Buffon’s Needle

Buffon[NumberOfNeedles, NeedleSize, SurfaceDimension, GridWidth, Show → True/ Parallelize]
e.g: Buffon[35,2,20,3,Show→True]

Options:
Show→...
True: Everything
Parallelize: Pi Estimation using Parallelization (fastest)

Kernels: Optional for even better parallelization
Launch amount of Logical CPU Cores (usually twice the physical cores/check in task manager)

```
launch[LogicalCores_] := Quiet@Which[LogicalCores > 8, Print["Input Maximum of 8"],
   $KernelCount == 8, Print["Done"], $KernelCount ≠ 0, LaunchKernels[LogicalCores - $KernelCount];
   Print["Done"], N[$KernelCount == 0], LaunchKernels[LogicalCores];
   Print["Done"]]
```

In[ ]:=
```
launch[8]
```

Done

```
(****************************************************************************************************************************)
```

In[ ]:=
```
ClearAll@Buffon;
Options[Buffon] = {Show → True};
Buffon::usage = "Buffon[NumberOfNeedles,NeedleSize,SurfaceDimension,GridWidth,Options_Show_True_Parallelize]";
FE`CacheTemplateAndUsage["Buffon"];
Buffon[iterate_, linesize_, plotrange_, width_, OptionsPattern[]] :=

 (*********************************************************************************************************************)
 (*LOCAL VARIABLES*)

 Quiet@Module[
   {a, b, h, bound, randomised, rules, equation, Plotting, FirstCoList, contour, endpoint, SecondCo, intersections, x, y,
    GridPlot, SecondPlot, SecondPlotCircle, SecondEndPlot, FirstPlot, NeedlePlot, calculate,
    ColourList = RandomColor[iterate]} (*List of Random Colours*),

   Catch[Check[
     (****************************************************************************************************************)
     (*PI ESTIMATE FOR PI ESTIMATION ONLY*)
     calculate := ParallelTable[

       ClearAll[a, b, h, equation, randomised] (*Clear variables every new iteration*);

       a = RandomReal[{0, N[plotrange]}] (*random x coordinate*);
       b = RandomReal[{0, N[plotrange]}] (*random y coordinate*);

       h = Part[First[Part[Quiet[NSolve[
             linesize^2 == (x - a) ^2 + (y - b) ^2, RandomChoice[{x, y}],
             Reals]], RandomInteger[{1, 2}]]], 2];

       randomised = RandomReal[Flatten[
          RegionBounds@ (*Transform region to bounds*)
            ImplicitRegion[#, x] & /@ (*Transform inequality to region *)
           ({Quiet@Reduce[ReplaceAll[Part[h, 2], y → x], x, Reals]}) (*Extract Domain of Semicircle Equation*)
         ]];

       (*Intersections*)
       MemberQ[ (*Give True if "True" is present in list*)
        IntervalMemberQ[ (*generate list for True where gridline⊂domain of line and false otherwise *)

          (*Domain of line*)
          Interval[
           If[Variables[h[[1]]] === {y}, (*if semicircle is in terms of y*)
```

