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Cooperation search algorithm: A novel metaheuristic evolutionary intelligence algorithm for numerical optimization and engineering optimization problems

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ABSTRACT

This paper develops a novel population-based evolutionary method called cooperation search algorithm (CSA) to address the complex global optimization problem. Inspired by the team cooperation behaviors in modern enterprise, the CSA method randomly generates a set of candidate solutions in the problem space, and then three operators are repeatedly executed until the stopping criterion is met: the team communication operator is used to improve the global exploration and determine the promising search area; the reflective learning operator is used to achieve a comprise between exploration and exploitation; the internal competition operator is used to choose solutions with better performances for the next cycle. Firstly, three kinds of mathematical optimization problems (including 24 famous test functions, 25 CEC2005 test problems and 30 CEC2014 test problems) are used to test the convergence speed and search accuracy of the CSA method. Then, several famous engineering optimization problems (like Gear train design, Welded beam design and Speed reducer design) are chosen to testify the engineering practicality of the CSA method. The results in different scenarios demonstrate that as compared with several existing evolutionary algorithms, the CSA method can effectively explore the decision space and produce competitive results in terms of various performance evaluation indicators. Thus, an effective tool is provided for solving the complex global optimization problems.

1. Introduction

In the following sections, three different aspects will be given in details: the first is to introduce the existing research results; the second is to demonstrate the motivation and contributions of this paper; and the last is to present the organizational structure of this research.

1.1. Background and related work

In practice, a large number of engineering problems are required to find the best solution in the search space [1–4]. Without loss of generality, the common constrained minimization optimization problems can be expressed as the following model:

$$\begin{aligned} &\min f(\boldsymbol{x}), \quad \boldsymbol{x} = \left[x_1, \ldots, x_j, \ldots, x_J\right] \in R^J \\ &s.t.\ g_e(\boldsymbol{x}) \le 0 \qquad e = 1, 2, \ldots, E \\ &\quad h_f(\boldsymbol{x}) = 0 \qquad f = 1, 2, \ldots, F \\ &\quad \underline{x}_j \le x_j \le \bar{x}_j \qquad j = 1, 2, \ldots, J \end{aligned} \tag{1}$$

where $f(\boldsymbol{x})$ is the objective value of the J-dimensional solution $\boldsymbol{x}$. x_j is the value of the jth variable in solution $\boldsymbol{x}$. $\bar{x}_j$ and $\underline{x}_j$ are the upper and lower limits of the jth variable. J is the number of decision variables. $g_e(\boldsymbol{x})$ is the eth inequality constraint. $h_f(\boldsymbol{x})$ is the fth equality constraint. E and F are the number of inequality and equality constraints.

As used to address the high-dimensional problems, the classical optimization methods may fail to yield feasible solutions because the computational burden usually grows exponentially with the number of decision variables [5–7]. For instance, the deep learning-based network is a popular and effective tool for addressing regression and classification problems [8–10], but usually fall into local minimum with a high probability because it is difficult for traditional method to determine the ideal combination of thousands of parameters [11–13]. In the past few decades, many scholars are devoted to developing the metaheuristic methods with the advantages of easy implementation, high flexibility, free gradient calculation and potential parallelization [14–16]. A variety of metaheuristic methods have been successfully developed for solving local versus global optimization problems, like task scheduling problem, strongly connected components problem, minimum vertex cover problem, and static scheduling. According to the employed search mechanism and thought

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origin, the existing metaheuristic methods can be roughly divided into three different categories [17–19]: evolution-based algorithms, physics-based algorithms and population-based algorithms. The evolution-based algorithms simulate the evolution laws of natural species [20], and the representatives include the genetic algorithm, differential evolution, evolution strategy, humpback whale optimization [21], sea lion optimization algorithm [22], and grey wolf optimization. The physics-based algorithms mimic the physical rules in universe [23], and its representatives include gravitational search algorithm [24], black hole algorithm, curved space optimization, and chemical reaction optimization [25,26]. The population-based algorithms imitate the behaviors of natural animals or human society [27], and its representatives include particle swarm optimization, artificial bee colony, teaching–learning optimization, and most valuable player algorithm [28].

1.2. Motivation and contributions

For almost all the metaheuristic algorithms, the search procedures are often composed of two aspects: exploration for traversing in the entire problem space, and exploitation for investigating the small promising areas [29,30]. However, it is not easy for the metaheuristic algorithms to achieve a comprise between exploration and exploitation due to the randomness rooted in the search process. By far, no universal methods are found to be suitable for all the problems. In other words, one method may produce satisfying solutions for some particular problems but fail to achieve it in other problems [31]. As a result, it is of great importance to further develop some new and effective metaheuristic methods for real-world problems with unknown decision spaces.

Motived by this practical necessity, this research successfully develops a novel metaheuristic method called Cooperation search algorithm (CSA). In CSA, three operators are designed to help seek out high-quality solutions: the team communication operator for global exploitation, the reflective learning operator for local exploration, and the dualistic competition operator for survival of the fittest. The practicability of the CSA method is fully verified by 79 mathematical optimization problems (24 classical benchmark functions, 25 CEC2005 test problems and 30 CEC2014 test problems) and several engineering optimization problems. The results show that the CSA method can yield better results than several existing methods with respect to the employed evaluation indicators, providing a new alternative for the complex global optimization problems.

1.3. Paper structure

The rest of this paper is organized as below. Section 2 gives the details of the developed method. Numerical experiments are used to verify the CSA performances in Section 3. The CSA method is used to solve engineering design problems in Section 4. The conclusions are given in the end.

2. Cooperation search algorithm (CSA)

In order to provide a novel alternative tool for global optimization problems, the details of the CSA method are given in this section: the team cooperation behaviors in modern enterprises is firstly introduced; and then the search principle of the proposed CSA method is given; finally, the computational complexity of the CSA method is briefly analyzed.

2.1. Inspiration from team cooperation behaviors in modern enterprises

In recent years, all kinds of companies are playing an increasingly important role in the healthy social and economic development of the world, and the team cooperation behavior is the key to the normal operation of one company. Generally, four different kinds of positions are often used in the company teamwork process, including the board of directors, the board of supervisors, the chairman and the staff. The board of directors, composed of the elected individuals to represent shareholders, externally conducts the company's business activities and internally manages the productive tasks. In other words, all the affairs and business of the company are conducted under the leadership of the board of directors. The board of supervisors is asked to supervise the executive directors and promote the shareholders' interests. Compared with the board of directors, the board of supervisors cannot internally participate in the company's business decision-making process and cannot externally represent the company. As an executive elected from the board of directors, the chairman is chiefly responsible for the scientific operations of the company. Besides, as the spokesperson of the company, the chairman often has a great or even conclusive influence on the company to guarantee the smooth and ordered running before achieving a consensus in the board decisions. The staff is asked to engage the specific works under the leadership of the board of directors, which usually have the right to select the members in the board of supervisors and the board of directors.

It is well known that human beings are one of the most important factors in raising productivity, which indicates that improving the staff strength is the key factor in the scientific development of the company. To realize this goal, it is necessary to help the individual staffs gain knowledge as much as possible. Generally, the staff knowledge can be simultaneously affected by a group of leaders in the board of directors, board of supervisors and chairman. The chairman on duty often has the largest influence due to its highest position in the team, while the members in the board of directors and supervisors provide abundant information to reduce or even avoid possible mistakes. After a period of time, each staff is encouraged to consider the self-improvement ways and given an opportunity to take the place of its superior leaders once its performance is better. In other words, the chairman, as well as the members in the board of directors and supervisors can be always dynamically updated to promote the company's market competitiveness. Fig. 1 shows the sketch map of the team cooperation relationship in modern enterprise. It can be found that there are close connections among the members of various social status; the under-performing staffs or even leaders may be replaced by the promising young people while the ordinary staffs also have the chances to enhance their knowledge and promote job positions by hard work. In this way, the enterprise can stay active to realize the sustainable development.

2.2. Search principle of the CSA method

In this section, this paper develops a novel CSA method for the global optimization problem. In CSA, the optimization process of the target problem is seen as the development of an enterprise; each solution is seen as a staff while a set of staffs form an enterprise team; the performance per staff is equal to the fitness value of the problem at hand; the board of supervisors are composed of the personal best-known solutions; the board of directors are composed of the external archive set (M global best-known solutions found by far); the chairman on duty is randomly chosen from the board of directors. Then, the population can

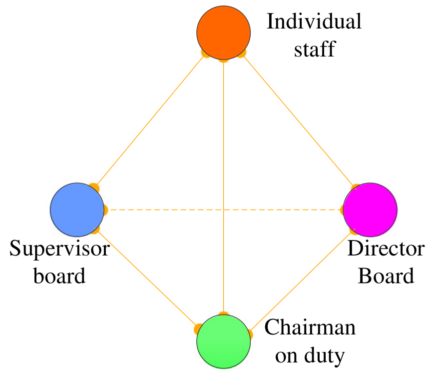

Fig. 1. Sketch map of the company team relationship.

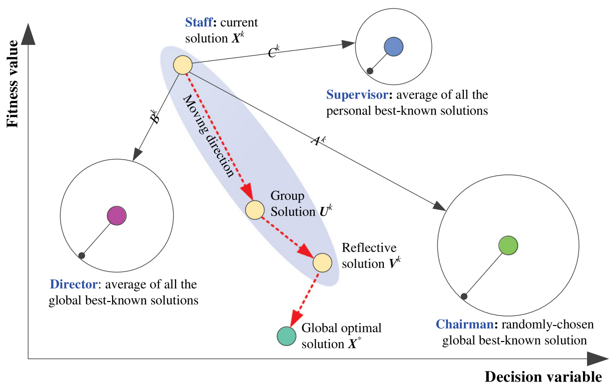

Fig. 2. Sketch map of the CSA method.

gradually find high-quality solutions by using three evolutionary operators by imitating the team cooperation behaviors in modern enterprise: the team communication operator is used to help the staff capture beneficial knowledge from leaders; the reflective learning operator is used to improve the staff's comprehensive strength by drawing lessons from the past; the internal competition operator is used to enhance the work experiences and leadership vision of the elite solutions. Next, the technical details of the CSA method are given as below:

(1) Team building phase. In this stage, all the staffs in the team are randomly determined by Eq. (2). After evaluating the performances of all the solutions, $M\in[1, I]$ leader solutions will be chosen from the initial swarm to form the external elite set.

$$x_{i,j}^{k}=\phi\left(\underline{x}_{j},\overline{x}_{j}\right),i\in[1,I],j\in[1,J],k=1 \tag{2}$$

where I is the number of solutions at the current swarm. $x_{i,j}^{k}$ is the jth value of the ith solution at the kth cycle. $\phi(L,U)$ is the function to generate a random number uniformly distributed in the range of $[L, U]$.

(2) Team communication operator. Each staff can gain new message by exchanging information with the leaders in the chairman as well as the board of directors and supervisors. As showed in Eq. (3), the team communication process involves three parts: the chairman's knowledge A, the collective knowledge B from the board of directors and the collective knowledge C from the board of supervisors. The chairman is randomly chosen from the board of directors to simulate the rotating mechanism, while all the members in the board of directors and supervisors are given the same positions in calculating B and C.

$$u_{i,j}^{k+1}=x_{i,j}^{k}+A_{i,j}^{k}+B_{i,j}^{k}+C_{i,j}^{k},\ i\in[1,I],j\in[1,J],k\in[1,K] \tag{3}$$

$$A_{i,j}^{k}=\log\left(1/\phi(0,1)\right)\cdot\left(gbest_{ind,j}^{k}-x_{i,j}^{k}\right) \tag{4}$$

$$B_{i,j}^{k}=\alpha\cdot\phi(0,1)\cdot\left[\frac{1}{M}\sum_{m=1}^{M}gbest_{m,j}^{k}-x_{i,j}^{k}\right] \tag{5}$$

$$C_{i,j}^{k}=\beta\cdot\phi(0,1)\cdot\left[\frac{1}{I}\sum_{i=1}^{I}pBest_{i,j}^{k}-x_{i,j}^{k}\right] \tag{6}$$

where $u_{i,j}^{k+1}$ is the jth value of the ith group solution at the k+1th cycle. $pBest_{i,j}^{k}$ is the jth value of the ith personal best-known solution at the kth cycle. $gBest_{ind,j}^{k}$ is the jth value of the indth global best-known solution from the beginning to the kth cycle. ind is the index randomly selected from the set of $\{1,2,\ldots, M\}$. $A_{i,j}^{k}$ denotes the knowledge gained from the chairman randomly chosen from the external elite set. $B_{i,j}^{k}$ and $C_{i,j}^{k}$ are the mean knowledge gained from M global best-known solutions found by far and I personal best-known solutions, respectively. α and β are the learning coefficients to adjust the influence degrees of $B_{i,j}^{k}$ and $C_{i,j}^{k}$.

(3) Reflective learning operator. Except for learning from the leader solutions, the staffs can also gain new knowledge by summing its own experience in their opposite direction, which can be expressed as below:

$$v_{i,j}^{k+1}=\begin{cases}r_{i,j}^{k+1} & \text{if } \left(u_{i,j}^{k+1}\geq c_j\right)\\ p_{i,j}^{k+1} & \text{if } \left(u_{i,j}^{k+1}<c_j\right)\end{cases},\ i\in[1,I],j\in[1,J],k\in[1,K] \tag{7}$$

$$r_{i,j}^{k+1}=\begin{cases}\phi\left(\overline{x}_j+\underline{x}_j-u_{i,j}^{k+1},c_j\right) & \text{if } \left(\left|u_{i,j}^{k+1}-c_j\right|<\phi(0,1)\cdot\left|\overline{x}_j-\underline{x}_j\right|\right)\\ \phi\left(\underline{x}_j,\overline{x}_j+\underline{x}_j-u_{i,j}^{k+1}\right) & \text{otherwise}\end{cases} \tag{8}$$

$$p_{i,j}^{k+1}=\begin{cases}\phi\left(c_j,\overline{x}_j+\underline{x}_j-u_{i,j}^{k+1}\right) & \text{if } \left(\left|u_{i,j}^{k+1}-c_j\right|<\phi(0,1)\cdot\left|\overline{x}_j-\underline{x}_j\right|\right)\\ \phi\left(\overline{x}_j+\underline{x}_j-u_{i,j}^{k+1},\overline{x}_j\right) & \text{otherwise}\end{cases} \tag{9}$$

$$c_j=\left(\overline{x}_j+\underline{x}_j\right)\cdot 0.5 \tag{10}$$

where $v_{i,j}^{k+1}$ is the jth value of the ith reflective solution at the k+1th cycle.

(4) Internal competition operator. The team gradually upgrades its market competitiveness by guaranteeing that all the staffs with better performances are always conserved, which can be expressed as below:

$$x_{i,j}^{k+1}=\begin{cases}u_{i,j}^{k+1} & \text{if } \left(F\left(\boldsymbol{u}_{i}^{k+1}\right)\leq F\left(\boldsymbol{v}_{i}^{k+1}\right)\right)\\ v_{i,j}^{k+1} & \text{if } \left(F\left(\boldsymbol{u}_{i}^{k+1}\right)>F\left(\boldsymbol{v}_{i}^{k+1}\right)\right)\end{cases},\ i\in[1,I],j\in[1,J],k\in[1,K] \tag{11}$$

where $F(\boldsymbol{x})$ is the fitness value of the solution $\boldsymbol{x}$. To effectively multiple physical constraints, all the variables in $\boldsymbol{x}$ are firstly modified to the feasible zone by Eq. (12), and then the penalty functions method in Eq. (13) is used to obtain the fitness value $F(\boldsymbol{x})$ by merging the constraint violation value into the objective value $f(\boldsymbol{x})$. Then, for feasible solutions, all the constraints are well met so that the fitness value is equal to the original objective value; for infeasible solutions, the constraint violation value becomes positive so that the fitness value is larger than the

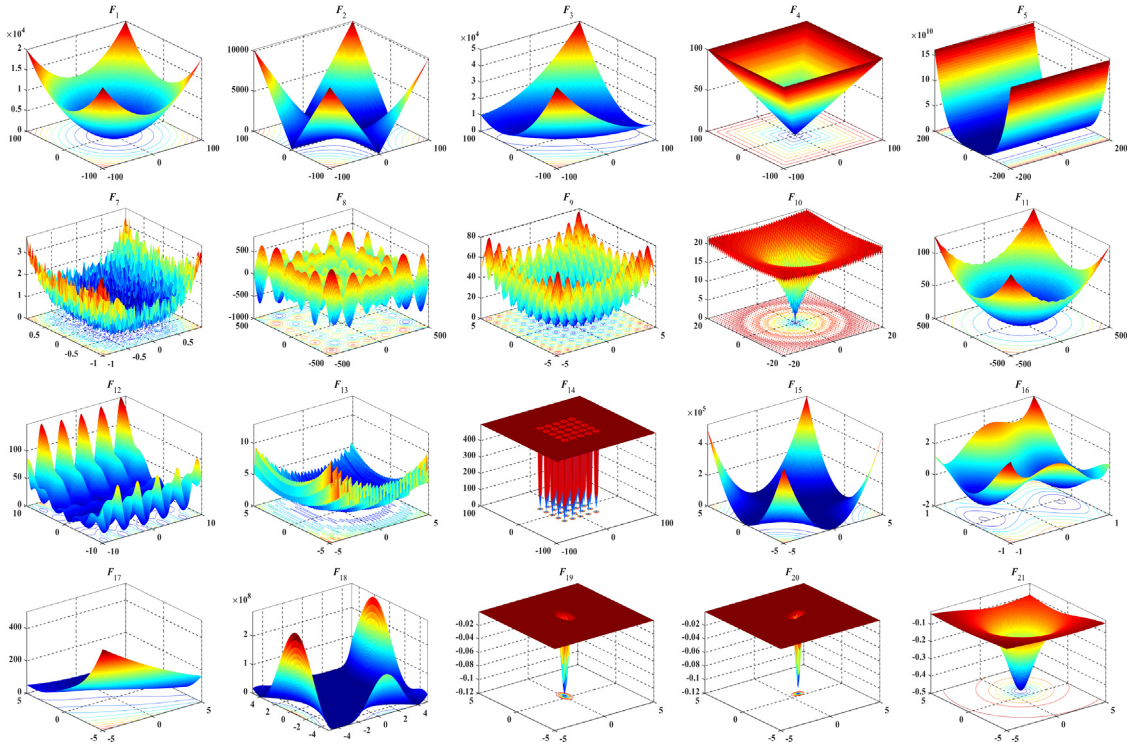

Fig. 3. 2-D shape of some selected benchmarks.

objective value. In this way, the swarm can be guided to feasible search area as far as possible.

$$x_j = \max\left\{\min\left\{\overline{x}_j, x_j\right\}, \underline{x}_j\right\} \tag{12}$$

$$F(\boldsymbol{x}) = f(\boldsymbol{x}) + \sum_{e=1}^{E} c_e^1 \cdot \max\{g_e(\boldsymbol{x}), 0\} + \sum_{f=1}^{F} c_f^2 \cdot \left|h_f(\boldsymbol{x})\right| \tag{13}$$

where x_j is the jth value in the solution $\boldsymbol{x}$ to be evaluated. c_e^1 is the penalty coefficient for the eth inequality constraint. c_f^2 is the penalty coefficient for the fth inequality constraint.

The pseudo-code of the CSA method is given as below:

Via the above carefully-designed operators, the CSA method in Fig. 2 can effectively improve the quality of all the obtained solutions to approximate the global optima. Next, the traits of the CSA method are summarized as below:

(1) Compared with the individual-based methods, the population-based evolutionary mechanism used in CSA creates multiple solutions in the search space, which can help find the promising regions and jump out of local optima.
(2) The swarm is able to achieve a balance between global exploitation and local exploration via the team communication and reflective learning operators, which can increase the probability of approximating the global optimal solution.
(3) With the aid of the internal competition operator, the better solutions found by far are stored and dynamically updated during the evolutionary process, which can effectively guarantee the global convergence of the population.
(4) The optimization problem is seen as a black box whose output is only related with its specific inputs, and then the CSA method can be theoretically applied to any optimization problems in the form of Eq. (1). Hence, operator can pay attention to the modeling process, rather than the development of optimization algorithm, which can effectively improve the work efficiency.
(5) In CSA, the original large swarm can be naturally divided into several small but independent subpopulations that can be implemented in multiple different computing units. In other words, it is easy to develop a parallel version of CSA to improve the execution time and solution quality under the high-performance computing environment.

