Here are some problem-solving hints I found on the web: "Try small cases. Plug in smaller numbers. Do examples. Divide into cases. Work backwards. Argue by contradiction. Consider extreme cases. Modify the problem. Generalize. Look for patterns. Look for symmetry. Draw pictures. Choose effective notation. Eat pizza. Don't be afraid of a little algebra. Use lots of paper. Don't give up after five minutes. Sleep on it if need be. Talk it over. Work in groups. Ask."

1. Find polynomials $f(x)$, $g(x)$, and $h(x)$, if they exist, such that for all real numbers x,

$$|f(x)| - |g(x)| + h(x) = \begin{cases} -1 & \text{if } x < -1 \\ 3x + 2 & \text{if } -1 \leq x \leq 0 \\ -2x + 2 & \text{if } x > 0 \end{cases}$$

2. Let $p(x)$ be a polynomial that is nonnegative for all real x. Prove that for some k, there are polynomials $f_1(x), \ldots, f_k(x)$ such that

$$p(x) = \sum_{j=1}^{k} (f_j(x))^2.$$

3. Prove that there is a constant C such that, if $p(x)$ is a polynomial of degree 1999, then

$$|p(0)| \leq C \int_{-1}^{1} |p(x)| \, dx.$$

4. Say that a polynomial with real coefficients in two variables x and y is balanced if the average value of the polynomial on each circle centered at the origin is 0. The balanced polynomials of degree at most 2009 form a vector space V over R. Find the dimension of V.