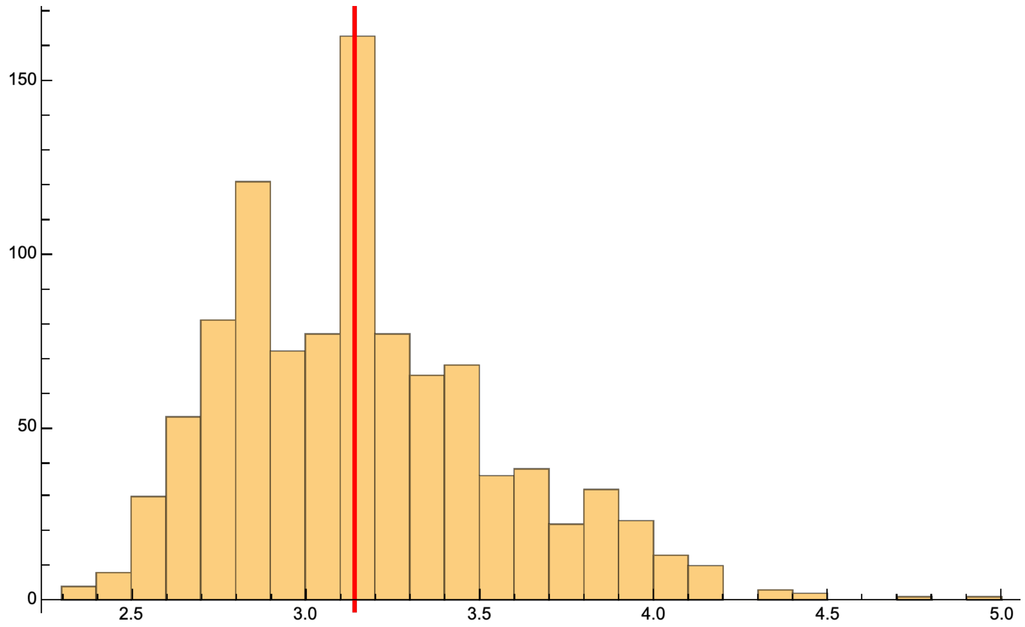
```
          Sort[{a, ReplaceAll[h⟦1⟧, y → randomised]}],
          Sort[{a, randomised}]]]],

       (*Interval of Gridlines*)
       Interval /@ Table[{a, a}, {a, 0, plotrange + width, width}]],
     True],

    iterate];
  (***************************************************************************************************************)
  (*PARALLELIZE*)

  If[OptionValue[Show] === Parallelize, Throw@
    N[linesize * ((2 * iterate) / (Count[Quiet[calculate], True] * width))]];
  (***************************************************************************************************************)
  (*DATA*)

  If[OptionValue[Show] === Data,
   Throw@N[calculate]];
  (***************************************************************************************************************)
  (*CALCULATIONS FOR PLOTTING*)
  If[OptionValue[Show] === True,
   rules = Dispatch[Part[(*Extract rules of Reap*)
       Reap[
        Quiet@Do[(*Reiterate for different random values a,b*)

          ClearAll[a, b, h, equation, randomised, bound](*Clear variables every new iteration*);

          a = RandomReal[{0, N[plotrange]}](*random x coordinate*);
          b = RandomReal[{0, N[plotrange]}](*random y coordinate*);

          (*Random Coordinates*)
          Sow[{a, b}, FirstCoList](*Append coordinate to FirstCoList*);

          (*Semicircle conditional expression*)
          equation = NSolve[
             Sow[linesize^2 == (x - a)^2 + (y - b)^2, Plotting], RandomChoice[{x, y}], (*solve for either x or y*)
             Reals]⟦RandomInteger[{1, 2}]⟧; (*choose top or bottom/left or right of semicircle*)

          (*Semicircle equation "="*)
          Sow[
           ReplaceAll[First[Normal[equation]], Rule → Equal], (*extract equation and transform → to = *)
           contour];

          (*Semicircle conditional expression Values only*)
          h = Part[First[equation], 2];

          (*(*Semicircle Equations "→"*)
          Sow[h⟦1⟧,EquationList](*Extract semicircle equation and append to "EquationList"*);*)

          (*Domain {x_min,x_max}*)
          bound =
           Flatten[
            RegionBounds@(*Transform region to bounds*)
               ImplicitRegion[#, x] & /@ (*Transform inequality to region *)
              ({Quiet@Reduce[ReplaceAll[Part[h, 2], y → x], x, Reals]})(*Extract Domain of Semicircle Equation*)
           ];

          (*Random x or y coordinate of second point to connect with midpoint*)
          randomised = RandomReal[bound];
          (*Generate random number with a domain of semicircle equation*)

          (*{x,y} Coordinate of second point connecting line*)
          Sow[
           If[Variables[h⟦1⟧] === {y}, (*if equation is in terms of Y*)
            {ReplaceAll[h⟦1⟧, y → randomised], randomised}, (*substitute random y to find x*)
            {randomised, ReplaceAll[h⟦1⟧, x → randomised]}], (*substitute random x to find y*)
           SecondCo];

          (*Endpoints of semicircle*)
          Sow[
           If[Variables[h⟦1⟧] === {y}, (*if equation is in terms of Y*)
```

```
            Table[{a, y}, {y, bound}], (* {midpoint_x , range_y}*)
            Table[{x, b}, {x, bound}] (* {Domain_x , midpoint_y}*)
           ], endpoint];

          (*Intersections*)
          Sow[MemberQ[(*Give True if "True" is present in list*)
            IntervalMemberQ[(*generate list for True where gridline∈domain of line and false otherwise *)

             (*Domain of line*)
             Interval[
              If[Variables[h[[1]]] === {y}, (*if semicircle is in terms of y*)
               Sort[{a, ReplaceAll[h[[1]], y → randomised]}],
               Sort[{a, randomised}]]],

             (*Interval of Gridlines*)
             Interval /@ Table[{a, a}, {a, 0, plotrange + width, width}]],
            True],
           intersections];,

          iterate] (*Reiterate "iterate" times*)
        , _, Rule] (*Reap as rules*)
       , 2]];

(****************************************************************************************************)
(*DEFINING PLOTS*)


(*Grid Lines*)
GridPlot = ContourPlot[(##1 &)[] (*Empty Plot*)
