

Question 8) $f(x) + f\left(1 - \frac{1}{x}\right) = 1 + x - \textcircled{1}$

$\therefore x \rightarrow 1 - \frac{1}{x}$

$\therefore f\left(1 - \frac{1}{x}\right) + f\left(1 - \frac{1}{1 - \frac{1}{x}}\right) = 1 + 1 - \frac{1}{x}$

$\therefore f\left(1 - \frac{1}{x}\right) + f\left(\frac{1}{1-x}\right) = 2 - \frac{1}{x} - \textcircled{2}$

$\therefore \textcircled{1} - \textcircled{2}$

$\therefore f(x) - f\left(\frac{1}{1-x}\right) = x + \frac{1}{x} - 1 - \textcircled{3}$

$\therefore x \rightarrow \frac{1}{1-x}$

$\therefore f\left(\frac{1}{1-x}\right) - f\left(\frac{1}{1 - \frac{1}{1-x}}\right) = \frac{1}{1-x} + 1 - x - 1$

$\therefore f\left(\frac{1}{1-x}\right) - f\left(1 - \frac{1}{x}\right) = \frac{1}{1-x} - x - \textcircled{4}$

$\therefore \textcircled{3} + \textcircled{4}$

$\therefore f(x) - f\left(1 - \frac{1}{x}\right) = \frac{1}{x} + \frac{1}{1-x} - 1 - \textcircled{5}$

$\therefore \textcircled{5} + \textcircled{1}$

$\therefore 2f(x) = x + \frac{1}{x} + \frac{1}{1-x}$

$$f(x) = \frac{1}{2} \cdot \left[x + \frac{1}{x} + \frac{1}{1-x} \right]$$