Pizza and Problem Solving 10/3/13

1. Next week I will order PapaJohn's famous 1 foot square party pizza. Before serving it, I will sprinkle on exactly 37 poppy seeds (my secret ingredient). The poppy seeds fall randomly onto the pizza dough (and let's suppose none roll off). What is the probability that at least two poppy seeds will land within 3 inches of each other?

2. If there are six people in a room, must there be at least three of them who are either all friends with each other or all non-friends with each other? (You should assume that "friendship" is a symmetric relation.)

3. Prove that if we select $n+1$ numbers from the set $S = \{1, 2, 3, \ldots, 2n\}$, among the numbers selected there are two such that one is a multiple of the other one.

4. (Putnam 1978) Let A be any set of 20 distinct integers chosen from the arithmetic progression $\{1, 4, 7, ..., 100\}$. Prove that there must be two distinct integers in A whose sum if 104.

5, (Putnam, 2006) Prove that, for every set $X = \{x_1, x_2, \ldots, x_n\}$ of n real numbers, there exists a non-empty subset S of X and an integer m such that

$$|m + \sum_{s \in S} s| \leq \frac{1}{n+1}.$$