Significant perturbations in observability graph

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Bandit framework:

- Sequence of losses $(l_t)_{1 \leq t \leq T}$ and actions $(i_t)_{1 \leq t \leq T}$
- Minimax regret
$R(G, T) = \min_S \max_{l_1,\dots,l_T} \mathbb{E} \sum_{t=1}^{T} l_t(i_t) - \min_i \sum_{t=1}^{T} l_t(i)$

Feedback graph

- Directed graph with K nodes corresponding to the possible actions.
- When a player chooses action i he gets feedback concerning some j: this represented by an edge.
- A self loop means that whenever the player chooses the action he gets a feedback on it.

Well-known configuration

Figure: Feedback with expert advice

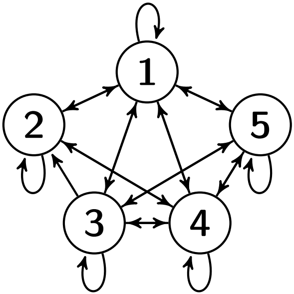
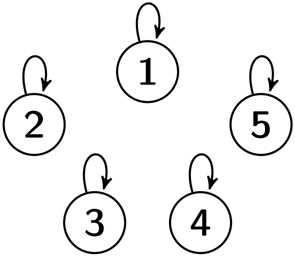


Figure: Bandit feedback



Previous results

Bounds on the minimax regret with respect of the geometry of the graph :

- Strongly observable with independence number α : $R(G, T) = \tilde{\Theta}(\alpha^{1/2} T^{1/2})$.
- Weakly observable with domination number δ : $R(G, T) = \tilde{\Theta}(\delta^{1/3} T^{2/3})$
- Unobservable : $R(G, T) = \tilde{\Theta}(T)$.

Definitions:

Definition:(Observability)

Let $G = (V, E)$ be a directed graph. A vertex v is called observable if $N_{in}(v) \neq \emptyset$.
v is strongly observable if it has a self loop or $V \setminus \{v\} \subset N_{in}(v)$. It is weakly observalble in case when it is observable but not strongly.
A graph G is observable if all its vertices are observable and it is strongly observable if all its vertices are strongly observable. A graph is weakly observable if it is observable but not strongly.

Definitions:

Definition:(Weak Domination)

In a directed graph $G = (V, E)$ with a set of weakly observable vertices $W \subset V$, a weakly dominating set $D \subset V$ is a set of vertices that dominates W. Namely, for any $w \in W$ there exists $d \in D$ such that $w \in N_{out}(d)$. The weak domination number of G, denoted by $\delta(G)$, is the size of the smallest weakly dominating set.

Definitions:

Definition:(Independence)

An independent set$S \subset V$ is a set of vertices that are not connected by any edges. Namely, for any $u, v \in S, u \neq v$ it holds that $(u, v) \notin E$. The independence number $\alpha(G)$ of G is the size of its largest independent set.

Stable graphs:

If we stay in the same class, adding or deleting one edge would result in:

- The independence number α varies at most by 2.
- The weak domination number δ varies at most by 1.

This means that, if the graph stays in the same class, the minimax regret does not change dramatically after perturbing one edge.

Unstable graphs

There are three possibilities:

(i) going from strongly observable graphs to weakly observable ones and back.

(ii) going from strongly observable graphs to non observable ones and back.

(iii) going from weakly observable graphs to non observable ones and back.

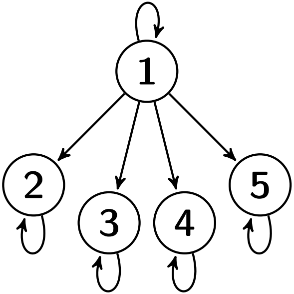
Unstable graphs

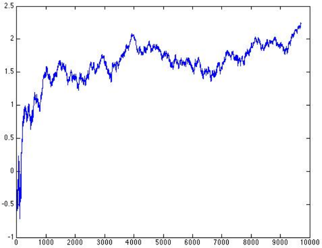
(i) In this case, we remove a self loop for an vertice that has other inward edges or we remove an edge for a loop less vertice that also has other inward edges.

(ii) When possible, we remove a loop for a vertice that has no other inward edge.

(iii) When possible, we remove the only inward edge, that is not a self loop, to such a vertice.

Strongly observable graph

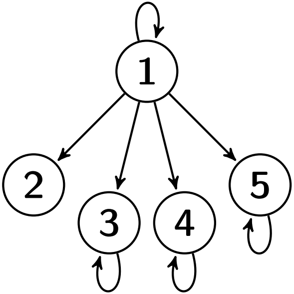


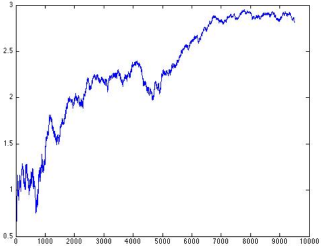


- Revealing action graph, $\alpha = 4$

Associated log-regret

Weakly observable graph

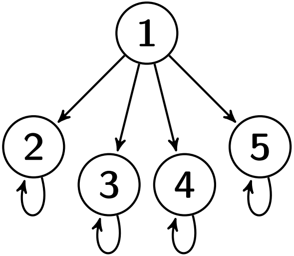




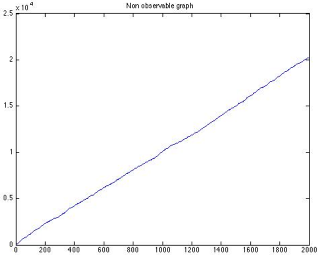
- Pertubed graph, $\delta = 1$

Associated log-regret

Non observable graph



- Node 1 non observable



Associated log-regret

Conclusion

- Unstable graphs : are the ones that changes class by adding or removing an edge.
- Experiments on the revealing action graph yield consistent results with $|\alpha - \delta|$ high and random losses.