# **BONMIN** Users' Manual

Pierre Bonami and Jon Lee

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# 1 Introduction

BONMIN (Basic Open-source Nonlinear Mixed INteger programming) is an open-source code for solving general MINLP (Mixed Integer NonLinear Programming) problems. It is distributed on COIN-OR(www.coin-or.org) under the CPL (Common Public License). The CPL is a license approved by the OSI<sup>1</sup>, (Open Source Initiative), thus BONMIN is OSI Certified Open Source Software.

There are several algorithmic choices that can be selected with BONMIN. B-BB is a NLP-based branch-and-bound algorithm, B-OA is an outer-approximation decomposition algorithm, B-iFP is an iterated feasibility pump algorithm, B-QG is an implementation of Quesada and Grossmann's branch-and-cut algorithm, B-Hyb is a hybrid outer-approximation based branch-and-cut algorithm and B-Ecp is a variant of B-QG based on adding additional ECP cuts.

Some of the algorithmic choices require the ability to solve MILP (Mixed Integer Linear Programming) problems and NLP (NonLinear Programming) problems. The default solvers for these are, respectively, the COIN-OR codes Cbc and Ipopt. In turn, Cbc uses further COIN-OR modules: Clp (for LP (Linear Programming) problems), Cgl (for generating MILP cutting planes), as well as various other utilities. It is also possible to step outside the open-source realm and use Cplex as the MILP solver and FilterSQP as the NLP solver.

Additional documentation can be found on the Bonmin homepage at

http://www.coin-or.org/Bonmin

and wiki at

https://projects.coin-or.org/Bonmin

## Types of problems solved

BONMIN solves MINLPs of the form

<sup>&</sup>lt;sup>1</sup>http://www.opensource.org

```
\begin{aligned} & \min f(x) \\ & \text{s.t.} \\ & g^L \leq g(x) \leq g^U, \\ & x^L \leq x \leq x^U, \\ & x \in \mathbb{R}^n, \ x_i \in \mathbb{Z} \ \forall i \in I, \end{aligned}
```

where the functions  $f: \{x \in \mathbb{R}^n : x^L \leq x \leq x^U\} \to \mathbb{R}$  and  $g: \{x \in \mathbb{R}^n : x^L \leq x \leq x^U\} \to \mathbb{R}^m$  are assumed to be twice continuously differentiable, and  $I \subseteq \{1,\ldots,n\}$ . We emphasize that BONMIN treats problems that are cast in *minimization* form.

The different methods that BONMIN implements are exact algorithms when the functions f and g are convex but are only heuristics when this is not the case (i.e., BONMIN is not a global optimizer).

#### Algorithms

BONMIN implements six different algorithms for solving MINLPs:

- B-BB: a simple branch-and-bound algorithm based on solving a continuous nonlinear program at each node of the search tree and branching on variables [7]; we also allow the possibility of SOS (Type 1) branching
- B-OA: an outer-approximation based decomposition algorithm [6, 8]
- B-QG: an outer-approximation based branch-and-cut algorithm [11]
- B-Hyb: a hybrid outer-approximation/nonlinear programming based branch-and-cut algorithm [2]
- B-Ecp: another outer-approximation based branch-and-cut inspired by the settings described in [1]
- B-iFP: an iterated feasibility pump algorithm [3].

In this manual, we will not go into a further description of these algorithms. Mathematical details of these algorithms and some details of their implementations can be found in [2] and [5].

Whether or not you are interested in the details of the algorithms, you certainly want to know which one of these six algorithms you should choose to solve your particular problem. For convex MINLPs, experiments we have made on a reasonably large test set of problems point in favor of using B-Hyb (it solved the most of the problems in our test set in 3 hours of computing time). Nevertheless, there are cases where B-OA is much faster than B-Hyb and others where B-BB is interesting. B-QG and B-ECP correspond mainly to a specific parameter setting

of B-Hyb but they can be faster in some case. B-iFP is more tailored at finding quickly good solutions to very hard convex MINLP. For nonconvex MINLPs, we strongly recommend using B-BB (the outer-approximation algorithms have not been tailored to treat nonconvex problems at this point). Although even B-BB is only a heuristic for such problems, we have added several options to try and improve the quality of the solutions it provides (see Section 5.3). Because it is appliable to more classes problem B-BB is the default algorithm in BONMIN.

# Required third party code

In order to run BONMIN, you have to download other external libraries (and pay attention to their licenses!):

- Lapack (Linear Algebra PACKage)
- Blas (Basic Linear Algebra Subroutines)
- a sparse linear solver that is supported by Ipopt, e.g., MA27 from the HSL (Harwell Subroutine Library), MUMPS, or Pardiso.

Note that Lapack and the Blas are free for commercial use from the Netlib Repository<sup>2</sup>, but they are not OSI Certified Open Source Software. The linear solver MA27 is freely available for noncommercial use.

The above software is sufficient to run BONMIN as a stand-alone C++ code, but it does not provide a modeling language. For functionality from a modeling language, BONMIN can be invoked from Ampl<sup>3</sup> (no extra installation is required provided that you have a licensed copy of Ampl installed), though you need the ASL (Ampl Solver Library) which is obtainable from the Netlib.

BONMIN can use FilterSQP [?] as an alternative to Ipopt for solving NLPs. Also, in the outer approximation methods B-OA and B-iFP, some MILP problems are solved. By default BONMIN uses Cbc to solve them, but it can also be set up to use the commercial solver Cplex<sup>4</sup>.

# Tested platforms

BONMIN has been installed on the following systems:

- Linux using g++ version 3.\* and 4.\* until 4.3 and Intel 9.\* and 10.\*
- Windows using version Cygwin 1.5.18
- Mac OS X using gcc 3.\* and 4.\* until 4.3 and Intel 9.\* and 10.\*
- SunOS 5 using gcc 4.3

<sup>&</sup>lt;sup>2</sup>http://www.netlib.org

<sup>&</sup>lt;sup>3</sup>http://www.ampl.com

 $<sup>^4</sup> http://www.ilog.com/products/cplex/product/mip.cfm\\$ 

# 2 Obtaining BONMIN

The BONMIN package consists of the source code for the BONMIN project but also source code from other COIN-OR projects:

- BuildTools
- Cbc
- Cgl
- Clp
- CoinUtils
- Ipopt
- Osi

When downloading the BONMIN package you will download the source code for all these and libraries of problems to test the codes.

Before downloading BONMIN you need to know which branch of Bonmin you want to download. In particular you need to know if you want to download the latest version from:

- the Stable branch, or from
- the Released branch.

These different version are made according to the guidelines of COIN-OR. The interpretation of these guidelines for the Bonmin project is explained on the wiki pages of Bonmin.

The main distinction between the Stable and Release branch is that a stable version that we propose to download may evolve over time to include bug fixes while a released version will never change. The released versions present an advantage in particular if you want to make experiments which you want to be able to reproduce the stable version presents the advantage that it is less work for you to update in the event where we fix a bug.

The easiest way to obtain the released version is by downloading a compressed archive from Bonmin archive directory. The latest release is Bonmin-1.3.2.

The only way to obain one of the stable versions is through subversion.

In Unix<sup>5</sup>-like environments, to download the latest stable version of Bonmin (1.3) in a sub-directory, say Bonmin-1.3 issue the following command

svn co https://projects.coin-or.org/svn/Bonmin/stable/1.3 Bonmin-1.3

<sup>&</sup>lt;sup>5</sup>UNIX is a registered trademark of The Open Group.

This copies all the necessary COIN-OR files to compile BONMIN to Bonmin-1.3. To download BONMIN using svn on Windows, follow the instructions provided at COIN-OR.

# 2.1 Obtaining required third party code

BONMIN needs a few external packages which are not included in the BONMIN package.

- Lapack (Linear Algebra PACKage)
- Blas (Basic Linear Algebra Subroutines)
- A sparse linear solver.
- Optionally ASL (the Ampl Solver Library), to be able to use BONMIN from Ampl.

Since these third-party software modules are released under licenses that are incompatible with the CPL, they cannot be included for distribution with BONMIN from COIN-OR, but you will find scripts to help you download them in the subdirectory ThirdParty of the BONMIN distribution. In most Linux distributions and CYGWIN, Lapack and Blas are available as prebuilt binary packages in the distribution (and are probably already installed on your machine).

Linear solvers are used by Ipopt. The most up-to-date information regarding the supported linear solvers and how to install them is found in Section 2.2 of the Ipopt manual.

