Problems for 2018 University of Texas Putnam Prep Session, week 6 (Oct 25)

Probability

1. Two coins are found in a fountain. One is a fair coin and the other has “heads” on both sides. One coin is chosen randomly and flipped 10 times. All 10 times it lands “heads” face up. What is the probability that the fair coin was chosen?

2. Sammy and Bevo each choose a real number at random between 1 and 10, inclusive. What is the probability that they differ by more than 4?

3. There are 1,000 points equally spaced on a circle of radius 10. Six points are chosen randomly; call them A, B, C, D, E and F (in some order). What is the probability that the triangles ADC and BEF do not intersect each other?

4. All thirteen spades in a deck of cards are shuffled uniformly and dealt in a line. Let S be a statement about the order of the thirteen cards and $P(S)$ be the probability that S is true. For example, suppose S is “The five appears before the nine”; then $P(S) = 1/2$. How many of the values $1, 1/2, 1/3, \ldots, 1/50$ can $P(S)$ attain?

5. Shanille OKeal shoots free throws on a basketball court. She hits the first and misses the second, and thereafter the probability that she hits the next shot is equal to the proportion of shots she has hit so far. What is the probability that she hits exactly 50 of her first 100 shots?

6. You have coins $C_1, C_2, \ldots, C_n$. For each k, coin C_k is biased so that, when tossed, it has probability $1/(2k + 1)$ of falling heads. If the n coins are tossed, what is the probability that the number of heads is odd? Express the answer as a rational function of n.

7. If α is an irrational number, $0 < \alpha < 1$, is there a finite game with an honest coin such that the probability of one player winning the game is α? (An honest coin is one for which the probability of heads and the probability of tails are both $1/2$. A game is finite if with probability 1 it must end in a finite number of moves.)