For what it's worth, we can actually compute the probability vectors v_n. Multiply each equation

$$(n+1)v_{n+1} = nv_n + R(v_n)$$

by x^n and sum from $n = 1$. Letting $V(x)$ be the vector-valued power series $\sum_{n=1}^{\infty} v_n x^n$, our sum may be written

$$V'(x) - v_1 = xV'(x) + R(V(x)) \qquad \textit{or} \qquad (1-x)V'(x) = R(V(x)) + v_1$$

Equivalently, we may separate out the components $V(x) = (A(x), B(x), C(x))$ and then present this as a system of three ODEs:

$$(1-x)A'(x) = C(x) + a_1, \qquad (1-x)B'(x) = A(x) + b_1, \qquad (1-x)C'(x) = B(x) + c_1$$

This can be made simpler: if $\bar{C}(x), \bar{A}(x), \bar{B}(x)$ are the three right sides of these equations and $u = 1 - x$, then

$$\bar{C} = -u\frac{d\bar{A}}{du}, \qquad \bar{A} = -u\frac{d\bar{B}}{du}, \qquad \bar{B} = -u\frac{d\bar{C}}{du}$$

which we may substitute back to get a linear ODE satisfied by $Y = \bar{A}, \bar{B}$, or $\bar{C}$:

$$u^3\frac{d^3Y}{du^3} + 3u^2\frac{d^2Y}{du^2} + u\frac{dY}{du} + Y = 0$$

From the Ansatz $Y = u^r$ we discover solutions of this type iff $r^3 = 1$. In real terms this means the general solution may be expressible in closed form:

$$Y = \frac{\alpha}{1-x} + \beta\sqrt{1-x}\sin\left(\frac{\sqrt{3}}{2}\log(1-x)\right) + \gamma\sqrt{1-x}\cos\left(\frac{\sqrt{3}}{2}\log(1-x)\right)$$

Matching the initial conditions we deduce the precise functions whose Taylor coefficients give the probabilities a_n, b_n, c_n:

$$\begin{array}{llllll}
A(x) & = \frac{2}{3} & +\frac{1/3}{1-x} & -\frac{\sqrt{1-x}}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}\ln(1-x)) & -\frac{\sqrt{1-x}}{3}\cos(\frac{\sqrt{3}}{2}\ln(1-x)), \\
B(x) & = \frac{-1}{3} & +\frac{1/3}{1-x} & +\frac{\sqrt{1-x}}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}\ln(1-x)) & -\frac{\sqrt{1-x}}{3}\cos(\frac{\sqrt{3}}{2}\ln(1-x)), \\
C(x) & = \frac{-1}{3} & +\frac{1/3}{1-x} & & +\frac{2\sqrt{1-x}}{3}\cos(\frac{\sqrt{3}}{2}\ln(1-x))
\end{array}$$

I don't see how exactly this helps with the Putnam problem but I can also use a computer while I'm here to run some numerical experiments. When $n = 1$ of course player A will win, and when $n = 2$ it's a tie between A and B. When $3 \le n \le 13$, the odds favor B; when $14 \le n \le 144$, the odds favor C; when $145 \le n \le 1607$, the odds favor A again, and finally when $1608 \le n \le 18040$, the odds favor B again.