Several options are available for linear solvers: MA27 from the Harwell Subroutine Library (and optionally, but strongly recommended, MC19 to enable automatic scaling in Ipopt), MA57 or Mumps. In our experiment MA27 and MA57 usually perform significantly better but they are freely available only for non-commercial, academic use. Note that linear solvers can also take advantage of Metis.

# 3 Installing BONMIN

The build process for BONMIN should be fairly automatic as it uses GNU autotools. It has been successfully compiled and run on the following platforms:

- Linux using g++ version 3.4 and 4.0
- Windows using version Cygwin 1.5.18
- Mac OS X using gcc 3.4 and 4.0

For Cygwin and OS X some specific setup has to be done prior to instalation. These step are described on the wiki pages of Bonmin CygwinInstall<sup>6</sup> and OsxInstall<sup>7</sup>.

BONMIN is compiled and installed using the commands:

```
./configure -C make make install
```

This installs the executable bonmin in Bonmin-1.3/bin. In what follows, we assume that you have put the executable bonmin on your path.

The configure script attempts to find all of the machine specific settings (compiler, libraries,...) necessary to compile and run the code. Although configure should find most of the standard ones, you may have to manually specify a few of the settings. The options for the configure script can be found by issuing the command

```
./configure --help
```

For a more in depth description of these options, the reader is invited to refer to the COIN-OR BuildTools trac page<sup>8</sup>.

## 3.1 Specifying the location of Cplex libraries

If you have Cplex installed on your machine, you may want to use it as the Mixed Integer Linear Programming subsolver in B-OA, B-Hyb and B-iFP. To do so you have to specify the location of the header files and libraries. You can either specify the location of the header files directory by passing it as an argument to the configure script or by writing it into a config.site.

In the former case, specify the location of the Cplex header files by using the argument --with-cplexincdir and the location of the Cplex library with --with-cplexlib (note that on the Linux platform you will also need to add -lpthread as an argument to --with-cplexlib).

For example, on a Linux machine if Cplex is installed in /usr/ilog , you would invoke configure with the arguments as follows:

```
./configure --with-cplex-incdir=/usr/ilog/cplex/include/ilcplex \
```

 $<sup>^6 {\</sup>rm https://projects.coin\text{-}or.org/Bonmin/wiki/CygwinInstall}$ 

<sup>&</sup>lt;sup>7</sup>https://projects.coin-or.org/Bonmin/wiki/OsxInstall

<sup>&</sup>lt;sup>8</sup>https://projects.coin-or.org/BuildTools

```
--with-cplex-lib="/usr/ilog/cplex/lib/libcplex.a -lpthread"
```

In the latter case, put a file called <code>config.site</code> in a subdirectory named <code>share</code> of the installation directory (if you do not specify an alternate installation directory to the <code>configure</code> script with the <code>--prefix</code> argument, the installation directory is the directory where you execute the <code>configure</code> script). To specify the location of <code>Cplex</code>, insert the following lines in the <code>config.site</code> file:

```
with_cplex_lib="/usr/ilog/cplex/lib/libcplex.a -lpthread"
with_cplex_incdir="/usr/ilog/cplex/include/ilcplex"
```

(You will find a config.site example in the subdirectory BuildTools of coin-Bonmin.)

#### 3.2 Compiling BONMIN in a external directory

It is recommended to compile BONMIN in a directory different from the source directory (Bonmin-1.3 in our case). This is convenient if you want to have several executables compiled for different architectures or have several executables compiled with different options (debugging and production, shared and static libraries) but also because you don't modify the directory where the sources are.

To do this just create a new directory, for example Bonmin-build in the parent directory of coin-Bonmin and run the configure command from Bonmin-build:

```
../Bonmin-\stableVersion/configure -C
```

This will create the makefiles in Bonmin-build, and you can then compile with the usual make and make install (in Bonmin-build).

## 3.3 Building the documentation

The documentation for BONMIN consists of a users' manual (this document) and a reference manual. You can build a local copy of the reference manual provided that you have Latex and Doxygen installed on your machine. Issue the command make doxydoc in coin-Bonmin. It calls Doxygen to build a copy of the reference manual. An html version of the reference manual can then be accessed in doc/html/index.html.

# 3.4 Running the test programs

By issuing the command  ${\tt make\ test}$  , you build and run the automatic test program for  ${\tt BONMIN}.$ 

# 4 Running BONMIN

BONMIN can be run

- (i) from a command line on a .nl file (see [10]),
- (ii) from the modeling language Ampl<sup>9</sup> (see [12]),
- (iii) from the Gams<sup>10</sup> modeling language,
- (iv) by invoking it from a C/C++ program.
- (v) remotely through the NEOS<sup>11</sup> web interface.

In the subsections that follow, we give some details about the various ways to run BONMIN.

#### 4.1 On a .nl file

BONMIN can read a .nl file which could be generated by Ampl (for example mytoy.nl in the Bonmin-dist/Bonmin/test subdirectory). The command line takes just one argument which is the name of the .nl file to be processed.

For example, if you want to solve mytoy.nl, from the Bonmin-dist directory, issue the command:

bonmin test/mytoy.nl

#### 4.2 From Ampl

To use BONMIN from Ampl you just need to have the directory where the bonmin executable is in your \$PATH and to issue the command

option solver bonmin;

in the Ampl environment. Then the next solve will use BONMIN to solve the model loaded in Ampl. After the optimization is finished, the values of the variables in the best-known or optimal solution can be accessed in Ampl. If the optimization is interrupted with <CTRL-C> the best known solution is accessible (this feature is not available in Cygwin).

<sup>&</sup>lt;sup>9</sup>http://www.ampl.com

<sup>&</sup>lt;sup>10</sup>http://www.gams.com/

<sup>&</sup>lt;sup>11</sup>http://neos.mcs.anl.gov/neos

#### 4.2.1 Example Ampl model

simple Ampl example model follows:

```
# An Ampl version of toy
reset;
var x binary;
var z integer >= 0 <= 5;
var y{1...2} >=0;
minimize cost:
    -x - y[1] - y[2];
subject to
    c1: (y[1] - 1/2)^2 + (y[2] - 1/2)^2 \le 1/4;
    c2: x - y[1] \le 0;
    c3: x + y[2] + z \le 2;
option solver bonmin; # Choose BONMIN as the solver (assuming that
                      # bonmin is in your PATH
solve;
                      # Solve the model
display x;
display y;
```

(This example can be found in the subdirectory Bonmin/examples/amplExamples/ of the BONMIN package.)

# 4.2.2 Setting up branching priorities, directions and declaring SOS1 constraints in ampl

Branching priorities, branching directions and pseudo-costs can be passed using Ampl suffixes. The suffix for branching priorities is "priority" (variables with a higher priority will be chosen first for branching), for branching direction is "direction" (if direction is 1 the  $\geq$  branch is explored first, if direction is -1 the  $\leq$  branch is explored first), for up and down pseudo costs "upPseudoCost" and "downPseudoCost" respectively (note that if only one of the up and down pseudo-costs is set in the Ampl model it will be used for both up and down).

For example, to give branching priorities of 10 to variables y and 1 to variable x and to set the branching directions to explore the upper branch first for all variables in the simple example given, we add before the call to solve:

```
suffix priority IN, integer, >=0, <= 9999;
y[1].priority := 10;
y[2].priority := 10;
x.priority := 1;

suffix direction IN, integer, >=-1, <=1;
y[1].direction := 1;
y[2].direction := 1;
x.direction := 1;</pre>
```

SOS Type-1 branching is also available in BONMIN from Ampl. We follow the conventional way of doing this with suffixes. Two type of suffixes should be declared:

Next, suppose that we wish to have variables

```
var X {i in 1..M, j in 1..N} binary;
and the "convexity" constraints:
subject to Convexity {i in 1..M}:
    sum {j in 1..N} X[i,j] = 1;
```

(note that we must explicitly include the convexity constraints in the  ${\tt Ampl}$  model).

