

Here are some Putnam- or Putnam-like questions that can at least be *stated* to a freshman Calculus student. (I would not expect a typical freshman to have much idea of how to solve them, though!)

1. Find the maximum value of the function

$$F(y) = \int_0^y \sqrt{x^4 + (y - y^2)^2} dx$$

2. Show that this improper integral converges:

$$\int_0^\infty \sin(x) \sin(x^2) dx$$

3. Evaluate

$$\sum_{n=2}^\infty \log \left(\frac{n^3 - 1}{n^3 + 1} \right)$$

4. Evaluate

$$\int_0^\infty \frac{\arctan(\pi x) - \arctan(x)}{x} dx$$

5. Suppose that f is differentiable and that $f'(x)$ is strictly increasing on $[0, \infty)$. Suppose further that $f(0) = 0$. Prove that $g(x) = f(x)/x$ is strictly increasing on $(0, \infty)$