3-3

The only way we can show a C such that this works is by defining the polynomial equation with degree 1999 such that the area under the curve from -1 to 1 is minimized and that the value of $p(0)$ is maximized, or constrained to a constant such as 1. We can do this by multiplicity of roots. Essentially, we need to constrain both the graph around $x = -1$ and $x = 1$ as close to zero as we can for both sides. By the rules of multiplicity, if we define $p(x)$ such that:

$p(x) = (x+1)^{1000}(x-1)^{999}$

(Technically, a better function to approximate what we would want is $p(x) = (x+1)^{999.5}(x-1)^{999.5}$, but this is not a polynomial function!) We achieve a function such at $x = \pm 1$, $p(x) = 0$ and at $x = 0$, $p(x) = 1$. On top of that, we find that if we take the area of any polynomial of degree 1999 such that $p(0) = 1$, the area is greater than our defined $p(x)$ (aside from the polynomial function $p(x) = (x+1)^{999}(x-1)^{1000}$, which has identical area) . Likewise, if we multiply $p(x)$ by any constant K, we can achieve the highest multiple of our function such that $p(0) = K$ and area is minimalized due to properties of integration. We have thus that:

$\dfrac{K}{K\displaystyle\int_{-1}^{1} |(x+1)^{1000}(x-1)^{999}|dx} = \dfrac{1}{\displaystyle\int_{-1}^{1} |(x+1)^{1000}(x-1)^{999}|dx} \leq C$, which should work for all cases of polynomial functions $p(x)$.