  , {x, 0 - linesize, plotrange + linesize}, {y, 0 - linesize, plotrange + linesize}, GridLines →
   {Range[0, plotrange + width, width], None}, GridLinesStyle → Directive[Black, Dotted], AspectRatio → Automatic];

(*Define Plot Circle of possible second points*)
SecondPlotCircle = ContourPlot[Evaluate@ReplaceAll[Plotting, rules], {x, -linesize, plotrange + linesize},
  {y, -linesize, plotrange + linesize}, AspectRatio → Automatic, ContourStyle → ColourList];

(*Define Plot Semicircle of possible second points*)
SecondPlot = ContourPlot[Evaluate@ReplaceAll[contour, rules], {x, -linesize, plotrange + linesize},
  {y, -linesize, plotrange + linesize}, AspectRatio → Automatic, ContourStyle → ColourList];

(*Define Plot of predefined Random Coordinates*)
FirstPlot = ListPlot[
  Tooltip[{#} & /@ ReplaceAll[FirstCoList, rules] (*wrap {} around each coordinate to allow for different colours*)
  ], PlotStyle → ColourList, AspectRatio → Automatic, Axes → False, Frame → True];

(*Define Plot of Semicircle Endpoints*)
SecondEndPlot = ListPlot[Tooltip[{#} & /@ (*wrap {} around each coordinate to allow for different colours*)
    Flatten[ReplaceAll[endpoint, rules], 1]],
  (*Create {x,y} coordinates by tranposing a list of X and Y coordinates*)
  PlotStyle → Map[Sequence @@ ConstantArray[#, 2] &, ColourList],
  (*Double every colour in list as semicircles have 2 endpoints*)
  Axes → False, Frame → True, AspectRatio → Automatic];

(*Define Plot of Needle*)
NeedlePlot = ListLinePlot[Tooltip@Transpose[{FirstCoList /. rules, SecondCo /. rules}],
  (*Create {x,y} by transposing a list of x and y coordinates*)
  Axes → False, Frame → True, AspectRatio → Automatic, PlotStyle → ColourList];

(****************************************************************************************************)
(*DISPLAY*)
(****************************************************************************************************)
(*Text*)

Manipulate[
 TableForm[{
   Style[Column[{

      Row[{"N: Number of Needles ", iterate}],
      Row[{"L: Needle Length: ", linesize}],
      Row[{"W : Grid width: ", width}],
      Row[{"I: Intersections: ", Count[ReplaceAll[intersections, rules], True]}],
```

```
      Row[{"Estimating Pi: 2 L N / I W → ", Row[{2, " x ", linesize , " x ", iterate}] / Row[
           {Count[ReplaceAll[intersections, rules], True], " x ", width}], " → ", Style[N[
           linesize * ((2 * iterate) / (Count[ReplaceAll[intersections, rules], True] * width))], FontSize → 13, Bold]}]

      }], FontSize → 14],
    (****************************************************************************************************)
    (*SHOW PLOTS*)

    Show[
     {(*GridLines*)
      If[lines == 1, GridPlot, ## &[]],

      (*Blank*)
      ContourPlot[(#1 &) [],
       {x, 0 - linesize, plotrange + linesize}, {y, 0 - linesize, plotrange + linesize}, AspectRatio → Automatic],

      (*Circle*)
      If[circle == 1, SecondPlotCircle, ## &[]],

      (*Semicircle*)
      If[Semicircle == 1, SecondPlot, ## &[]],

      (*Random Coordinates*)
      If[RandomCo == 1, FirstPlot, ## &[]],

      (*Semicircle Endpoints*)
      If[Endpoint == 1, SecondEndPlot, ## &[]],

      (*Needle Line*)
      If[Needle == 1, NeedlePlot, ## &[]]},

     ImageSize → {400, 400}
    ]
   }],
   (****************************************************************************************************)
   (*CHECKBOXES*)

   (*GridLines*)
   {{lines, 1, Style["GridLines", FontSize → 14]}, {1, 0}},

   (*Random Coordinates*)
   {{RandomCo, 0, Style["Random First Points", FontSize → 14]}, {1, 0}},

   (*Circle*)
   {{circle, 0, Style["Possible Second Points Circle", FontSize → 14]}, {1, 0}},

   (*Semicircle*)
   {{Semicircle, 0, Style["Possible Second Points SemiCircle", FontSize → 14]}, {1, 0}},

   (*Endpoint*)
   {{Endpoint, 0, Style["Semicircle Endpoints", FontSize → 14]}, {1, 0}},

   (*Needle*)
   {{Needle, 0, Style["Needle", FontSize → 14]}, {1, 0}},
   ControlPlacement → {Top}(*CheckBox Position*)

  ]


 ]
 (****************************************************************************************************)
 , Throw[0]]](*give 0 if no intersections*)


]
```

