：设两个多项式的项数分别是m和n，则总的比较次数为O(m+n)

最坏情况下：两个多项式的指数项都不等且交叉递增，如

A(x): $a_5x^5+a_3x^3+a_1x+a_0$ m=4 比较m+n-1次

B(x): $b_4x^4+b_2x^2+b_0$ n=3

2009年考研统考题

(15分)已知一个带有表头结点的单链表,结点结构为 | data | link | ,假设该链表只给出了头指针 list. 在不改变链表的前提下, 请设计一个尽可能高效的算法,查找链表中倒数第k个位置上的结点(k为正整数). 若查找成功,算法输出该结点的data域的值, 并返回1; 否则返回0. 要求:

1) 描述算法的基本设计思想;
2) 描述算法的详细实现步骤;
3) 根据设计思想和实现步骤, 采用程序设计语言描述算法(使用C或C++或JAVA语言实现), 关键之处请给出简要注释.

3.2.6. Cursor implementation of Linked Lists

use array to implement linked list:

cursorSpace

	element	next
0		1
1		2
2		3
3		4

		element	next
header →	0		6
	1	30	2
p →	2	50	3
	3	67	8
	4	15	7
	5	81	0
	6	10	4
	7	20	1
	8	78	5

P = p.next p = cursorSpace[p].next

p.data cursorSpace[p].data

3.2.6. Cursor implementation of Linked Lists

1)Node and iterator for cursor implemetation of linked lists

```
class CursorNode
{   CursorNode( object theElement )
        {   this( theElement, 0 );  }
     CursorNode( object theElement, int n )
        {   element = theElement;
            next = n;
         }

        object element;
        int next;
}
```

3.2.6. Cursor implementation of Linked Lists

```
public class CursorListItr
{   CursorListItr( int theNode ) {  current = theNode; }
     public boolean isPastEnd( ) { return current = = 0; }
     public object retrieve( )
     {  return isPastEnd( ) ? null:
                              CursorList.cursorSpace[ current ].element;
     }
     public void advance( )
     {   if( !isPastEnd( ))
              current = CursorList.cursorSpace[ current ].next;
      }

      int current;
}
```

3.2.6. Cursor implementation of Linked Lists

2) Class skeleton for CursorList

```
public class CursorList
{   private static int alloc( )
    private static void free( int p)
    public CursorList( )
       {  header = alloc( );   cursorSpace[ header ].next = 0; }
    public boolean isEmpty( )
        {  return cursorSpace[ header ].next = = 0; }
    public void makeEmpty( )
    public CursorListItr zeroth( )
         {   return new CursorListItr( header ); }
    public CursorListItr first( )
         {   return new CursorListItr( cursorSpace[ header ].next ); }
```

```
public CursorListItr find( object x )
public void insert( object x, CursorListItr p)
public void remove( object x )
public CursorListItr findPrevious( object x )

private int header;
static CursorNode [ ] cursorSpace;

private static final int SPACE-SIZE = 100;

static
{   cursorSpace = new CursorNode[ SPACE-SIZE ];
    for( int i = 0; i<SPACE-SIZE; i++)
        cursorSpace[ i ] = new CursorNode( null, i + 1 );
     cursorSpace[ SPACE-SIZE-1].next = 0;
 }
}
```

3.2.6. Cursor implementation of Linked Lists

Some Routines:

- Alloc and free

```
private static int alloc( )
{   int p = cursorSpace[ 0 ].next;
    cursorSpace[0].next = cursorSpace[p].next;
    if( p = = 0 )
        throw new OutOfMemoryError( );
     return p;
}
private static void free( int p )
{   cursorSpace[p].element = null;
    cursorSpace[p].next = cursorSpace[0].next;
    cursorSpace[0].next = p;
}
```

3.2.6. Cursor implementation of Linked Lists

- Find routine----cursor implementation

```
public CursorListItr find( object x )
{   int itr = cursorSpace[ header ].next;
    while( itr != 0 && !cursorSpace[ itr ].element.equals( x ) )
          itr = cursorSpace[ itr ].next;
     return new CursorListItr( itr );
}
```

