Here are some more Putnam-ish problems that have a calculus theme.

1. Compute the first three terms $a_0 + a_1 x + a_2 x^2$ of the Taylor series of

$$\frac{3x^2 - 12x + 11}{(x-1)(x-2)(x-3)}$$

2. Determine whether or not this series converges:

$$\sum_{n \geq 1} \frac{(-1)^{n(n-1)/2}}{n}$$

3. Evaluate $\displaystyle\sum_{n=2}^{\infty} \log\left(\frac{n^3 - 1}{n^3 + 1}\right)$

4. For each $n = 0, 1, 2, \ldots$ let D_n be the n-th derivative of $f(x) = e^{(-1/x^2)}$. Show that for every n,

$$\lim_{x \to 0} D_n(x) = 0$$

5. The four points A = (-6;-2; 3); B = (-6; 8; 3); C = (-7; 5; 3); D = (4;-6; 5) are all equally far from a point P. Find P.