Putnam geometry problems for your enjoyment, to be completed on/by Nov 15

1. Three parallel chords are drawn across a circle; in each case, joining the center to the two endpoints of the chord produces an isoceles triangle. Call the central angles α_i ($i = 1, 2, 3$), and call the lengths of the corresponding chords L_i. Suppose that $L_1 = 2, L_2 = 3, L_3 = 4$ and that $\alpha_3 = \alpha_1 + \alpha_2 < \pi$. Compute $\cos(\alpha_1)$.

2. Let R and S be two squares in the plane of equal side length. Find a decomposition of R into pieces which can be moved by *translation alone* to give a decomposition of S. Try to find a decomposition with as few pieces as possible.

3. [1996-A2]: Let C_1 and C_2 be circles whose centers are 10 units apart and whose radii are 1 and 3. Find, with proof, the locus of all points M for which there exist points X on C_1 and Y on C_2 such that M is the midpoint of the line segment XY.

4. Show that there exist tetrahedra of arbitrarily large volume whose vertices lie at integer points and which do not contain any other lattice points (neither on their boundaries nor in their interiors).

5. What is the smallest α such that two squares with total area 1 can always be placed inside a rectangle area α with sides parallel to those of the rectangle and with no overlap (of their interiors)?

6. On the unit circle centered at the origin ($x^2 + y^2 = 1$) we pick three points at random. We cut the circle into three arcs at those points. What is the expected length of the arc containing the point $(1, 0)$?