2.3. Computational complexity of the CSA method

Generally, the computational complexity of the metaheuristic methods is an important element considered in practice. For the sake of simplicity, it is assumed that I solutions and K iterations are involved in the optimization process of a J-variable problem, while the fitness value evaluation per solution is much larger than the other calculations. In CSA, the total number of evaluations is I+2IK because I solutions are evaluated in the initial phase while I group and I reflective solutions are evaluated per cycle. Thus, the time complexity of the CSA method is about $O(I+2IK)$, which can be expressed as $O(IK)$ when K or I becomes infinite. Besides, in the whole iterative process, I solutions in 3 versions (original, group and reflective), I personal best-known solutions and M global best-known solutions should be stored, which means that the memory requirement is about $4IJ+MJ$. Given that $1 \le M \le I$, the space complexity of the CSA method in the worst case is about $O(5IJ)$, which can be expressed as $O(IJ)$ when J or I becomes infinite.

3. Numerical experiments to verify the CSA performance

In this section, three different test problems composed of 79 functions are employed to verify the performances of the CSA method.

```
Define the objective function and all the physical constraints.
Initialization
    Generate randomly the initial swarm in the feasible space by Eq. (2).
    Compute the fitness values of the initial solutions by Eq. (13).
    Make the group and reflective solutions be equal to its initial one.
End Initialization
Repeat search process
    Update I personal best-known solutions for the current swarm.
    Update M global best-known solutions found by far.
    Obtain I group solutions by Eqs. (3)~(6) for global exploitation.
    Obtain I reflective solutions by Eqs.(7)~(10) for local exploration.
    Compute the fitness values of the group and reflective solutions by Eq. (13).
    Use Eq. (11) to choose I better solutions for the next cycle.
While (the counter k≤ maximum iterations K)
Export the global best-known individual as the final solution for the problem.
```

3.1. Benchmark functions set I: 24 famous test problems

3.1.1. Benchmark functions

The selected problems can be roughly divided into three different kinds of categories: unimodal functions with changing variables, multimodal functions with changing variables and multimodal functions with fixed variables. Generally, unimodal functions with one global optimum can verify the convergence rate of evolutionary algorithms, multimodal functions with multiple local optima can test the ability of avoiding premature convergence. Table 1 shows the detailed information of 24 benchmarks functions, while the 2-D shapes of some selected functions are given in Fig. 3. It can be found that the selected functions have obvious differences in the response surface of the objective functions, which can help fully check the performance of various algorithms.

3.1.2. Parameters setting

To fully verify the feasibility of the CSA method, the detailed computation results of several existing evolutionary methods are introduced, including the lightning search algorithm (LSA) [32], differential search algorithm (DSA) [33], backtracking search algorithm (BSA) [34], firefly algorithm (FA) [35], harmony search (HS) [36], particle swarm optimization (PSO), gravitational search algorithm (GSA), and sine cosine algorithm (SCA). Four methods (PSO, SCA, GSA and CSA) developed in JAVA language are obtained in 20 independent runs while the results of other methods are taken from previous literature. For the sake of fairness, the parameters of all the developed methods should be defined as the same value as much as possible. Thus, the number of solutions and maximum iterations in PSO, SCA, GSA and CSA are set as 50 and 1000, while the values of other parameters are chosen from the previous literatures:

PSO: The inertia weight w is linearly decreased from the initial value 0.9 to the final value 0.1, while two learning factors (c_1 and c_2) are set as 2.0, which is similar to that of Ref. [37].

GSA: The attenuation factor a and initial gravitational constant G_0 are set as 20 and 100 based on Ref. [24].

SCA: The computational constant a is set as 2.0 as mentioned in Ref. [38].

CSA: The computational coefficients (a, β and M) are set as 0.10, 0.15 and 3.

3.1.3. Statistical results analysis

(1) Unimodal functions

As mentioned above, the unimodal functions ($F_1 \sim F_{13}$ and F_{24}) have only one global minimum and evaluates the exploitation ability of the developed method. Table 2 gives the statistical results for unimodal benchmark functions by different methods in 20 independent runs, including the best, median, mean, worst and standard deviation (STD) values of objective functions. From Table 2, it can be found that for almost all the unimodal functions, the CSA method is superior to the control methods in terms of all the measures. Hence, the CSA method is able to produce satisfying results for unimodal functions by effectively exploiting the search space.

(2) Multimodal functions

The multimodal functions ($F_{14} \sim F_{23}$) with a mass of local minimum can sharply increase the risk of premature convergence, which can evaluate the exploration ability of evolutionary algorithms. The statistical results of multimodal functions by different methods are given in Table 3. It can be found that as compared with the existing evolutionary methods, the proposed method can produce solutions with better approximations to the optimal objective value. Thus, it can be concluded that the CSA method has satisfying performances in finding the global optimal solution.

(3) Computational time

The computation time of several methods for test functions are given in Table 4. It can be found that the computational time of the proposed method is smaller than the GSA method and slightly larger than PSO and SCA, demonstrating its satisfying execution efficiency.

3.1.4. Wilcoxon nonparametric test results

To obtain a valuable statistical conclusion, the famous Wilcoxon nonparametric test method is introduced to compare the performances of the CSA method and other methods. Table 5 shows the Wilcoxon signed rank test results for 24 famous benchmark functions at alpha = 0.05, where the average objective value per function is chosen as the samples for testing. In Table 5, the symbol R^+ or R^- indicate that the CSA method has better or worse performances than the control one; the "Better", "Equal" and "Worse" indicate the numbers of the test functions where the CSA method is better, equal or worse than the control one; the value of P indicates the significance level, and when it is less than 0.05, it can be said that two methods have obvious differences; the symbol "+", and "≈" means that the CSA performance is better than or equal to the control method. From the data in Table 5, it can be found that the CSA method can obtain a larger number of two symbols ("Better" and R^+), while the P values are less than 0.05 in most cases. Hence, the above analysis proves the conclusion that the CSA method is statistically significant than the other methods.

3.1.5. Box plot analysis

In this section, the Box and whisker is chosen to show the distributions of the objective values obtained in different runs

Table 1
Detailed information of 24 benchmarks functions.

Function	*Dim*	*Range*	$f_{\min}$	Type
$F_1(x)=\sum_{i=1}^{n} x_i^2$	30	[−100, 100]	0	Unimodal
$F_2(x)=\sum_{i=1}^{n}\lvert x_i\rvert+\prod_{i=1}^{n}\lvert x_i\rvert$	30	[−10, 10]	0	Unimodal
$F_3(x)=\sum_{i=1}^{n}(\sum_{j=1}^{i}x_j)^2$	30	[−100, 100]	0	Unimodal
$F_4(x)=\max\{\lvert x_i\rvert, 1\le i\le n\}$	30	[−100, 100]	0	Unimodal
$F_5(x)=\sum_{i=1}^{n-1}[100(x_{i+1}-x_i^2)^2+(x_i-1)^2]$	30	[−30, 30]	0	Unimodal
$F_6(x)=\sum_{i=1}^{n}(\lfloor x_i+0.5\rfloor)^2$	30	[−100, 100]	0	Unimodal
$F_7(x)=\sum_{i=1}^{n} ix_i^4+random[0,1)$	30	[−1.28, 1.28]	0	Unimodal
$F_8(x)=\sum_{i=1}^{n}-x_i\sin(\sqrt{\lvert x_i\rvert})$	30	[−500, 500]	−12567	Multimodal
$F_9(x)=\sum_{i=1}^{n}[x_i^2-10\cos(2\pi x_i)+10]$	30	[−5.12, 5.12]	0	Multimodal
$F_{10}(x)=-20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2})-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i))+20+e$	30	[−32, 32]	0	Multimodal
$F_{11}(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	30	[−600, 600]	0	Multimodal
$F_{12}(x)=\frac{\pi}{n}\{10\sin 2(\pi y_1)+\sum_{i=1}^{n-1}(y_i-1)^2[1+10\sin 2(\pi y_{i+1})]$ $+(y_n-1)^2\}+\sum_{i=1}^{n}u(x_i,10,100,4)$ $y_i=1+\frac{x_i+1}{4},\ u(x_i,a,k,m)=\begin{cases}k(x_i-a)^m & x_i>a\\ 0 & -a\le x_i<a\\ k(-x_i-a)^m & x_i<-a\end{cases}$	30	[−50, 50]	0	Multimodal
$F_{13}(x)=0.1\{\sin 2(3\pi x_1)+\sum_{i=1}^{n}(x_i-1)^2[1+\sin 2(3\pi x_{i+1}+1)]$ $+(x_n-1)^2[1+\sin 2(2\pi x_n)]\}+\sum_{i=1}^{n}u(x_i,5,100,4)$	30	[−50, 50]	0	Multimodal
$F_{14}(x)=[\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6}]^{-1}$	2	[−65.536, 65.536]	1	Fixed
$F_{15}(x)=\sum_{i=1}^{11}\left[a_i-\frac{x_1(b_i^2+b_ix_2)}{b_i^2+b_ix_3+x_4}\right]^2$	4	[−5, 5]	0.0003075	Fixed
$F_{16}(x)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	2	[−5, 5]	−1.0316285	Fixed
$F_{17}(x)=(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6)^2+10(1-\frac{1}{8\pi})\cos x_1+10$	2	[−5, 10] × [0, 15]	0.398	Fixed
$F_{18}(x)=[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2$ $+6x_1x_2+3x_1^2)]\times[30+(2x_1-3x_2)^2(18-32x_1$ $+12x_1^2+48x_2-36x_1x_2+27x_2^2)]$	2	[−2, 2]	3	Fixed

(*continued on next page*)

because it can provide abundant information (like the minimum, median, second or third quartile, maximum) of the testing samples. Fig. 4 shows the Box plot of several methods for 24 classic benchmark functions. It can be found that the objective distributions of the CSA method are obviously smaller than the other methods, demonstrating its robust performances in the test problems.

3.1.6. *Convergence trajectory comparison*

Fig. 5 shows the convergence corves of several methods for 24 benchmark functions, where the horizontal axis and vertical axis

Table 1 (*continued*).

Function	Dim	Range	$f_{\min}$	Type
$F_{19}(x) = -\sum_{i=1}^{4} c_i \exp[-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2]$	3	[0, 1]	−3.86	Fixed
$F_{20}(x) = -\sum_{i=1}^{4} c_i \exp[-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2]$	6	[0, 1]	−3.32	Fixed
$F_{22}(x) = -\sum_{i=1}^{5} [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	[0, 10]	−10.1532	Fixed
$F_{23}(x) = -\sum_{i=1}^{7} [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	[0, 10]	−10.4028	Fixed
$F_{23}(x) = -\sum_{i=1}^{10} [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	[0, 10]	−10.5363	Fixed
$F_{24}(x) = \sum_{i=1}^{n} \lvert x_i \rvert^{i+1}$	30	[−100, 100]	0	Unimodal

Table 2
Statistical values of several methods for 30-variable unimodal functions.

Function	Item	LSA	DSA	BSA	FA	HS	PSO	SCA	GSA	CSA
F_1	Best	1.06E−19	9.40E−01	1.43E+00	5.03E−03	1.13E+01	9.07E−02	1.61E−07	9.08E−12	**0.00E+00**
	Median	1.86E−15	9.50E+00	7.03E+00	1.15E−02	2.39E+01	3.71E−01	2.68E−05	3.00E−11	**0.00E+00**
	Mean	4.81E−08	1.16E+01	9.97E+00	1.20E−02	2.47E+01	4.27E−01	4.61E−04	2.97E−11	**0.00E+00**
	Worst	2.41E−06	3.72E+01	5.36E+01	2.65E−02	3.92E+01	1.27E+00	3.08E−03	7.37E−11	**0.00E+00**
	STD	3.40E−07	6.94E+00	9.81E+00	4.30E−03	6.67E+00	2.59E−01	8.82E−04	1.52E−11	**0.00E+00**
F_2	Best	2.22E−07	4.21E−01	4.55E−01	1.72E−01	9.53E−01	4.74E−01	1.47E−08	1.94E−06	**0.00E+00**
	Median	1.07E−03	9.21E−01	1.14E+00	3.70E−01	1.43E+00	1.82E+00	1.24E−06	5.19E−06	**0.00E+00**
	Mean	3.68E−02	1.01E+00	1.20E+00	3.73E−01	1.46E+00	5.89E+00	6.42E−06	5.09E−06	**0.00E+00**
	Worst	9.70E−01	1.94E+00	2.88E+00	6.54E−01	2.25E+00	2.05E+01	7.36E−05	9.34E−06	**0.00E+00**
	STD	1.56E−01	3.58E−01	5.29E−01	1.01E−01	2.68E−01	5.89E+00	1.64E−05	1.82E−06	**0.00E+00**
F_3	Best	9.20E+00	7.18E+03	7.18E+02	7.16E+02	2.98E+03	4.83E+01	7.28E+00	1.36E+02	**0.00E+00**
	Median	3.48E+01	2.16E+04	2.54E+03	1.87E+03	6.70E+03	1.17E+02	2.35E+03	3.00E+02	**0.00E+00**
	Mean	4.32E+01	2.09E+04	2.72E+03	1.81E+03	6.88E+03	1.16E+02	2.69E+03	3.24E+02	**0.00E+00**
	Worst	1.26E+02	3.56E+04	7.43E+03	4.06E+03	1.26E+04	1.77E+02	1.25E+04	5.17E+02	**0.00E+00**
	STD	2.99E+01	6.91E+03	1.18E+03	6.60E+02	1.94E+03	3.45E+01	2.82E+03	9.81E+01	**0.00E+00**
F_4	Best	1.18E−01	1.20E+01	5.86E+00	5.50E−02	7.39E+00	1.79E+00	1.65E+00	4.43E−06	**7.64E−300**
	Median	8.86E−01	2.71E+01	9.23E+00	7.34E−02	9.13E+00	2.20E+00	9.67E+00	7.59E−06	**7.64E−300**
	Mean	1.49E+00	2.78E+01	9.83E+00	7.67E−02	9.39E+00	2.23E+00	1.11E+01	9.47E−02	**7.64E−300**
	Worst	5.98E+00	4.38E+01	1.53E+01	1.09E−01	1.20E+01	2.67E+00	2.83E+01	1.78E+00	**7.64E−300**
	STD	1.30E+00	7.08E+00	2.27E+00	1.46E−02	1.23E+00	2.32E−01	8.11E+00	3.97E−01	**0.00E+00**
F_5	Best	**5.60E−01**	3.54E+02	2.08E+02	2.79E+01	3.38E+02	1.84E+02	2.82E+01	2.54E+01	2.22E+01
	Median	7.35E+01	1.00E+03	3.94E+02	2.94E+01	6.99E+02	5.10E+02	3.19E+01	2.76E+01	**2.26E+01**
	Mean	6.43E+01	1.11E+03	4.72E+02	1.28E+02	8.30E+02	6.90E+02	4.28E+01	4.35E+01	**2.26E+01**
	Worst	2.02E+02	2.35E+03	1.24E+03	1.85E+03	2.81E+03	2.29E+03	8.87E+01	2.07E+02	**2.31E+01**
	STD	4.38E+01	5.72E+02	2.31E+02	2.79E+02	4.74E+02	5.22E+02	1.92E+01	4.95E+01	**2.12E−01**
F_6	Best	**0.00E+00**	2.00E+00	2.00E+00	**0.00E+00**	1.20E+01	1.10E−01	3.91E+00	1.09E−11	1.05E−31
	Median	3.00E+00	1.20E+01	1.05E+01	**0.00E+00**	2.40E+01	2.77E−01	4.28E+00	4.45E−11	6.36E−31
	Mean	3.34E+00	1.57E+01	1.39E+01	**0.00E+00**	2.51E+01	3.87E−01	4.35E+00	5.51E−11	1.95E−25
	Worst	8.00E+00	4.90E+01	1.15E+02	**0.00E+00**	4.70E+01	9.47E−01	4.87E+00	2.05E−10	3.89E−24
	STD	2.09E+00	1.13E+01	1.73E+01	**0.00E+00**	7.53E+00	2.59E−01	3.05E−01	4.42E−11	8.69E−25
F_7	Best	1.63E−02	4.50E−02	2.27E−02	8.30E−03	2.48E−01	3.04E−01	5.18E−03	7.86E−02	**2.81E−06**
	Median	2.27E−02	1.05E−01	5.21E−02	3.16E−02	4.33E−01	3.41E+00	2.51E−01	5.27E−01	**1.58E−05**
	Mean	2.41E−02	1.23E−01	5.45E−02	3.52E−02	4.63E−01	5.00E+00	3.70E−01	4.78E−01	**2.43E−05**
	Worst	4.08E−02	3.59E−01	1.03E−01	1.16E−01	7.61E−01	1.69E+01	9.45E−01	8.66E−01	**1.24E−04**
	STD	**5.73E−03**	6.53E−02	1.61E−02	2.40E−02	1.13E−01	4.97E+00	3.27E−01	3.08E−01	2.99E−01
F_8	Best	−9.19E+03	**−1.06E+04**	−1.04E+04	−7.40E+03	−8.18E+03	−8.30E+03	−4.62E+03	−3.34E+03	−1.04E+04
	Median	−8.06E+03	**−1.00E+04**	−9.61E+03	−5.87E+03	−7.39E+03	−7.10E+03	−4.09E+03	−2.73E+03	−9.62E+03
	Mean	−8.00E+03	**−1.00E+04**	−9.62E+03	−5.90E+03	−7.40E+03	−7.07E+03	−4.11E+03	−2.72E+03	−9.51E+03
	Worst	−6.49E+03	**−9.44E+03**	−9.10E+03	−4.34E+03	−6.61E+03	−5.74E+03	−3.59E+03	−2.04E+03	−8.42E+03
	STD	6.69E+02	2.79E+02	2.53E+02	6.56E+02	3.50E+02	7.35E+02	**2.49E+02**	3.86E+02	5.80E+02

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represent the iteration number and fitness values. It can be found that in most unimodal or multimodal functions, the CSA method can quickly seek out satisfying solutions at the early evolutionary stage while the other methods fail to make it; as the iteration number is close to the maximum, the quality of the solutions obtained by the CSA approach is higher than the other methods.

Fig. 6 shows the searching histories of the CSA method for 4 test functions. It can be found that at the early evolutionary stage, all the solutions are uniformly distributed in the problem space;

Table 2 (*continued*).

Function	Item	LSA	DSA	BSA	FA	HS	PSO	SCA	GSA	CSA
	Best	4.08E+01	3.35E+01	5.32E+01	1.14E+01	5.97E+01	1.13E+02	4.81E−04	8.95E+00	**0.00E+00**
	Median	5.97E+01	4.67E+01	6.64E+01	2.46E+01	8.56E+01	2.04E+02	2.38E−01	1.89E+01	**0.00E+00**
F_9	Mean	6.28E+01	4.71E+01	6.58E+01	2.63E+01	8.70E+01	2.06E+02	1.17E+01	1.93E+01	**0.00E+00**
	Worst	1.05E+02	6.35E+01	8.26E+01	5.65E+01	1.07E+02	2.87E+02	1.05E+02	3.08E+01	**0.00E+00**
	STD	1.49E+01	7.20E+00	8.03E+00	9.15E+00	1.08E+01	3.96E+01	3.06E+01	4.76E+00	**0.00E+00**
	Best	8.73E−08	1.03E+00	6.55E−01	2.23E−02	7.28E+00	3.17E−01	2.70E−04	1.43E−06	**4.44E−16**
	Median	2.54E+00	3.65E+00	2.92E+00	5.07E−02	8.80E+00	1.20E+00	1.95E+01	3.28E−06	**4.44E−16**
F_{10}	Mean	2.69E+00	4.05E+00	3.15E+00	5.12E−02	8.82E+00	1.19E+00	1.19E+01	3.28E−06	**4.44E−16**
	Worst	5.59E+00	1.05E+01	7.85E+00	8.74E−02	9.89E+00	2.18E+00	2.02E+01	7.07E−06	**4.44E−16**
	STD	9.11E−01	1.98E+00	1.49E+00	1.37E−02	6.12E−01	5.44E−01	9.98E+00	1.33E−06	**0.00E+00**
	Best	2.22E−16	8.68E−01	6.79E−01	2.46E−03	1.13E+00	4.65E−03	6.79E−06	1.66E+00	**0.00E+00**
	Median	7.40E−03	1.09E+00	1.07E+00	5.65E−03	1.24E+00	2.54E−02	2.69E−02	4.13E+00	**0.00E+00**
F_{11}	Mean	7.24E−03	1.11E+00	1.07E+00	5.84E−03	1.24E+00	3.17E−02	1.43E−01	4.52E+00	**0.00E+00**
	Worst	2.46E−02	1.47E+00	1.35E+00	8.96E−03	1.46E+00	7.23E−02	7.43E−01	7.98E+00	**0.00E+00**
	STD	6.75E−03	9.39E−02	8.69E−02	1.43E−03	6.41E−02	1.74E−02	2.22E−01	1.50E+00	**0.00E+00**
	Best	2.23E−15	1.15E−02	1.13E−02	9.15E−05	4.57E−02	4.72E−04	3.13E−01	5.83E−14	**1.57E−32**
	Median	1.04E−01	1.71E−01	7.59E−02	2.31E−04	1.59E−01	2.29E−03	7.22E−01	1.01E−02	**2.22E−32**
F_{12}	Mean	3.58E−01	2.92E−01	1.35E−01	2.38E−04	1.80E−01	1.85E−02	4.95E+01	1.29E−01	**3.68E−32**
	Worst	4.38E+00	1.47E+00	8.57E−01	5.48E−04	4.11E−01	1.07E−01	6.68E+02	8.70E−01	**2.98E−31**
	STD	7.44E−01	3.37E−01	1.53E−01	1.00E−04	9.74E−02	3.78E−02	1.60E+02	2.17E−01	**6.22E−32**

Table 3
Statistical values of objective functions for 30-variable multimodal functions.

