Calculus!

1. Compute $\int_0^\infty \lfloor x \rfloor e^{-x}\,dx$, where $\lfloor x \rfloor$ is the floor function (the *greatest integer* function).

2. Let C be the curve defined by the equation $y^2 = 2x(x+2)(x+8)$, that is,

$$C = \{(x,y)\,;\, y^2 = 2x(x+2)(x+8)\}$$

Find all lines that are tangent to the curve C and which also pass through the origin.

3. Find the integer part of $\sum_{n=1}^{40000} \frac{1}{\sqrt{n}}$. (That is, if the sum is evaluated numerically, what are the digits to the left of the decimal point?)

4. Evaluate the following limit (or explain why the limit does not exist):

$$\lim_{(x,y)\to(0,0)} \frac{\cos(x) + \frac{1}{2}x^2 - 1}{x^4 + y^4}$$

5. Compute the first four terms $a_0 + a_1 x + a_2 x^2 + a_3 x^3$ of the Maclaurin series (i.e. the Taylor series at 0) for

$$f(x) = \frac{5x - 7}{(x-1)(x-2)}$$

6. Compute

$$\int_0^{\pi/2} \frac{dx}{\left(\sqrt{\sin(x)} + \sqrt{\cos(x)}\right)^4}$$

7. Suppose A is a positive real number. Among all sequences of positive numbers x_i with $\sum_{n=0}^\infty x_i = A$, what are the possible values of $\sum_{n=0}^\infty x_i^2$?

8. Let s be any arc of the unit circle lying entirely in the first quadrant. Let A be the area of the region lying below s and above the x-axis and let B be the area of the region lying to the right of the y-axis and to the left of s. Prove that $A + B$ depends only on the arc length, and not on the position, of s.