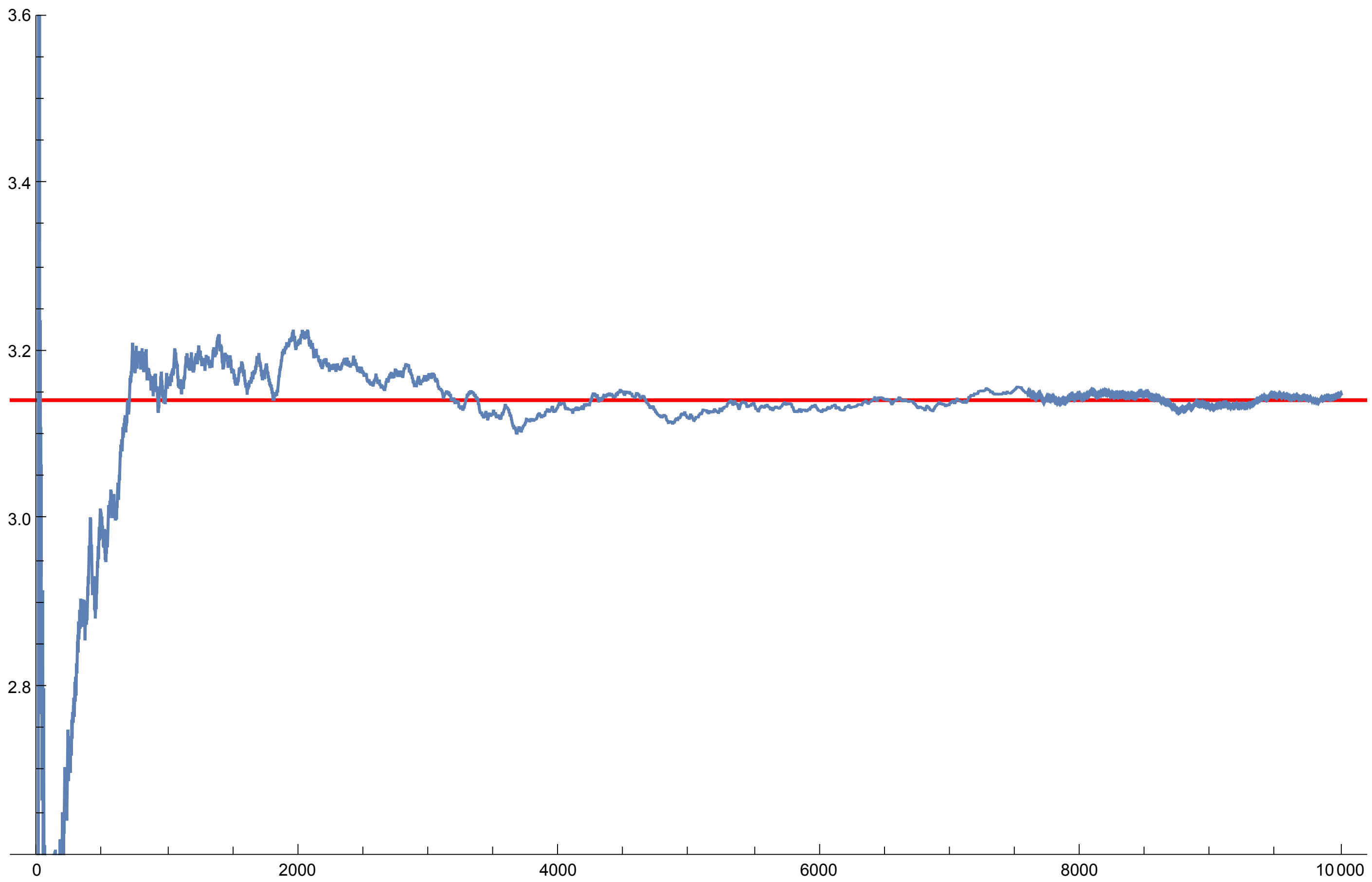
```
In[ ]:= Histogram[ParallelTable[Buffon[100, 2, 100, 3, Show → Parallelize], 1000], {0.1},
         GridLines → {{π}, None}, Method → {"GridLinesInFront" → True}, GridLinesStyle → Directive[Red, Thick]]
```

