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NOTE

Refined Filtering of Image Noise Using Local Statistics

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An effective algorithm for digital image noise filtering is presented in this paper. Most noise filtering techniques such as Kalman filter and transform domain methods require extensive image modeling and produce filtered images with considerable contrast loss. The algorithm proposed in this paper is an extension of Lee's local statistics method modified to utilize local gradient information. It does not require image modeling, and it will not smear edges and subtle details. For both the additive and multiplicative noise cases, the local mean and variance are computed from a reduced set of pixels depending on the orientation of the edge. Consequently, noise along the edge is removed, and the sharpness of the edge is enhanced. For practical applications when the noise variance is spatially varying and unknown, an adaptive filtering algorithm is developed. Experiments show its good potential for processing real-life images. Examples on images containing 256 × 256 pixels are given to substantiate the theoretical development.

1. INTRODUCTION

Recently, Lee [1] has developed noise filtering algorithms for both the additive and multiplicative noise case. The techniques based on the use of local mean and local variance do not require image modeling as do other methods using Kalman or Wiener filtering techniques [2–4]. The only assumption is that the sample mean and variance of a pixel is equal to its local mean and variance based on pixels within a fixed neighborhood surrounding it. In the additive noise filtering case, the a priori mean (variance) of an image is calculated as the difference between the local mean (variance) of the noise-corrupted image and the mean (variance) of the noise. It is well known that once the a priori mean and variance are given, it is straightforward to compute the optimal mean square estimate of it. It is found in reference [1] that the filtering algorithm is a linear weighted sum of the local mean and the image itself. The distinct characteristic is that in very low contrast areas, the estimated pixel approaches the local mean whereas in the high-contrast areas (edge areas), the estimated pixel favors the corrupted image pixel, thus retaining the edge information. It is generally claimed that human vision is more sensitive to noise in a flat area than in an edge area. However, it is still desirable to reduce noise in the edge area without sacrificing the edge sharpness. This is the objective which we wish to achieve in this paper.

The basic idea is to redefine the neighborhood (i.e., the area where local mean and variance are computed) near the high-contrast region taking into account the orientation of the edge. In other words, we incorporated the local gradient into the local mean and variance filtering algorithm. For each high local variance pixel (i.e., high-contrast point) over a threshold, a gradient will be computed for the local area to obtain the orientation of the edge. Then a subset of pixels in the local area on either side of the edge is defined. Because the subset only contains pixels on either

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side of the edge, the local mean and variance computed in the subset is a more precise representation of the a priori mean and variance of the pixel under consideration. From another angle, the local variance will be greatly reduced, and hence the noise along the edge will be removed.

There are many other noise-smoothing methods which do not require image modeling. Median filters [5] of one or two dimensions are effective in removing discrete impulse noise but at the cost of greater signal suppression. Nagao and Matsuyama [6] proposed an algorithm which selects the most homogeneous neighborhood and replaces the pixel by its neighborhood average. Wang *et al.* [7] applied a 3×3 window and computed for each pixel its weighted average of inverse gradients with its neighboring pixels. The idea is to weigh less on those pixels having greater absolute differences with their center pixel. Lev *et al.* [8] applied a template matching technique to detect edges and lines and then replaced the pixel by the weighted average corresponding to the particular pattern detected. These aforementioned algorithms are most suitable for smoothing noise in images with strong edges as preprocessing for future segmentation. However, subtle details such as thin lines, and small but distinguishable objects (i.e., a cluster of three or four pixels) are very likely to be washed out. Loosely speaking, the algorithm to be presented in this paper is a combination of a mean square filtering technique and an edge smoothing technique [6]. It preserves subtle details and lines while removing noise along edges as well as in flat areas. Furthermore the computational load is less as compared with these previously discussed algorithms, since most of them require several iterations to smooth noise.

In the next section, the local mean and variance method will be reviewed briefly, and the refined algorithm will be given in detail. In the third section, an adaptive algorithm is developed for the case of an unknown noise variance. Extension of the refined algorithm to a multiplicative noise-corrupted image is discussed in Section 4. Remarks and conclusion are given in the final two sections. Experimental results for images of dimension 256×256 are given for each case.

