Lecture 12

Friedman's Permanent Income Hypothesis

> "The Permanent Income Hypothesis (PIH) says that the permanent consumption C_i^P of a individual i is a function of that individual's permanent income Y_i^P : $C_i^P = f\left(Y_i^P\right)$."
>
> "For Friedman, the permanent income was the interest rate r times the individual's wealth W_i (human capital plus non-human wealth): $Y_i^P = rW_i$."
>
> "Now, actual income Y_i is the sum of the permanent income Y_i^P and the temporary income Y_i^T (fluctuations at each period of time): $Y_i = Y_i^P + Y_i^T$. Actual consumption C_i is also given by the sum of the permanent consumption C_i^P and temporary consumption C_i^T: $C_i = C_i^P + C_i^T$."
>
> "Actual income and consumption are what we, in fact, observe. Whereas an error in the LHS variable of the model C_i^P does not cause big problem, errors in the measurement of the RHS variable Y_i^P indeed does."
>
> "Both C_i^T and Y_i^T are assumed to be uncorrelated random shocks to consumption and income, respectively, and, as such, random variables uncorrelated between themselves and also with C_i^P and Y_i^P."

Under this hypothesis, the true model would be $C_i^P = \beta_1 + \beta_2 Y_i^P$, with $\beta_1 = 0$ (the consumption curve, in a consumption *versus* disposable income plane, should start from the origin).

To test, we would like to run $C_i^P = \beta_1 + \beta_2 Y_i^P + v_i$ (v_i is the stochastic error), under the hypotheses:

$$\begin{aligned} H_0 &: \beta_1 = 0 \\ H_1 &: \beta_1 \neq 0 \end{aligned}$$

Since what we truly observe is the actual consumption C_i^P and actual income Y_i^P, the model we are supposed to run is $C_i - C_i^P = \beta_1 + \beta_2\left(Y_i - Y_i^P\right) + v_i$, which can be rewritten as:

$$C_i = \beta_1 + \beta_2 Y_i + u_i,$$

where $u_i = v_i + C_i^T - \beta_2 Y_i^T$.

Thus, there is correlation between two variables on the RHS: Y_i and Y_i^T. That will bias down the estimated coefficient $\hat{\beta}_2$.

- We have seen that $\hat{\beta}_2 = \beta_2 + \dfrac{\sum(Y_i - \bar{Y})u_i}{\sum(Y_i - \bar{Y})^2}$.
- We have also seen that, as $N \to \infty$, $\operatorname{plim} \hat{\beta}_2 = \beta_2 + \dfrac{\operatorname{Cov}(Y_i, u_i)}{\operatorname{Var}(Y_i)}$.
- Thus, $\operatorname{plim} \hat{\beta}_2 = \beta_2 + \dfrac{\operatorname{Cov}\left(Y_i^P + Y_i^T,\ v_i + C_i^T - \beta_2 Y_i^T\right)}{\operatorname{Var}\left(Y_i^P + Y_i^T\right)}$.
    - Since the only correlated term inside the covariance is Y_i^T (with itself), then that probability limit can be rewritten as $\operatorname{plim} \hat{\beta}_2 = \beta_2 - \beta_2 \dfrac{\operatorname{Var}(Y_i^T)}{\operatorname{Var}(Y_i^P) + \operatorname{Var}(Y_i^T)}$.

- Remember that $\text{Cov}(Y, Y) = \text{Var}(Y)$.
- And, since Y_i^P and Y_i^T are uncorrelated, $\text{Var}(Y_i^P + Y_i^T) = \text{Var}(Y_i^P) + \text{Var}(Y_i^T)$.

- Thus, $\text{plim}\ \hat{\beta}_2 = \beta_2 \left(1 - \dfrac{\text{Var}(Y_i^T)}{\text{Var}(Y_i^P) + \text{Var}(Y_i^T)}\right)$.
  - Since the variances are always positive values, then $\text{plim}\ \hat{\beta}_2 < \beta_2$.

Having $\hat{\beta}_2$ to be biased downward causes $\hat{\beta}_1$ to be biased upward.

- From OLS, we know that $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}$.
- As $N \to \infty$, $\bar{y} \to \mu_y$ and $\bar{x} \to \mu_x$.
  - Thus, for an unbiased and consistent estimate $\hat{\beta}_2$, $\text{plim}\ \hat{\beta}_1 = \mu_y - \beta_2 \mu_x = \beta_1$.
- Suppose $\hat{\beta}_2$ is biased down by a constant $c > 0$, so that $\text{plim}\ \hat{\beta}_2 = \beta_2 - c$.
  - Then, $\text{plim}\ \hat{\beta}_1 = \mu_y - (\beta_2 - c)\mu_x = \mu_y - \beta_2 \mu_x + c\mu_x$.
  - Given that $\beta_1 = \mu_y - \beta_2 \mu_x$, then $\text{plim}\ \hat{\beta}_1 = \beta_1 + c\mu_x$.
  - For $\mu_x > 0$, $\text{plim}\ \hat{\beta}_1 > \beta_1$.

If the hypothesis is true, then, rather than a curve that starts from the origin, the measurement errors would cause the consumption function would to form a curve with too flat a slope (biased down $\hat{\beta}_2$) and a non-zero intercept (biased up $\hat{\beta}_1$).

More on the PIH [here](#).

Measurement error example:

True model: $\boldsymbol{Q} = \beta_1 + \beta_2 \boldsymbol{Z} + \boldsymbol{v}$.

Suppose there is a common measurement error ω :

$$
\begin{aligned}
y_i &= Q_i + \omega_i \\
x_i &= Z_i + \omega_i
\end{aligned}
$$

Substituting: $y_i - \omega_i = \beta_1 + \beta_2 (x_i - \omega_i) + v_i$.

