Week 1, Problem 4.

We discussed the use of "Summation by Parts" as a tool to estimate the partial sums of this series, and thus to deduce its convergence.

Dustan L sent me an (informal) proof that the series does NOT converge absolutely (with other details presented a the Putnam meeting). Here's his argument:

`By the proportion of terms in a subsequence, I mean the limit of the proportion of terms included in the subsequence up to the kth term of the original sequence as k goes to infinity. In this context, we could also take the proportion to be the liminf of the truncated proportions, to allow for more general subsequences.`

`It's not too difficult to show that when you add up a nonzero proportion of the reciprocals of natural numbers, you get a divergent series; you simply need to bound the series below by sum(1/(Kn)) for a sufficiently large K. If N is a number such that after the Nth term, the truncated proportion is always at least 1/p, then K=N+p will easily be sufficient (and overkill, but who cares?).`

`So to show that the terms of sin(n)/n captured by a window around the point (0,1) in the unit circle are divergent, you simply need the result that those terms represent a nonzero proportion of the set of all terms. This might also be overkill: I think that would probably have to involve showing that the natural number angles are equidristributed in the unit circle, which is a tricky but interesting proof in its own right.`

There is still the question of what this series converges to. I remember now how to do that one. We can talk more on Thursday.