2. LOCAL STATISTICS METHOD AND SUBSET SELECTION

For the sake of completeness a brief review of the local statistics filtering algorithm [1] is given in this section. Then, attention will be focused on how to redefine the subset of a neighborhood.

Let $z_{i,j}$ be the brightness of the pixel (i,j) in a two-dimensional $N \times N$ image and $x_{i,j}$ be the pixel before degradation. Then, for the additive noise case,

$$z_{i,j} = x_{i,j} + \omega_{i,j}, \tag{1}$$

where $\omega_{i,j}$ is the white random sequence with zero mean and σ^2 variance. In most filtering algorithms the a priori mean and variance of $x_{i,j}$ are derived from an assumed correlation model. The local statistics method deviates from this by assuming that the a priori mean and variance ($\bar{x}_{i,j}$ and $Q_{i,j}$) are approximated by the local mean and variance of all pixels in the neighborhood surrounding $z_{i,j}$. From Eq. (1), we have

$$\bar{x}_{i,j} = \bar{z}_{i,j},$$

$$Q_{i,j} = E\left[\left(z_{i,j} - \bar{z}_{i,j}\right)^2\right] - \sigma_1^2, \tag{2}$$

where $\bar{z}_{i,j}$ and $E[(z_{i,j} - \bar{z}_{i,j})^2]$ are approximated by local mean and variance. Under this assumption, it is very easy to obtain the minimum mean square filter [1]. The estimated $x_{i,j}, \hat{x}_{i,j}$, is given by

$$\hat{x}_{i,j} = \bar{x}_{i,j} + k_{i,j}(z_{i,j} - \bar{x}_{i,j}), \tag{3}$$

where

$$k_{i,j} = \frac{Q_{i,j}}{Q_{i,j} + \sigma_1^2}. \tag{4}$$

Since $Q_{i,j}$ and σ_1^2 are both positive, $k_{i,j}$ will lie between 0 and 1. For a flat or low-contrast area, $Q_{i,j}$ is small and $\hat{x}_{i,j} \approx \bar{x}_{i,j}$, whereas for an edge or high-contrast area, $Q_{i,j}$ is much larger than σ_1^2 and $\hat{x}_{i,j} \approx z_{i,j}$. For most noisy images this algorithm produces quite satisfactory results since, as stated earlier, human vision is more sensitive to noise in a flat area than in an edge region. In many cases it is desirable, however, to smooth out the noise around the edge area.

The window used in the local statistics method is typically a (7×7) pixel region with $x_{i,j}$ being at the center. The neighborhood of $x_{i,j}$ could be altered to improve its statistics. For illustration, Fig. 1 shows the neighborhood of $x_{i,j}$. It is apparent that $x_{i,j}$ is more likely to be a member of the subset of pixels in the unshaded area of the window rather than the whole neighborhood. If the local mean and variance are computed based on pixels in this subset, the new $Q_{i,j}$ for this subset will be considerably smaller than the $Q_{i,j}$ computed for the whole set. Consequently from Eqs. (3) and (4), $\hat{x}_{i,j} \approx \bar{x}_{i,j}$, where $\bar{x}_{i,j}$ is the local mean of the subset. In other words noise at the edge will be smoothed. Our computational experience shows that this procedure will also enhance the edges.

To determine the subset, we have to know the orientation of the edge and on which side of the edge $x_{i,j}$ lies. A 3×3 pixel local gradient mask is used to determine the edge orientation. To minimize the noise effect on the local gradient, the window is divided into nine subareas and the local mean of each subarea is computed. Then the 3×3 gradient mask is applied to the local means of these areas, as shown in Fig. 2. Once the edge orientation is computed, the subarea means in the direction orthogonal to the edge are compared to determine to which side of the edge $x_{i,j}$ belongs, and hence to which subset. For the case shown in Fig. 2, compare $|m_{31} - m_{22}|$ and $|m_{13} - m_{22}|$ to determine the subset.