Then after reading in the data, we set the suffix values:

```
# The numbers 'val[i,j]' are chosen typically as
# the values 'represented' by the discrete choices.
let {i in 1..M, j in 1..N} X[i,j].ref := val[i,j];
# These identify which SOS constraint each variable belongs to.
let {i in 1..M, j in 1..N} X[i,j].sosno := i;
```

## 4.3 From Gams

Thanks to the GAMSlinks<sup>12</sup> project, Bonmin is available in Gams since release 22.5 of the GAMS<sup>13</sup> modeling system. The system is available for download from

 $<sup>^{12} \</sup>rm http://projects.coin-or.org/GAMS links$ 

 $<sup>^{13} \</sup>mathrm{http://www.gams.com/}$ 

GAMS<sup>14</sup>. Without buying a license it works as a demo with limited capabilities. Documentation for using BONMIN in GAMS is available at

http://www.gams.com/solvers/coin.pdf

# 4.4 From a C/C++ program

BONMIN can also be run from within a C/C++ program if the user codes the functions to compute first- and second-order derivatives. An example of such a program is available in the subdirectory CppExample of the examples directory. For further explanations, please refer to this example and to the reference manual.

# 5 Options

## 5.1 Passing options to BONMIN

Options in BONMIN can be set in several different ways.

First, you can set options by putting them in a file called bonmin.opt in the directory where bonmin is executing. If you are familiar with the file ipopt.opt (formerly named PARAMS.DAT) in Ipopt, the syntax of the bonmin.opt is similar. For those not familiar with ipopt.opt, the syntax is simply to put the name of the option followed by its value, with no more than two options on a single line. Anything on a line after a # symbol is ignored (i.e., treated as a comment).

Note that BONMIN sets options for Ipopt. If you want to set options for Ipopt (when used inside BONMIN) you have to set them in the file bonmin.opt (the standard Ipopt option file ipopt.opt is not read by BONMIN.) For a list and a description of all the Ipopt options, the reader may refer to the documentation of Ipopt<sup>15</sup>.

Since bonmin.opt contains both Ipopt and BONMIN options, for clarity all BONMIN options should be preceded with the prefix "bonmin." in bonmin.opt. Note that some options can also be passed to the MILP subsolver used by BONMIN in the outer approximation decomposition and the hybrid (see Subsection ??).

The most important option in BONMIN is the choice of the solution algorithm. This can be set by using the option named bonmin.algorithm which can be set to B-BB, B-OA, B-QG, or B-Hyb (it's default value is B-BB). Depending on the value of this option, certain other options may be available or not. Table 1 gives the list of options together with their types, default values and availability in each of the four algorithms. The column labeled 'type' indicates the type of the parameter ('F' stands for float, 'I' for integer, and 'S' for string). The column labeled default indicates the global default value. Then for each of the

<sup>&</sup>lt;sup>14</sup>http://download.gams-software.com/

<sup>&</sup>lt;sup>15</sup>http://www.coin-or.org/Ipopt/documentation/node54.html

four algorithm B-BB, B-OA, B-QG, and B-Hyb, '+' indicates that the option is available for that particular algorithm while '-' indicates that it is not.

An example of a bonmin.opt file including all the options with their default values is located in the Test sub-directory.

A small example is as follows:

```
bonmin.bb_log_level 4
bonmin.algorithm B-BB
print_level 6
```

This sets the level of output of the branch-and-bound in BONMIN to 4, the algorithm to branch-and-bound and the output level for Ipopt to 6.

When BONMIN is run from within Ampl, another way to set an option is through the internal Ampl command options. For example

has the same affect as the bonmin.opt example above. Note that any BONMIN option specified in the file bonmin.opt overrides any setting of that option from within  $\hat{A}mpl$ .

A third way is to set options directly in the C/C++ code when running BONMIN from inside a C/C++ program as is explained in the reference manual.

A detailed description of all of the BONMIN options is given in Appendix A. In the following, we give some more details on options for the MILP subsolver and on the options specifically designed for nonconvex problems.

Table 1: List of options and compatibility with the different algorithms.

Table 1: List of options and									
Option	type	B-BB	B-OA	B-QG	B-Hyb	В-Еср	B-iFP	Cbc_Par	
Algorithm choice									
algorithm	S						<b>√</b>		
Branch-and-bound options									
allowable_fraction_gap	F	<b>√</b>				<b>√</b>			
allowable_gap	F	v	$\checkmark$	√.	$\sqrt{}$	√,	, ,	· √	
cutoff	F	\	V		$\sqrt{}$	, v	, ,	, √	
cutoff_decr	F	V	V	√.	$\sqrt{}$	, √	l v √	, ,	
enable_dynamic_nlp	S		_	√	√	√	· -	_	
integer_tolerance	F	<b>√</b>	$\checkmark$	·	√	, √	√		
iteration_limit	I	l v∕	V	√	$\sqrt{}$	√	\ \ \	, v	
nlp_failure_behavior	S	l √	_		_	_	· -	_	
node_comparison	S	V	$\checkmark$	√	$\checkmark$	√,	<b>√</b>	_	
node_limit	I	\	V	$$	$\sqrt{}$	, v	\ \ \sqrt{}	<b>1</b> /	
num_cut_passes	I	_	_	$$	V	V,	· -	_	
num_cut_passes_at_root	I	_	_	$\sqrt{}$	V	V	_	_	
number_before_trust	I	<b>√</b>	$\checkmark$	$\sqrt{}$	V	V		$\checkmark$	
number_strong_branch	I	V	V	$$	$\sqrt{}$	V,	l v √	1	
solution_limit	I		V	$$	V	V	\ \\ \\	\ \ \\ \\ \\ \\	
sos_constraints	S	V	V	$\sqrt{}$	V	V	V	_	
time_limit	F	\ \\ \/	V	$\sqrt{}$	V	\ \sqrt{}	\	<b>1</b> /	
tree_search_strategy	s	1 1/	$\sqrt{}$	$$	V	V	$\downarrow$	_	
variable_selection	S	1 1/	_	_	-	_	\ <u>'</u> _	_	
MILP cutting planes in hybrid									
2mir_cuts	I	<u> </u>		$\sqrt{}$		<b>√</b>		$\sqrt{}$	
Gomory_cuts	l I	_	V	$\sqrt{}$	V	V	$\downarrow$	V √	
clique_cuts	I	_	$\sqrt{}$	$\sqrt{}$	V	V,	V	<b>√</b>	
cover_cuts	I	_	V	$\sqrt{}$	V	V	V	V √	
flow_cover_cuts	I	_	V	$\sqrt{}$	V	V	\ \\ \\	V	
lift_and_project_cuts	l I	_	V	$\sqrt{}$	V	V	V	V √	
mir_cuts	I	_	V	$\sqrt{}$	V	l v √	V		
reduce_and_split_cuts	I	_	1/	√ V	V	V	\ \\ \\	$\sqrt{}$	
MINLP Heuristics									
feasibility_pump_objective_norm	I		√	<b>√</b>		<b>√</b>		_	
heuristic_RINS	s	\ \sqrt{}	$\sqrt{}$	$\sqrt{}$	V	V	l	_	
heuristic_dive_MIP_vectorLength	s	\	$\sqrt{}$	\ \ \ \ \ \	V	V	V √	_	
heuristic_dive_fractional	S	V	V	$\sqrt{}$	V	V	V	_	
heuristic_dive_vectorLength	S	V	$\sqrt{}$	$\sqrt{}$	$\checkmark$	V,	V	_	
heuristic_feasibility_pump	s	\ \\ \\ \/	$\sqrt{}$	$\sqrt{}$	V	V	V,	_	
pump_for_minlp	s	\ \\ \\ \\ \\	v 1/	\ \\ \\ \/ \	1/	1/	\ \/	_	
MINLP heuristics									
continued on next page									
continued on next page									

