Problem 1. If n is a positive integer, then define

$$f(n) = 1! + 2! + ... + n!$$

Find polynomials $P(n)$ and $Q(n)$ such that

$$f(n+2) = P(n)f(n+1) + Q(n)f(n)$$

for all $n \geq 1$.

Solution. Notice that $n! = n * (n-1) * ... * (2) * (1) = f(n) - f(n-1)$. Solving for $f(n+2) = (n+2)! + (n+1)! + f(n)$, we get $f(n+2) = (n+2)(f(n+1) - f(n)) + f(n+1) - f(n) + f(n)$, and we distribute to get $f(n+2) = (n+3)f(n+1) - (n+2)f(n) \implies P(n) = n+3$ and $Q(n) = -n-2$.