

DDT Math Write Up

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$$\begin{aligned}
\frac{dT_h}{dt} &= -\frac{\dot{Q}_h}{m_h C_{ph}} + \frac{V^2 R_h}{R_{sys}^2} \\
\frac{dT_{a1}}{dt} &= \frac{\dot{Q}_h}{2m_{a1} C_{pc}} - \frac{\dot{m}_a}{m_{a1}} T_{a1} + \frac{\dot{m}_a}{m_{a1}} T_{a4} \\
\frac{dT_{a2}}{dt} &= \frac{\dot{Q}_h}{2m_{a2} C_{pc}} - \frac{\dot{m}_a}{m_{a2}} T_{a2} + \frac{\dot{m}_a}{m_{a2}} T_{a1} \\
\frac{dT_{a3}}{dt} &= -\frac{\dot{Q}_{ab}}{2m_{a3} C_{pc}} - \frac{\dot{m}_a}{m_{a3}} T_{a3} + \frac{\dot{m}_a}{m_{a3}} T_{a2} \\
\frac{dT_{a4}}{dt} &= -\frac{\dot{Q}_{ab}}{2m_{a4} C_{pc}} - \frac{\dot{m}_a}{m_{a4}} T_{a4} + \frac{\dot{m}_a}{m_{a4}} T_{a3}
\end{aligned}$$

$$\begin{aligned}
\frac{dT_{b1}}{dt} &= \frac{\dot{Q}_{ab}}{2m_{b1} C_{pc}} - \frac{\dot{m}_b}{m_{b1}} T_{b1} + \frac{\dot{m}_b}{m_{b1}} T_{b4} \\
\frac{dT_{b2}}{dt} &= \frac{\dot{Q}_{ab}}{2m_{b2} C_{pc}} - \frac{\dot{m}_b}{m_{b2}} T_{b2} + \frac{\dot{m}_b}{m_{b2}} T_{b1} \\
\frac{dT_{b3}}{dt} &= -\frac{\dot{Q}_{bc}}{2m_{b3} C_{pc}} - \frac{\dot{m}_b}{m_{b3}} T_{b3} + \frac{\dot{m}_b}{m_{b3}} T_{b2} \\
\frac{dT_{b4}}{dt} &= -\frac{\dot{Q}_{bc}}{2m_{b4} C_{pc}} - \frac{\dot{m}_b}{m_{b4}} T_{b4} + \frac{\dot{m}_b}{m_{b4}} T_{b3}
\end{aligned}$$

$$\begin{aligned}
\frac{dT_{c1}}{dt} &= \frac{\dot{Q}_{bc}}{2m_{c1} C_{pc}} - \frac{\dot{m}_c}{m_{c1}} T_{c1} + \frac{\dot{m}_c}{m_{c1}} T_{in} \\
\frac{dT_{c2}}{dt} &= \frac{\dot{Q}_{bc}}{2m_{c2} C_{pc}} - \frac{\dot{m}_c}{m_{c2}} T_{c2} + \frac{\dot{m}_c}{m_{c2}} T_{c1}
\end{aligned}$$

$$\dot{Q}_h = -h_h A_h l_H \frac{(T_{a1}-T_{a2})}{\ln(\frac{T_h-T_{a1}}{T_h-T_{a2}})}$$

$$\dot{Q}_{ab} = -h_{ab} \pi d_{ab} l_{ab} \frac{(T_{a3}-T_{b2})-(T_{a4}-T_{b1})}{\ln(\frac{T_{a3}-T_{b2}}{T_{a4}-T_{b1}})}$$

$$\dot{Q}_{bc} = -h_{bc} \pi d_{bc} l_{bc} \frac{(T_{b3}-T_{c2})-(T_{b4}-T_{c1})}{\ln(\frac{T_{b3}-T_{c2}}{T_{b4}-T_{c1}})}$$