Calculus Problems

October 10, 2020

Problem 2

Assuming $a > 0$ and $b > 0$, evaluate

$$\lim_{n \to \infty} n^{b-a} \frac{\sum_{k=1}^{\infty} k^a}{\sum_{k=1}^{\infty} k^b}$$

$$= \lim_{n \to \infty} \frac{\frac{1}{n^a} \sum_{k=1}^{\infty} k^a}{\frac{1}{n^b} \sum_{k=1}^{\infty} k^b}$$

$$= \lim_{n \to \infty} \frac{\frac{1}{n^{a+1}} \sum_{k=1}^{\infty} k^a}{\frac{1}{n^{b+1}} \sum_{k=1}^{\infty} k^b}$$

$$= \frac{\lim_{n \to \infty} \frac{1}{n^{a+1}} \sum_{k=1}^{\infty} k^a}{\lim_{n \to \infty} \frac{1}{n^{b+1}} \sum_{k=1}^{\infty} k^b}$$

$$= \frac{\int_0^1 x^a dx}{\int_0^1 x^b dx}$$

$$= \boxed{\frac{b+1}{a+1}}$$