Here are some problems, some of them taken from actual Putnam exams, all of them really "just" calculus. Work on them for until say 6:45, and we will come together to discuss solutions. (I have to leave tonight around 7:30.)

1. Find the maximum value of the function

$$F(y) = \int_0^y \sqrt{x^4 + (y - y^2)^2}\, dx$$

2. Evaluate

$$\Sigma_{n=2}^{\infty} \log\left(\frac{n^3 - 1}{n^3 + 1}\right)$$

3. Let A be a positive real number. What are the possible values of $\sum_{j=0}^{\infty} x_j^2$ given that the x_i are positive real numbers and $\sum_{j=0}^{\infty} x_j = A$?

4. Show that this improper integral converges:

$$\int_0^{\infty} \sin(x)\sin(x^2)\, dx$$

5. Let $f(x)$ be a continuous function such that $f(2x^2 - 1) = 2x\, f(x)$ for all x. Show that $x = 0$ for $x \in [-1, 1]$.

6. Let $p(x)$ be a polynomial that is non-negative for all real x. Prove that p is a sum of squares, that is, for some integer k there are polynomials $f_1(x)$, $f_2(x), \ldots, f_k(x)$ such that

$$p(x) = \sum_{j=1}^{k} (f_j(x))^2$$

There will be no Putnam meeting next week; the department is going to honor Professor Uhlenbeck at a reception. See you October 30!