3.2.6. Cursor implementation of Linked Lists

- Insertion routine for linked lists---cursor implementation

```
public void insert( object x, CursorListItr p )
{   if( p != null && p.current != 0)
    {   int pos = p.current;
        int tmp = alloc( );

        cursorSpace[ tmp ].element = x;
        cursorSpace[ tmp ].next = cursorSpace[ pos ].next;
        cursorSpace[ pos ].next = tmp;
    }
}
```

3.2.6. Cursor implementation of Linked Lists

- Deletion routine for linked lists----cursor implementation

```
public void remove( object x )
{    CursorListItr p = findPrevious( x );
     int pos = p.current;

      if( cursorSpace[ pos ].next != 0 )
      {   int tmp = cursorSpace[ pos ].next;
          cursorSpace[ pos ].next = cursorSpace[ tmp ].next;
          free( tmp );
       }
}
```

- makeEmpty for cursor implementation

```
public void makeEmpty( )
{   while( !isEmpty( ) )
        remove( first( ).retrieve( ) );
}
```

3.2.7. Examples

2. Radix Sort

64, 8, 216, 512, 27, 729, 0, 1, 343, 125

288 371 260 531 287 235 56 299 18 23

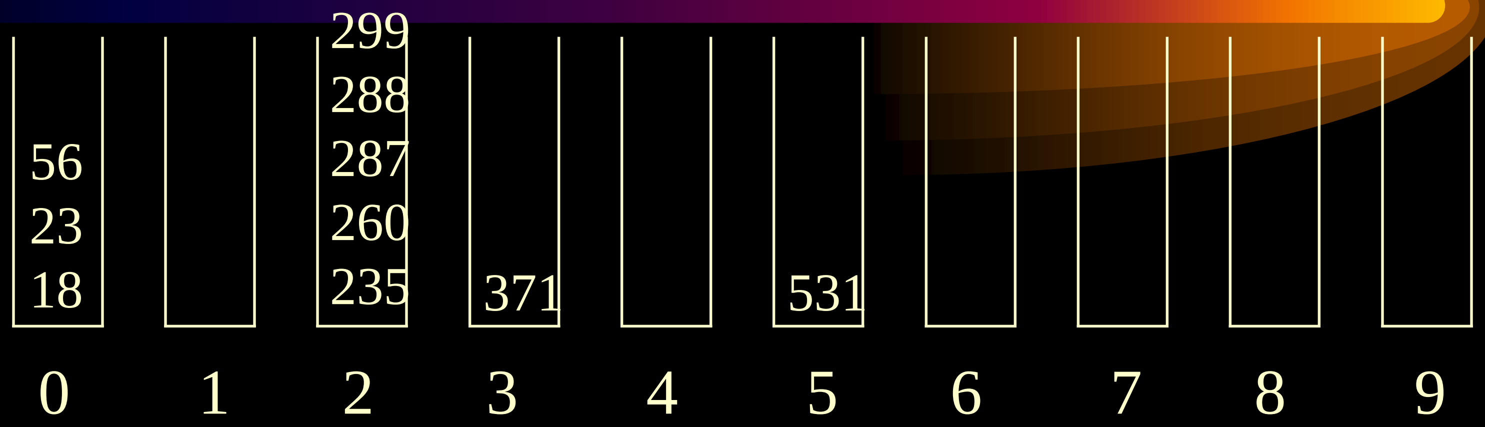
	531							18	
260	371		23		235	56	287	288	299

260 371 531 23 235 56 287 288 18 299

			235					288	
	18	23	531		56	260	371	287	299
0	1	2	3	4	5	6	7	8	9

18 23 531 235 56 260 371 287 288 299

3.2.7. Examples



18 23 56 235 260 287 288 299 371 531

如何实现：原始要排序的数据、桶中的数据都用链表来实现。

Chapter 3

exercises:

1. Swap two adjacent elements by adjusting only the links(and not the data) using:

   a. Singly linked lists.

   b. Doubly linked lists.

2. Given two sorted lists L_1 and L_2,write a procedure to compute $L_1 \cap L_2$ using only the basic list operations.

3. Given two sorted lists, L_1 and L_2, write a procedure to compute $L_1 \cup L_2$ using only the basic list operations.

4. Write a nonrecursive method to reverse a singly linked List in O(N) time.

Chapter 3

上机实习题：

3. 多项式相加，　用链表实现。
4. Josephus(n, m), 用数组、链表实现。