6.1 Theorem. *In every set S of $n+1$ distinct positive integers, each less than $2n$, where $n \in \mathbb{Z}_+$, $\exists a, b \in S$ with $a|b$.*

Proof. We divide S into subsets C_k, where $C_k = \{a \in S | a = k2^n,\ n \geq 0,\ \textit{and } k \textit{ is odd}\}$. We see that these sets are disjoint, for if $a \in C_k$ and $a \in C_l$, $a = l2^n = k2^m$, which gives that $l = k2^{m-n}$. If $m - n > 0$, then l is even, a contradiction. If $m - n < 0$, then l isn't an integer, a contradiction. So, $m = 0$, which gives that $k = l$, so $C_k = C_l$.

It's trivial to see that each integer is in some C_k, so these subsets divide S into equivalence classes (you could probably phrase this in a cleaner way to show that the "odd part" of an integer gives an equivalence relation, but I think this does the trick).

Now, we see that if $a, b \in C_k$, and $a \neq b$, then either $a|b$ or $b|a$. Without loss of generality, let $a < b$. So, $a = k2^m$, $b = k2^n$, and $n > m$. This gives that $b = a2^{n-m}$, and $a|b$.

Therefore, in order for a collection of integers to have the property that no integer in the collection divides another integer in the collection, the collection can only contain at most representative from each equivalence class C_k. But there are exactly n odd integers in $\{1, ...2n\}$, so there are only n distinct equivalence classes. Because we have $n+1$ integers in S, at least two of them must be from the same equivalence class, so $\exists a, b \in S$ with $a|b$. $\square$