Function	Item	LSA	DSA	BSA	FA	HS	PSO	SCA	GSA	CSA
	Best	2.75E−19	2.58E−01	1.49E−01	7.28E−04	1.05E+00	5.27E−02	2.07E+00	6.68E−13	**4.68E−32**
	Median	1.10E−02	1.02E+00	5.27E−01	1.90E−03	1.70E+00	1.67E−01	2.57E+00	**1.93E−11**	1.10E−02
F_{13}	Mean	2.41E−02	1.17E+00	7.56E−01	**2.24E−03**	1.83E+00	2.19E−01	4.30E+00	3.33E−02	4.55E−02
	Worst	2.33E−01	3.68E+00	3.85E+00	**5.35E−03**	3.93E+00	6.74E−01	2.42E+01	6.44E−01	2.95E−01
	STD	4.73E−02	7.85E−01	7.49E−01	**1.04E−03**	5.50E−01	1.65E−01	5.22E+00	1.44E−01	7.31E−02
	Best	**0.998004**	**0.998004**	**0.998004**	**0.998004**	**0.998004**	**0.998004**	**0.998004**	1.891408	**0.998004**
	Median	**0.998004**	**0.998004**	**0.998004**	1.992088	**0.998004**	1.992031	0.998014	2.982127	2.982105
F_{14}	Mean	1.077447	**0.998004**	**0.998004**	1.875598	1.605938	2.579809	1.295645	4.285704	2.875134
	Worst	2.982105	**0.998004**	**0.998004**	3.068711	5.988898	6.903336	2.982105	10.770589	10.763181
	STD	0.33798	**3.36E−16**	**3.36E−16**	0.668873	1.371648	1.873362	0.726859	3.175621	2.051951
	Best	**0.000307**	0.000602	0.000379	0.000443	0.000739	0.000581	0.000414	0.001686	**0.000307**
	Median	0.000309	0.000865	0.000632	0.000984	0.000934	0.001356	0.000731	0.004105	**0.000308**
F_{15}	Mean	**0.000535**	0.00096	0.000626	0.001062	0.000963	0.005941	0.00086	0.004355	0.002359
	Worst	0.001594	0.001954	**0.000817**	0.001901	0.0015	0.020363	0.001452	0.008215	0.020363
	STD	0.000424	0.000268	**0.000108**	0.000359	0.000176	0.00855	0.000378	0.001923	0.006161
	Best	**−1.031628**	**−1.031628**	**−1.031628**	**−1.031628**	−1.031627	**−1.031628**	**−1.031628**	**−1.031628**	**−1.031628**
	Median	**−1.031628**	**−1.031628**	**−1.031628**	**−1.031628**	−1.02233	**−1.031628**	−1.031616	**−1.031628**	**−1.031628**
F_{16}	Mean	**−1.031628**	**−1.031628**	**−1.031628**	**−1.031628**	−0.996514	**−1.031628**	−1.031615	**−1.031628**	**−1.031628**
	Worst	**−1.031628**	**−1.031628**	**−1.031628**	**−1.031628**	−0.668653	**−1.031628**	−1.031588	**−1.031628**	**−1.031628**
	STD	**0**	**0**	**0**	2.83E−09	0.06811	2.22E−16	0.0000106	7.36E−16	2.22E−16
	Best	**0.397887**	**0.397887**	**0.397887**	**0.397887**	**0.397887**	**0.397887**	0.397891	**0.397887**	**0.397887**
	Median	**0.397887**	**0.397887**	**0.397887**	**0.397887**	0.39834	**0.397887**	0.398157	**0.397887**	**0.397887**
F_{17}	Mean	**0.397887**	**0.397887**	**0.397887**	**0.397887**	0.407773	**0.397887**	0.398417	**0.397887**	**0.397887**
	Worst	**0.397887**	**0.397887**	**0.397887**	**0.397887**	0.495021	**0.397887**	0.401634	**0.397887**	**0.397887**
	STD	1.68E−16	7.8E−12	1.43E−12	3.13E−09	0.021638	**0**	0.00082	8.28E−14	**0**
	Best	**3**	**3**	**3**	3.000000001	3.000000457	**3**	3.000000431	**3**	**3**
	Median	**3**	**3**	**3**	3.000000024	3.000029069	**3**	3.000006115	**3**	**3**
F_{18}	Mean	**3**	3.000000008	**3**	3.000000029	3.000052375	**3**	3.000015858	**3**	**3**
	Worst	**3**	3.00000038	**3**	3.000000111	3.000258898	**3**	3.000102102	**3**	**3**
	STD	3.34E−15	5.38E−08	3.51E−15	2.56E−08	0.0000538	1.1E−15	0.0000249	7.05E−13	**1.02E−15**
	Best	**−3.862782**	**−3.862782**	**−3.862782**	**−3.862782**	−3.86272	**−3.862782**	−3.862132	**−3.862782**	**−3.862782**
	Median	**−3.862782**	**−3.862782**	**−3.862782**	**−3.862782**	−3.861372	**−3.862782**	−3.854523	−3.539936	**−3.862782**
F_{19}	Mean	**−3.862782**	**−3.862782**	**−3.862782**	**−3.862782**	−3.860211	−3.862388	−3.855231	−3.514778	**−3.862782**
	Worst	**−3.862782**	**−3.862782**	**−3.862782**	**−3.862782**	−3.843937	−3.854901	−3.853671	−3.08581	**−3.862782**
	STD	**0**	**0**	**0**	9.16E−10	0.003257	0.001762	0.002327	0.245722	2.26E−15
	Best	**−3.321995**	**−3.321995**	**−3.321995**	**−3.321995**	−3.303125	**−3.321995**	−3.126322	**−3.321995**	**−3.321995**
	Median	**−3.321995**	**−3.321995**	**−3.321995**	**−3.321995**	−3.155958	−3.198477	−3.011903	−2.019945	−3.203102
F_{20}	Mean	−3.27206	**−3.321995**	**−3.321995**	−3.267674	−3.121637	−3.198721	−2.906018	−2.048445	−3.256604
	Worst	−3.203102	**−3.321995**	−3.321978	−3.185407	−2.664424	−2.840422	−1.456282	−1.254658	−3.203102
	STD	0.059276	2.32E−08	**2.63E−60**	0.061983	0.133471	0.118604	0.411373	0.587789	0.060685

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as the iteration proceeds, all the solutions search in the most potential area; in the end, most of the agents converge to the global optimal solutions. Thus, the CSA method can improve the quality of the randomly-placed solutions by balancing exploration and exploitation.

3.1.7. Comparison with other existing evolutionary algorithms

In this section, the CSA method is further compared with several other existing methods on 23 typical test functions. The statistical results obtained by different methods are given in Table 6, including whale optimization algorithm (WOA) [39],

Table 3 (*continued*).

Function	Item	LSA	DSA	BSA	FA	HS	PSO	SCA	GSA	CSA
F_{21}	Best	**−10.1532**	**−10.1532**	**−10.1532**	−10.153199	−9.127235	**−10.1532**	−8.10853	−5.055198	**−10.1532**
	Median	−5.100772	−10.153195	−10.153199	−10.153197	−2.167447	−5.100772	−2.375601	−5.055198	**−10.1532**
	Mean	−7.02732	−10.152834	−10.153094	−8.427091	−2.782671	−7.11035	−3.012834	−5.055198	**−10.1532**
	Worst	−2.630472	−10.144972	−10.149128	−2.630472	−1.141857	−5.055198	−0.498192	−5.055198	**−10.1532**
	STD	3.156152	0.001254	0.000584	3.120294	1.813603	2.549082	2.362323	1.72E−12	**3.58E−15**
F_{22}	Best	**−10.402941**	**−10.402941**	**−10.402941**	−10.40294	−9.367837	**−10.402941**	−6.587977	**−10.402941**	**−10.402941**
	Median	**−10.402941**	−10.40294	−10.40294	−10.402938	−2.485769	**−10.402941**	−4.658063	**−10.402941**	**−10.402941**
	Mean	−7.136702	−10.393586	**−10.402917**	−10.278482	−3.045779	−8.814533	−3.846918	−8.542597	−9.735077
	Worst	−1.837593	−10.101115	**−10.402242**	−4.180111	−1.259142	−5.087672	−0.906982	−5.087672	−3.7243
	STD	3.514977	0.04417	**0.000107**	0.880041	1.6455	2.489392	1.905963	2.601082	2.055642
F_{23}	Best	**−10.53641**	**−10.53641**	**−10.53641**	−10.536409	−10.151938	**−10.53641**	−7.998784	**−10.53641**	**−10.53641**
	Median	**−10.53641**	−10.536396	**−10.53641**	−10.536408	−2.682462	**−10.53641**	−5.060989	**−10.53641**	**−10.53641**
	Mean	−7.910438	−10.53317	−10.536394	−10.536408	−4.204342	−8.925823	−5.538952	**−10.53641**	−9.125809
	Worst	−1.85948	−10.458195	−10.535979	−10.536404	−2.629258	−5.128481	−4.512439	**−10.53641**	−2.427335
	STD	3.596043	0.012505	**6.42E−50**	0.00000112	3.008542	2.524144	0.94944	3.46E−12	2.907981
F_{24}	Best	N/A	N/A	N/A	N/A	N/A	2.58E−19	1.32E−20	4.05E−11	**0.00E+00**
	Median	N/A	N/A	N/A	N/A	N/A	2.09E−16	3.47E−14	9.69E−09	**0.00E+00**
	Mean	N/A	N/A	N/A	N/A	N/A	2.25E−15	2.29E−07	3.78E−08	**0.00E+00**
	Worst	N/A	N/A	N/A	N/A	N/A	9.71E−15	4.20E−06	2.55E−07	**0.00E+00**
	STD	N/A	N/A	N/A	N/A	N/A	3.62E−15	9.39E−07	6.70E−08	**0.00E+00**

Table 4
Computational time of several methods for test functions (second).

Function	PSO	SCA	GSA	CSA	Function	PSO	SCA	GSA	CSA
F_1	0.08	0.13	1.69	0.28	F_{13}	0.12	0.19	1.81	0.30
F_2	0.08	0.14	1.66	0.24	F_{14}	0.09	0.08	0.18	0.15
F_3	0.08	0.13	1.67	0.29	F_{15}	0.04	0.05	0.24	0.05
F_4	0.11	0.17	1.73	0.31	F_{16}	0.04	0.04	0.13	0.05
F_5	0.09	0.14	1.69	0.22	F_{17}	0.04	0.04	0.15	0.04
F_6	0.09	0.14	1.72	0.24	F_{18}	0.04	0.04	0.15	0.05
F_7	0.12	0.18	1.77	0.31	F_{19}	0.04	0.05	0.19	0.06
F_8	0.12	0.18	1.79	0.30	F_{20}	0.05	0.05	0.38	0.09
F_9	0.11	0.17	1.77	0.30	F_{21}	0.04	0.04	0.25	0.10
F_{10}	0.11	0.18	1.78	0.31	F_{22}	0.04	0.04	0.26	0.07
F_{11}	0.13	0.19	1.84	0.34	F_{23}	0.04	0.04	0.27	0.08
F_{12}	0.11	0.19	1.75	0.30	F_{24}	0.10	0.18	1.64	0.30

Table 5
Wilcoxon signed rank test results for 24 30-variable benchmark functions at alpha = 0.05.

Method	Better	Equal	Worse	R^+	R^-	P-value	Symbol
CSA-LSA	14	5	5	237.5	62.5	8.90E−03	+
CSA-DSA	13	4	7	206	94	1.00E−01	≈
CSA-BSA	12	5	7	203.5	96.5	9.90E−02	≈
CSA-FA	12	4	8	190	110	2.32E−01	≈
CSA-HS	19	1	4	279.5	20.5	2.33E−04	+
CSA-PSO	19	3	2	285	15	2.14E−04	+
CSA-SCA	21	0	3	272	28	4.91E−04	+
CSA-GSA	19	3	2	268	32	1.30E−03	+

genetic algorithm (GA) [40], grey wolf optimizer (GWO) [41], salp swarm algorithm (SSA) [42], evolution strategy with covariance matrix adaptation (CMA-ES) [43], cuckoo search (CS) [44], modified cuckoo search (MCS) [45], evolution strategy (ES) [46], ant lion optimizer (ALO) [47], gbest-guided gravitational search algorithm GGSA) [48] and improved gravitational search algorithm (IGSA) [49]. It can be clearly seen that for most functions, the CSA method outperforms its competitor with respect to the mean and standard deviation of objective values.

To demonstrate the superiority of the CSA method, Table 7 shows the Wilcoxon signed rank test results between CSA and the existing methods at alpha = 0.05. For almost all the comparisons in Table 7, the number of two symbols ("Better" and R^+) is larger than the contrast symbols while the values of P in different tests are smaller than 0.05. It can be concluded that the CSA method has better performances than the existing methods to find the optima of the benchmark functions.

3.1.8. Comparison with other algorithm in high-dimensional problems

In this section, four methods (PSO, SCA, GSA and CSA) are used to resolve 13 test functions with changing variables. The statistical values of four methods for 100-variable and 300-variable unimodal functions are given in Tables 8–9. It can be found that the CSA method can always find the best solutions and the smallest deviations in various cases. Then, Table 10 gives the Wilcoxon signed rank test results for 13 benchmark functions at alpha = 0.05. It can be found that the CSA method produces the larger values of R^+ and Better symbols. Hence, the robustness and feasibility of the CSA method in large-scale optimization problem is proved here.

3.2. Benchmark functions set II: 25 CEC2005 test problems

3.2.1. Benchmark functions

In this section, 25 CEC2005 problems with 10 variables are used to verify the performance of the CSA method. Table 11 shows the brief information of 25 CEC2005 benchmark functions

Table 6
Statistical results of CSA and existing evolutionary methods for 23 30-variable benchmark functions.

Function	Item	WOA	GWO	GA	SSA	CMA-ES	CS	MCS	ES	ALO	GGSA	IGSA	CSA
F_1	Mean	1.41E−30	6.59E−28	5.55E−01	1.58E−07	1.42E−18	9.06E+01	1.01E+00	2.04E+03	6.21E−06	2.78E−18	1.60E−17	**0.00E+00**
	STD	4.91E−30	1.58E−28	1.23E+00	1.71E−07	3.13E−18	2.62E+01	2.72E−01	1.61E+03	1.16E−05	2.48E−18	4.71E−18	**0.00E+00**
F_2	Mean	1.06E−21	7.18E−17	5.66E−03	2.66E+00	2.98E−07	9.70E+00	1.81E−01	1.18E+01	6.32E+00	9.53E−09	1.57E−08	**0.00E+00**
	STD	2.39E−21	7.28E−17	1.44E−02	1.67E+00	1.79E+00	1.98E+00	3.31E−02	3.30E+00	4.01E+01	7.39E−09	3.20E−09	**0.00E+00**
F_3	Mean	5.39E−07	3.29E−06	8.46E+02	1.71E+03	1.59E−05	3.84E+03	4.62E+02	1.33E+04	1.08E+03	4.77E+02	1.35E+03	**0.00E+00**
	STD	2.93E−06	1.61E−05	1.61E+02	1.12E+04	2.21E−05	7.24E+02	1.23E+02	5.21E+03	6.58E+02	1.66E+02	6.67E+02	**0.00E+00**
F_4	Mean	7.26E−02	5.61E−07	4.56E+00	1.17E+01	2.01E−06	7.23E+00	1.73E+00	4.79E+01	1.39E+01	7.00E−01	7.34E+00	**7.64E−300**
	STD	3.97E−01	1.04E−06	5.92E−01	4.18E+00	1.25E−06	6.76E−01	5.12E−01	8.52E+00	3.62E+00	8.56E−01	3.49E+00	**0.00E+00**
F_5	Mean	2.79E+01	2.68E+01	2.68E+02	2.96E+02	3.68E+01	**4.98E+00**	5.81E+01	2.05E+05	2.88E+01	3.71E+01	5.22E+01	2.26E+01
	STD	7.64E−01	7.93E−01	3.38E+02	5.09E+02	3.35E+01	1.75E+00	3.31E+01	2.12E+05	1.20E+02	2.91E+01	4.47E+01	**2.12E−01**
F_6	Mean	3.12E+00	8.17E−01	5.63E−01	1.80E−07	6.83E−19	4.31E+04	4.44E+03	2.16E+03	6.72E−06	3.08E−18	3.48E−17	**1.95E−25**
	STD	5.32E−01	4.82E−01	1.72E+00	3.00E−07	6.71E−19	7.21E+03	7.52E+02	1.13E+03	4.66E−06	2.90E−18	1.16E−17	**8.69E−25**
F_7	Mean	**1.43E−03**	2.21E−03	4.29E−02	1.76E−01	2.75E−02	2.46E−02	9.10E−03	5.46E+01	9.79E−02	3.73E−01	1.49E−03	4.77E−01
	STD	1.15E−03	2.00E−03	5.94E−03	6.29E−02	7.90E−03	7.90E−03	2.20E−03	5.56E+01	3.87E−02	1.48E+00	**6.98E−04**	2.99E−01
F_8	Mean	−5.08E+03	−6.12E+03	**−1.05E+04**	−7.46E+03	−7.01E+03	−8.98E+03	−9.80E+03	−8.55E+03	−5.46E+03	−2.90E+03	−6.62E+03	−9.51E+03
	STD	6.96E+02	9.10E+02	3.53E+02	7.73E+02	7.74E+02	**1.98E+02**	5.31E+02	7.41E+02	1.72E+03	5.01E+02	9.73E+02	5.80E+02
F_9	Mean	**0.00E+00**	3.11E−01	3.08E+01	5.84E+01	2.53E+01	2.94E+02	1.35E+02	2.45E+02	7.71E+01	3.31E+01	3.54E+01	**0.00E+00**
	STD	**0.00E+00**	3.52E−01	7.57E+00	2.00E+01	8.55E+00	1.43E+01	2.16E+01	2.88E+01	2.26E+01	1.19E+01	9.24E+00	**0.00E+00**
F_{10}	Mean	7.40E+00	1.06E−13	1.64E+00	2.68E+00	1.56E+01	1.93E+01	1.21E+01	7.25E+00	2.17E+00	1.18E−09	3.13E−09	**4.44E−16**
	STD	9.90E+00	2.24E−13	4.62E−01	8.28E−01	7.93E+00	3.50E−01	7.52E−01	1.66E+00	1.17E+00	4.68E−10	4.24E−10	**0.00E+00**
F_{11}	Mean	2.89E−04	4.48E−03	5.61E−01	1.60E−02	5.76E−15	2.12E+02	8.32E+00	7.62E+01	2.50E−03	7.28E+00	1.49E−02	**0.00E+00**
	STD	1.59E−03	6.65E−03	2.69E−01	1.12E−02	6.18E−15	3.97E+01	1.54E+00	2.90E+01	7.91E−03	3.45E+00	2.23E−02	**0.00E+00**
F_{12}	Mean	3.40E−01	5.34E−02	3.09E−02	6.99E+00	2.87E−16	1.47E+00	1.38E−01	3.89E+04	8.40E+00	2.14E−01	7.50E−01	**3.68E−32**
	STD	2.15E−01	2.07E−02	4.09E−02	4.42E+00	5.64E−16	3.61E−01	2.86E−01	1.32E+05	2.90E+00	3.52E−01	1.29E+00	**6.22E−32**
F_{13}	Mean	1.89E+00	6.54E−01	3.62E−01	1.59E+01	3.66E−04	9.33E−01	3.90E−03	5.81E+05	1.10E−02	2.20E−03	**8.87E−32**	4.55E−02
	STD	2.66E−01	4.47E−03	3.10E−01	1.61E+01	2.00E−03	2.13E−01	5.30E−03	1.81E+06	1.29E+00	4.47E−03	**1.11E−31**	7.31E−02
F_{14}	Mean	2.111973	4.042493	0.998004	1.1965	10.237	**0.998**	2.344	8.06397	0.998004	3.59199	1.428222	2.875134
	STD	2.498594	4.252799	**4.23E−12**	0.5467	7.5445	1.466E−06	2.1186	5.877871	1.306659	2.780351	0.722411	2.051951
F_{15}	Mean	0.000572	0.00337	0.005206	0.000886	0.0057	0.000556	**0.0005487**	0.003	0.000782	0.002066	0.000757	0.002359
	STD	0.000324	0.00625	0.00703	0.000257	0.0121	**0.0000749**	0.000154	0.005902	0.005973	0.00091	0.000187	0.006161
F_{16}	Mean	**−1.03163**	**−1.03163**	−1.03162	**−1.03163**	−1.03162	−1.0316	−1.0316	**−1.03163**	**−1.03163**	**−1.03163**	**−1.03163**	−1.031628
	STD	0.00000042	2.13E−08	0.00000134	6.13E−14	6.77E−16	7.56E−12	3.65E−10	6.71E−16	5.63E−14	5.68E−16	4.88E−16	**2.22E−16**
F_{17}	Mean	0.397914	0.397889	0.39789	**0.397887**	**0.397887**	0.3979	0.3979	55.6	0.398	**0.397887**	**0.397887**	**0.397887**
	STD	0.000027	0.000213	0.0000108	3.41E−14	**0**	1.12E−08	3.7E−09	2.89E−14	5.92E−14	**0**	**0**	**0**
F_{18}	Mean	**3**	3.000028	3.000002	**3**	8.4	**3**	**3**	**3**	**3**	**3**	**3**	**3**
	STD	4.22E−15	0.000424	0.00000406	2.2E−13	20.55	2.99E−09	7.85E−08	**8.61E−16**	2.75E−13	2.83E−15	2.64E−15	1.02E−15
F_{19}	Mean	−3.85616	−3.86263	−3.86278	−3.86278	−3.86278	**−3.8628**	**−3.8628**	−3.86278	−3.86278	−3.86147	−3.86278	−3.862782
	STD	0.00271	0.00273	1.63E−07	1.47E−10	2.7E−15	0.00000869	9.71E−08	2.71E−15	3.29E−14	0.002987	2.39E−15	**2.26E−15**
F_{20}	Mean	−2.98105	−3.28654	−3.27443	−3.2304	−3.2903	−3.3213	−3.2744	−3.22367	**−3.3219**	−3.09862	−3.23084	−3.256604
	STD	0.376653	0.10556	0.05924	0.0616	0.0535	**0.0004875**	0.0614	0.050751	0.058277	0.498156	0.051146	0.060685
F_{21}	Mean	−7.04918	−8.7214	−5.72536	−9.6334	−5.6642	−10.143	−3.4299	−4.99059	−5.10077	−4.7287	−7.97499	**−10.1532**
	STD	3.629551	2.6914	3.32622	1.8104	3.3543	0.0132	2.3623	3.465861	2.702862	1.667901	2.998983	**3.58E−15**
F_{22}	Mean	−8.18178	−9.2415	−6.94349	−9.0295	−8.4434	−10.391	−8.3021	−7.07562	**−10.4029**	−7.92248	−9.71753	−9.735077
	STD	3.829202	1.61254	3.56118	2.3911	3.3388	**0.0082**	3.393	3.64907	3.424583	2.697054	2.122013	2.055642
F_{23}	Mean	−9.34238	−10.5343	−7.0208	−9.0333	−8.075	−10.4927	−5.9976	−7.02909	**−10.5364**	−10.3561	−9.65303	−9.125809
	STD	2.414737	**0.00125**	3.85233	2.9645	3.5964	0.0293	3.916	3.838	2.939	0.987348	2.33909	2.907981

