Here is a grab bag of problems for you to try. Note that there are plenty of problems from previous weeks that we have not shared good solutions for! Practice, practice!

1. Show that the sum of all terms in the nth row of Pascal's Triangle is a multiple of 2^n.

2. Prove that

$$\frac{22}{7} - \pi = \int_0^1 \frac{x^4(1-x)^4}{1+x^2}\, dx$$

Remark: This proves that π is NOT equal to 22/7 !

3. Prove that the decimal part of

$$(5 + \sqrt{26})^n$$

begins with either n zeros or n nines, for all positive integers n.

4. Let A and B be 2×2 matrices with integer entries such that $A, A+B, A+2B, A+3B$, and $A+4B$ are all invertible matrices whose inverses have integer entries. Show that $A+5B$ is also invertible and its inverse has integer entries.

5. Prove that there exists a unique function f from the set $R+$ of positive real numbers to $R+$ such that

$$f(f(x)) = 6x - f(x) \text{ and } f(x) > 0 \qquad \text{for all } x > 0.$$

6. Suppose you are given a set of n red dots and n blue dots in the plane. Show that there is a way to join all the dots in red-blue pairs using straight line segments that do not intersect. (That is, there is a permutation σ such that when each red point r_i is joined to the blue point $b_{\sigma(i)}$ with a line segment L_i, then the line segments are pairwise disjoint: $L_i \cap L_j = \phi$ for every $i \neq j$.)

No meeting next week (Nov 23). We will have one more practice the following week (Nov 30), when I will add problems from more advanced topics: linear algebra, group theory, real analysis, probability, etc. The contest itself is the following Saturday, Dec 4 2010, in CPE 2.210. I will be there around 8:30; we will start promptly at 9am.