

Putnam practice – Oct 4, 2012

More number theory!

1. How many zeros are at the end of $1000!$?
2. If p and $p^2 + 2$ are primes, show that $p^3 + 2$ is prime.
3. Show that $\gcd(2^a - 1, 2^b - 1) = 2^{\gcd(a,b)} - 1$ for positive integers a, b .
4. Suppose that a, b, c are distinct integers and that $p(x)$ is a polynomial with integer coefficients. Show that it is not possible to have $p(a) = b$, $p(b) = c$, and $p(c) = a$.
5. A triangular number is a positive integer of the form $n(n+1)/2$. Show that m is a sum of two triangular numbers iff $4m + 1$ is a sum of two squares. (A-1, Putnam 1975)