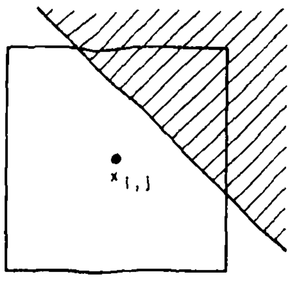

FIGURE 1

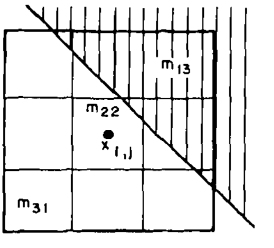

FIGURE 2

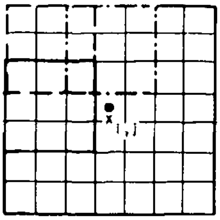

FIGURE 3

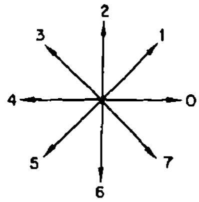

FIGURE 4

In practical implementation, a 7 × 7 window is used, and each subarea contains 3 × 3 pixels. For the sake of easier implementation, subareas are overlapped with neighbors as indicated in Fig. 3. The subarea means are computed and the simple 3 × 3 gradient masks [9] are applied to the subarea means. Only four directional gradient masks are required, because masks in opposite directions complement each other. The direction of absolute maximum of the gradients is used as the direction of the edge. For convenience, a directional index is used as shown in Fig. 4. Suppose the gradient direction labeled 2, is the maximum. Subarea means (Fig. 5) m_{12} and m_{32} are compared with m_{22} to determine whether the subset is in direction 2 or 6. The subset corresponding to its direction is shown in Fig. 6 for a 7 × 7 window. If $|m_{32} - m_{22}| > |m_{12} - m_{22}|$, subset 6 will be chosen and all pixels in the unshaded area are used in the computation of local mean and variance.

For clarity, a numerical example of a vertical noisy edge in a 7 × 7 window is given below.

99	105	124	138	128	34	62
105	91	140	98	114	63	31
107	94	128	138	96	61	82
137	129	136	105	100	55	85
144	145	113	132	119	39	50
102	97	102	110	103	34	53
107	146	115	123	101	76	56
					edge boundary	

The center pixel valued at 105 is the pixel to be filtered. If the original local statistics filter is used, the 7 × 7 mean = 99 and the 7 × 7 variance = 1029.24. Then, assuming noise variance $\sigma_1^2 = 300$, we have $k_{i,j} = 0.708$ and $\hat{x}_{i,j} = 103.25$. Since the local variance is high, we assign a considerably higher weight to the observed pixel, valued at 105. If the improved algorithm is used, and applied to the

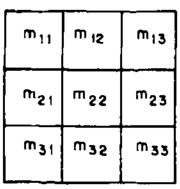

FIG. 5. Subarea mean.

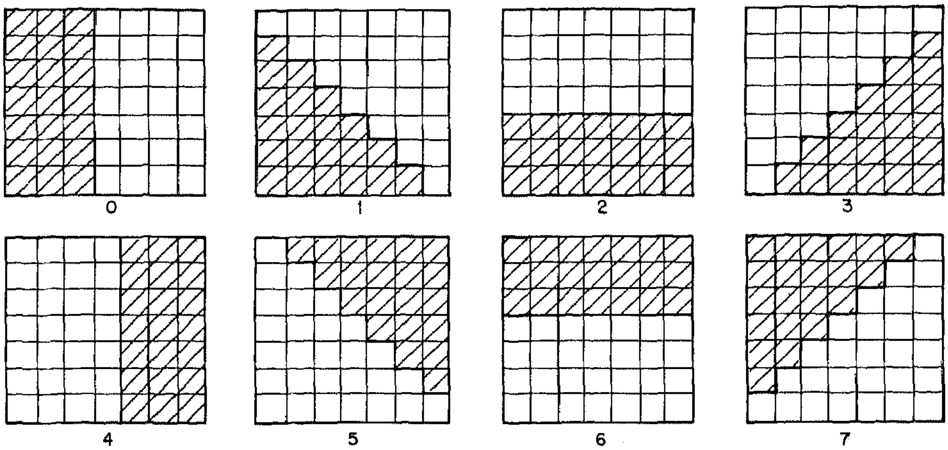

FIG. 6. Directional subset (unshaded area).