Option	type	B-BB	B-OA	B-QG	B-Hyb	В-Еср	B-iFP	Cbc_Par	
heuristic_dive_MIP_fractional	S	1/	1/	1/	1/	1/	1/	-	
Nlp solution robustness									
max_consecutive_failures	I	<b>√</b>	_	_	_	_	_	_	
max_random_point_radius	F	v	-	_	-	_	_	_	
num_iterations_suspect	I	Ĭ ,	√	<b>√</b>	√	<b>√</b>		√ √	
num_retry_unsolved_random_point	I	\ \ \	\ \ \	\ \ \	\	\ \ \		\ \ \	
random_point_perturbation_interval	F	,	· -	· -	· -	· -	_	· -	
random_point_type	S	l √	-	-	-	-	-	-	
Nlp solve options in B-Hyb									
nlp_solve_frequency	I	-	-	-		-	-	-	
nlp_solve_max_depth	I	-	-	-		-	-	-	
nlp_solves_per_depth	F	-	-	-	√	-	-	-	
Options for MILP solver									
milp_log_level	I	-	-	-	-	-	-	√	
milp_solver	S	-	-	-	-	-	-	√	
Options for OA decomposition									
oa_decomposition	S	-	-	√			-	-	
oa_log_frequency	F	√	-	-	√	√	-	-	
oa_log_level	I		-	-			-	-	
Options for ecp cuts generation									
ecp_abs_tol	F	-	-	√	√	-	-	-	
ecp_max_rounds	I	-	-	√	√	-	-	-	
ecp_propability_factor	F	-	-	<b> </b> √	<b>√</b>	-	-	-	
ecp_rel_tol	F	-	-	√	√	-	-	-	
filmint_ecp_cuts	I	-	-			-	-	-	
		ns for fe	asibility	checker	r using O	A cuts			
feas_check_cut_types	S	-	-	<b> </b> √,	√,	<b>√</b>	-	-	
feas_check_discard_policy	S	-	-	<b> </b> √	√.	<b>√</b>	-	-	
generate_benders_after_so_many_oa	I	-	-		<u></u> √	$\sqrt{}$	-	-	
		ns for fe	asibility	pump					
fp_log_frequency	F	-	-	√ <sub>.</sub>	√,	-	-	-	
fp_log_level	I		-	<u> √</u>	<b>√</b>	-	-	-	
Options for non-convex problems									
coeff_var_threshold	F	√,	-	-	-	-	-	-	
dynamic_def_cutoff_decr	S	√,	-	-	-	-	-	-	
first_perc_for_cutoff_decr	F	√,	-	-	-	-	-	-	
max_consecutive_infeasible	I	<b> </b> √,	-	-	-	-	-	-	
num_resolve_at_infeasibles	I	\ \frac{1}{2}	-	-	-	_	-	-	
num_resolve_at_node	I	\ \frac{1}{2}	-	-	_	_	_	-	
num_resolve_at_root	I	√,	-	-	-	_	-	-	
second_perc_for_cutoff_decr	F	<u>√</u>			-	_	_	_	
add only violated		Approx	imation	cuts ge	eneration	/	/	/	
add_only_violated_oa	S	· -	<u> </u>	√	√	V	√	√	
continued on next page									

Option	type	B-BB	B-OA	B-QG	В-Нур	В-Еср	B-iFP	Cbc_Par	
oa_cuts_log_level	I	-							
oa_cuts_scope	S	-						$\checkmark$	
tiny_element	F	-				$\checkmark$		$\checkmark$	
very_tiny_element	F	-			$\sqrt{}$	$\checkmark$		$\checkmark$	
Output ond log-levels options									
bb_log_interval	I	√	√					√	
bb_log_level	I					$\checkmark$		$\checkmark$	
lp_log_level	I	-			√	$\checkmark$		$\checkmark$	
nlp_log_at_root	I					$\checkmark$		-	
Strong branching setup									
candidate_sort_criterion	S							-	
maxmin_crit_have_sol	F					$\checkmark$		-	
maxmin_crit_no_sol	F					$\checkmark$	$$	-	
min_number_strong_branch	I					$\checkmark$	$\sqrt{}$	-	
number_before_trust_list	I					$\checkmark$		-	
number_look_ahead	I	$$	$$	$$	√	$\checkmark$	-	-	
number_strong_branch_root	I	$$	$$		√			-	
setup_pseudo_frac	F				√	$\checkmark$		-	
trust_strong_branching_for_pseudo_cost	S					$\checkmark$		-	
nlp interface option									
file_solution	S								
nlp_log_level	I	√	√	√	√		√		
nlp_solver	S	√	√			$\checkmark$	$$		
warm_start	S		-	-	-	-	-	-	

# 5.2 Passing options to local search based heuristics and oa generators

Several parts of the algorithms in BONMIN are based on solving a simplified version of the problem with another instance of BONMIN: Outer Approximation Decomposition (called in B-Hyb at the root node) and Feasibility Pump for MINLP (called in B-Hyb or B-BB at the root node), RINS, RENS, Local Branching.

In all these cases, one can pass options to the sub-algorithm used through the bonmin.opt file. The basic principle is that the "bonmin." prefix is replaced with a prefix that identify the sub-algorithm used:

- to pass options to Outer Approximation Decomposition: oa\_decomposition.,
- to pass options to Feasibility Pump for MINLP: pump\_for\_minlp.,
- to pass options to RINS: rins.,
- to RENS: rens.,
- to Local Branching: local\_branch.

For example, we may want to run a maximum of 60 seconds of FP for MINLP until 6 solutions are found at the beginning of the hybrid algorithm. To do so we set the following option in bonmin.opt

bonmin.algorithm B-Hyb

bonmin.pump\_for\_minlp yes #Tells to run fp for MINLP pump\_for\_minlp.time\_limit 60 #set a time limit for the pump pump\_for\_minlp.solution\_limit 6 # set a solution limit

Note that the actual solution and time limit will be the minimum of the global limits set for  ${\tt BONMIN}$ .

A slightly more complicated set of options may be used when using RINS. Say for example that we want to run RINS inside B-BB. Each time RINS is called we want to solve the small-size MINLP generated using B-QG (we may run any algorithm available in bonmin for solving an MINLP) and want to stop as soon as B-QG found 1 solution. We set the following options in bonmin.opt

bonmin.algorithm B-BB

bonmin.rins yes
rins.algorithm B-QG
rins.solution\_limit 1

This example shows that it is possible to set any option used in the sub=algorithm to be different than the one used for the main algorithm.

In the context of OA and FP for MINLP, a standard MILP solver is used. Several option are available for configuring this MILP solver. BONMIN allows a choice of different MILP solvers through the option bonmin.milp\_subsolver. Values for this option are: Cbc\_D which uses Cbc with its default settings, Cplex which uses Cplex with its default settings, and Cbc\_Par which uses a version of Cbc that can be parameterized by the user. The options that can be set in Cbc\_Par are the number of strong-branching candidates, the number of branches before pseudo costs are to be trusted, and the frequency of the various cut generators (these options are signaled in Table 1).

# 5.3 Getting good solutions to nonconvex problems

To solve a problem with non-convex constraints, one should only use the branch-and-bound algorithm B-BB.

A few options have been designed in BONMIN specifically to treat problems that do not have a convex continuous relaxation. In such problems, the solutions obtained from Ipopt are not necessarily globally optimal, but are only locally optimal. Also the outer-approximation constraints are not necessarily valid inequalities for the problem.

No specific heuristic method for treating nonconvex problems is implemented yet within the OA framework. But for the pure branch-and-bound B-BB, we implemented a few options having in mind that lower bounds provided by Ipopt should not be trusted, and with the goal of trying to get good solutions. Such options are at a very experimental stage.

First, in the context of nonconvex problems, Ipopt may find different local optima when started from different starting points. The two options num\_resolve\_at\_root and num\_resolve\_at\_node allow for solving the root node or each node of the tree, respectively, with a user-specified number of different randomly-chosen starting points, saving the best solution found. Note that the function to generate a random starting point is very naïve: it chooses a random point (uniformly) between the bounds provided for the variable. In particular if there are some functions that can not be evaluated at some points of the domain, it may pick such points, and so it is not robust in that respect.

Secondly, since the solution given by Ipopt does not truly give a lower bound, we allow for changing the fathoming rule to continue branching even if the solution value to the current node is worse than the best-known solution. This is achieved by setting allowable\_gap and allowable\_fraction\_gap and cutoff\_decr to negative values.

#### 5.4 Notes on Ipopt options

Ipopt has a very large number of options, to get a complete description of them, you should refer to the Ipopt manual. Here we only mention and explain some of the options that have been more important to us, so far, in developing and using BONMIN.

#### 5.4.1 Default options changed by BONMIN

Ipopt has been tailored to be more efficient when used in the context of the solution of a MINLP problem. In particular, we have tried to improve Ipopt's warm-starting capabilities and its ability to prove quickly that a subproblem is infeasible. For ordinary NLP problems, Ipopt does not use these options by default, but BONMIN automatically changes these options from their default values.

