

Putnam practice! Here are some challenging (but not quite Putnam-worthy) calculus problems. How are you at solving them?

1091 Evaluate $\lim_{x \rightarrow \infty} \frac{\sqrt{x^3 - x^2 + 3x}}{\sqrt{x^3} - \sqrt{x^2} + \sqrt{3x}}.$

1093 Compute $\lim_{x \rightarrow 0} \frac{\cos(2x) + 2\sin(x^2) - 1}{x^4}.$

1092 Determine whether these series converge or diverge. (Be sure to justify your answer.)

$$(a) \sum_{n=2}^{\infty} \frac{n^8 - 1}{n^9 - 1} \qquad (b) \sum_{n=2}^{\infty} \frac{1}{\ln(n!)}$$

1094 The four points

$$A = (-6, -2, 3), \quad B = (-6, 8, 3), \quad C = (-7, 5, 3), \quad D = (4, -6, 5)$$

are all equally far from a point P . Find P .

1095 Compute the minimum value of the function

$$f(u, v) = \left(u - v\right)^2 + \left((3 - u) - \left(\frac{5}{v}\right)\right)^2$$

on the region where $v > 0$.

1121 Determine whether these series converge or diverge. (Be sure to justify your answer.)

$$(a) \sum_{n=1}^{\infty} \frac{1}{(3n-2)^{n+(1/2)}} \qquad (b) \sum_{n=1}^{\infty} \frac{(-1)^{n(n-1)/2}}{n} = 1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots$$

1122 Compute the following limit, or show that it does not exist:

$$\lim_{x \rightarrow 0} \frac{x^3 \sin(\frac{1}{x})}{\ln(1+x^2)}$$

1123 Compute the first three terms $a_0 + a_1x + a_2x^2$ of the Maclaurin series (i.e. the Taylor series at 0) for

$$f(x) = \frac{5x - 7}{(x - 1)(x - 2)}$$

1124 Find a point which is equidistant from all four planes

$$x = 0 \quad y = 0 \quad z = 0 \quad 2x + 3y + 6z = 36.$$

1125 Find all the critical points of the function below, and state whether they are local minima, local maxima, or saddle points:

$$f(x, y) = 1 - (x^2 - 1)^2 - (x^2y - x - 1)^2.$$