Table 7
Wilcoxon signed rank test results between CSA and existing evolutionary methods for several 30-variable benchmark functions at alpha = 0.05.

Method	Better	Equal	Worse	R^+	R^-	P-value	Symbol
CSA-WOA	16	3	5	234	66	1.90E−02	+
CSA-GWO	18	1	5	242.5	57.5	9.73E−03	+
CSA-GA	17	2	5	239.5	60.5	1.42E−02	+
CSA-SSA	17	4	3	253	47	3.19E−03	+
CSA-CMA-ES	18	3	3	255	45	4.14E−03	+
CSA-CS	14	2	8	218.5	81.5	4.24E−02	+
CSA-MCS	15	2	7	228.5	71.5	2.21E−02	+
CSA-ES	21	3	0	293	7	6.90E−05	+
CSA-ALO	13	3	8	210	90	6.80E−02	≈
CSA-GGSA	16	3	5	240	60	1.06E−02	+
CSA-IGSA	15	4	5	213	87	7.93E−02	≈

with 10 variables, including 5 unimodal functions ($F_1 \sim F_5$), 9 multimodal functions ($F_6 \sim F_{14}$) and 11 hybrid functions ($F_{15} \sim F_{25}$). Due to the existence of matrix manipulation, the calculated amount involved in the optimization process of the second function set is sharply increased compared with the first test function set. Next, the results of PSO, SCA, GSA and CSA in 20 independent runs are fully compared.

3.2.2. Statistical results analysis

Table 12 gives the statistical results of 25 CEC2005 test problems by several methods. The unimodal functions ($F_1 \sim F_5$) can test the exploiting ability in the entire space; the multimodal functions ($F_6 \sim F_{14}$) can test the exploration ability in the promising areas; the hybrid functions ($F_{15} \sim F_{25}$) can test the exploration ability with the increasing balance between exploration and exploitation. As showed in Table 12, the following interesting phenomenon can be found: ① for unimodal functions, the CSA method outperforms three other methods in most statistics, proving the efficacy of three operators in exploiting the problem space; ② for multimodal functions, the CSA method is able to produce better results than the control methods, proving the feasibility of guaranteeing exploration ability of the population; ② for hybrid functions, the CSA method can guarantee the solution quality and avoid the premature convergence, proving the effectiveness of achieving a compromise between exploration and exploitation; ③ the computation time of the CSA method is obviously better than GSA and close to SCA and PSO. Thus, it can be concluded that the CSA method is an effective and reliable method for global optimization.

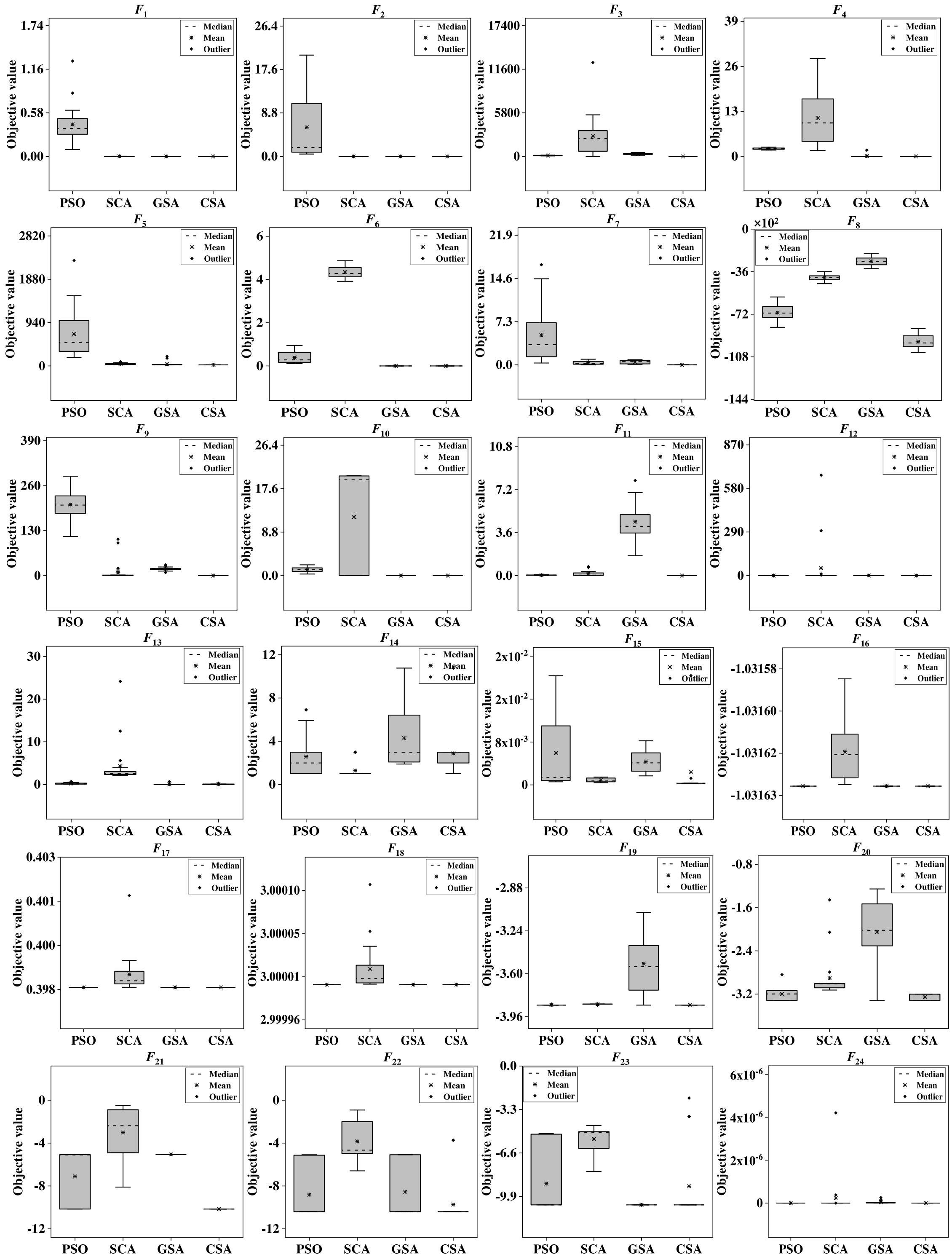

Fig. 4. Box plot of several methods for the 30-variable test functions.

3.2.3. Wilcoxon nonparametric statistical test

In this section, the Wilcoxon signed rank test is adopted to verify the improvement of the CSA method, while the detailed computational results deduced from the mean objective values of all the test functions are given in Table 13. It can be found that the number of the "Better" symbols is larger than that of two other symbols; the R^- values are larger than the R^+ value while the P values are less than 0.05. Thus, the CSA method can produce better results than the competitor methods by improving the dynamic search performance of the swarm in the evolutionary process.

3.3. Benchmark functions set II: 30 CEC2014 test problems

3.3.1. Benchmark functions

In this section, 30 CEC2014 test problems with 10 variables in Table 14 are considered. This function set can be divided into 4 groups: 3 unimodal functions ($F_1 \sim F_3$), 13 multimodal functions

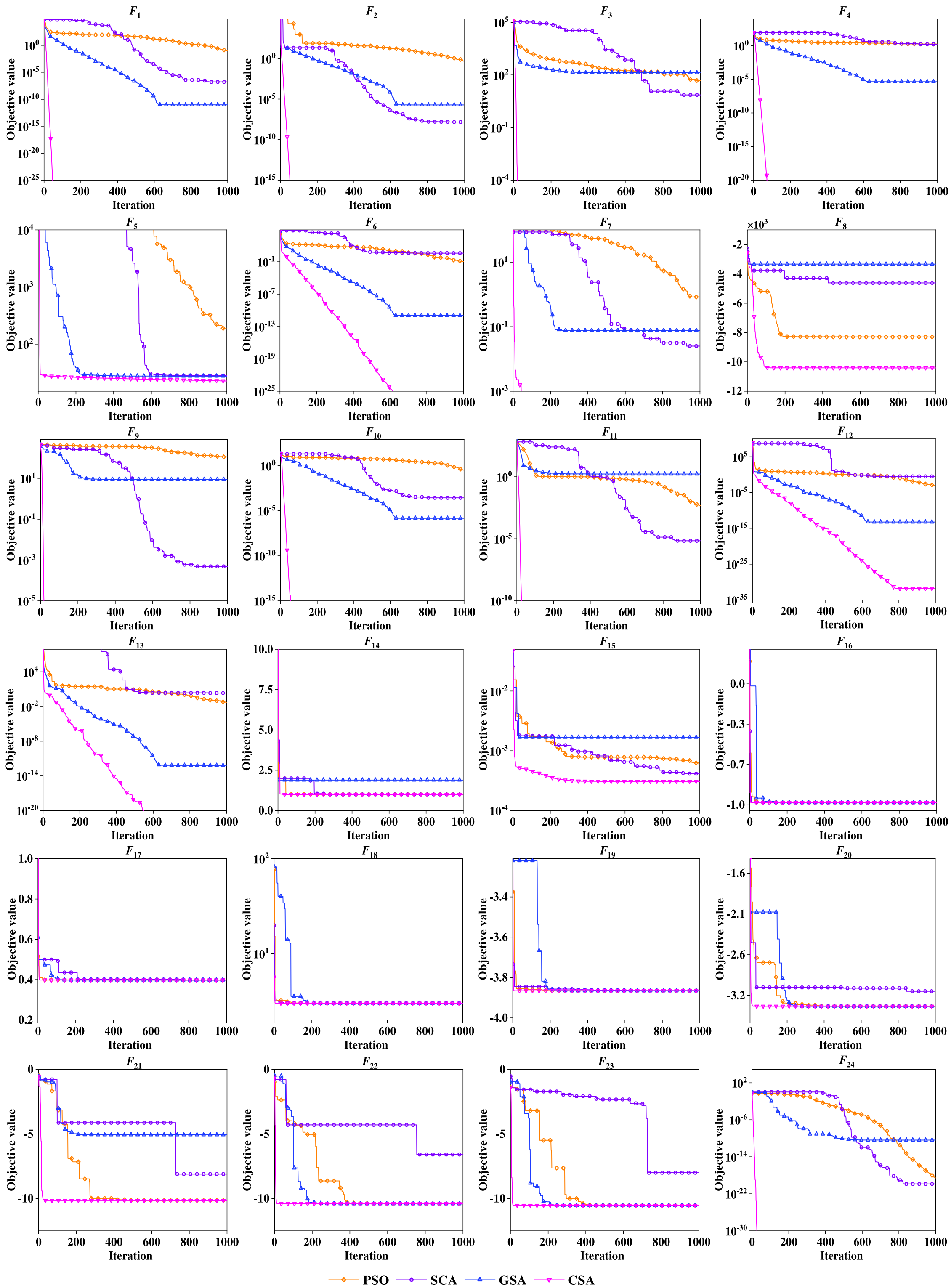

Fig. 5. Convergence corves of several methods for several 30-variable benchmark functions.

(F_4∼F_{16}), 6 hybrid functions (F_{17}∼F_{22}) and 8 composite functions (F_{23}∼F_{30}). Similar to the above two function sets, the results of PSO, SCA, GSA and CSA in 20 independent runs are compared.

3.3.2. Statistical results analysis

Table 15 shows the statistical results of 30 10-variable CEC2014 test functions obtained by several methods. It can be found that the CSA method can yield better methods than three other methods in most of the indexes. For instance, the CSA method can find the best solution and the smallest standard deviation in terms of the objective values of the unimodal function F_1. Besides, the average time of the CSA method is obviously smaller than GSA and slightly larger than SCA and PSO. Thus, this case

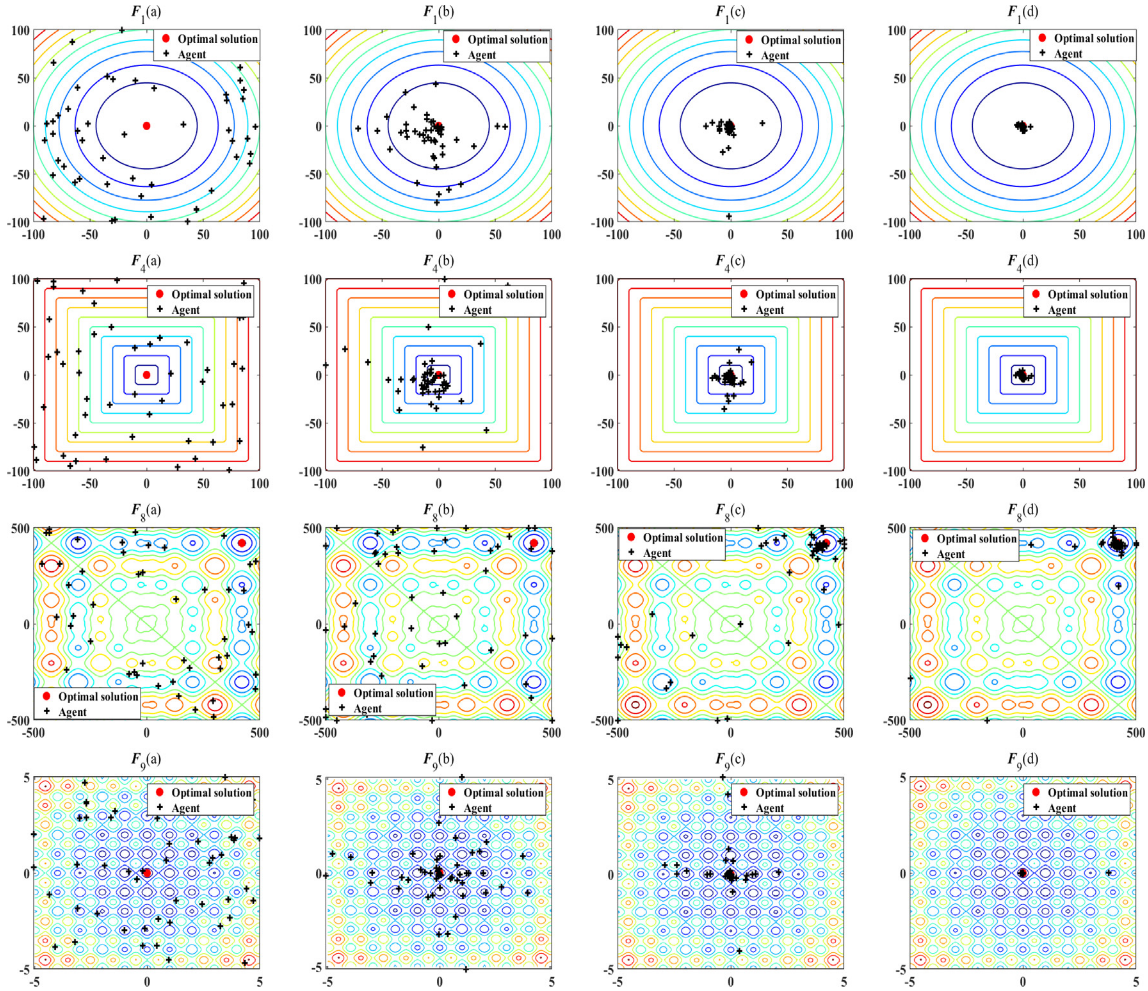

Fig. 6. Searching histories of the CSA method for several test functions with two variables.

demonstrates that the CSA method can effectively address the global optimization problems.