original 7 × 7 window then the subarea means are formed as shown below.

$$\left.\begin{matrix} 99 & 105 & 124 \\ 105 & 91 & 140 \\ 107 & 94 & 128 \end{matrix}\right\} 110 \qquad \left.\begin{matrix} 124 & 138 & 128 \\ 140 & 98 & 114 \\ 128 & 138 & 96 \end{matrix}\right\} 123$$

$$\left.\begin{matrix} 107 & 94 & 128 \\ 137 & 129 & 136 \\ 144 & 145 & 113 \end{matrix}\right\} 126 \qquad \left.\begin{matrix} 128 & 138 & 96 \\ 136 & 105 & 100 \\ 113 & 132 & 119 \end{matrix}\right\} 119$$

and so forth yielding the values

$$\begin{matrix} 110 & 123 & 75 \\ 126 & 119 & 76 \\ 119 & 113 & 70 \end{matrix}$$

After applying a 3 × 3 simple gradient mask, we find the maximum gradient mask is in direction 0. Comparing $|126 - 119|$ to $|76 - 119|$, we know the pixel is on the left side of the edge. Hence, subset 4 is chosen with the subset mean = 118 and the subset variance = 303, which represents a reduction by a factor 3. The new $\hat{x} = 117.8$, which is much closer to the average of the left side of edge.

It should be noted that this improved filtering algorithm should not be applied to every pixel of the image, because improvement in noise filtering is more significant in edge areas (or high local variance areas) than in flat areas. A local variance threshold is set up and only those pixels with local variance exceeding it are processed with this more sophisticated algorithm. Consequently, only a moderate increase in computation time is expected.

Figure 7a shows a test image, for which the difference in gray levels between the dark and bright area is 40, and uniformly distributed (between −30 and 30) white noise is added to the image. Images filtered by the original algorithm and the improved algorithm are shown in Figs. 7b and c, respectively. It is evident that the edge area is noisier in Fig. 7b than in Fig. 7c. This improvement is also shown in the intensity profiles along a scan line for all three images (see Figs. 7d, e and f).

