Thus, we have $X_N = A_N B_N = \frac{2}{N(N+1)} \frac{N^2+N+1}{3}$

taking the limit as $N \to \infty$ we see X_N converges to

$$\lim_{N \to \infty} \left(\frac{2}{3} \frac{N^2+N+1}{N^2+N} \right) = \frac{2}{3} \lim_{N \to \infty} \frac{N^2+N+1}{N^2+N} = \underline{\frac{2}{3}}$$

we have that $X_N = \exp Y_N \longrightarrow Y_N = \log X_N$

since $\lim_{N \to \infty} Y_N = \lim_{N \to \infty} \log X_N = \log \lim_{N \to \infty} X_N = \log \frac{2}{3}$

(log(x) is continuous as $x \to \infty$)   so $\sum_{n=2}^{\infty} \log\left(\frac{n^3-1}{n^3+1}\right) = \log \frac{2}{3}$