

Here are some more Putnam (–like) challenges, including both calculus and other topics.

1. Show that the value of this integral exceeds $\frac{3\pi}{2}$:

$$\int_0^\pi e^{\sin^2 x} dx$$

2. Evaluate this integral:

$$\int_0^{\pi/2} \frac{dx}{1 + (\tan(x))^{1/3}}$$

3. Let A be an $a \times b \times c$ brick in R^3 . Then let B be the set of points in R^3 which are outside of A but whose distance from A is less than 1. What is the volume of B ?

4. Compute (all?) the terms of the Taylor series of a function f , just from the information that

$$f\left(\frac{1}{n}\right) = \frac{n^2}{n^2 + 1} \quad \text{for } n = 1, 2, 3, \dots$$

and from the guarantee that f does indeed have a convergent Taylor series.

5. Suppose P is a polynomial whose coefficients are all integers less than 100 in magnitude. Show that all the real roots of P are also less than 100 in magnitude.