Out[ ]=




```
(**************************************************************************************************)

BuffonData[iterate_, linesize_, plotrange_, width_] := Module[{a = 0},
  ListLinePlot[Transpose[{Range[iterate], N[MapThread[Quiet@Catch[Check[(2 linesize * #2) / (#1 width), Throw[0]]] &,
       {Accumulate@Boole@Buffon[iterate, linesize, plotrange, width, Show → Data], Range[iterate]}]]}],
   GridLines → {None, {π}}, GridLinesStyle → Directive[Red, Thick], PlotRange → {2, 4}]]

BuffonData[10 000, 2, 100, 3]
```

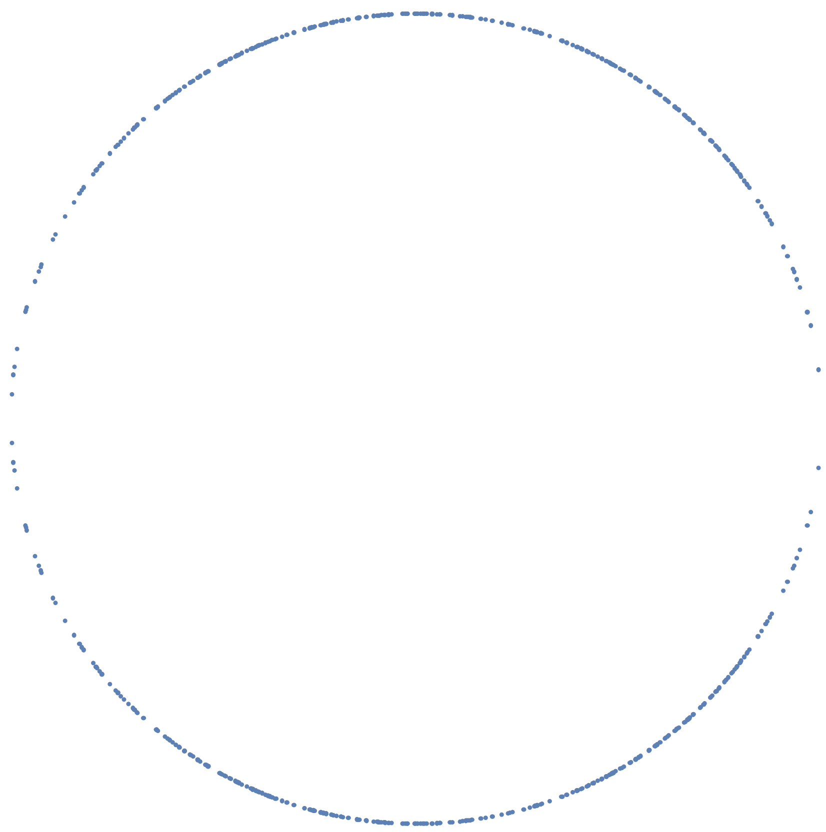
Out[ ]=




```
(**************************************************************************************************)
```