3.3.3. Wilcoxon nonparametric statistical test

To verify the performance of the CSA method, the Wilcoxon signed rank test is conducted for 10-variable CEC2014 functions and the obtained results are given in Table 16. It can be found that for all the comparisons, the R^+ symbols are far larger than the R^- symbols while the P values are smaller than 0.05. Thus, the CSA method can improve the global search ability in the evolutionary process.

4. CSA for solving engineering optimization problems

In this section, the CSA method is used to find the optimal solution of engineering optimization problems which are subjected to numerous equality constraints and inequality constraints.

4.1. Gear train design problem

As showed in Eq. (14), the gear train design problem is a typical integer optimization problem with 4 discrete integer variables [50]. The goal is to find the optimal number of teeth to minimize the cost of the gear ratio (x_1x_2/x_3x_4) of the train. During the evolutionary process, each variable will be rounded to the nearest integer number. As showed in Fig. 7, This problem with the variable vector $\boldsymbol{x} = (x_1, x_2, x_3, x_4) = \left(T_d, T_b, T_a, T_f\right)$ can be mathematically described as below:

$$\begin{aligned} &\min \quad f(\boldsymbol{x}) = \left(\frac{1}{6.931} - \frac{x_1x_2}{x_3x_4}\right)^2 \\ &s.t.\ \ 12 \le x_i \le 60, i = 1, 2, 3, 4 \\ &\qquad x_i \in Z^+, i = 1, 2, 3, 4 \end{aligned} \tag{14}$$

Table 17 shows the results of the CSA method and several methods in previous literature. It can be found that as compared with the existing methods, the CSA method is capable of providing the scheme with better objective for the gear train design problem.

4.2. Parameter estimation for frequency-modulated (FM) sound waves problem

For a complicated multimodal problem, the goal is to find the best parameter combinations of 6 variables for the frequency-modulated synthesizer. As a multimodal problem, this problem with the variable vector $\boldsymbol{x} = (x_1, x_2, x_3, x_4, x_5, x_6) = (a_1, w_1, a_2, w_2, a_3, w_3)$ can be described as below:

$$\begin{aligned} &\min \ f(\boldsymbol{x}) = \sum_{t=0}^{100} [y(t) - y_0(t)]^2 \\ &s.t. -6.4 \le a_1, w_1, a_2, w_2, a_3, w_3 \le 6.35 \end{aligned} \tag{15}$$

Table 8
Statistical values of several methods for 100-variable test functions.

Function	Method	Best	Median	Mean	Worst	STD	Time(s)
F_1	PSO	1.05E+02	1.48E+02	1.49E+02	2.11E+02	3.46E+01	0.26
	SCA	3.75E+02	2.32E+03	4.00E+03	1.55E+04	4.17E+03	0.41
	GSA	2.14E+01	1.53E+02	1.63E+02	4.01E+02	9.84E+01	5.20
	CSA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	0.79
F_2	PSO	2.18E+02	2.80E+02	2.90E+02	3.96E+02	5.19E+01	0.25
	SCA	2.86E−02	6.79E−01	1.17E+00	4.08E+00	1.20E+00	0.46
	GSA	1.15E+00	2.83E+00	2.90E+00	5.79E+00	1.11E+00	5.37
	CSA	**4.51E−286**	**1.30E−284**	**3.24E−284**	**2.45E−283**	**0.00E+00**	0.74
F_3	PSO	1.01E+04	1.42E+04	1.41E+04	2.07E+04	2.64E+03	0.27
	SCA	1.23E+05	1.61E+05	1.69E+05	2.49E+05	3.90E+04	0.45
	GSA	3.50E+03	5.98E+03	6.17E+03	1.04E+04	1.69E+03	5.27
	CSA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	0.87
F_4	PSO	9.11E+00	1.08E+01	1.11E+01	1.62E+01	1.65E+00	0.39
	SCA	7.67E+01	8.46E+01	8.44E+01	9.05E+01	3.73E+00	0.58
	GSA	9.82E+00	1.16E+01	1.19E+01	1.47E+01	1.48E+00	5.45
	CSA	**9.34E−258**	**6.73E−256**	**1.52E−255**	**1.43E−254**	**0.00E+00**	1.01
F_5	PSO	1.40E+05	3.01E+05	2.81E+05	4.24E+05	7.84E+04	0.27
	SCA	1.08E+07	3.34E+07	3.81E+07	8.80E+07	2.30E+07	0.44
	GSA	1.18E+03	3.38E+03	4.52E+03	1.20E+04	3.22E+03	5.43
	CSA	**9.43E+01**	**9.46E+01**	**9.47E+01**	**9.53E+01**	**2.35E−01**	0.77
F_6	PSO	1.02E+02	1.41E+02	1.45E+02	2.16E+02	3.10E+01	0.26
	SCA	1.08E+02	2.85E+03	4.17E+03	2.01E+04	4.67E+03	0.45
	GSA	2.90E+01	1.39E+02	1.42E+02	2.47E+02	6.50E+01	5.44
	CSA	**7.68E−07**	**1.39E−05**	**4.98E−05**	**7.10E−04**	**1.56E−04**	0.76
F_7	PSO	3.63E+02	6.71E+02	6.66E+02	1.09E+03	2.02E+02	0.36
	SCA	1.06E+00	4.08E+01	5.03E+01	1.26E+02	3.83E+01	0.58
	GSA	1.52E+00	3.20E+00	3.43E+00	8.09E+00	1.57E+00	5.59
	CSA	**8.19E−07**	**2.31E−05**	**3.09E−05**	**1.05E−04**	**2.45E−05**	0.99
F_8	PSO	−2.60E+04	−2.23E+04	−2.23E+04	−1.93E+04	1.69E+03	0.33
	SCA	−8.55E+03	−7.43E+03	−7.36E+03	−6.27E+03	**5.73E+02**	0.53
	GSA	−6.05E+03	−4.98E+03	−4.87E+03	−3.94E+03	5.81E+02	5.56
	CSA	**−3.36E+04**	**−3.11E+04**	**−3.10E+04**	**−2.90E+04**	1.19E+03	0.91
F_9	PSO	1.21E+03	1.38E+03	1.36E+03	1.52E+03	9.34E+01	0.32
	SCA	7.81E+01	1.97E+02	2.29E+02	5.30E+02	1.32E+02	0.51
	GSA	7.90E+01	1.00E+02	1.02E+02	1.25E+02	1.42E+01	5.61
	CSA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	0.90
F_{10}	PSO	5.76E+00	6.49E+00	6.50E+00	7.57E+00	4.56E−01	0.33
	SCA	5.47E+00	2.06E+01	1.92E+01	2.06E+01	4.35E+00	0.57
	GSA	1.01E+00	1.53E+00	1.46E+00	2.05E+00	3.17E−01	5.62
	CSA	**−3.11E−15**	**−3.11E−15**	**−3.11E−15**	**−3.11E−15**	**0.00E+00**	0.91
F_{11}	PSO	8.56E−01	9.48E−01	9.49E−01	1.02E+00	4.31E−02	0.35
	SCA	2.55E+00	3.72E+01	4.54E+01	1.59E+02	4.15E+01	0.55
	GSA	4.37E+01	6.03E+01	5.98E+01	7.66E+01	9.83E+00	5.62
	CSA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	0.94
F_{12}	PSO	3.76E+00	7.52E+00	7.90E+00	1.20E+01	2.59E+00	0.37
	SCA	2.97E+06	1.59E+08	1.61E+08	3.29E+08	9.74E+07	0.61
	GSA	1.38E+00	2.58E+00	2.61E+00	4.28E+00	7.74E−01	5.50
	CSA	**8.09E−09**	**5.42E−08**	**1.21E−06**	**2.12E−05**	**4.70E−06**	0.92
F_{13}	PSO	1.06E+02	3.13E+02	4.05E+02	1.19E+03	2.57E+02	0.39
	SCA	3.98E+07	1.62E+08	1.84E+08	3.81E+08	9.97E+07	0.62
	GSA	6.92E+01	8.71E+01	9.01E+01	1.36E+02	1.81E+01	5.80
	CSA	**7.78E−05**	**2.75E−02**	**5.61E−02**	**2.03E−01**	**6.68E−02**	0.91

Table 9
Statistical values of several methods for 300-variable test functions.

Function	Method	Best	Median	Mean	Worst	STD	Time(s)
F_1	PSO	2.33E+03	2.97E+03	3.02E+03	3.56E+03	3.43E+02	1.18
	SCA	1.19E+04	6.14E+04	6.77E+04	1.40E+05	3.48E+04	2.00
	GSA	7.40E+03	9.87E+03	9.75E+03	1.18E+04	1.09E+03	25.86
	CSA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	3.67
F_2	PSO	1.02E+03	1.11E+03	1.11E+03	1.24E+03	4.94E+01	1.21
	SCA	5.22E+00	3.54E+01	3.43E+01	7.24E+01	1.96E+01	2.09
	GSA	5.16E+01	5.95E+01	5.93E+01	7.11E+01	5.70E+00	25.85
	CSA	**4.73E−260**	**2.27E−258**	**1.32E−257**	**1.83E−256**	**0.00E+00**	3.61
F_3	PSO	1.32E+05	1.94E+05	1.98E+05	3.06E+05	4.99E+04	1.37
	SCA	1.39E+06	1.86E+06	1.92E+06	3.55E+06	4.83E+05	2.14
	GSA	3.71E+04	5.77E+04	5.86E+04	7.76E+04	9.62E+03	25.49
	CSA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	3.99

(*continued on next page*)

Table 9 (*continued*).

Function	Method	Best	Median	Mean	Worst	STD	Time(s)
F_4	PSO	2.31E+01	2.57E+01	2.60E+01	2.94E+01	1.59E+00	2.20
	SCA	9.64E+01	9.74E+01	9.74E+01	9.84E+01	5.21E−01	2.89
	GSA	1.63E+01	1.79E+01	1.77E+01	1.93E+01	8.96E−01	25.98
	CSA	**5.14E−234**	**3.37E−232**	**1.71E−231**	**7.98E−231**	**0.00E+00**	5.45
F_5	PSO	1.39E+07	1.67E+07	1.77E+07	2.34E+07	2.99E+06	1.27
	SCA	3.25E+08	6.53E+08	6.67E+08	9.72E+08	1.72E+08	2.11
	GSA	3.95E+05	5.79E+05	5.86E+05	1.04E+06	1.43E+05	25.40
	CSA	**2.94E+02**	**2.95E+02**	**2.95E+02**	**2.96E+02**	**4.73E−01**	3.67
F_6	PSO	2.65E+03	3.06E+03	3.05E+03	3.43E+03	2.10E+02	1.30
	SCA	3.09E+04	6.28E+04	6.23E+04	1.05E+05	2.46E+04	2.12
	GSA	7.54E+03	9.26E+03	9.32E+03	1.10E+04	8.89E+02	25.59
	CSA	**3.76E+00**	**5.74E+00**	**5.70E+00**	**7.21E+00**	**9.36E−01**	3.62
F_7	PSO	1.62E+04	1.89E+04	1.88E+04	2.04E+04	1.15E+03	1.87
	SCA	1.86E+03	3.23E+03	3.11E+03	4.25E+03	6.32E+02	2.72
	GSA	1.68E+02	2.70E+02	2.80E+02	4.13E+02	6.37E+01	26.24
	CSA	**1.54E−06**	**3.71E−05**	**4.15E−05**	**2.04E−04**	**4.25E−05**	4.69
F_8	PSO	−7.33E+04	−6.37E+04	−6.38E+04	−5.63E+04	5.14E+03	1.65
	SCA	−1.39E+04	−1.28E+04	−1.29E+04	−1.17E+04	**6.51E+02**	2.55
	GSA	−1.26E+04	−9.08E+03	−9.64E+03	−7.66E+03	1.57E+03	26.18
	CSA	**−9.18E+04**	**−8.74E+04**	**−8.63E+04**	**−6.94E+04**	5.21E+03	4.32
F_9	PSO	4.44E+03	4.78E+03	4.77E+03	5.01E+03	1.75E+02	1.59
	SCA	2.72E+02	6.10E+02	7.18E+02	2.21E+03	4.38E+02	2.48
	GSA	5.77E+02	7.82E+02	7.80E+02	8.90E+02	7.75E+01	26.07
	CSA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	4.25
F_{10}	PSO	1.13E+01	1.16E+01	1.17E+01	1.24E+01	2.99E−01	1.64
	SCA	9.93E+00	2.07E+01	1.92E+01	2.08E+01	3.62E+00	2.51
	GSA	4.88E+00	5.22E+00	5.30E+00	6.16E+00	3.48E−01	26.37
	CSA	**1.11E−14**	**1.47E−14**	**1.34E−14**	**1.47E−14**	**1.74E−15**	4.28
F_{11}	PSO	1.69E+00	1.82E+00	1.80E+00	1.89E+00	6.37E−02	1.76
	SCA	3.04E+02	5.12E+02	5.87E+02	1.02E+03	2.03E+02	2.65
	GSA	1.34E+03	1.40E+03	1.40E+03	1.51E+03	3.90E+01	26.50
	CSA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	4.38
F_{12}	PSO	8.22E+04	1.48E+05	1.46E+05	2.28E+05	4.77E+04	1.88
	SCA	8.43E+08	2.00E+09	1.92E+09	3.20E+09	5.28E+08	3.05
	GSA	8.85E+00	9.81E+00	1.05E+01	1.60E+01	1.83E+00	26.57
	CSA	**1.13E−02**	**1.69E−02**	**1.72E−02**	**2.98E−02**	**4.03E−03**	4.36
F_{13}	PSO	8.26E+05	1.52E+06	1.68E+06	2.62E+06	5.65E+05	2.02
	SCA	1.79E+09	3.57E+09	3.46E+09	4.88E+09	8.81E+08	2.96
	GSA	4.32E+03	1.42E+04	2.09E+04	6.22E+04	1.75E+04	26.82
	CSA	**1.21E+01**	**1.41E+01**	**1.40E+01**	**1.52E+01**	**8.96E−01**	4.28

Table 10
Wilcoxon signed rank test results between CSA and existing evolutionary methods for 13 benchmark functions at alpha = 0.05.

Variable	Method	Better	Equal	Worse	R^+	R^-	P-value	Symbol
100	CSA *vs* PSO	13	0	0	91	0	2.44E−04	+
	CSA *vs* SCA	13	0	0	91	0	2.44E−04	+
	CSA *vs* GSA	13	0	0	91	0	2.44E−04	+
300	CSA *vs* PSO	13	0	0	91	0	2.44E−04	+
	CSA *vs* SCA	13	0	0	91	0	2.44E−04	+
	CSA *vs* GSA	13	0	0	91	0	2.44E−04	+

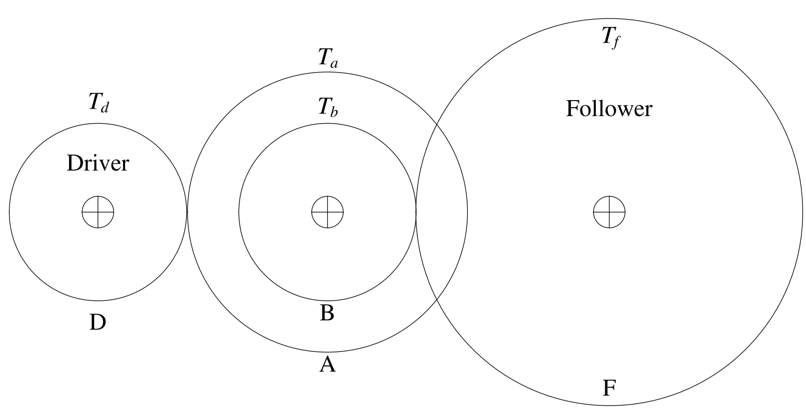

Fig. 7. Sketch map of the Gear train design problem.

Table 11
Brief information of 25 CEC2005 benchmark functions with 10 variables.

Function	*Dim*	*Range*	$f_{\min}$	Type
F_1: Shifted Sphere Function	10	[−100, 100]	−450	Unimodal
F_2: Shifted Schwefel's Problem 1.2	10	[−100, 100]	−450	Unimodal
F_3: Shifted Rotated High Conditioned Elliptic Function	10	[−100, 100]	−450	Unimodal
F_4: Shifted Schwefel's Problem 1.2 with Noise in Fitness	10	[−100, 100]	−450	Unimodal
F_5: Schwefel's Problem 2.6 with Global Optimum on Bounds	10	[−100, 100]	−310	Unimodal
F_6: Shifted Rosenbrock's Function	10	[−100, 100]	390	Multimodal
F_7: Shifted Rotated Griewank's Function without Bounds	10	[−600, 600]	−180	Multimodal
F_8: Shifted Rotated Ackley's Function with Global Optimum on Bounds	10	[−32, 32]	−140	Multimodal
F_9: Shifted Rastrigin's Function	10	[−5, 5]	−330	Multimodal
F_{10}: Shifted Rotated Rastrigin's Function	10	[−5, 5]	−330	Multimodal
F_{11}: Shifted Rotated Weierstrass Function	10	[−0.5, 0.5]	90	Multimodal
F_{12}: Schwefel's Problem 2.13	10	[-π, π]	−460	Multimodal
F_{13}: Expanded Extended Griewank's plus Rosenbrock's Function (F_8F_2)	10	[−3, 1]	−130	Multimodal
F_{14}: Shifted Rotated Expanded Scaffer's F_6	10	[−100, 100]	−300	Multimodal
F_{15}: Hybrid Composition Function	10	[−5, 5]	120	Hybrid
F_{16}: Rotated Hybrid Composition Function	10	[−5, 5]	120	Hybrid
F_{17}: Rotated Hybrid Composition Function with Noise in Fitness	10	[−5, 5]	120	Hybrid
F_{18}: Rotated Hybrid Composition Function	10	[−5, 5]	10	Hybrid
F_{19}: Rotated Hybrid Composition Function with a Narrow Basin for Global Optimum	10	[−5, 5]	10	Hybrid
F_{20}: Rotated Hybrid Composition Function with the Global Optimum on the Bounds	10	[−5, 5]	10	Hybrid
F_{21}: Rotated Hybrid Composition Function	10	[−5, 5]	360	Hybrid
F_{22}: Rotated Hybrid Composition Function with High Condition Number Matrix	10	[−5, 5]	360	Hybrid
F_{23}: Non-Continuous Rotated Hybrid Composition Function	10	[−5, 5]	360	Hybrid
F_{24}: Rotated Hybrid Composition Function	10	[−5, 5]	260	Hybrid
F_{25}: Rotated Hybrid Composition Function without Bounds	10	[−5, 5]	260	Hybrid

Table 12
Statistical results of 25 10-variable CEC2005 benchmark functions by several methods.

