UT Putnam Prep 2017-10-30 — Probability Problems

1. Two evenly matched teams play in the world series, a best of seven competition in which the competition stops as soon as one team has won four games. Is the world series more likely to end in six or seven games?

2. Shanille O'Keal shoots free throws on a basketball court. She hits the first and misses the second, and thereafter the probability that she hits the next shot is equal to the proportion of shots she has hit so far. What is the probability that she hits exactly 50 of her first 100 shots?

3. A dart, thrown at random, hits a square target. Assuming that any two parts of the target of equal area are equally likely to be hit, find the probability that the point hit is nearer to the center than to any edge. Express your answer in the form $(a\sqrt{b}+c)/d$, where a, b, c, d are integers.

4. Let k be a positive integer. Suppose that the integers $1, 2, 3, \ldots, 3k+1$ are written down in random order. What is the probability that at no time during the process, the sum of the integers that have been written up to that time is a positive integer divisible by 3? Your answer should be in closed form, but may include factorials.

5. Let r and s be given positive real numbers with $r < s$. If two points are selected at random from a straight line segment of length s, what is the probability that the distance between them is at least r?

6. Three closed boxes lie on a table. One box (you don't know which) contains a \$1000 bill. The others are empty. After paying an entry fee, you play the following game with the owner of the boxes: you point to a box but do not open it; the owner then opens one of the two remaining boxes and shows you that it is empty; you may now open either the box you first pointed to or else the other unopened box, but not both. If you find the \$1000, you get to keep it. Does it make any difference which box you choose? What is a fair entry fee for this game?

7. NCAA basketball pool. There are 64 teams who play single elimination tournament, hence 6 rounds, and you have to predict all the winners in all 63 games. Your score is then computed as follows: 32 points for correctly predicting the final winner, 16 points for each correct finalist, and so on, down to 1 point for every correctly predicted winner for the first round. (The maximum number of points you can get is thus 192.) Knowing nothing about any team, you flip fair coins to decide every one of your 63 bets. Compute the expected number of points.