Here is a grab bag of problems for you to try.

1. Show that the sum of all terms in the nth row of Pascal's Triangle is a multiple of 2^n.

2. Prove that

$$\frac{22}{7} - \pi = \int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$$

Remark: This proves that π is NOT equal to $22/7$!

3. Prove that the decimal part of

$$(5+\sqrt{26})^n$$

begins with either n zeros or n nines, for all positive integers n.

4. Let A and B be 2×2 matrices with integer entries such that $A, A+B, A+2B, A+3B$, and $A+4B$ are all invertible matrices whose inverses have integer entries. Show that $A+5B$ is also invertible and its inverse has integer entries.

5. Prove that there exists a unique function f from the set $R+$ of positive real numbers to $R+$ such that

$$f(f(x)) = 6x - f(x) \text{ and } f(x) > 0 \qquad \text{for all } x > 0.$$

6. Suppose you are given a set of n red dots and n blue dots in the plane. Show that there is a way to join all the dots in red-blue pairs using straight line segments that do not intersect. (That is, there is a permutation σ such that when each red point r_i is joined to the blue point $b_{\sigma(i)}$ with a line segment L_i, then the line segments are pairwise disjoint: $L_i \cap L_j = \phi$ for every $i \neq j$.)