5)Suppose P is a polynomial whose coefficients are all integers less than 100 in magnitude. Show that all the real roots of P are also less than 100 in magnitude.

Pf:
(Note: integer cooef. implies magnitude ≥ 1)
Take x s.t. $|x| \geq 100$. Then $|P(x)| =$

$$|a_0 x^n + a_1 x^{n-1} + ...| =$$
$$| \pm (a_0 x^n + a_1 x^{n-1} + ...)|$$
(Because we can make a_0 positive,) $\geq$
$$|x^n| + |a_1 x^{n-1} + ...| \geq$$
$$|x^n| - |a_1 x^{n-1} + ...|$$
(Proof by extremes, the biggest magnitude on the second term is achieved only if all cooeficients have the same sign, so assume this,)$\geq$
$$|x^n| - |99x^{n-1} + 99x^{n-2} + ...| =$$
$$|x|^n - 99 \sum_{0 \to i}^{n-1} |x|^i =$$
(By the finite geometric sum formula)
$$|x|^n - 99 \frac{1-|x|^{(n-1)+1}}{1-|x|} \geq$$
(Because $1 - |x| < -99$)
$$|x|^n - 99 \frac{1-|x|^{(n-1)+1}}{-99} =$$
$$|x|^n + 1 - |x|^n = 1 \geq 0$$
Thus, $P(x)$ has no roots having magnitude ≥ 100.