**Week 2, Problem 10:**

Proof:

Let a,b,c > 0. By the symmetry of the terms on the left hand side of the inequality, we may assume, without loss of generality, that

(a²b + b²c + c²a) ≥ (ab² + bc² + ca²)

So we have

(a²b + b²c + c²a)(ab² + bc² + ca²) ≥ (ab² + bc² + ca²)²

We will show that

(ab² + bc² + ca²)² ≥ 9abc

This is,

(ab² + bc² + ca²)² - (3abc)² ≥ 0

We see that this is a difference of squares, so after factoring, we get

(ab² + bc² + ca² +3abc)(ab² + bc² + ca² -3abc) ≥ 0

The left hand term is clearly positive, so it suffices to show that

abc((b/c) + (c/a) + (a∕b) -3) ≥ 0

Clearly abc > 0, so we must show that

(1/3)[(b/c) + (c/a) +(a/b)] ≥ 1

We see that this is just the arithmetic mean of the terms inside the brackets, and since the geometric mean is bounded by the arithmetic mean, we have,

(1/3)[(b/c) + (c/a) +(a/b)] ≥ ∛[(b/c)(c/a)(a/b)] = ∛1 = 1

as required. We conclude that the inequality holds for all a,b,c > 0.