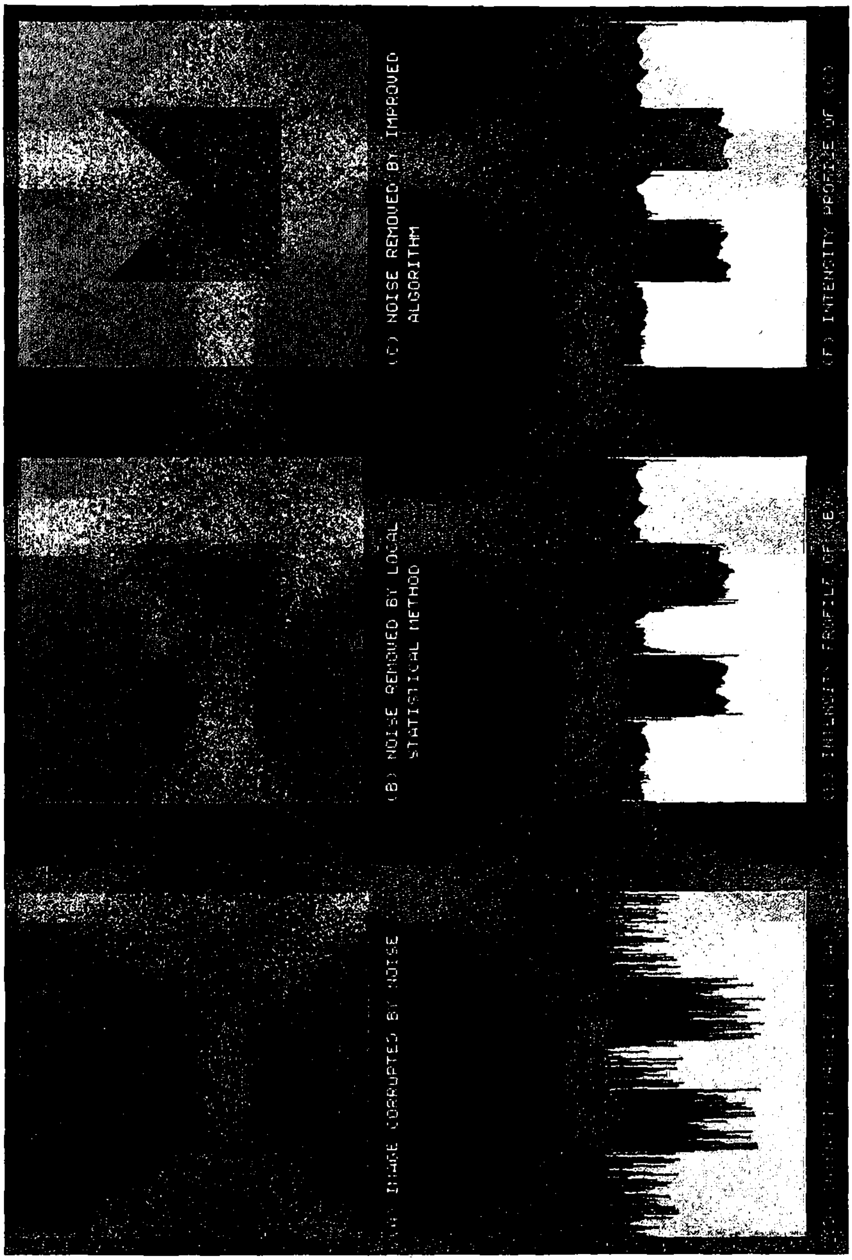

FIG. 7. Test images and their line intensity profiles.