Note that options set by the user in bonmin.opt will override these settings.

mu\_strategy and mu\_oracle are set, respectively, to adaptive and probing by default (these are newly implemented strategies in **Ipopt** for updating the barrier parameter [13] which we have found to be more efficient in the context of MINLP).

gamma\_phi and gamma\_theta are set to  $10^{-8}$  and  $10^{-4}$  respectively. This has the effect of reducing the size of the filter in the line search performed by Ipopt.

required\_infeasibility\_reduction is set to 0.1. This increases the required infeasibility reduction when **Ipopt** enters the restoration phase and should thus help detect infeasible problems faster.

expect\_infeasible\_problem is set to yes which enables some heuristics to
detect infeasible problems faster.

warm\_start\_init\_point is set to yes when a full primal/dual starting point is available (generally all the optimizations after the continuous relaxation has been solved).

print\_level is set to 0 by default to turn off Ipopt output.

#### 5.4.2 Some useful Ipopt options

**bound\_relax\_factor** is by default set to  $10^{-8}$  in <code>Ipopt</code>. All of the bounds of the problem are relaxed by this factor. This may cause some trouble when constraint functions can only be evaluated within their bounds. In such cases, this option should be set to 0.

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# A List of BONMIN options

# A.1 Algorithm choice

sec:algorithm

algorithm: Choice of the algorithm.

This will preset some of the options of bonmin depending on the algorithm choice. The default value for this string option is "B-BB". Possible values:

- B-BB: simple branch-and-bound algorithm,
- B-OA: OA Decomposition algorithm,
- B-QG: Quesada and Grossmann branch-and-cut algorithm,
- B-Hyb: hybrid outer approximation based branch-and-cut,
- B-Ecp: ecp cuts based branch-and-cut a la FilMINT.
- B-iFP: Iterated Feasibility Pump for MINLP.

# A.2 Branch-and-bound options

sec:allowable\_fraction\_gap

allowable\_fraction\_gap: Specify the value of relative gap under which the algorithm stops.

Stop the tree search when the gap between the objective value of the best known solution and the best bound on the objective of any solution is less than this fraction of the absolute value of the best known solution value. The valid range for this real option is  $-1 \cdot 10^{+20} \leq \texttt{allowable\_fraction\_gap} \leq 1 \cdot 10^{+20}$  and its default value is 0.

sec:allowable\_gap

allowable\_gap: Specify the value of absolute gap under which the algorithm stops.

Stop the tree search when the gap between the objective value of the best known solution and the best bound on the objective of any solution is less than this. The valid range for this real option is  $-1 \cdot 10^{+20} \le \mathtt{allowable\_gap} \le 1 \cdot 10^{+20}$  and its default value is 0.

sec:cutoff

**cutoff:** Specify cutoff value.

cutoff should be the value of a feasible solution known by the user (if any). The algorithm will only look for solutions better than cutoof. The valid range for this real option is  $-1 \cdot 10^{+100} \le \text{cutoff} \le 1 \cdot 10^{+100}$  and its default value is  $1 \cdot 10^{+100}$ .

 $sec:cutoff\_decr$ 

cutoff\_decr: Specify cutoff decrement.

Specify the amount by which cutoff is decremented below a new best upper-bound (usually a small positive value but in non-convex problems it may be a negative value). The valid range for this real option is  $-1 \cdot 10^{+10} \le \text{cutoff\_decr} \le 1 \cdot 10^{+10}$  and its default value is  $1 \cdot 10^{-05}$ .

sec:enable\_dynamic\_nlp

**enable\_dynamic\_nlp:** Enable dynamic linear and quadratic rows addition in nlp

The default value for this string option is "no". Possible values:

- no:
- yes:

sec:integer\_tolerance

**integer\_tolerance:** Set integer tolerance.

Any number within that value of an integer is considered integer. The valid range for this real option is  $0 < integer_tolerance < +inf$  and its default value is  $1 \cdot 10^{-06}$ .

sec:iteration\_limit

iteration\_limit: Set the cumulated maximum number of iteration in the algorithm used to process nodes continuous relaxations in the branch-and-bound. value 0 deactivates option. The valid range for this integer option is  $0 \le iteration_limit < +inf$  and its default value is 2147483647.

sec:nlp\_failure\_behavior

nlp\_failure\_behavior: Set the behavior when an NLP or a series of NLP are unsolved by Ipopt (we call unsolved an NLP for which Ipopt is not able to guarantee optimality within the specified tolerances).

If set to "fathom", the algorithm will fathom the node when Ipopt fails to find a solution to the nlp at that node whithin the specified tolerances. The algorithm then becomes a heuristic, and the user will be warned that the solution might not be optimal. The default value for this string option is "stop". Possible values:

- stop: Stop when failure happens.
- fathom: Continue when failure happens.

sec:node\_comparison

**node\_comparison:** Choose the node selection strategy.

Choose the strategy for selecting the next node to be processed. The default value for this string option is "best-bound".

Possible values:

- best-bound: choose node with the smallest bound,
- depth-first: Perform depth first search,
- breadth-first: Perform breadth first search,
- dynamic: Cbc dynamic strategy (starts with a depth first search and turn to best bound after 3 integer feasible solutions have been found).
- best-guess: choose node with smallest guessed integer solution

sec:node\_limit

**node\_limit:** Set the maximum number of nodes explored in the branch-and-bound search.

The valid range for this integer option is  $0 \le \mathtt{node\_limit} < +\mathtt{inf}$  and its default value is 2147483647.

sec:num\_cut\_passes

**num\_cut\_passes:** Set the maximum number of cut passes at regular nodes of the branch-and-cut.

The valid range for this integer option is  $0 \le num\_cut\_passes < +inf$  and its default value is 1.

 $sec:num\_cut\_passes\_at\_root$ 

**num\_cut\_passes\_at\_root:** Set the maximum number of cut passes at regular nodes of the branch-and-cut.

The valid range for this integer option is  $0 \le \text{num\_cut\_passes\_at\_root} < +\text{inf}$  and its default value is 20.

sec:number\_before\_trust

**number\_before\_trust:** Set the number of branches on a variable before its pseudo costs are to be believed in dynamic strong branching.

A value of 0 disables pseudo costs. The valid range for this integer option is  $0 \le number\_before\_trust < +inf$  and its default value is 8.

sec:number\_strong\_branch

**number\_strong\_branch:** Choose the maximum number of variables considered for strong branching.

Set the number of variables on which to do strong branching. The valid range for this integer option is  $0 \le \text{number\_strong\_branch} < +\text{inf}$  and its default value is 20.

sec:solution\_limit

**solution\_limit:** Abort after that much integer feasible solution have been found by algorithm

value 0 deactivates option The valid range for this integer option is  $0 \le \text{solution\_limit} < + \text{inf}$  and its default value is 2147483647.

sec:sos\_constraints

sos\_constraints: Wether or not to activate SOS constraints.

(only type 1 SOS are supported at the moment) The default value for this string option is "enable".

Possible values:

- enable:
- disable:

sec:time\_limit

**time\_limit:** Set the global maximum computation time (in secs) for the algorithm.

The valid range for this real option is  $0 \le \texttt{time\_limit} < +\texttt{inf}$  and its default value is  $1 \cdot 10^{+10}$ .

sec:tree\_search\_strategy

tree\_search\_strategy: Pick a strategy for traversing the tree

All strategies can be used in conjunction with any of the node comparison functions. Options which affect dfs-dive are max-backtracks-in-dive and max-dive-depth. The dfs-dive won't work in a non-convex problem where objective does not decrease down branches. The default value for this string option is "probed-dive".

Possible values:

- top-node: Always pick the top node as sorted by the node comparison function
- dive: Dive in the tree if possible, otherwise pick top node as sorted by the tree comparison function.
- probed-dive: Dive in the tree exploring two childs before continuing the dive at each level.

