5) Suppose that f is differentiable and that $f'(x)$ is strictly increasing on $[0, 1)$. Suppose further that $f(0) = 0$. Prove that $g(x) = \frac{f(x)}{x}$ is strictly increasing on (0;1).

Pf:
On any interval $[a, b]$ in the domain of f, $\exists c \in (a, b) | f'(c) = \frac{f(b)-f(a)}{b-a}$. (By the Mean Value Theorem)
But because f' is increasing, we also have $f'(c) < f'(b)$.
Take the interval $[0, x], x > 0$. Then,

$$\exists c \in (0, x) | f'(c) = \frac{f(x)-f(0)}{x-0} = \frac{f(x)}{x} = f'(c) < f'(x)$$
$$xf'(x) - f(x) > 0$$
$$\frac{xf'(x)-f(x)*(1)}{x^2} > 0$$
$$\frac{d}{dx}\left(\frac{f(x)}{x}\right) > 0$$

So $g'(x)$ is increasing for positive x