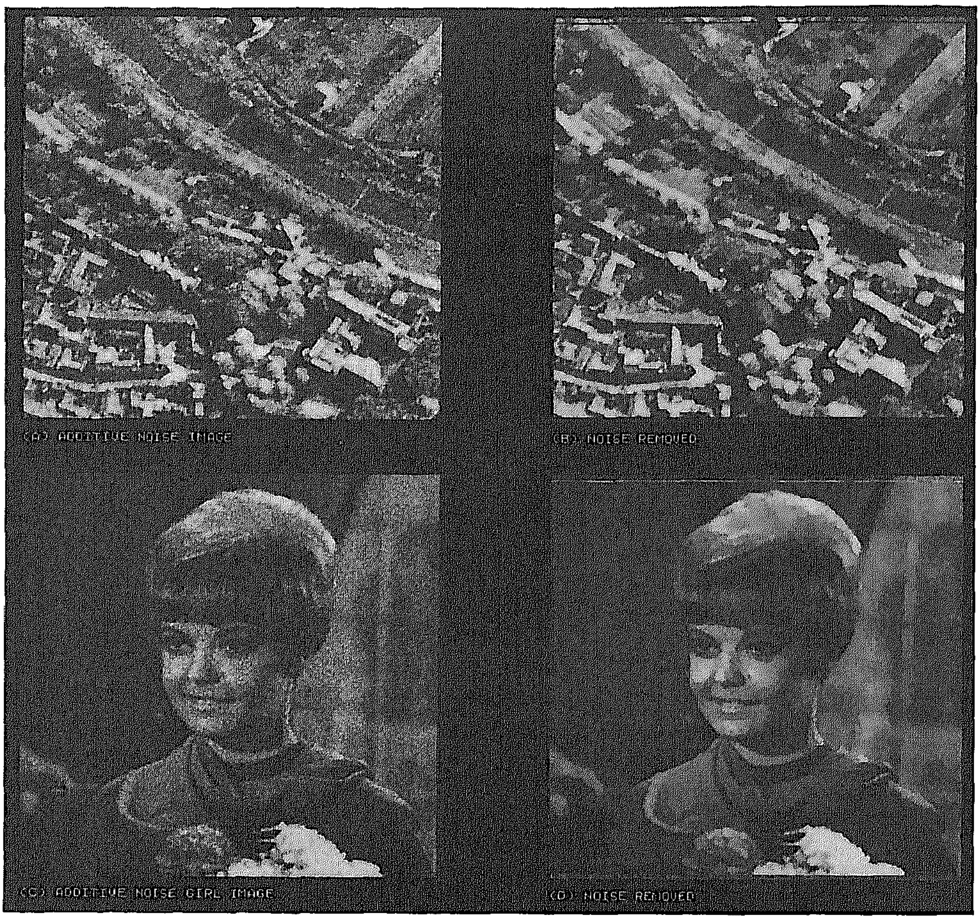

FIG. 8. Refined additive noise filtering algorithm.

Two other images contaminated by additive noise with $\sigma_I^2 = 300$ and their filtered versions are shown in Fig. 8. In both images, local variance thresholds are set at 500. Both images display sharp edges and also preserve subtle details. In particular, edges in the filtered image of the girl are enhanced to the extent that the desirable softness of the image is somewhat damaged.

3. ADAPTIVE FILTERING ALGORITHM FOR ADDITIVE NOISE

In most practical applications, the noise variance is unknown and spatially variant. It is well known that the noise variance of a local area can be estimated by the local variance of a flat area. Based on this idea, an adaptive algorithm is devised to estimate the local noise variance to be used in the algorithm developed in Section 2. Theoretically, after the local variances associated with each pixel in a small block are computed, the minimum of the variances in this block is a good estimate of error variance. However, because of the small sample size involved in computing the local variance, an average of the five smallest variances is a better estimate when the block size is 7×7. By incorporating this procedure into the

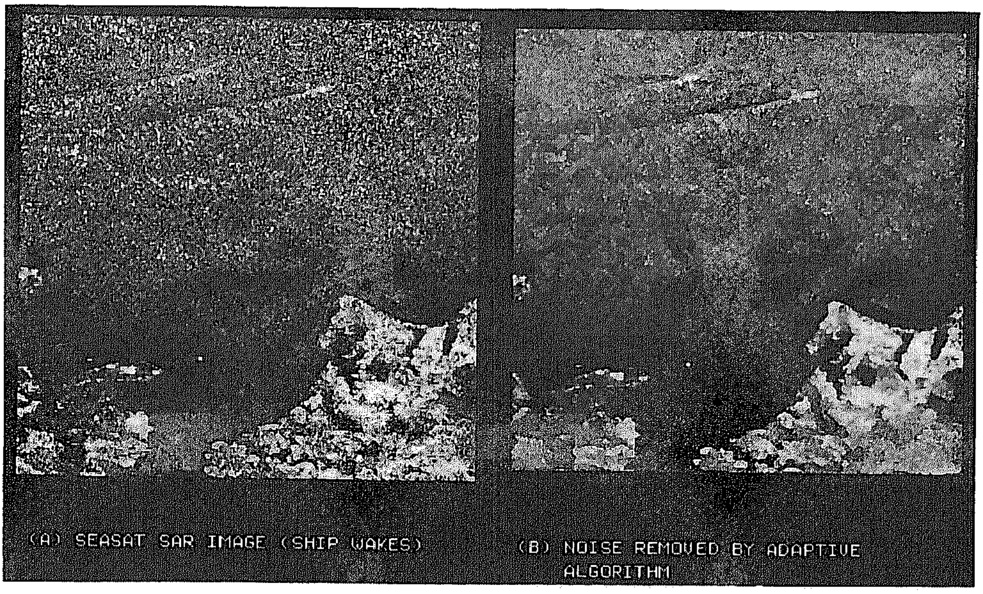

FIGURE 9

aforementioned algorithm, we obtain a very practical noise filtering algorithm without any necessity for a priori image modeling or evaluation of the noise statistics. This algorithm requires very small additional computation, since the major computational load is in the calculating local mean and variance for each picture element.

