Problems, problems, problems! Oct 26 2010.

1. (Polynomials again!) If n is a positive integer, then define

$$f(n) = 1! + 2! + \ldots + n!$$

Find polynomials $P(n)$ and $Q(n)$ such that

$$f(n+2) = P(n)f(n+1) + Q(n)f(n)$$

for all $n \geq 1$.

2. (Let's try geometry!) Let C be a circle of radius 1, and let D be a diameter of C. Let P be the set of all points inside or on C which are closer to D than to the circumference of C. What is the area of P?

3. Let P be a convex polygon with n sides, $n \geq 3$. Any set of $n-3$ diagonals of P that do not intersect in the interior of the polygon determine a *triangulation* of P into $n-2$ triangles. Find all the possible values of n such that when P is regular there is a triangulation of P consisting of only isosceles triangles.

4. (And now for something completely different...) A game starts with four heaps of beans, containing 3, 4, 5, and 6 beans, respectively. Agamemnon and Brunhilde move alternately. The set M of legal moves consists of taking either

(a) one bean from a heap, provided at least two beans are left behind in that heap, or

(b) a complete heap of two or three beans.

The player who takes the last heap wins. To win the game, do you want to move first or second? Give a winning strategy.