- $y_i = \beta_1 + \beta_2 x_i + u_i$, where $u_i = (v_i + \omega_i - \beta_2 \omega_i)$.
- We have seen that $\text{plim}\ \hat{\beta}_2 = \beta_2 + \dfrac{\text{Cov}(x_i, u_i)}{\text{Var}(x_i)}$.
  - $\text{plim}\ \hat{\beta}_2 = \beta_2 + \dfrac{\text{Cov}\Big(\omega_i, \omega_i(1 - \beta_2)\Big)}{\text{Var}(Z_i) + \text{Var}(\omega_i)}$.
  - $\text{plim}\ \hat{\beta}_2 = \beta_2 + (1 - \beta_2)\dfrac{\text{Var}(\omega_i)}{\text{Var}(Z_i) + \text{Var}(\omega_i)}$.
    - $\beta_2 > 1 \to \text{plim}\ \hat{\beta}_2 < \beta_2$.
    - $\beta_2 < 1 \to \text{plim}\ \hat{\beta}_2 > \beta_2$.

<u>Solution</u>: **Instrumental variables**

The model: $y_i = \beta_1 + \beta_2 x_i + u_i$.

Now, suppose that x_i and u_i are somehow correlated.

> Measurement errors induce correlation between the error and the RHS variable. However, not every correlation between those terms stems, necessarily, from a measurement error. That being so, the following solution is more general than the issue of errors of measurement.

Suppose we have another variable $\boldsymbol{z}$ with three properties:

1. $\boldsymbol{z}$ is correlated with $\boldsymbol{x}$;
2. $\boldsymbol{z}$ is uncorrelated with the error;
3. $\boldsymbol{z}$ is no one of the $\boldsymbol{x}$'s (multiple RHS variables).

From OLS, we know $\hat{\beta}_2^{OLS} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$.

- We know $\hat{\beta}_2^{OLS}$ is inconsistent if $\boldsymbol{x}$ and $\boldsymbol{u}$ are correlated.
- We will introduce a new estimator $\hat{\beta}_2^{IV} = \frac{\sum(z_i - \bar{z})(y_i - \bar{y})}{\sum(z_i - \bar{z})(x_i - \bar{x})}$.
  - $\hat{\beta}_2^{IV} = \frac{\sum(z_i - \bar{z})\left[(\beta_1 + \beta_2 x_i + u_i) - (\beta_1 + \beta_2 \bar{x} + \bar{u})\right]}{\sum(z_i - \bar{z})(x_i - \bar{x})}$.
    - $\bar{u}$, as always, is assumed to be zero.
  - $\hat{\beta}_2^{IV} = \frac{\sum(z_i - \bar{z})(\beta_2 x_i - \beta_2 \bar{x} + u_i)\big]}{\sum(z_i - \bar{z})(x_i - \bar{x})}$.
    - This can be rewritten as $\hat{\beta}_2^{IV} = \beta_2 \frac{\sum(z_i - \bar{z})(x_i - \bar{x})}{\sum(z_i - \bar{z})(x_i - \bar{x})} + \frac{\sum(z_i - \bar{z})u_i}{\sum(z_i - \bar{z})(x_i - \bar{x})}$.
    - $\hat{\beta}_2^{IV} = \beta_2 + \frac{\sum(z_i - \bar{z})u_i}{\sum(z_i - \bar{z})(x_i - \bar{x})}$.
  - As $N \to \infty$, $\text{plim } \hat{\beta}_2^{IV} = \beta_2 + \frac{\text{Cov}(z_i, u_i)}{\text{Cov}(z_i, x_i)}$.
    - Since we assumed z_i and u_i are uncorrelated, $\text{Cov}(z_i, u_i) = 0$.
    - Given that z_i and x_i were assumed to be correlated, $\text{Cov}(z_i, x_i) \neq 0$.
  - Then, $\text{plim } \hat{\beta}_2^{IV} = \beta_2$.

Usually, the variance of the estimator $\hat{\beta}_2^{OLS}$ is given by: $\text{Var}\left(\hat{\beta}_2^{OLS}\right) = \left(\frac{\text{Var}(u_i)}{\sum(x_i - \bar{x})}\right)$.

- When using an instrumental variable, $\text{Var}\left(\hat{\beta}_2^{IV}\right) = \left(\frac{\text{Var}(u_i)}{\sum(x_i - \bar{x})}\right)\left(\frac{1}{r_{xz}^2}\right)$.
  - r_{xz} is the Pearson correlation coefficient between $\boldsymbol{x}$ and $\boldsymbol{z}$.
    - $-1 \leq r_{xz} \leq 1 \to 0 \leq r_{xz}^2 \leq 1$. Thus, $\text{Var}\left(\hat{\beta}_2^{IV}\right) \geq \text{Var}\left(\hat{\beta}_2^{OLS}\right)$.
    - That means that using an instrumental variable comes with a price: the estimated coefficient for the instrumental variable, while consistent, has a higher variance. Hence, the model loses precision.
  - The lower the correlation between $\boldsymbol{x}$ and $\boldsymbol{z}$, the higher the variance of $\hat{\beta}_2^{IV}$.

Example:

In macroeconomics, when modeling consumption $\boldsymbol{C}$ as a function of income $\boldsymbol{Y}$:
$C_t = a + bY_t + u_t$.

- If we worry that Y_t and u_t are correlated, Y_{t-1} is a good choice for an instrumental variable.
  - Y_t and Y_{t-1} are - most likely - closely correlated.
  - Y_{t-1} cannot be correlated with a posterior random error u_t.
- However, we must beware of serial correlation between u_{t-1} and u_t.

More on instrumental variables estimation here.