Assignment 1

Instructions

1. This assignment is due at 5pm on 11th Feb 2014.
2. There are two portions to hand in for this assignment: Submit the derivations on paper to the course mailbox at the DSAP office, and submit a file that contains `R` code via the workbin on IVLE.
3. `R` has to be submitted for questions 1 and 2, not any others.
4. Name your `R` file according to this convention. If your matriculation number is a999999u, then name the file `a999999u_assignment_01.R`.

Questions

1. Suppose that U_1 and U_2 are $Unif[0,1]$ random variables. Find the density of the following random variables. For each of them, write simple code to generate 1000 samples, and to plot their histograms, along with the actual density that you have derived.
   (a) $2\min(U_1, U_2)$.
   (b) $2|U_1 + U_2 - 1|$.
   (c) $2(1 - \sqrt{U_1})$.
2. This question involves the use of the `Vectorize` function in `R` . Suppose we observe $U_1 = 3, U_2 = 5$ independent, from $Unif[0, \theta]$. Write down the likelihood function $L(\theta)$. The following function is an implementation of the function L:

```
L <- function(theta) {
  if(theta <5)
    return(0) else
    return(1/theta^2)
}
```

   Provide a snippet of code that uses the `Vectorize` function to plot the function over a grid from 5 to 100, and read off the MLE (Maximum Likelihood Estimate) of θ.
3. Suppose that X has pdf
$$f(x) = ce^x, \quad 0 < x < 1$$
   where c is the value that ensures that $\int_0^1 f(x)\,dx = 1$. Determine c, and then derive $Var(X)$.
4. Let U be $Unif[0,1]$. Show that $\min(U, 1-U)$ is uniform on $[0, 1/2]$, and that $\max(U, 1-U)$ is uniform on $[1/2, 1]$.
5. Without actually computing any Z_i's, determine which of the following LCGs have full period:
   (a) $Z_i = (13Z_{i-1} + 13) \mod 16$
   (b) $Z_i = (12Z_{i-1} + 13) \mod 16$
   (c) $Z_i = (13Z_{i-1} + 12) \mod 16$
   (d) $Z_i = (Z_{i-1} + 12) \mod 16$