Assignment 4

Instructions

1. This assignment is due at 5pm on Friday 04th Apr 2014.
2. There are two portions to hand in for this assignment: Submit the derivations on paper to the course mailbox at the DSAP office, and submit a file that contains `R` code via the workbin on IVLE.
3. Include the final answers for ALL parts in your handwritten portion.
4. You may use the code in `assignment_04_template.R` to assist you for part (3), or you may write your own. Replace the `....` sections in the template with your code.
5. Name your `R` file according to this convention: If your matriculation number is a999999u, then name the file `a999999u_assignment_04.R`.

Question

Daily demand for newspaper is approximately gamma distributed, with mean 10,000 and variance 1,000,000. The newspaper prints and distributes 11,000 copies each day. The profit on each newspaper sold is \$1, and the loss on each unsold (complete) newspaper is \$0.25. Formally, the daily profit function h is

$$h(X) = \begin{cases} 11000 & \text{if } X \geq 11000 \\ \lfloor X \rfloor + (11000 - \lfloor X \rfloor)(-0.25) & \text{if } X < 11000 \end{cases}$$

where X represents the daily demand. In this assignment, we shall estimate the expected daily profit using simulation, and then use a control variable to reduce the variance of our estimate. You are allowed to use `dgamma` and `dexp` for computing the densities f and g when evaluating the acceptance criterion in this assignment.

1. Write down the shape and scale parameters of the Gamma distribution corresponding to the daily demand.
2. Derive a rejection algorithm for simulating from the distribution in part (1), using an Exp random variable with the same mean as the distribution in part (1). In other words, our target pdf is:

   $$f(x) = \frac{1}{\Gamma(a)b^a} x^{a-1} e^{-x/b}$$

   Our candidate pdf is:

   $$g(x) = \frac{1}{ab} e^{-x/ab}$$

   Work out the optimal rejection constant c.
3. Implement this rejection algorithm in `R` . Generate 10000 samples from X using this implementation and form a 95% confidence interval for $E(h(X))$. Use $z_{\alpha/2} = 1.96$.
4. Identify a control variable for reducing the variance of the estimate of X. Denote it as Y, and write down its expectation.
5. Estimate the values of $Cov(h(X), Y)$, $Var(Y)$ and c^* using the sample you have generated.
6. Form the new confidence interval based on the control variable you have identified.