"Gee! I'm a tree!" (Putnam Nov 02 2015)

1. A rectangle $HOMF$ has sides $HO = 11$ and $OM = 5$. A triangle ABC has H as the intersection of its altitudes, O as the center of its circumscribed circle, M as the midpoints of BC, and F as the foot of the altitude from A. What is the length of BC?

2. Let $d_1, d_2, \ldots, d_{12}$ be real numbers in the open interval $(1, 12)$. Show that there exist distinct indices i, j, k such that d_i, d_j, d_k are the side lengths of an acute triangle.

3. What is the maximum number of rational points that can be on a circle in $\mathbf{R}^2$ whose center is not a rational point? (A *rational point* is a point both of whose coordinates are rational numbers.)

4. Show that for any set of five points on a sphere there is a set of four of them that lie on a closed hemisphere.

5. Can an arc of a parabola inside a circle of radius 1 have length greater than 4?

6. A unit cube is positioned in R^3 in some orientation and projected projected onto the coordinate plane $\{x_1 = 0\}$. What is the largest possible area of this projection?