Function	Method	Best	Median	Mean	Worst	STD	Time(s)
F_1	PSO	**−4.50E+02**	**−4.50E+02**	**−4.50E+02**	**−4.50E+02**	5.65E−11	0.05
	SCA	−1.81E+02	8.01E+01	1.94E+02	1.42E+03	3.50E+02	0.05
	GSA	**−4.50E+02**	**−4.50E+02**	**−4.50E+02**	**−4.50E+02**	1.48E−12	0.57
	CSA	**−4.50E+02**	**−4.50E+02**	**−4.50E+02**	**−4.50E+02**	**2.26E−14**	0.11
F_2	PSO	**−4.50E+02**	**−4.50E+02**	−4.19E+02	−3.04E+02	5.85E+01	0.08
	SCA	−8.94E+01	3.08E+02	3.63E+02	9.49E+02	3.26E+02	0.05
	GSA	−2.70E+02	−1.06E+02	−5.13E+01	2.74E+02	1.71E+02	0.60
	CSA	**−4.50E+02**	**−4.50E+02**	**−4.50E+02**	**−4.50E+02**	**1.83E−13**	0.09
F_3	PSO	1.62E+04	1.25E+05	1.71E+05	5.01E+05	1.27E+05	0.08
	SCA	2.11E+06	4.35E+06	4.78E+06	1.05E+07	1.97E+06	0.06
	GSA	9.47E+04	3.88E+05	4.82E+05	1.19E+06	2.87E+05	0.59
	CSA	**9.97E+03**	**1.15E+05**	**1.38E+05**	**3.94E+05**	**9.47E+04**	0.11
F_4	PSO	−4.31E+02	5.31E+03	5.22E+03	1.36E+04	3.61E+03	0.05
	SCA	9.35E+01	7.65E+02	9.45E+02	2.52E+03	7.50E+02	0.05
	GSA	7.67E+03	1.22E+04	1.18E+04	1.64E+04	2.69E+03	0.60
	CSA	**−4.50E+02**	**−4.26E+02**	**−2.31E+02**	**8.35E+02**	**3.47E+02**	0.09
F_5	PSO	**−3.10E+02**	2.99E+01	5.71E+02	2.77E+03	1.06E+03	0.05
	SCA	1.54E+02	6.39E+02	7.15E+02	1.64E+03	4.58E+02	0.05
	GSA	3.01E+02	1.89E+03	1.76E+03	4.43E+03	1.01E+03	0.64
	CSA	**−3.10E+02**	**−3.10E+02**	**−2.87E+02**	**−3.98E+01**	**6.30E+01**	0.10
F_6	PSO	3.94E+02	3.95E+02	4.32E+02	9.64E+02	1.29E+02	0.05
	SCA	1.48E+06	4.36E+06	5.64E+06	1.74E+07	4.03E+06	0.05
	GSA	3.97E+02	4.42E+02	9.13E+02	3.93E+03	8.99E+02	0.61
	CSA	**3.90E+02**	**3.92E+02**	**4.04E+02**	**5.80E+02**	**4.25E+01**	0.09
F_7	PSO	−3.78E+01	1.54E+02	1.61E+02	4.59E+02	1.48E+02	0.05
	SCA	−1.65E+02	−1.58E+02	−1.54E+02	−1.18E+02	1.22E+01	0.06
	GSA	−3.99E+01	1.08E+02	1.22E+02	2.58E+02	7.95E+01	0.60
	CSA	**−1.80E+02**	**−1.80E+02**	**−1.79E+02**	**−1.78E+02**	**3.76E−01**	0.12
F_8	PSO	**−1.20E+02**	**−1.20E+02**	**−1.20E+02**	−1.19E+02	8.91E−02	0.05
	SCA	**−1.20E+02**	**−1.20E+02**	**−1.20E+02**	−1.19E+02	**6.33E−02**	0.06
	GSA	**−1.20E+02**	**−1.20E+02**	**−1.20E+02**	−1.19E+02	1.29E−01	0.57
	CSA	**−1.20E+02**	**−1.20E+02**	**−1.20E+02**	**−1.20E+02**	6.75E−02	0.12
F_9	PSO	−3.26E+02	−3.08E+02	−3.06E+02	−2.72E+02	1.19E+01	0.05
	SCA	−3.11E+02	−3.00E+02	−3.00E+02	−2.93E+02	4.35E+00	0.06
	GSA	**−3.28E+02**	**−3.27E+02**	**−3.27E+02**	**−3.22E+02**	**1.42E+00**	0.58
	CSA	**−3.28E+02**	−3.19E+02	−3.20E+02	−3.13E+02	4.49E+00	0.13
F_{10}	PSO	−3.13E+02	−2.90E+02	−2.92E+02	−2.74E+02	1.16E+01	0.05
	SCA	−2.91E+02	−2.80E+02	−2.80E+02	−2.68E+02	5.96E+00	0.07
	GSA	**−3.29E+02**	**−3.27E+02**	**−3.27E+02**	**−3.25E+02**	**1.04E+00**	0.63
	CSA	−3.27E+02	−3.23E+02	−3.22E+02	−3.15E+02	3.36E+00	0.12

(*continued on next page*)

Table 12 (*continued*).

Function	Method	Best	Median	Mean	Worst	STD	Time(s)
F_{11}	PSO	9.42E+01	9.72E+01	9.73E+01	1.00E+02	1.56E+00	0.92
	SCA	9.64E+01	9.88E+01	9.87E+01	1.00E+02	**9.04E−01**	0.91
	GSA	**9.00E+01**	**9.00E+01**	**9.08E+01**	**9.31E+01**	1.10E+00	1.44
	CSA	9.14E+01	9.45E+01	9.49E+01	9.90E+01	1.95E+00	1.70
F_{12}	PSO	−4.56E+02	−9.51E+01	6.89E+03	5.26E+04	1.40E+04	0.18
	SCA	4.69E+03	1.02E+04	1.11E+04	2.17E+04	3.91E+03	0.19
	GSA	−4.40E+02	−1.84E+02	1.26E+03	1.07E+04	3.07E+03	0.76
	CSA	**−4.60E+02**	**−4.41E+02**	**3.26E+02**	**5.38E+03**	**1.63E+03**	0.36
F_{13}	PSO	−1.29E+02	−1.28E+02	−1.27E+02	−1.23E+02	1.70E+00	0.05
	SCA	−1.28E+02	−1.26E+02	−1.26E+02	−1.25E+02	7.21E−01	0.06
	GSA	−1.29E+02	**−1.29E+02**	−1.28E+02	−1.27E+02	5.17E−01	0.62
	CSA	**−1.30E+02**	**−1.29E+02**	**−1.29E+02**	**−1.29E+02**	**2.80E−01**	0.15
F_{14}	PSO	**−2.97E+02**	**−2.97E+02**	**−2.97E+02**	**−2.96E+02**	4.29E−01	0.06
	SCA	**−2.97E+02**	−2.96E+02	−2.96E+02	**−2.96E+02**	**1.75E−01**	0.07
	GSA	−2.96E+02	−2.96E+02	−2.96E+02	−2.95E+02	2.76E−01	0.58
	CSA	**−2.97E+02**	**−2.97E+02**	**−2.97E+02**	**−2.96E+02**	3.41E−01	0.20
F_{15}	PSO	2.42E+02	6.32E+02	5.64E+02	9.55E+02	2.38E+02	1.69
	SCA	4.03E+02	5.94E+02	6.07E+02	7.54E+02	**1.25E+02**	2.07
	GSA	2.43E+02	**4.13E+02**	4.79E+02	1.02E+03	1.74E+02	2.19
	CSA	**2.09E+02**	5.33E+02	**4.35E+02**	**6.49E+02**	1.51E+02	3.42
F_{16}	PSO	2.47E+02	3.33E+02	3.29E+02	4.30E+02	4.32E+01	1.78
	SCA	2.81E+02	3.34E+02	3.42E+02	4.19E+02	3.46E+01	1.85
	GSA	2.18E+02	2.43E+02	**2.44E+02**	2.83E+02	**1.66E+01**	2.24
	CSA	**2.14E+02**	**2.42E+02**	2.47E+02	**2.79E+02**	2.13E+01	3.51
F_{17}	PSO	2.83E+02	3.86E+02	4.08E+02	5.69E+02	7.57E+01	1.78
	SCA	3.12E+02	4.13E+02	4.26E+02	6.46E+02	7.88E+01	1.83
	GSA	2.52E+02	3.01E+02	3.25E+02	6.33E+02	8.23E+01	2.26
	CSA	**2.35E+02**	**2.76E+02**	**2.81E+02**	**3.62E+02**	**3.30E+01**	3.50
F_{18}	PSO	7.29E+02	1.05E+03	9.83E+02	1.19E+03	1.25E+02	1.80
	SCA	7.35E+02	**9.55E+02**	**9.21E+02**	1.08E+03	1.17E+02	1.96
	GSA	9.10E+02	1.03E+03	9.94E+02	1.10E+03	**7.33E+01**	2.16
	CSA	**6.35E+02**	9.79E+02	9.27E+02	**1.04E+03**	1.22E+02	3.56
F_{19}	PSO	7.24E+02	1.05E+03	9.76E+02	1.16E+03	1.15E+02	1.79
	SCA	7.32E+02	9.44E+02	9.23E+02	1.07E+03	1.04E+02	1.91
	GSA	9.10E+02	1.04E+03	1.01E+03	1.13E+03	**6.84E+01**	2.15
	CSA	**4.28E+02**	**8.56E+02**	**8.68E+02**	**1.05E+03**	1.47E+02	3.63
F_{20}	PSO	7.20E+02	9.62E+02	9.44E+02	1.10E+03	1.02E+02	2.07
	SCA	**7.07E+02**	**8.89E+02**	**8.98E+02**	**1.08E+03**	1.21E+02	1.86
	GSA	9.10E+02	1.04E+03	9.84E+02	**1.08E+03**	**6.92E+01**	2.16
	CSA	7.36E+02	9.66E+02	9.37E+02	**1.08E+03**	1.05E+02	3.66
F_{21}	PSO	1.17E+03	1.56E+03	1.55E+03	1.70E+03	**1.17E+02**	1.99
	SCA	1.12E+03	**1.34E+03**	**1.39E+03**	1.65E+03	1.63E+02	1.87
	GSA	8.60E+02	1.53E+03	1.45E+03	**1.61E+03**	2.08E+02	2.15
	CSA	**6.60E+02**	1.50E+03	**1.39E+03**	1.62E+03	2.20E+02	3.64
F_{22}	PSO	1.18E+03	1.26E+03	1.26E+03	1.32E+03	3.99E+01	2.13
	SCA	1.15E+03	1.22E+03	1.23E+03	1.29E+03	3.31E+01	2.05
	GSA	1.10E+03	**1.11E+03**	**1.12E+03**	**1.16E+03**	**2.25E+01**	2.31
	CSA	**6.60E+02**	1.12E+03	1.13E+03	1.31E+03	1.30E+02	3.97
F_{23}	PSO	1.19E+03	**1.55E+03**	1.52E+03	1.69E+03	1.43E+02	1.91
	SCA	1.34E+03	1.58E+03	1.56E+03	1.66E+03	1.01E+02	1.87
	GSA	1.45E+03	1.58E+03	1.57E+03	**1.64E+03**	**6.58E+01**	2.05
	CSA	**9.16E+02**	**1.55E+03**	**1.47E+03**	1.65E+03	2.20E+02	3.69
F_{24}	PSO	1.27E+03	1.29E+03	1.32E+03	1.56E+03	**8.78E+01**	1.15
	SCA	6.56E+02	9.71E+02	9.39E+02	**1.09E+03**	1.12E+02	1.27
	GSA	**4.60E+02**	1.25E+03	1.14E+03	1.53E+03	3.53E+02	1.85
	CSA	**4.60E+02**	**4.60E+02**	**5.10E+02**	1.16E+03	1.67E+02	2.17
F_{25}	PSO	1.27E+03	1.28E+03	1.30E+03	1.50E+03	**7.00E+01**	1.17
	SCA	7.76E+02	9.71E+02	9.75E+02	**1.25E+03**	9.09E+01	1.18
	GSA	**4.60E+02**	1.16E+03	1.15E+03	1.56E+03	3.41E+02	1.73
	CSA	**4.60E+02**	**4.60E+02**	**6.41E+02**	1.27E+03	2.43E+02	2.23

Table 13
Wilcoxon signed rank test results for 10-variable CEC2005 functions at alpha = 0.05.

Item	Better	Equal	Worse	R^+	R^-	P-value	Symbol
CSA *vs* PSO	25	0	0	325	0	1.23E−05	+
CSA *vs* SCA	22	0	3	305	20	1.26E−04	+
CSA *vs* GSA	19	0	6	288	37	7.33E−04	+

Table 14
Brief information of 25 CEC2014 benchmark functions with 10 variables.

Function	*Dim*	*Range*	$f_{\min}$	Type
F_1: Rotated High Conditioned Elliptic Function	10	[−100, 100]	100	Unimodal
F_2: Rotated Bent Cigar Function	10	[−100, 100]	200	Unimodal
F_3: Rotated Discus Function	10	[−100, 100]	300	Unimodal
F_4: Shifted and Rotated Rosenbrock's Function	10	[−100, 100]	400	Multimodal
F_5: Shifted and Rotated Ackley's Function	10	[−100, 100]	500	Multimodal
F_6: Shifted and Rotated Weierstrass Function	10	[−100, 100]	600	Multimodal
F_7: Shifted and Rotated Griewank's Function	10	[−100, 100]	700	Multimodal
F_8: Shifted Rastrigin's Function	10	[−100, 100]	800	Multimodal
F_9: Shifted and Rotated Rastrigin's Function	10	[−100, 100]	900	Multimodal
F_{10}: Shifted Schwefel's Function	10	[−100, 100]	1000	Multimodal
F_{11}: Shifted and Rotated Schwefel's Function	10	[−100, 100]	1100	Multimodal
F_{12}: Shifted and Rotated Katsuura Function	10	[−100, 100]	1200	Multimodal
F_{13}: Shifted and Rotated HappyCat Function	10	[−100, 100]	1300	Multimodal
F_{14}: Shifted and Rotated HGBat Function	10	[−100, 100]	1400	Multimodal
F_{15}: Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	10	[−100, 100]	1500	Multimodal
F_{16}: Shifted and Rotated Expanded Scaffer's F_6Function	10	[−100, 100]	1600	Multimodal
F_{17}: Hybrid Function 1 ($N = 3$)	10	[−100, 100]	1700	Hybrid
F_{18}: Hybrid Function 2 ($N = 3$)	10	[−100, 100]	1800	Hybrid
F_{19}: Hybrid Function 3 ($N = 4$)	10	[−100, 100]	1900	Hybrid
F_{20}: Hybrid Function 4 ($N = 4$)	10	[−100, 100]	2000	Hybrid
F_{21}: Hybrid Function 5 ($N = 5$)	10	[−100, 100]	2100	Hybrid
F_{22}: Hybrid Function 6 ($N = 5$)	10	[−100, 100]	2200	Hybrid
F_{23}: Composition Function 1 ($N = 5$)	10	[−100, 100]	2300	Composite
F_{24}: Composition Function 2 ($N = 3$)	10	[−100, 100]	2400	Composite
F_{25}: Composition Function 3 ($N = 3$)	10	[−100, 100]	2500	Composite
F_{26}: Composition Function 4 ($N = 5$)	10	[−100, 100]	2600	Composite
F_{27}: Composition Function 5 ($N = 5$)	10	[−100, 100]	2700	Composite
F_{28}: Composition Function 6 ($N = 5$)	10	[−100, 100]	2800	Composite
F_{29}: Composition Function 7 ($N = 3$)	10	[−100, 100]	2900	Composite
F_{30}: Composition Function 8 ($N = 3$)	10	[−100, 100]	3000	Composite

Table 15
Statistical results of 30 10-variable CEC2014 benchmark functions by several methods.

Function	Item	Best	Median	Mean	Worst	STD	Time(s)
F_1	PSO	2.37E+03	**5.74E+04**	9.33E+04	2.10E+05	8.93E+04	0.05
	SCA	2.70E+06	7.03E+06	8.52E+06	1.52E+07	3.57E+06	0.07
	GSA	9.60E+06	1.51E+07	1.46E+07	2.23E+07	3.11E+06	0.60
	CSA	**1.62E+03**	7.99E+04	**7.05E+04**	**1.25E+05**	**3.93E+04**	0.12
F_2	PSO	2.23E+02	9.69E+02	1.57E+03	5.76E+03	1.53E+03	0.06
	SCA	2.37E+08	5.26E+08	5.82E+08	1.02E+09	2.37E+08	0.06
	GSA	2.25E+02	**4.17E+02**	**6.33E+02**	**1.50E+03**	**4.19E+02**	0.61
	CSA	**2.00E+02**	1.86E+03	4.01E+03	1.14E+04	4.01E+03	0.11
F_3	PSO	**3.11E+02**	**9.45E+02**	1.13E+03	2.90E+03	7.57E+02	0.05
	SCA	1.99E+03	3.05E+03	4.42E+03	1.64E+04	3.41E+03	0.06
	GSA	1.15E+04	1.64E+04	1.64E+04	2.38E+04	2.93E+03	0.58
	CSA	6.63E+02	9.46E+02	**9.63E+02**	**1.18E+03**	**1.57E+02**	0.11
F_4	PSO	4.00E+02	**4.35E+02**	4.30E+02	**4.35E+02**	**9.40E+00**	0.04
	SCA	4.36E+02	4.56E+02	4.58E+02	5.14E+02	1.63E+01	0.05
	GSA	4.01E+02	**4.35E+02**	4.34E+02	4.55E+02	1.16E+01	0.62
	CSA	**4.00E+02**	**4.35E+02**	**4.26E+02**	**4.35E+02**	1.50E+01	0.12
F_5	PSO	5.20E+02	5.20E+02	5.20E+02	5.21E+02	1.00E−01	0.05
	SCA	5.20E+02	5.20E+02	5.20E+02	5.21E+02	1.00E−01	0.06
	GSA	**5.20E+02**	**5.20E+02**	**5.20E+02**	**5.20E+02**	**0.00E+00**	0.59
	CSA	5.20E+02	5.20E+02	5.20E+02	5.20E+02	1.00E−01	0.11
F_6	PSO	**6.00E+02**	6.03E+02	6.03E+02	6.06E+02	1.60E+00	1.58
	SCA	6.04E+02	6.06E+02	6.06E+02	6.09E+02	1.20E+00	1.61
	GSA	6.04E+02	6.05E+02	6.05E+02	6.08E+02	**1.10E+00**	1.95
	CSA	**6.00E+02**	**6.02E+02**	**6.02E+02**	**6.04E+02**	1.30E+00	2.84
F_7	PSO	**7.00E+02**	7.00E+02	7.00E+02	7.01E+02	1.00E−01	0.06
	SCA	7.06E+02	7.10E+02	7.11E+02	7.20E+02	3.60E+00	0.08
	GSA	7.00E+02	7.01E+02	7.02E+02	7.09E+02	2.80E+00	0.58
	CSA	**7.00E+02**	**7.00E+02**	**7.00E+02**	**7.00E+02**	**0.00E+00**	0.13
F_8	PSO	8.11E+02	8.23E+02	8.22E+02	8.37E+02	7.80E+00	0.06
	SCA	8.25E+02	8.41E+02	8.41E+02	8.59E+02	7.10E+00	0.07
	GSA	8.24E+02	8.36E+02	8.37E+02	8.50E+02	7.70E+00	0.64
	CSA	**8.03E+02**	**8.10E+02**	**8.10E+02**	**8.19E+02**	**4.90E+00**	0.12
F_9	PSO	**9.06E+02**	9.21E+02	9.20E+02	**9.33E+02**	7.30E+00	0.06
	SCA	9.29E+02	9.43E+02	9.43E+02	9.63E+02	7.50E+00	0.08
	GSA	9.18E+02	9.31E+02	9.32E+02	9.40E+02	**6.00E+00**	0.64
	CSA	**9.06E+02**	**9.13E+02**	**9.16E+02**	9.37E+02	8.70E+00	0.13

(*continued on next page*)

Table 15 (*continued*).

