Here is a solution to problem B1 of the the 2018 Putnam exam.

Comment: the construction I provide seems to require an inordinate amount of annoying fiddling with detail.

There are $3 \times 101 = 303$ vectors in $\mathcal{P}$, and the sum of all of them is

$$\sum \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 101(0+1+2) \\ 3(0+1+\ldots+100) \end{pmatrix} == \begin{pmatrix} 303 \\ 3 \cdot (100 \cdot 101)/2 \end{pmatrix} = \begin{pmatrix} 303 \\ 15150 \end{pmatrix}$$

So we must partition all but one of these vectors into two sets $\mathcal{S}$ and $\mathcal{T}$, each with 151 elements, and each summing to $((303-a)/2, (15150-b)/2)$, where $v = (a, b)$ is the vector to be removed.

Note that a must be odd, and hence $a = 1$. Similarly b must be even.

Conversely, for each $k = 0, 1, \ldots, 50$, we will show how to partition $\mathcal{P} \backslash \{(1, 2k)\}$ into two such sets.

When k itself is odd (and hence $k \leq 25$) we will let $\mathcal{S}$ consist of the following vectors:

(1) All 51 vectors $(0, b)$ with b even, except $(0, k+1)$

(2) All 50 vectors $(1, b)$ with b odd, together with (1,0)

(3) All 51 vectors $(2, b)$ with b even, except (2,24)

Then $\mathcal{S}$ contains 151 vectors (so $\mathcal{T} = \mathcal{P} \backslash \mathcal{S} \backslash \{v\}$ will also contain 151 vectors.) The sum of the elements in $\mathcal{S}$ is $(0, 50 \cdot 51) + (50, 50^2) + (2 \cdot 51, 50 \cdot 51) - (0, k+1) + (1, 0) - (2, 24) = (151, 7575 - k)$. That will leave 152 other vectors, which sum to $(303, 15150) - (151, 7575 - k) = (152, 7575 + k)$, so if $\mathcal{T}$ is formed by removing $v = (1, 2k)$, then $\mathcal{S}$ and $\mathcal{T}$ will have the equal cardinality and equal sum.

In a similar way, when k itself is even (and hence less than 25), let $\mathcal{S}$ contain:

(1) All 51 vectors $(0, b)$ with b even, except $(0, k)$ and (0,50), together with (0,49)

(2) All 50 vectors $(1, b)$ with b odd, together with (1,2)

(3) All 51 vectors $(2, b)$ with b even, except (2,26)

Then $\mathcal{S}$ contains 151 vectors which sum to $(0, 50 \cdot 51) + (50, 50^2) + (2 \cdot 51, 50 \cdot 51) - (0, k) - (0, 50) + (0, 49) + (1, 2) - (2, 26) = (151, 7575 - k)$, so as before $\mathcal{T}$ will have the correct cardinality and sum after $(1, 2k)$ is removed.

Note that care has been taken to include only one vector of the form $(1, 2k)$ in the set $\mathcal{S}$, and that the parity of this k differs from that of the vector v we wish to remove from $\mathcal{S} \cup \mathcal{T}$.