Tutorial 2

Question 1

Algorithm:

1) Generate U~U[0,1]
2) Let X=(-log(U)/alpha)^(1/beta)

Question 2

Algorithm 1:

1) Let i=0, Y=p, p=pi^r,
2) Generate U~U[0,1]
3) If U<Y, X=i, Else
   p=p*(i+r)*(1-pi)/(i+1)
   Y=Y+p;i=i+1
4) return to step 3

Algorithm 2:

Using the geometrical random generator packages:

Generate X1, X2, ..., Xr ~Geometrical(p)

X1+X2+..+Xr ~Negative Binominal (r,p)

Question 3

a) $p_n = \left(1-\sum_{j=1}^{n-1} p_j\right)\lambda_n$

$= \left(1-\sum_{j=1}^{n-2} p_j\right)(1-\lambda_{n-1})\lambda_n$

$= \left(1-\sum_{j=1}^{n-3} p_j\right)(1-\lambda_{n-2})(1-\lambda_{n-1})\lambda_n$

$= \dots$

$= (1-p_1)\dots(1-\lambda_{n-2})(1-\lambda_{n-1})\lambda_n$

$= (1-\lambda_1)(1-\lambda_2)\dots(1-\lambda_{n-1})\lambda_n$ (shown)

b) for p_n,

$U_i \geq \lambda_i$, $i = 1, \cdots, n-1$

means $U_n < \lambda_n$ and $P(X = n) = \lambda_n$.

$p_n = P(X = n) = (1-\lambda_1)(1-\lambda_2)\cdots(1-\lambda_{n-1})(\lambda_n)$ (shown).

as desired pmf.

c) $X \sim$ Geometric (p)

$$p_n = (1-p)^{n-1} p$$
$$= (1-p)\cdots(1-p)p$$

$$\lambda_n = \frac{p_n}{1 - \sum_{j=1}^{n-1} p_j}$$
$$= \frac{(1-p)^{n-1}p}{1 - \sum_{j=1}^{n-1}(1-p)^{j-1}p}$$
$$= \frac{(1-p)^{n-1}p}{(1-p)^{n-1}} = p$$

$$\therefore P(X=n) = (1-p)^{n-1} p$$

Question 4

a) Given a<=x<=b
b) Rejection method:
   c=sup (f(x)/g(x))=1/(G(b)-G(a)).
   1) Generate V~ g(x)
   2) Generate U~U[0,1]
   3) If U<=f(V)/(c*g(V)), set X=V.

Question 5

Algorithm:

1) Generate Z~N(0,1), using standard normal distribution packages
2) Set V=mu+sigma*Z
3) If a<=V<=b, set X=V; else go back to 1)