Function	Item	Best	Median	Mean	Worst	STD	Time(s)
F_{10}	PSO	1.13E+03	1.47E+03	1.52E+03	1.99E+03	2.23E+02	0.08
	SCA	1.62E+03	2.09E+03	2.05E+03	2.42E+03	2.14E+02	0.11
	GSA	1.46E+03	1.96E+03	1.95E+03	2.54E+03	3.05E+02	0.66
	CSA	**1.05E+03**	**1.25E+03**	**1.27E+03**	**1.69E+03**	**1.72E+02**	0.17
F_{11}	PSO	1.37E+03	1.68E+03	1.75E+03	2.22E+03	2.60E+02	0.08
	SCA	1.98E+03	2.46E+03	2.39E+03	2.59E+03	**1.74E+02**	0.10
	GSA	1.65E+03	2.14E+03	2.15E+03	2.59E+03	2.99E+02	0.68
	CSA	**1.13E+03**	**1.48E+03**	**1.50E+03**	**1.89E+03**	2.34E+02	0.17
F_{12}	PSO	1.20E+03	1.20E+03	1.20E+03	1.20E+03	5.00E−01	0.46
	SCA	1.20E+03	1.20E+03	1.20E+03	1.20E+03	**2.00E−01**	0.54
	GSA	1.20E+03	1.20E+03	1.20E+03	1.20E+03	7.00E−01	0.96
	CSA	**1.20E+03**	**1.20E+03**	**1.20E+03**	**1.20E+03**	5.00E−01	0.99
F_{13}	PSO	1.30E+03	1.31E+03	1.31E+03	1.31E+03	9.00E−01	0.05
	SCA	1.30E+03	1.30E+03	1.30E+03	1.30E+03	**1.00E−01**	0.07
	GSA	1.30E+03	1.30E+03	1.30E+03	1.31E+03	7.00E−01	0.59
	CSA	**1.30E+03**	**1.30E+03**	**1.30E+03**	**1.30E+03**	**1.00E−01**	0.11
F_{14}	PSO	1.43E+03	1.46E+03	1.46E+03	1.48E+03	1.60E+01	0.05
	SCA	1.40E+03	1.40E+03	1.40E+03	1.40E+03	6.00E−01	0.06
	GSA	1.40E+03	1.40E+03	1.40E+03	**1.40E+03**	**0.00E+00**	0.69
	CSA	**1.40E+03**	**1.40E+03**	**1.40E+03**	1.40E+03	2.00E−01	0.11
F_{15}	PSO	7.14E+03	1.38E+05	2.07E+05	7.32E+05	1.99E+05	0.06
	SCA	1.50E+03	1.51E+03	1.51E+03	1.51E+03	1.90E+00	0.07
	GSA	1.50E+03	1.50E+03	1.50E+03	1.50E+03	8.00E−01	0.58
	CSA	**1.50E+03**	**1.50E+03**	**1.50E+03**	**1.50E+03**	**5.00E−01**	0.12
F_{16}	PSO	1.60E+03	1.60E+03	1.60E+03	1.60E+03	**1.00E−01**	0.05
	SCA	1.60E+03	1.60E+03	1.60E+03	1.60E+03	2.00E−01	0.07
	GSA	1.60E+03	1.60E+03	1.60E+03	1.60E+03	2.00E−01	0.59
	CSA	**1.60E+03**	**1.60E+03**	**1.60E+03**	**1.60E+03**	3.00E−01	0.11
F_{17}	PSO	1.43E+06	8.54E+06	1.21E+07	4.59E+07	1.11E+07	0.08
	SCA	8.17E+03	1.79E+04	4.08E+04	3.66E+05	7.82E+04	0.07
	GSA	3.30E+05	6.26E+05	6.20E+05	9.94E+05	1.86E+05	0.63
	CSA	**2.47E+03**	**3.62E+03**	**3.94E+03**	**8.41E+03**	**1.59E+03**	0.14
F_{18}	PSO	6.48E+06	1.15E+08	1.44E+08	4.37E+08	1.26E+08	0.06
	SCA	6.65E+03	1.79E+04	2.54E+04	6.49E+04	1.62E+04	0.07
	GSA	**5.92E+03**	**9.25E+03**	**9.43E+03**	**1.32E+04**	**2.07E+03**	0.62
	CSA	8.06E+03	1.20E+04	1.20E+04	2.01E+04	2.96E+03	0.13
F_{19}	PSO	1.92E+03	2.00E+03	2.00E+03	2.11E+03	4.52E+01	0.35
	SCA	1.90E+03	1.91E+03	1.91E+03	1.91E+03	**8.00E−01**	0.37
	GSA	1.90E+03	1.91E+03	1.91E+03	1.91E+03	1.40E+00	0.89
	CSA	**1.90E+03**	**1.90E+03**	**1.90E+03**	**1.90E+03**	1.20E+00	0.73
F_{20}	PSO	9.74E+03	6.35E+06	1.25E+07	7.63E+07	2.05E+07	0.06
	SCA	**2.09E+03**	**3.40E+03**	**4.44E+03**	**1.15E+04**	**2.72E+03**	0.08
	GSA	6.38E+03	1.51E+04	1.74E+04	6.42E+04	1.22E+04	0.63
	CSA	3.95E+03	7.26E+03	7.94E+03	1.25E+04	2.75E+03	0.13
F_{21}	PSO	1.59E+05	4.55E+06	9.99E+06	4.70E+07	1.18E+07	0.06
	SCA	5.48E+03	9.91E+03	1.14E+04	**2.11E+04**	**4.75E+03**	0.08
	GSA	9.82E+04	8.00E+05	8.10E+05	1.90E+06	5.10E+05	0.63
	CSA	**2.32E+03**	**6.96E+03**	**1.03E+04**	2.52E+04	7.96E+03	0.15
F_{22}	PSO	2.53E+03	2.81E+03	2.82E+03	3.19E+03	1.60E+02	0.14
	SCA	2.23E+03	2.26E+03	**2.26E+03**	**2.27E+03**	**1.16E+01**	0.13
	GSA	2.36E+03	2.49E+03	2.49E+03	2.57E+03	5.76E+01	0.68
	CSA	**2.20E+03**	**2.24E+03**	2.27E+03	2.36E+03	6.43E+01	0.25
F_{23}	PSO	2.70E+03	2.93E+03	2.98E+03	3.39E+03	2.06E+02	0.16
	SCA	2.64E+03	2.64E+03	2.64E+03	2.65E+03	4.40E+00	0.18
	GSA	**2.50E+03**	**2.55E+03**	**2.58E+03**	2.76E+03	8.58E+01	0.66
	CSA	2.63E+03	2.63E+03	2.63E+03	**2.63E+03**	**0.00E+00**	0.32
F_{24}	PSO	2.62E+03	2.64E+03	2.64E+03	2.67E+03	1.16E+01	0.13
	SCA	2.55E+03	2.55E+03	2.55E+03	**2.56E+03**	4.90E+00	0.15
	GSA	2.60E+03	2.60E+03	2.60E+03	2.61E+03	**1.40E+00**	0.68
	CSA	**2.52E+03**	**2.53E+03**	**2.55E+03**	2.60E+03	3.82E+01	0.26
F_{25}	PSO	2.71E+03	2.73E+03	2.73E+03	2.76E+03	1.23E+01	0.14
	SCA	**2.65E+03**	2.70E+03	**2.70E+03**	2.70E+03	1.37E+01	0.17
	GSA	2.69E+03	**2.70E+03**	2.70E+03	2.70E+03	1.40E+00	0.70
	CSA	2.70E+03	2.70E+03	2.70E+03	**2.70E+03**	**0.00E+00**	0.30
F_{26}	PSO	2.70E+03	2.71E+03	2.71E+03	2.71E+03	2.90E+00	1.56
	SCA	2.70E+03	2.70E+03	2.70E+03	2.70E+03	**1.00E−01**	1.61
	GSA	2.70E+03	2.71E+03	2.71E+03	2.74E+03	1.13E+01	2.12
	CSA	**2.70E+03**	**2.70E+03**	**2.70E+03**	**2.70E+03**	**1.00E−01**	3.15

(*continued on next page*)

Table 15 (*continued*).

Function	Item	Best	Median	Mean	Worst	STD	Time(s)
F_{27}	PSO	3.26E+03	3.47E+03	3.45E+03	3.56E+03	**7.98E+01**	1.61
	SCA	2.71E+03	3.10E+03	**3.01E+03**	**3.11E+03**	1.71E+02	1.66
	GSA	2.95E+03	3.34E+03	3.27E+03	3.57E+03	2.21E+02	2.15
	CSA	**2.70E+03**	**3.07E+03**	3.03E+03	3.14E+03	1.17E+02	3.32
F_{28}	PSO	4.01E+03	4.86E+03	4.87E+03	5.67E+03	4.59E+02	0.21
	SCA	3.22E+03	3.25E+03	3.27E+03	**3.37E+03**	**5.00E+01**	0.22
	GSA	3.32E+03	3.77E+03	3.72E+03	4.30E+03	3.29E+02	0.80
	CSA	**3.17E+03**	**3.18E+03**	**3.24E+03**	3.37E+03	8.11E+01	0.45
F_{29}	PSO	8.14E+04	3.90E+07	4.66E+07	1.15E+08	3.51E+07	0.46
	SCA	3.95E+03	7.93E+03	7.65E+03	1.36E+04	2.83E+03	0.44
	GSA	**3.10E+03**	**3.10E+03**	**3.15E+03**	**3.41E+03**	**1.03E+02**	0.93
	CSA	3.16E+03	3.34E+03	3.36E+03	3.75E+03	1.64E+02	0.87
F_{30}	PSO	4.06E+04	5.70E+05	6.93E+05	1.92E+06	6.00E+05	0.18
	SCA	4.12E+03	4.62E+03	4.79E+03	5.71E+03	5.52E+02	0.19
	GSA	**3.20E+03**	1.25E+04	1.39E+04	2.96E+04	7.03E+03	0.79
	CSA	3.54E+03	**4.10E+03**	**4.16E+03**	**5.26E+03**	**4.10E+02**	0.38

Table 16
Wilcoxon signed rank test results for 10-variable CEC2014 functions at alpha = 0.05.

Item	Better	Equal	Worse	R^+	R^-	P-value	Symbol
CSA *vs* PSO	23	0	7	442	23	1.64E−05	+
CSA *vs* SCA	21	0	9	403	62	4.53E−04	+
CSA *vs* GSA	20	0	10	379	86	2.58E−03	+

Table 17
Comparison results of the best solutions.

Item	Sandgren [50]	Kannan and Kramer [51]	Deb and Goya [52]	Gandomi et al. [53]	CSA
$T_d(x_1)$	18	13	19	19	19
$T_b(x_2)$	22	15	16	16	16
$T_a(x_3)$	45	33	49	43	43
$T_f(x_4)$	60	41	43	49	49
Gear ratio	0.146667	0.144124	0.144281	0.144281	0.144281
$f(\boldsymbol{x})$	5.712×10^{-6}	2.146×10^{-8}	2.701×10^{-12}	2.701×10^{-12}	2.701×10^{-12}

Table 18
Best result by various methods for parameter estimation for frequency-modulated sound waves problem.

Method	x_1	x_2	x_3	x_4	x_5	x_6	$f(\boldsymbol{x})$
SCA	−0.63712547	−0.01634062	4.09479208	4.88484151	0.17317635	0.00187278	12.22982793
GSA	0.45811358	−5.17371979	3.71487656	−4.83003278	0.05808168	−1.24201621	21.38559436
CSA	−0.62390921	−0.01120452	−4.32808097	−4.85660001	1.22183050	−0.02602830	11.55831235

Table 19
Statistical results of different methods for the speed reducer design problem.

Algorithms	Best	Median	Mean	Worst	STD
Akhtar et al. [54]	3008.08	N/A	3012.12	3028	N/A
Montes et al. [55]	3025.005	N/A	3088.7778	3078.5918	N/A
Ray and Liew [56]	2994.744241	3001.758264	3001.758226	3009.964736	4.0091423
Montes et al. [57]	2996.356689	N/A	2996.36722	N/A	8.20×10^{-3}
Cagnina et al. [58]	2996.348165	N/A	2996.3482	N/A	0
Gandomi et al. [53]	3000.981	N/A	3007.1997	N/A	4.9634
CSA	2996.34816496853	2996.34816496853	2996.348164968529	2996.3481649685305	9.90×10^{-13}

where

$$\begin{cases} y(t) = a_1 \sin\left(w_1 t\theta + a_2 \sin\left(w_2 t\theta + a_3 \sin\left(w_3 t\theta\right)\right)\right) \\ y_0(t) = \sin\left(5t\theta - 1.5 \sin\left(4.8t\theta + 2.0 \sin\left(4.9t\theta\right)\right)\right) \end{cases}, \theta = 2\pi/100 \quad (16)$$

This problem is address by three methods (SCA, GSA and CSA), and the best results are given in Table 18. It can be clearly seen that the CSA method can produce better solutions than the other method, demonstrating its superiority and feasibility in engineering problems.

4.3. Speed reducer design problem

Fig. 8 shows the sketch map of the speed reducer design problem [59]. In this problem, the goal is to determine the optimal parameters to minimize the weight of the speed reducer, including the face width b, teeth module m, number of teeth on pinion z, length of shaft 1 between bearings l_1, length of shaft 2 between bearings l_2, diameter of shaft 1 d_1, and diameter of shaft 2 d_2. Then, this problem with variable vector $\boldsymbol{x}$ =

Table 20
Optimal results of different methods for the speed reducer design problem.

Variable	Akhtar et al. [54]	Montes et al. [55]	Ray and Liew [56]	Montes et al. [57]	Cagnina et al. [58]	Gandomi et al. [53]	CSA
x_1	3.506122	3.506163	3.50000681	3.50001	3.5	3.5015	3.5
x_2	0.700006	0.700831	0.70000001	0.7	0.7	0.7	0.7
x_3	17	17	17	17	17	17	17
x_4	7.549126	7.460181	7.32760205	7.300156	7.3	7.605	7.3
x_5	7.85933	7.962143	7.71532175	7.800027	7.8	7.8181	7.8
x_6	3.365576	3.3629	3.35026702	3.350221	3.350214	3.352	3.35021467
x_7	5.289773	5.309	5.2866545	5.286685	5.286683	5.2875	5.28668323
g_1	−0.0755	−0.0777	−0.0739171	−0.073918	−0.073915	−0.0743	−0.07391528
g_2	−0.1994	−0.2013	−0.1980001	−0.198001	−0.197998	−0.1983	−0.19799853
g_3	−0.4562	−0.4741	−0.9999967	−0.499144	−0.499172	−0.4349	−0.49917225
g_4	−0.8994	−0.8971	−0.9999995	−0.901471	−0.901471	−0.9008	−0.90147170
g_5	−0.0132	−0.011	−0.6667294	−0.000005	0	−0.0011	0
g_6	−0.0017	−0.0125	−1.95E−08	−0.000001	−5.00E−16	−0.0004	−4.44E−16
g_7	−0.7025	−0.7022	−0.7024999	−0.7025	−0.7025	−0.7025	−0.7025
g_8	−0.0017	−0.0006	−0.0000019	−0.000003	−1.00E−16	−0.0004	0
g_9	−0.5826	−0.5831	−0.5833325	−0.583332	−0.583333	−0.5832	−0.58333333
g_{10}	−0.0796	−0.0691	−0.0548885	−0.051345	−0.051325	−0.089	−0.05132575
g_{11}	−0.0179	−0.0279	−2.33E−07	−0.010856	−0.010852	−0.013	−0.01085237
$f(\boldsymbol{x})$	3008.08	3025.005	2994.744241	2996.356689	2996.348165	3000.981	2996.348165

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (b, m, z, l_1, l_2, d_1, d_2)$ is described as:

$$
\begin{aligned}
\min\quad & f(\boldsymbol{x}) = 0.7854x_1x_2^2\left(3.3333x_3^2 + 14.9334x_3 - 43.0934\right) \\
& -1.508x_1\left(x_6^2 + x_7^2\right) + 7.4777\left(x_6^3 + x_7^3\right) + 0.7854\left(x_4x_6^2 + x_5x_7^2\right) \\
s.t.\quad & g_1(\boldsymbol{x}) = \frac{27}{x_1x_2^2x_3} - 1 \le 0 \\
& g_2(\boldsymbol{x}) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \le 0 \\
& g_3(\boldsymbol{x}) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \le 0 \\
& g_4(\boldsymbol{x}) = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \le 0 \\
& g_5(\boldsymbol{x}) = \frac{\sqrt{(745x_4/x_2x_3)^2 + 16.9\times 10^6}}{110x_6^3} - 1 \le 0 \\
& g_6(\boldsymbol{x}) = \frac{\sqrt{(745x_5/x_2x_3)^2 + 157.5\times 10^6}}{85x_7^3} - 1 \le 0 \\
& g_7(\boldsymbol{x}) = \frac{x_2x_3}{40} - 1 \le 0 \\
& g_8(\boldsymbol{x}) = \frac{5x_2}{x_1} - 1 \le 0 \\
& g_9(\boldsymbol{x}) = \frac{x_1}{12x_2} - 1 \le 0 \\
& g_{10}(\boldsymbol{x}) = \frac{1.56x_6 + 1.9}{x_4} - 1 \le 0 \\
& g_{11}(\boldsymbol{x}) = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0 \\
& 2.6 \le x_1 \le 3.6 \\
& 0.7 \le x_2 \le 0.8 \\
& 17 \le x_3 \le 28 \\
& 7.3 \le x_4 \le 8.3 \\
& 7.8 \le x_5 \le 8.3 \\
& 2.9 \le x_6 \le 3.9 \\
& 5.0 \le x_7 \le 5.5
\end{aligned} \tag{17}
$$

This problem is addressed by several evolutionary algorithms, and the statistical results and best result obtained by several methods for the speed reducer design problem are given in Tables 19–20. It can be seen that the CSA method not only outperforms the existing methods in most of statistical indexes, but also find the best solution to minimize the total weight of the speed reducer.

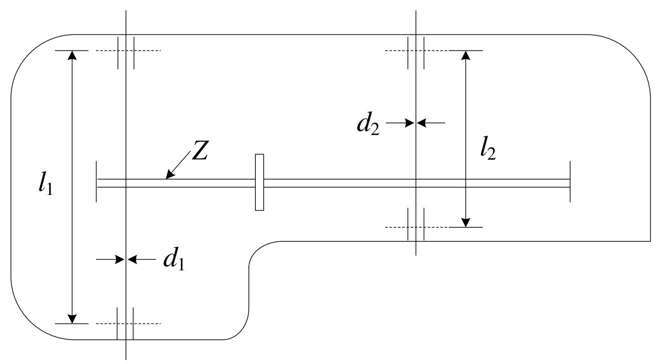

Fig. 8. Sketch map of the speed reducer design problem.

4.4. Himmelblau's nonlinear problems

As a famous nonlinear constrained optimization problem [60], the goal is to find out the best parameters to minimize the objective function. Then, this problem with the variable vector $\boldsymbol{x} = (x_1, x_2, x_3, x_4, x_5)$ can be mathematically described as below:

$$
\begin{aligned}
\min\ & f(\boldsymbol{x}) = 5.3578547x_3^2 + 0.8356891x_2x_5 \\
& +37.293239x_1 - 40792.141 \\
s.t.\ & 0 \le g_1(\boldsymbol{x}) = 85.334407 + 0.0056858x_2x_5 \\
& +0.0006262x_1x_4 - 0.0022053x_3x_5 \le 92 \\
& 90 \le g_2(\boldsymbol{x}) = 80.51249 + 0.0071317x_2x_5 \\
& +0.0029955x_1x_2 - 0.0021813x_3^2 \le 110 \\
& 20 \le g_3(\boldsymbol{x}) = 9.300961 + 0.0047026x_3x_5 \\
& +0.0012547x_1x_3 + 0.0019085x_3x_4 \le 25 \\
& 78 \le x_1 \le 102 \\
& 33 \le x_2 \le 45 \\
& 27 \le x_3, x_4, x_5 \le 45
\end{aligned} \tag{18}
$$

This problem is solved by the CSA method and several existing methods. Then, the statistical results and best results by the employed methods are given in Tables 21–22. It can be found that standard variation of the CSA method is obviously smaller than the other methods; besides, the CSA method works well to find better solution as compared with conventional approaches.

4.5. Welded beam design problem

Fig. 9 shows the sketch map of the welded beam design problem. The goal is to minimize the fabricating cost by carefully choosing 4 variables of the welded beam [64], including the weld thickness h, the length of the welded joint l, the beam width t and the beam thickness b. Next, two different versions of this problem with the variable vector $\boldsymbol{x} = (x_1, x_2, x_3, x_4) = (h, l, t, b)$ are used.

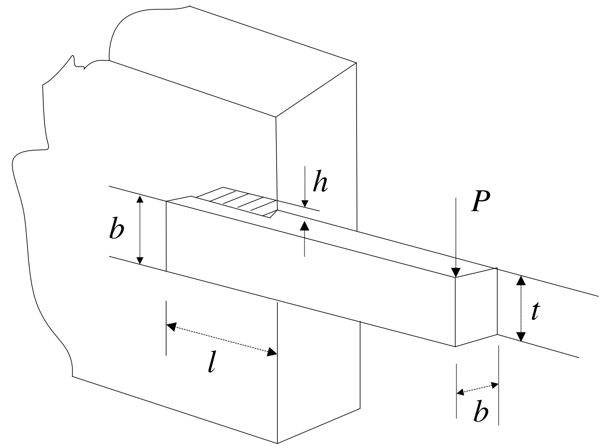

Fig. 9. Sketch map of the welded beam design problem.