A practical example is given in Fig. 9. A Seasat Synthetic Aperture Radar image is shown in Fig. 9A and the noise filtered version using this adaptive algorithm is shown in Fig. 9B.

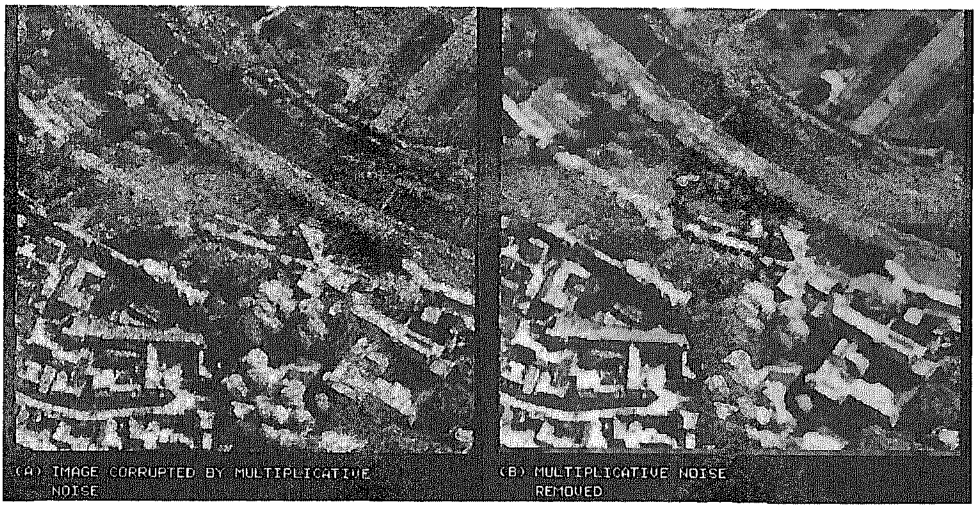

FIGURE 10

4. EXTENSION TO MULTIPLICATIVE NOISE

The refined noise filtering algorithm of Section 2 can be extended directly to filtering problems involving multiplicative noise. In Lee's local statistics algorithm [1] a linear approximation is made for multiplicative noise. The use of partial area to compute local mean and variance of Section 2 in the multiplicative noise filtering algorithm [1] produces an improved algorithm. An example is given in Fig. 10. Figure 10A shows an image corrupted by multiplicative noise which is uniformly distributed between 0.8 and 1.2. A special characteristic of this noise is that the brighter the area, the noisier it is. This characteristic is shown in Fig. 10A. Figure 10B shows the filtered image with the improved algorithm.

5. REMARKS AND DISCUSSION

1. It is mentioned in Section 2 that the subareas of a (7×7) array are overlapped for easier implementation. Intuitively, however, nonoverlapped subareas are more desirable in reducing the error in the location of an edge direction.
2. The 3×3 simple gradient mask is used in this paper. Other 3×3 gradient masks may perform better than the one used. We did experiment with Robinson's gradient mask [9] and found that it makes more errors in locating the direction of a noisy edge than the simple gradient mask.
3. The main computational load is in calculating the local mean and local variance. This paper does not attempt to optimize the efficiency of this algorithm, but rather to present the basic idea. More efficient algorithms, either by approximation [10] or by interaction, can be devised depending on individual computer configurations.
4. It has been mentioned in Section 2 that the girl image in Fig. 8d is harsher in appearance compared with Fig. 8c. Other methods aiming at filtering noise and preserving edge sharpness will also introduce distortions. While it is impossible to restore the image without distortion, we believe that the method presented in this paper represents a good compromise between mean square filtering algorithms and edge smoothing algorithms.

6. CONCLUSION

Improved noise filtering algorithms based on local statistics are presented here by incorporating local gradient information. The local mean and variance are computed from a reduced set of neighborhood pixels. The reduced set contains only those pixels which are found on one side of an edge. Examples show great improvement compared with the previous algorithms. Future research in the image processing area favors the use of local operators, because they are naturally suitable for parallel processing.

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