- dfs-dive: Dive in the tree if possible doing a depth first search. Backtrack on leaves or when a prescribed depth is attained or when estimate of best possible integer feasible solution in subtree is worst than cutoff. Once a prescribed limit of backtracks is attained pick top node as sorted by the tree comparison function
- dfs-dive-dynamic: Same as dfs-dive but once enough solution are found switch to best-bound and if too many nodes switch to depth-first.

sec:variable\_selection

variable\_selection: Chooses variable selection strategy

The default value for this string option is "strong-branching". Possible values:

- most-fractional: Choose most fractional variable
- strong-branching: Perform strong branching
- reliability-branching: Use reliability branching
- curvature-estimator: Use curvature estimation to select branching variable
- qp-strong-branching: Perform strong branching with QP approximation
- lp-strong-branching: Perform strong branching with LP approximation
- nlp-strong-branching: Perform strong branching with NLP approximation
- osi-simple: Osi method to do simple branching
- osi-strong: Osi method to do strong branching
- random: Method to choose branching variable randomly

## A.3 MILP cutting planes in hybrid

sec:2mir\_cuts

**2mir\_cuts:** Frequency (in terms of nodes) for generating 2-MIR cuts in branch-and-cut

If k  $\downarrow$  0, cuts are generated every k nodes, if -99  $\uparrow$  k  $\uparrow$  0 cuts are generated every -k nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if k=-99 generate cuts only at the root node, if k=0 or 100 do not generate cuts. The valid range for this integer option is  $-100 \leq 2 \text{mir\_cuts} < + \text{inf}$  and its default value is 0.

sec:Gomory\_cuts

**Gomory\_cuts:** Frequency k (in terms of nodes) for generating Gomory cuts in branch-and-cut.

If k  $\stackrel{.}{,}$  0, cuts are generated every k nodes, if -99  $\stackrel{.}{,}$  k  $\stackrel{.}{,}$  0 cuts are generated every -k nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if k=-99 generate cuts only at the root node, if k=0 or 100 do not generate cuts. The valid range for this integer option is  $-100 \le \texttt{Gomory\_cuts} < + \texttt{inf}$  and its default value is -5. sec:clique\\_cuts

clique\_cuts: Frequency (in terms of nodes) for generating clique cuts in branchand-cut

If k  $\downarrow$  0, cuts are generated every k nodes, if -99  $\mid$  k  $\mid$  0 cuts are generated every -k nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if k=-99 generate cuts only at the root node, if k=0 or 100 do not generate cuts. The valid range for this integer option is  $-100 \leq \texttt{clique\_cuts} < +\texttt{inf}$  and its default value is -5.

 ${f cover\_cuts:}$  Frequency (in terms of nodes) for generating cover cuts in branch-and-cut

If k  $\downarrow$  0, cuts are generated every k nodes, if -99  $\mid$  k  $\mid$  0 cuts are generated every -k nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if k=-99 generate cuts only at the root node, if k=0 or 100 do not generate cuts. The valid range for this integer option is  $-100 \leq \text{cover\_cuts} < +\text{inf}$  and its default value is -5. sec:flow\_cover\_cuts

 $\begin{tabular}{ll} {\bf flow\_cover\_cuts:} & {\bf Frequency~(in~terms~of~nodes)~for~generating~flow~cover~cuts} \\ {\bf in~branch-and-cut} & {\bf flow}. \\ \end{tabular}$ 

If k  $\downarrow$  0, cuts are generated every k nodes, if -99  $\mid$  k  $\mid$  0 cuts are generated every -k nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if k=-99 generate cuts only at the root node, if k=0 or 100 do not generate cuts. The valid range for this integer option is  $-100 \leq \texttt{flow\_cover\_cuts} < + \texttt{inf}$  and its default value is -5. sec:lift\_and\_project\_cuts

lift\_and\_project\_cuts: Frequency (in terms of nodes) for generating lift-andproject cuts in branch-and-cut

If k  $\downarrow$  0, cuts are generated every k nodes, if -99  $\uparrow$  k  $\uparrow$  0 cuts are generated every -k nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if k=-99 generate cuts only at the root node, if k=0 or 100 do not generate cuts. The valid range for this integer option is  $-100 \leq \texttt{lift\_and\_project\_cuts} < +\texttt{inf}$  and its default value is 0.

 $sec:mir\_cuts$ 

mir\_cuts: Frequency (in terms of nodes) for generating MIR cuts in branch-and-cut

If k  $\downarrow$  0, cuts are generated every k nodes, if -99  $_{\rm i}$  k  $_{\rm i}$  0 cuts are generated every -k nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if k=-99 generate cuts only at the root node, if k=0 or 100 do not generate cuts. The valid range for this integer option is  $-100 \leq \text{mir\_cuts} < +\text{inf}$  and its default value is -5.

sec:reduce\_and\_split\_cuts

reduce\_and\_split\_cuts: Frequency (in terms of nodes) for generating reduce-and-split cuts in branch-and-cut

If k  $\downarrow$  0, cuts are generated every k nodes, if -99  $\rm j$  k  $\rm j$  0 cuts are generated every -k nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if k=-99 generate cuts only at the root node, if k=0 or 100 do not generate cuts. The valid range for this integer option is  $-100 \leq \tt reduce\_and\_split\_cuts < +inf$  and its default value is 0.

#### A.4 MINLP Heuristics

sec:feasibility\_pump\_objective\_norm

**feasibility\_pump\_objective\_norm:** Norm of feasibility pump objective function

The valid range for this integer option is  $1 \le \texttt{feasibility\_pump\_objective\_norm} \le 2$  and its default value is 1.

sec:heuristic\_RINS

heuristic\_RINS: if yes runs the RINS heuristic

The default value for this string option is "no". Possible values:

- no: don't run it
- yes: runs the heuristic

sec:heuristic\_dive\_MIP\_vectorLength

heuristic\_dive\_MIP\_vectorLength: if yes runs the Dive MIP VectorLength heuristic

The default value for this string option is "no". Possible values:

- no: don't run it
- yes: runs the heuristic

sec:heuristic\_dive\_fractional

heuristic\_dive\_fractional: if yes runs the Dive Fractional heuristic

The default value for this string option is "no". Possible values:

- no: don't run it
- yes: runs the heuristic

sec:heuristic\_dive\_vectorLength

heuristic\_dive\_vectorLength: if yes runs the Dive VectorLength heuristic

The default value for this string option is "no". Possible values:

- no: don't run it
- yes: runs the heuristic

sec:heuristic\_feasibility\_pump

heuristic\_feasibility\_pump: whether the heuristic feasibility pump should be used

The default value for this string option is "no". Possible values:

- no: don't use it
- yes: use it

sec:pump\_for\_minlp

pump\_for\_minlp: if yes runs FP for MINLP

The default value for this string option is "no". Possible values:

- no: don't run it
- yes: runs the heuristic

## A.5 MINLP heuristics

 $sec:heuristic\_dive\_MIP\_fractional$ 

heuristic\_dive\_MIP\_fractional: if yes runs the Dive MIP Fractional heuristic

The default value for this string option is "no". Possible values:

• no: don't run it

• yes: runs the heuristic

# A.6 Nlp solution robustness

sec:max\_consecutive\_failures

 $max\_consecutive\_failures$ : (temporarily removed) Number n of consecutive unsolved problems before aborting a branch of the tree.

When n > 0, continue exploring a branch of the tree until n consecutive problems in the branch are unsolved (we call unsolved a problem for which Ipopt can not guarantee optimality within the specified tolerances). The valid range for this integer option is  $0 \le \mathtt{max\_consecutive\_failures} < +\mathtt{inf}$  and its default value is 10.

sec:max\_random\_point\_radius

max\_random\_point\_radius: Set max value r for coordinate of a random point.

When picking a random point coordinate i will be in the interval  $[\min(\max(l, r), u-r), \max(\min(u,r), l+r)]$  (where l is the lower bound for the variable and u is its upper bound) The valid range for this real option is  $0 < \max_{random_point_radius} < +\inf$  and its default value is 100000.

sec:num\_iterations\_suspect

num\_iterations\_suspect: Number of iterations over which a node is considered "suspect" (for debugging purposes only, see detailed documentation). When the number of iterations to solve a node is above this number, the subproblem at this node is considered to be suspect and it will be outputed in a file (set to -1 to deactivate this). The valid range for this integer option is  $-1 \le \text{num\_iterations\_suspect} < +\text{inf}$  and its default value is -1. sec:num\_retry\_unsolved\_random\_point

**num\_retry\_unsolved\_random\_point:** Number k of times that the algorithm will try to resolve an unsolved NLP with a random starting point (we call unsolved an NLP for which Ipopt is not able to guarantee optimality within the specified tolerances).