4.5.1. Version I

The optimization model for the welded beam design problem on version I is given as below:

$$\begin{aligned}
\min \quad & f(\boldsymbol{x}) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2) \\
s.t. \quad & g_1(\boldsymbol{x}) = \tau(\boldsymbol{x}) - \tau_{\max} \le 0 \\
& g_2(\boldsymbol{x}) = \sigma(\boldsymbol{x}) - \sigma_{\max} \le 0 \\
& g_3(\boldsymbol{x}) = x_1 - x_4 \le 0; \\
& g_4(\boldsymbol{x}) = 0.125 - x_1 \le 0 \\
& g_5(\boldsymbol{x}) = \delta(\boldsymbol{x}) - 0.25 \le 0 \\
& g_6(\boldsymbol{x}) = P - P_c \le 0 \\
& 0.1 \le x_1, x_4 \le 2 \\
& 0.1 \le x_2, x_3 \le 10
\end{aligned} \tag{19}$$

where $\tau_{\max} = 13\,600$ psi is the maximum shear stress of the weld. σ is the normal stress in the beam. $\sigma_{\max} = 30\,000$ psi is the maximum normal stress in the beam. $P = 6000$ lb is the load. P_c is the bar buckling load. δ is the beam end deflection. τ is the shear stress in the weld, which is obtained by

$$\tau = \sqrt{\tau_1^2 + 2\tau_1\tau_2\left(\frac{x_2}{2R}\right) + \tau_2^2};\ \tau_1 = \frac{P}{x_1x_2\sqrt{2}};\ \tau_2 = \frac{MR}{J} \tag{20}$$

$$M = P\left(L + \frac{x_2}{2}\right);\ J = 2\left\{\frac{x_1x_2}{\sqrt{2}}\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\} \tag{21}$$

Where

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2};\ \sigma = \frac{6PL}{x_4x_3^2};\ \delta = \frac{4PL^3}{Ex_3^3x_4} \tag{22}$$

$$P_c = \frac{4.013\sqrt{\frac{EGx_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \tag{23}$$

$$G = 12 \times 10^6\ \text{psi},\ E = 30 \times 10^6\ \text{psi},\ P = 6000\ \text{lb},\ L = 14\text{in} \tag{24}$$

This problem is solved by a variety of methods. The statistical results and best results obtained by the above methods are given in Tables 23–24. It can be found that the CSA can provide the near-optimal solutions and outperforms its competitor with respect to almost all the indexes. Thus, the CSA method proves to be an effective method for engineering design problem.

4.5.2. Version II

In the version II, an extra constraint g_7 is considered in the modeling process, which the terms different from the Version I are also given as below:

$$g_7(\boldsymbol{x}) = 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5 \le 0 \tag{25}$$

$$J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\};\ \sigma = \frac{6PL^3}{x_1x_3^3x_4} \tag{26}$$

$$P_c = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \tag{27}$$

For the welded beam design problem on version II, the results of various methods are introduced for comparison. From the data listed in Tables 25–26, it can be found that the CSA method can provide the best solution among all the selected methods. Thus, this case implies that the CSA method has a stronger ability to obtain the global optima than the control methods.

4.6. Tabular column design problem

Fig. 10 shows the sketch map of the uniform tabular column design problem [64]. In this problem, the goal is to use the minimal cost to obtain a uniform column of tabular section that can carry a compressive load $P = 2500$ kgf. The mean diameter d and the thickness t vary in the range of [2,14] and [0.2,0.8]. The characteristic parameters in the constituent materials of the column are set as: the yield stress $\sigma_y = 500$ kgf/cm^2, the elasticity modulus $E = 0.85 \times 10^6$ kgf/cm^2 and the density $\rho = 0.0025$ kgf/cm^3. The length L of the column is 250 cm. Then, this problem with the variable vector $\boldsymbol{x} = (x_1, x_2) = (d, t)$ can be mathematically described as below:

$$\begin{aligned}
\min \quad & f(\boldsymbol{x}) = 9.82dt + 2d \\
s.t. \quad & g_1(\boldsymbol{x}) = \frac{P}{\pi dt\sigma_y} - 1 \le 0; \\
& g_2(\boldsymbol{x}) = \frac{8PL^2}{\pi^3Edt\left(d^2 + t^2\right)} - 1 \le 0 \\
& g_3(\boldsymbol{x}) = 2.0/d - 1 \le 0 \\
& g_4(\boldsymbol{x}) = d/14 - 1 \le 0 \\
& g_5(\boldsymbol{x}) = 0.2/t - 1 \le 0 \\
& g_6(\boldsymbol{x}) = t/0.8 - 1 \le 0 \\
& 2 \le d \le 14 \\
& 0.2 \le t \le 0.8
\end{aligned} \tag{28}$$

The tabular column design problem is solved by different methods. Then, the statistical results are given in Table 27 while

Table 21
Optimal results of different methods for the Himmelblau's nonlinear problems.

Methods	x_1	x_2	x_3	x_4	x_5	$f(\boldsymbol{x})$	g_1	g_2	g_3
Himmelblau [60]	N/A	N/A	N/A	N/A	N/A	−30373.949	N/A	N/A	N/A
Deb [61]	N/A	N/A	N/A	N/A	N/A	−30665.539	N/A	N/A	N/A
He et al. [62]	78	33	29.995256	45	36.775813	−30665.539	93.28536a	100.40478	20
Dimopoulos [63]	78	33	29.995256	45	36.775813	−30665.54	92	98.8405	20
Gandomi et al. [53]	78	33	29.99616	45	36.77605	−30665.233	91.99996	98.84067	20.0003
CSA	78	33	29.995256	45	36.775813	−30665.538672	92	94.915402	20

Table 22
Statistical results of different methods for the Himmelblau's nonlinear problems.

Method	Best	Median	Mean	Worst	STD
Deb [61]	−30665.537	−30665.535	NA	−29846.654	NA
He et al. [62]	−30665.539	NA	−30643.989	NA	70.043
Dimopoulos [63]	−30665.54	NA	NA	NA	NA
Gandomi et al. [53]	−30665.2327	NA	NA	NA	11.6231
Mehta and Dasgupta [65]	−30665.53874	NA	NA	NA	NA
CSA	−30665.53867	−30627.72462	−30625.97242	−30561.19298	35.52345

Table 23
Statistical results of various methods for the welded beam design problem on version I.

Method	Best	Median	Mean	Worst	Std-dev
Ragsdell and Phillips [66]	2.385937	N/A	N/A	N/A	N/A
Rao [64]	2.386	N/A	N/A	N/A	N/A
Deb [61]	2.38119	N/A	N/A	N/A	N/A
Ray and Liew [56]	2.3854347	3.0025883	3.2551371	6.3996785	0.959078
CSA	2.3828427	2.6153739	2.6365241	3.0794703	0.2112024

Table 24
Best results of various methods for the welded beam design problem on version I.

Methods	x_1	x_2	x_3	x_4	$f(\boldsymbol{x})$
Ragsdell and Phillips [66]	0.2455	6.196	8.273	0.2455	2.385937
Rao [64]	0.2455	6.196	8.273	0.2455	2.386
Deb [61]	NA	NA	NA	NA	2.38119
Ray and Liew [56]	0.244438276	6.237967234	8.288576143	0.244566182	2.3854347
CSA	0.243905569	6.223303469	8.303331578	0.244328939	2.382842737

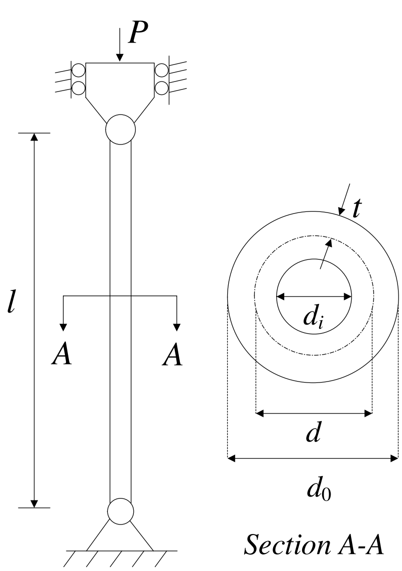

Fig. 10. Sketch map of the tabular column design problem.

the best results are given Table 28. It can be found that variation of the CSA method is obviously smaller than the other methods; besides, the best result of the CSA method has the best performances. Thus, this case proves the superiority of the developed method.

4.7. Minimize the vertical deflection of an I-beam problem

In this problem, the goal is to find the best variables that can minimize the vertical deflection of an I-beam while satisfying the cross-sectional area and stress constraints under the preset loads [53]. As showed in Fig. 11, this problem with the variable vector $\boldsymbol{x} = (x_1, x_2, x_3, x_4) = (h, b, t_w, t_f)$ can be described as:

$$\begin{aligned}
\min \quad & f(\boldsymbol{x}) = \frac{5000}{\dfrac{t_w\left(h-2t_f\right)^3}{12} + \dfrac{bt_f^3}{6} + 2bt_f\left(\dfrac{h-t_f}{2}\right)^2} \\
s.t. \quad & g_1(\boldsymbol{x}) = 2bt_w + t_w\left(h-2t_f\right) \le 300 \\
& g_2(\boldsymbol{x}) = \frac{18h\times 10^4}{t_w\left(h-2t_f\right)^3 + 2bt_w\left(4t_f^2 + 3h\left(h-2t_f\right)\right)} \\
& \quad + \frac{15b\times 10^3}{\left(h-2t_f\right)t_w^3 + 2t_w b^3} \le 6 \\
& 10 \le h \le 80 \\
& 10 \le b \le 50 \\
& 0.9 \le t_w \le 5 \\
& 0.9 \le t_f \le 5
\end{aligned} \tag{29}$$

This nonlinear optimization problem is solved by several methods. The statistical results and best results obtained by different methods are given in Tables 29–30, respectively. It can be found that the CSA method is superior to the other methods, demonstrating its engineering practicability.

Table 25
Statistical results of various methods for the welded beam design problem on version II.

Method	Best	Median	Mean	Worst	STD
Coello [67]	1.748309	NA	1.771973	1.785835	0.01122
Coello and Montes [68]	1.728226	NA	1.792654	1.993408	0.07471
Dimopoulos [63]	1.731186	NA	NA	NA	NA
He and Wang [69]	1.72802	NA	1.748831	1.782143	0.012926
Montes et al. [57]	1.724852	NA	1.725	NA	1.00E−15
Montes and Coello [70]	1.7373	NA	1.81329	1.994651	0.0705
Cagnina et al. [58]	1.724852	NA	2.0574	NA	0.2154
Kaveh and Talatahari [71]	1.724849	NA	1.727564	1.759522	0.008254
Kaveh and Talatahari [72]	1.724918	NA	1.729752	1.775961	0.0092
Gandomi et al. [73]	1.7312065	NA	1.878656	2.3455793	0.2677989
Mehta and Dasgupta [65]	1.724855	1.724861	1.724865	1.72489	NA
Akay and Karaboga [74]	1.724852	NA	1.741913	NA	0.031
CSA	1.695505466	1.700841829	1.724051281	1.87392175	0.047618

Table 26
Best results of various methods for the welded beam design problem on version II.

Methods	x_1	x_2	x_3	x_4	$f(\boldsymbol{x})$
Coello [67]	0.2088	3.4205	8.9975	0.21	1.748309
Coello and Montes [68]	0.205986	3.471328	9.020224	0.20648	1.728226
Dimopoulos [63]	0.2015	3.562	9.041398	0.205706	1.731186
He and Wang [69]	0.202369	3.544214	9.04821	0.205723	1.728024
Mahdavi et al. [75]	0.20573	3.47049	9.03662	0.20573	1.7248
Montes et al. [57]	0.20573	3.470489	9.036624	0.20573	1.724852
Montes and Coello [70]	0.199742	3.61206	9.0375	0.206082	1.7373
Cagnina et al. [58]	0.205729	3.470488	9.036624	0.205729	1.724852
Kaveh and Talatahari [71]	0.205729	3.469875	9.036805	0.205765	1.724849
Kaveh and Talatahari [72]	0.2057	3.471131	9.036683	0.205731	1.724918
Gandomi et al. [73]	0.2015	3.562	9.0414	0.2057	1.73121
Mehta and Dasgupta [65]	0.20572885	3.47050567	9.03662392	0.20572964	1.724855
Akay and Karaboga [74]	0.20573	3.470489	9.036624	0.20573	1.724852
CSA	0.205692017	3.254453177	9.036360313	0.205753289	1.695505466

Table 27
Best results of various methods for the tabular column design problem.

Item	Hsu and Liu [76]	Rao [64]	Gandomi et al. [53]	CSA
d	5.4507	5.44	5.45139	5.451163397
t	0.292	0.293	0.29196	0.291965509
g_1	−7.80E−05	−0.8579	−0.0241	$-1.4230944 \times 10^{-6}$
g_2	**0.1317**	**0.0026**	−0.1095	$-4.0441652 \times 10^{-6}$
g_3	−0.6331	−0.8571	−0.6331	−0.633105843
g_4	−0.6107	0	−0.6106	−0.610631186
g_5	−0.3151	−0.75	−0.315	−0.314987581
g_6	−0.635	0	−0.6351	−0.635043113
$f(\boldsymbol{x})$	25.5316(infeasible)	26.5323(infeasible)	26.5321	26.531364472

Table 28
Statistical results of various methods for the tabular column design problem.

Methods	Best	Median	Mean	Worst	STD
Gandomi et al. [53]	26.53217	NA	26.53504	26.53972	1.93×10^{-3}
CSA	26.5313645	26.5315394	26.5315766	26.5319781	1.66×10^{-4}

Table 29
Best results of various methods for the vertical deflection of an I-beam problem.

Item	ARSM[77]	Improved ARSM[77]	CS[53]	CSA
h	80	79.99	80	80
b	37.05	48.42	50	49.99999999
t_w	1.71	0.9	0.9	0.9
t_f	2.31	2.4	2.3216715	2.3217923
$f(\boldsymbol{x})$	0.0157	0.131	0.0130747	0.013074119

Table 30
Statistical results of various methods for the vertical deflection of an I-beam problem.

Method	Best	Median	Mean	Worst	STD
CS[53]	0.0130747	N/A	0.0132165	0.01353646	1.345×10^{-4}
CSA	0.013074119	0.013091263	0.013139695	0.013393231	9.3686×10^{-5}

Table 31
Best results of various methods for the Three bar truss design problem.

Methods	A_1	A_2	g_1	g_2	g_3	$f(\boldsymbol{x})$
Hernández [78]	0.788	0.408	NA	NA	NA	263.9
Ray and Saini [79]	0.795	0.395	−0.00169	−0.26124	−0.74045	264.3
Ray and Liew [56]	0.788621037	0.408401334	−8.275×10−9	−1.46392765	−0.536072358	263.8958466
Raj et al. [80]	0.789764410	0.405176050	−7.084×10−9	−1.4675992	−0.53240078	263.89671
Tsai [81]	0.788	0.408	**0.00082**	−0.2674	−0.73178	263.68(infeasible)
Zhang et al. [82]	0.788675136	0.408248287	−2.10E−11	−1.46410161	−0.5358983	263.8958434
Gandomi et al. [83]	0.78867	0.40902	−0.00029	−0.26853	−0.73176	263.9716
CSA	0.788638976	0.408350573	0	−1.463985345	−0.536014655	263.895844337

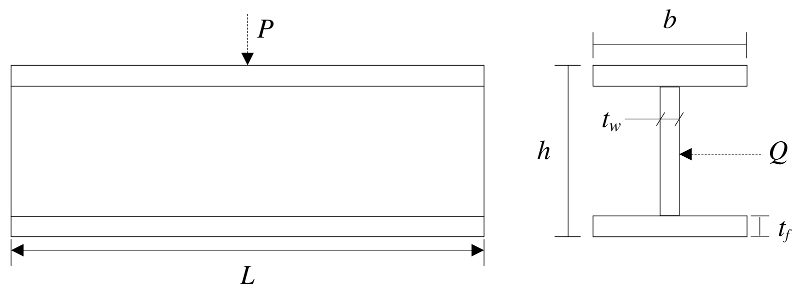

Fig. 11. Sketch map of the vertical deflection of an I-beam problem.

4.8. Three bar truss design problem

Fig. 12 shows the sketch map of the three bar truss design problem minimizing the volume while satisfying the stress constraints on each side of the truss members. Then, this problem with the variable vector $\boldsymbol{x} = (x_1, x_2) = (A_1, A_2)$ can be described as:

$$\begin{aligned} \min \quad & f(\boldsymbol{x}) = \left(2\sqrt{2}A_1 + A_2\right) \times l \\ s.t. \quad & g_1 = \frac{\sqrt{2}A_1 + A_2}{\sqrt{2}A_1^2 + 2A_1A_2}P - \sigma \le 0 \\ & g_2 = \frac{A_2}{\sqrt{2}A_1^2 + 2A_1A_2}P - \sigma \le 0 \\ & g_3 = \frac{1}{A_1 + \sqrt{2}A_2}P - \sigma \le 0 \\ & 0 \le A_1, A_2 \le 1 \end{aligned} \tag{30}$$

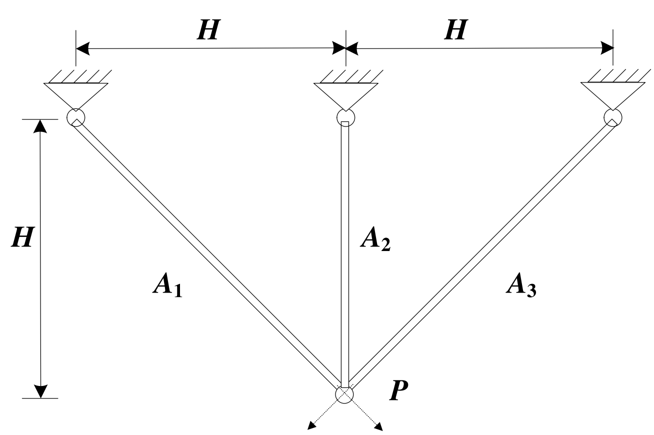

Fig. 12. Sketch map of the three bar truss design problem.

Table 31 shows the best results of various methods for the Three bar truss design problem. It can be seen that the results of Raj et al. [80] violates the constraints; while the CSA method can obtain the near-optimal solution for the target problem. Hence, the performance of the presented method in engineering optimization problem is proved again.

5. Conclusions and discussions

In this research, a novel Cooperation search algorithm (CSA) is presented to solve the complex global optimization problems. In CSA, three practical operators are designed to guarantee the global convergence of the population, including the team communication operator, the reflective learning operator and the internal competition operator. Based on 79 mathematical optimization problems (24 classical benchmark functions, 25 CEC2005 test problems and 30 CEC2014 test problems) and several engineering design problems, an extensive comparative experiments are used to fully evaluate the exploration, exploitation and convergence performances of the presented CSA method. The results demonstrate that the CSA method outperforms several conventional evolutionary algorithms regardless of the problem features in the search space. Taking the 23th CEC2005 function as the example, the best results obtained by the proposed method are improved by 23.03%, 31.64% and 36.83% compared with PSO, SCA and GSA.

The superiority of the CSA method lies in the following points. Firstly, as a population-based metaheuristic method, the CSA method can avoid falling into local minimum by extensively searching in the entire state space. Secondly, with the aid of team communication and reflective learning operators, the CSA method can achieve a good balance between global exploitation and local exploration, which can help the swarm quickly converge to the promising areas to yield high-quality solutions. Thirdly, the CSA method can update the solutions' position around the neighborhood area of the destination point and then guarantee the survival of better solutions via internal competition operator. Finally, the CSA method treats the target problem as a black box requiring no gradient information and then work with various constraint handling techniques to handle the complex constrained optimization problem. Hence, a novel and effective method is proposed to solve the global optimization problem.

To be mentioned, all the evolutionary algorithms based on probabilistic search strategies often require common controlling parameters, like population size, number of generations, and elite size. The proper tuning of the algorithm-specific parameters is crucial and affects the performance of evolutionary algorithms. An improper tuning of algorithm-specific parameters either increases the computation effort or yields local optimal solutions. In order to avoid this problem, one way is to enhance the performance of the developed search strategies while another way is to reduce the number of parameters as much as possible. For the current CSA version, it is found that the values of elite solutions ($M = 3$), two learning coefficients ($a = 0.1$ and $\beta = 0.15$) can provide satisfying solutions for most test problems in this study. In other words, only two parameters (the number of individuals and iterations) need to be tuned in CSA, which can effectively reduce the workload in practice. Meanwhile, in the future, the rigorous mathematical proof concerning the convergence property of the CSA method deserve further investigations, while the binary and multi-objective versions of the CSA method with applications in engineering problems under uncertainty should be carefully studied.

CRediT authorship contribution statement

Zhong-kai Feng: Idea, Modeling, Programming implementation, Finalized the manuscripts. **Wen-jing Niu:** Idea, Modeling, Programming implementation, Finalized the manuscripts. **Shuai Liu:** Programming implementation, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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