In[ ]:= ListPlot[Flatten[Table[{x, #} & /@ SolveValues[$x^2 + y^2 == 1$, y] /. x → RandomReal[{-1, 1}], 300], 1],
Axes → False, AspectRatio → Automatic]

Out[ ]=



(***)

In[ ]:= Assuming[{L > 0, w > 0, 0 < θ < Pi / 2},
Integrate[Integrate[Piecewise[{{$\frac{4}{w\,\pi}$, 0 ≤ x ≤ w / 2}}], {x, 0, (L * Sin[θ]) / 2}] , {θ, 0, Pi / 2}] // FullSimplify]

Out[ ]=

$$\begin{cases} \frac{2L}{\pi w} & L \le w \\ \frac{2\left(L-\sqrt{(L-w)\ (L+w)}+w\,\mathrm{ArcCos}\left[\frac{w}{L}\right]\right)}{\pi w} & \mathrm{True} \end{cases}$$

(***)

FullSimplify[ReplaceAll[$\begin{cases} \frac{2l}{\pi w} & l \le w \\ \frac{2\left(l-\sqrt{(l-w)\ (l+w)}+w\,\mathrm{ArcCos}\left[\frac{w}{l}\right]\right)}{\pi w} & l > w \end{cases}$, {w → l / z}], z > 1]

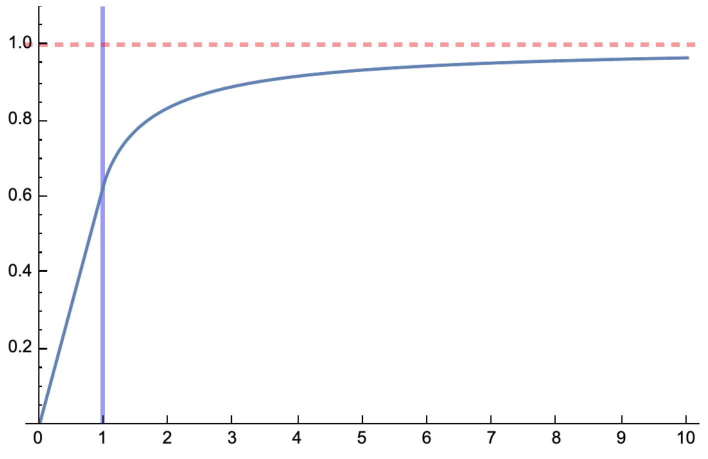
Out[ ]=

$$\begin{cases} \frac{2z}{\pi} & z \le 1 \\ \frac{2\left(z-\sqrt{-1+z^2}+\mathrm{ArcSec}[z]\right)}{\pi} & z > 1 \end{cases}$$

(***)

In[ ]:= Plot[$\begin{cases} \frac{2z}{\pi} & z \le 1 \\ \frac{2\left(z-\sqrt{-1+z^2}+\mathrm{ArcSec}[z]\right)}{\pi} & z > 1 \end{cases}$, {z, 0, 10}, GridLines → {{{1, Directive[Blue, Thick]}}, {{1, Directive[Dashed, Red, Thick]}}},
PlotRange → {0, 1.1}, Ticks → {Range[0, 14], Automatic}]

Out[ ]=



(***)

Buffon[NumberOfNeedles , NeedleSize , SurfaceDimension , GridWidth , Options_Show_True_Parallelize]

In[ ]:= Buffon[30, 2, 25, 3, Show → True]

In[ ]:= Buffon[10 000, 2, 25, 3, Show → Parallelize]