When Ipopt fails to solve a continuous NLP sub-problem, if k > 0, the algorithm will try again to solve the failed NLP with k new randomly chosen starting points or until the problem is solved with success. The valid range for

this integer option is  $0 \le num\_retry\_unsolved\_random\_point < +inf and its default value is 0.$ 

sec:random\_point\_perturbation\_interval

 ${f random\_point\_perturbation\_interval:}$  Amount by which starting point is perturbed when choosing to pick random point by perturbating starting point. The valid range for this real option is  $0 < {f random\_point\_perturbation\_interval} < + {f inf}$  and its default value is 1.

sec:random\_point\_type

random\_point\_type: method to choose a random starting point

The default value for this string option is "Jon". Possible values:

- Jon: Choose random point uniformly between the bounds
- Andreas: perturb the starting point of the problem within a prescribed interval
- Claudia: perturb the starting point using the perturbation radius suffix information

# A.7 Nlp solve options in B-Hyb

sec:nlp\_solve\_frequency

nlp\_solve\_frequency: Specify the frequency (in terms of nodes) at which NLP relaxations are solved in B-Hyb.

A frequency of 0 amounts to to never solve the NLP relaxation. The valid range for this integer option is  $0 \le \texttt{nlp\_solve\_frequency} < + \texttt{inf}$  and its default value is 10.

sec:nlp\_solve\_max\_depth

nlp\_solve\_max\_depth: Set maximum depth in the tree at which NLP relaxations are solved in B-Hyb.

A depth of 0 amounts to to never solve the NLP relaxation. The valid range for this integer option is  $0 \le \texttt{nlp\_solve\_max\_depth} < +\texttt{inf}$  and its default value is 10

 $sec:nlp\_solves\_per\_depth$ 

nlp\_solves\_per\_depth: Set average number of nodes in the tree at which NLP relaxations are solved in B-Hyb for each depth.

The valid range for this real option is  $0 \le \texttt{nlp\_solves\_per\_depth} < + \texttt{inf}$  and its default value is  $1 \cdot 10^{+100}$ .

# A.8 Options for MILP solver

sec:milp\_log\_level

milp\_log\_level: specify MILP solver log level.

Set the level of output of the MILP subsolver in OA: 0 - none, 1 - minimal, 2 - normal low, 3 - normal high The valid range for this integer option is  $0 \le \texttt{milp\_log\_level} \le 3$  and its default value is 0.

sec:milp\_solver

milp\_solver: Choose the subsolver to solve MILP sub-problems in OA decompositions.

To use Cplex, a valid license is required and you should have compiled OsiCpx in COIN-OR (see Osi documentation). The default value for this string option is "Cbc\_D".

Possible values:

- Cbc\_D: Coin Branch and Cut with its default
- Cbc\_Par: Coin Branch and Cut with passed parameters
- Cplex: Ilog Cplex

# A.9 Options for OA decomposition

sec:oa\_decomposition

oa\_decomposition: If yes do initial OA decomposition

The default value for this string option is "no". Possible values:

- no:
- yes:

sec:oa\_log\_frequency

oa\_log\_frequency: display an update on lower and upper bounds in OA every n seconds

The valid range for this real option is  $0 < oa\_log\_frequency < +inf$  and its default value is 100.

 $sec:oa\_log\_level$ 

oa\_log\_level: specify OA iterations log level.

Set the level of output of OA decomposition solver : 0 - none, 1 - normal, 2 - verbose The valid range for this integer option is  $0 \le \mathtt{oa\_log\_level} \le 2$  and its default value is 1.

# A.10 Options for ecp cuts generation

sec:ecp\_abs\_tol

ecp\_abs\_tol: Set the absolute termination tolerance for ECP rounds.

The valid range for this real option is  $0 \le \text{ecp\_abs\_tol} < +\text{inf}$  and its default value is  $1 \cdot 10^{-06}$ .

sec:ecp\_max\_rounds

**ecp\_max\_rounds:** Set the maximal number of rounds of ECP cuts.

The valid range for this integer option is  $0 \le \texttt{ecp\_max\_rounds} < +\texttt{inf}$  and its default value is 5.

sec:ecp\_propability\_factor

ecp\_propability\_factor: Factor appearing in formula for skipping ECP cuts.

Choosing -1 disables the skipping. The valid range for this real option is -inf < ecp\_propability\_factor < +inf and its default value is 10. sec:ecp\_rel\_tol

**ecp\_rel\_tol:** Set the relative termination tolerance for ECP rounds.

The valid range for this real option is  $0 \le \texttt{ecp\_rel\_tol} < + \texttt{inf}$  and its default value is 0.

sec:filmint\_ecp\_cuts

filmint\_ecp\_cuts: Specify the frequency (in terms of nodes) at which some a la filmint ecp cuts are generated.

A frequency of 0 amounts to to never solve the NLP relaxation. The valid range for this integer option is  $0 \le \texttt{filmint\_ecp\_cuts} < + \texttt{inf}$  and its default value is 0.

#### A.11 Options for feasibility checker using OA cuts

sec:feas\_check\_cut\_types

**feas\_check\_cut\_types:** Choose the type of cuts generated when an integer feasible solution is found

If it seems too much memory is used should try Benders to use less The default value for this string option is "outer-approximations".

Possible values:

- $\bullet$  outer-approximations: Generate a set of Outer Approximations cuts.
- Benders: Generate a single Benders cut.

sec:feas\_check\_discard\_policy

feas\_check\_discard\_policy: How cuts from feasibility checker are discarded Normally to avoid cycle cuts from feasibility checker should not be discarded in the node where they are generated. However Cbc sometimes does it if no care is taken which can lead to an infinite loop in Bonmin (usualy on simple problems). To avoid this one can instruct Cbc to never discard a cut but if we do that for all cuts it can lead to memory problems. The default policy here is to detect cycles and only then impose to Cbc to keep the cut. The two other alternative are to instruct Cbc to keep all cuts or to just ignore the problem and hope for the best The default value for this string option is "detect-cycles". Possible values:

- detect-cycles: Detect if a cycle occurs and only in this case force not to discard.
- keep-all: Force cuts from feasibility checker not to be discarded (memory hungry but sometimes better).
- treated-as-normal: Cuts from memory checker can be discarded as any other cuts (code may cycle then)

sec:generate\_benders\_after\_so\_many\_oa

generate\_benders\_after\_so\_many\_oa: Specify that after so many oa cuts have been generated Benders cuts should be generated instead. It seems that sometimes generating too many oa cuts slows down the optimization compared to Benders due to the size of the LP. With this option we specify that after so many OA cuts have been generated we should switch to Benders cuts. The valid range for this integer option is  $0 \le \text{generate\_benders\_after\_so\_many\_oa} < + \text{inf}$  and its default value is 5000.

#### A.12 Options for feasibility pump

sec:fp\_log\_frequency

**fp\_log\_frequency:** display an update on lower and upper bounds in FP every n seconds

The valid range for this real option is  $0 < \text{fp\_log\_frequency} < +\text{inf}$  and its default value is 100.

sec:fp\_log\_level

**fp\_log\_level:** specify FP iterations log level.

Set the level of output of OA decomposition solver : 0 - none, 1 - normal, 2 - verbose The valid range for this integer option is  $0 \le \texttt{fp\_log\_level} \le 2$  and its default value is 1.

#### A.13 Options for non-convex problems

sec:coeff\_var\_threshold

**coeff\_var\_threshold:** Coefficient of variation threshold (for dynamic definition of cutoff\_decr).

Coefficient of variation threshold (for dynamic definition of cutoff\_decr). The valid range for this real option is  $0 \le \texttt{coeff\_var\_threshold} < + \texttt{inf}$  and its default value is 0.1.

sec:dynamic\_def\_cutoff\_decr

**dynamic\_def\_cutoff\_decr:** Do you want to define the parameter cutoff\_decr dynamically?

The default value for this string option is "no". Possible values:

- no: No, define it statically
- yes: Yes, define it dynamically

sec:first\_perc\_for\_cutoff\_decr

first\_perc\_for\_cutoff\_decr: The percentage used when, the coeff of variance is smaller than the threshold, to compute the cutoff\_decr dynamically. The percentage used when, the coeff of variance is smaller than the threshold, to compute the cutoff\_decr dynamically. The valid range for this real option is -inf < first\_perc\_for\_cutoff\_decr < +inf and its default value is -0.02. sec:max\_consecutive\_infeasible

max\_consecutive\_infeasible: Number of consecutive infeasible subproblems before aborting a branch.

Will continue exploring a branch of the tree until "max\_consecutive\_infeasible" consecutive problems are infeasibles by the NLP sub-solver. The valid range for this integer option is  $0 \le max\_consecutive\_infeasible < +inf and its default value is 0. sec:num\_resolve\_at\_infeasibles$ 

 $num_resolve_at_infeasibles:$  Number k of tries to resolve an infeasible node (other than the root) of the tree with different starting point.

