Problems, problems, problems! Oct 31 2013.

1. (Geometry) Show that four points on the parabola $y = x^2$, say $(a, a^2), \ldots (d, d^2)$ (with a, b, c, d distinct) are concylic if and only if $a + b + c + d = 0$.

2. Let C be a circle of radius 1, and let D be a diameter of C. Let P be the set of all points inside or on C which are closer to D than to the circumference of C. What is the area of P?

3. Let P be a convex polygon with n sides, $n \geq 3$. Any set of $n - 3$ diagonals of P that do not intersect in the interior of the polygon determine a *triangulation* of P into $n - 2$ triangles. Find all the possible values of n such that when P is regular there is a triangulation of P consisting of only isosceles triangles.

4. Show that each number in the sequence 49, 4489, 444889, 44448889, ... is a perfect square.

5. Do there exist one million consecutive integers, each of which is divisible by a perfect square (larger than 1)?

6. (And now for something completely different...) A game starts with four heaps of beans, containing 3, 4, 5, and 6 beans, respectively. Agamemnon and Brunhilde move alternately. The set M of legal moves consists of taking either

(a) one bean from a heap, provided at least two beans are left behind in that heap, or

(b) a complete heap of two or three beans.

The player who takes the last heap wins. To win the game, do you want to move first or second? Give a winning strategy.