The algorithm will solve all the infeasible nodes with k different random starting points and will keep the best local optimum found. The valid range for this integer option is  $0 \le \text{num\_resolve\_at\_infeasibles} < +\text{inf}$  and its default value is 0.

 $sec:num\_resolve\_at\_node$ 

**num\_resolve\_at\_node:** Number k of tries to resolve a node (other than the root) of the tree with different starting point.

The algorithm will solve all the nodes with k different random starting points and will keep the best local optimum found. The valid range for this integer option is  $0 \le num\_resolve\_at\_node \le +inf$  and its default value is 0.

 $sec:num\_resolve\_at\_root$ 

 $num_resolve_at_root$ : Number k of tries to resolve the root node with different starting points.

The algorithm will solve the root node with k random starting points and will keep the best local optimum found. The valid range for this integer option is  $0 \le num\_resolve\_at\_root < +inf$  and its default value is 0.

 $sec:second\_perc\_for\_cutoff\_decr$ 

second\_perc\_for\_cutoff\_decr: The percentage used when, the coeff of variance is greater than the threshold, to compute the cutoff\_decr dynamically. The percentage used when, the coeff of variance is greater than the threshold, to compute the cutoff\_decr dynamically. The valid range for this real option is -inf < second\_perc\_for\_cutoff\_decr < +inf and its default value is -0.05.

# A.14 Outer Approximation cuts generation

sec:add\_only\_violated\_oa

add\_only\_violated\_oa: Do we add all OA cuts or only the ones violated by current point?

The default value for this string option is "no". Possible values:

- no: Add all cuts
- yes: Add only violated Cuts

sec:oa\_cuts\_log\_level

oa\_cuts\_log\_level: level of log when generating OA cuts.

0: outputs nothing, 1: when a cut is generated, its violation and index of row from which it originates, 2: always output violation of the cut. 3: output generated cuts incidence vectors. The valid range for this integer option is  $0 \le oa\_cuts\_log\_level < +inf$  and its default value is 0.

sec:oa\_cuts\_scope

oa\_cuts\_scope: Specify if OA cuts added are to be set globally or locally valid

The default value for this string option is "global". Possible values:

- local: Cuts are treated as globally valid
- global: Cuts are treated as locally valid

sec:tiny\_element

tiny\_element: Value for tiny element in OA cut

We will remove "cleanly" (by relaxing cut) an element lower than this. The valid range for this real option is  $-0 \le \texttt{tiny\_element} < +\texttt{inf}$  and its default value is  $1 \cdot 10^{-08}$ .

sec:very\_tiny\_element

very\_tiny\_element: Value for very tiny element in OA cut

Algorithm will take the risk of neglecting an element lower than this. The valid range for this real option is  $-0 \le \text{very\_tiny\_element} < +\text{inf}$  and its default value is  $1 \cdot 10^{-17}$ .

# A.15 Output ond log-levels options

sec:bb\_log\_interval

**bb\_log\_interval:** Interval at which node level output is printed.

Set the interval (in terms of number of nodes) at which a log on node resolutions (consisting of lower and upper bounds) is given. The valid range for this integer option is  $0 \le \mathtt{bb\_log\_interval} < +\mathtt{inf}$  and its default value is 100.

sec:bb\_log\_level

**bb\_log\_level:** specify main branch-and-bound log level.

Set the level of output of the branch-and-bound : 0 - none, 1 - minimal, 2 - normal low, 3 - normal high The valid range for this integer option is  $0 \le bb\_log\_level \le 5$  and its default value is 1.

sec:lp\_log\_level

lp\_log\_level: specify LP log level.

Set the level of output of the linear programming sub-solver in B-Hyb or B-QG : 0 - none, 1 - minimal, 2 - normal low, 3 - normal high, 4 - verbose The valid range for this integer option is  $0 \le \texttt{lp\_log\_level} \le 4$  and its default value is 0. sec:nlp\_log\_at\_root

nlp\_log\_at\_root: Specify a different log level for root relaxtion.

The valid range for this integer option is  $0 \le nlp\_log\_at\_root \le 12$  and its default value is 0.

#### A.16 Strong branching setup

sec:candidate\_sort\_criterion

**candidate\_sort\_criterion:** Choice of the criterion to choose candidates in strong-branching

The default value for this string option is "best-ps-cost". Possible values:

- best-ps-cost: Sort by decreasing pseudo-cost
- worst-ps-cost: Sort by increasing pseudo-cost
- most-fractional: Sort by decreasing integer infeasibility
- least-fractional: Sort by increasing integer infeasibility

sec:maxmin\_crit\_have\_sol

maxmin\_crit\_have\_sol: Weight towards minimum in of lower and upper branching estimates when a solution has been found.

The valid range for this real option is  $0 \le \mathtt{maxmin\_crit\_have\_sol} \le 1$  and its default value is 0.1.

sec:maxmin\_crit\_no\_sol

maxmin\_crit\_no\_sol: Weight towards minimum in of lower and upper branching estimates when no solution has been found yet.

The valid range for this real option is  $0 \leq \mathtt{maxmin\_crit\_no\_sol} \leq 1$  and its default value is 0.7.

sec:min\_number\_strong\_branch

min\_number\_strong\_branch: Sets minimum number of variables for strong branching (overriding trust)

The valid range for this integer option is  $0 \le \min_{\text{number\_strong\_branch}} < +\inf$  and its default value is 0.

sec:number\_before\_trust\_list

**number\_before\_trust\_list:** Set the number of branches on a variable before its pseudo costs are to be believed during setup of strong branching candidate list.

The default value is that of "number\_before\_trust" The valid range for this integer option is  $-1 \leq \text{number_before_trust_list} < +\text{inf}$  and its default value is 0.

 $sec:number\_look\_ahead$ 

number\_look\_ahead: Sets limit of look-ahead strong-branching trials The valid range for this integer option is  $0 \le number_look_ahead < +inf$  and its default value is 0.

 $sec:number\_strong\_branch\_root$ 

**number\_strong\_branch\_root:** Maximum number of variables considered for strong branching in root node.

The valid range for this integer option is  $0 \le number\_strong\_branch\_root < +inf and its default value is 2147483647.$ 

sec:setup\_pseudo\_frac

**setup\_pseudo\_frac:** Proportion of strong branching list that has to be taken from most-integer-infeasible list.

The valid range for this real option is  $0 \le \texttt{setup\_pseudo\_frac} \le 1$  and its default value is 0.5.

 $sec:trust\_strong\_branching\_for\_pseudo\_cost$ 

**trust\_strong\_branching\_for\_pseudo\_cost:** Whether or not to trust strong branching results for updating pseudo costs.

The default value for this string option is "yes". Possible values:

- no:
- yes:

# A.17 nlp interface option

sec:file\_solution

file\_solution: Write a file bonmin.sol with the solution

The default value for this string option is "no". Possible values:

- yes:
- no:

sec:nlp\_log\_level

**nlp\_log\_level:** specify NLP solver interface log level (independent from ipopt print\_level).

Set the level of output of the OsiTMINLPInterface : 0 - none, 1 - normal, 2 - verbose The valid range for this integer option is  $0 \le \texttt{nlp\_log\_level} \le 2$  and its default value is 1.

sec:nlp\_solver

nlp\_solver: Choice of the solver for local optima of continuous nlp's Note that option will work only if the specified solver has been installed. Ipopt will usually be installed with Bonmin by default. For FilterSQP please see <a href="http://www-unix.mcs.anl.gov/leyffer/solvers.html">http://www-unix.mcs.anl.gov/leyffer/solvers.html</a> on how to obtain it and <a href="https://projects.coin-or.org/Bonmin/wiki/HintTric-">https://projects.coin-or.org/Bonmin/wiki/HintTric-</a> ks on how to configure Bonmin to use it. The default value for this string option is "Ipopt".

Possible values:

- Ipopt: Interior Point OPTimizer (https://projects.coin-or.org/Ipopt)
- filterSQP: Sequential quadratic programming trust region algorithm (http://www-unix.mcs.anl.gov/ leyffer/solvers.h- tml)
- all: run all available solvers at each node

sec:warm\_start

warm\_start: Select the warm start method

This will affect the function getWarmStart(), and as a consequence the warm starting in the various algorithms. The default value for this string option is "none".

Possible values:

• none: No warm start

• optimum: Warm start with direct parent optimum

• interior\_point: Warm start with an interior point of direct parent