5.3 Adaptive Control Based on Popov Hyperstability Theory

■ Basic concepts of hyperstability theory

The early research publications on adaptive control using hyperstability theory:

① I.D. Landau, *Adaptive Control: The Model Reference Approach*, New York: Marcel Dekker, 1979.

② B. Anderson, A Simplified Viewpoint of Hyperstability, *IEEE Transactions on Automatic Control*, Vol. 13, No. 3, 292-294, June 1968.

③ K.S. Narendra, L.S. Valavani, A Comparison of Lyapunov and Hyperstability Approaches to Adaptive Control of Continuous Systems, *IEEE Transactions On Automatic Control*, Vol. AC-25, No. 2, 243-247, April, 1980.

♦ Introduction to the concept of hyperstability

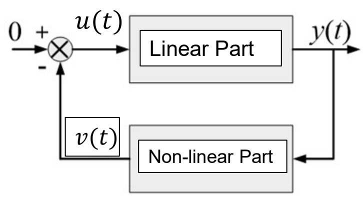
Integral inequality composed of an input (Popov integral inequality):

$$\int_0^t y^T(\tau)\, v(\tau) d\tau \geq -\gamma_0^2, \qquad \forall t \geq 0 \tag{5.3.1}$$

or

$$\int_0^T u^T(t)\, y(t) dt \leq \delta[\|x(0)\|] \sup_{0 \leq t \leq T} \|x(t)\| \tag{5.3.2}$$

System H:



where $x(t)$: the state vector of the linear part,

γ_0: a real number,

δ: a normal number that depends on the initial value of the system but is independent of time (all time),

$\|.\|$: Euclidean norm.

♦ **Hyperstable**

If there is any input satisfying inequality (5.3.1) and for all time t, the following inequality holds:

$$\|x(t)\| \leq K(\|x(0)\| + \delta)$$

System H is said to be hyperstable.

♦ **Asymptotically hyperstable**

For any "bounded" input that satisfies inequality (5.3.1), on the basis of hyperstability, if the following holds

$$\lim_{t\to\infty} x\,(t) = 0$$

System H is said to be "asymptotically hyperstable".

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■ Definition of positive real and strictly positive real matrices

♦ **Hermite matrix**

For a complex matrix G, if

$$G = G^* \qquad \text{for } G \in C^{n\times n}$$

where * means "conjugate transpose", then G is a Hermite matrix.

♦ **System passivity**

A system with input u and output y is passive if $\langle y, u\rangle \geq 0$. (Note: $\langle y, u\rangle := y^T u$)

The system is input strictly passive (ISP) if there exists $\varepsilon > 0$ such that $\langle y, u\rangle \geq \varepsilon u^2$.

It is output strictly passive (OSP) if there exists $\varepsilon > 0$ such that $\langle y, u\rangle \geq \varepsilon y^2$.

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♦ **Positive real**

A rational transfer function $G(s)$ with real coefficients is positive real (PR) if $\mathrm{Re}(G(s)) \geq 0$, for $\mathrm{Re}(s) \geq 0$.

A transfer function $G(s)$ is strictly positive real (SPR) if $G(s-\varepsilon)$ is positive real for some real $\varepsilon > 0$.

♦ **Characterizing positive real transfer functions**

Theorem

A rational transfer function $G(s)$ with real coefficients is PR if and only if the following conditions hold.

a) The function has no poles in the right half-plane.

b) If the function has poles on the imaginary axis or at infinity, they are simple poles.

c) The real part of $G(s)$ is nonnegative along the $j\omega$ axis, that is, $\mathrm{Re}(G(j\omega)) \geq 0$.

A transfer function is SPR if conditions a) and c) hold and if condition b) is replaced by the condition that $G(s)$ has no poles or zeros on the imaginary axis.

■ Definition of positive real rational transfer function matrices

The rational transfer function matrix $G(s)$ is called positive real if the following three conditions are true:

(1) When s is real, all elements of $G(s)$ are real;

(2) Each element of $G(s)$ has no pole in the "open right half complex plane, that is, s>0".
If there is a pole on the imaginary axis, it is required to be simplex, and the corresponding residue matrix is a "nonnegative definite" Hermite matrix;

(3) For all Re(s)>0, as long as it is not the pole of an element of $G(s)$

$$G(s) + G^*(s)$$

is a nonnegative definite Hermite matrix.

Note: For the real rational transfer function matrix, the above conditions (1) and (3) can be combined into the following conditions: For $Re(s) > 0$, $G(s) + G^*(s)$ is required to be a nonnegative definite Hermite matrix.

■ Strictly positive real rational transfer function matrix

"The rational transfer function matrix $G(s)$ is called strictly positive real, which means

(1) $G(s)$ satisfies the positive real condition;

(2) All poles of $G(s)$ are in the open left semi-complex plane;

(3) For all ω, $G(j\omega) + G^*(j\omega)$ is a positive definite Hermite matrix.

♦ Hyperstability theorems

For the following linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

where $x(t)$, $y(t)$, $u(t)$ are the state, output and input vectors of the system,
A, B, C, D are system matrices.

Theorem 1: System H with the linear part determined by $G(s) = D + C(sI - A)^{-1}B$ is "hyperstable" if and only if $G(s)$ is "positive real".

Theorem 2: System H with the linear part determined by $G(s) = D + C(sI - A)^{-1}B$ is "asymptotically hyperstable" is if and only if $G(s)$ is "strictly positive real".

Proof: omitted

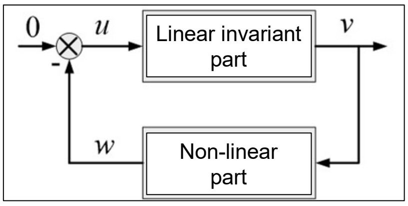
■ Application to adaptive controller design

Consider the following feedback system:

$$\dot{x}(t) = Ax(t) + Bu(t) = Ax(t) - Bw(t)$$

$$v(t) = Cx(t)$$

$$w(t) = f(v, t)$$



Nonlinear feedback system

The linear part is strictly positive real.

The feedback part satisfies the following Popov inequality:

$$\int_0^T u^T(t)v(t) \le r_0^2 \quad \text{or} \quad \int_0^T w^T(t)v(t) \ge -r_0^2$$

Then

$$\|x(t)\| \le K[\|x(0)\| + r_1] \quad \text{and} \quad \lim_{t\to\infty} x\,(t) = 0$$

where K and r_1 are positive constants.

■ Design steps

Step 1: Convert the model reference adaptive system into an equivalent feedback system consisting of two parts, one is a forward part and the other is a feedback part.

Step 2: The adaptive law in the feedback part of the equivalent system is obtained according to the Popov integral inequality.

Step 3: Make the forward part strictly positive, and calculate the corresponding adaptive parameters.

Step 4: Return to the original system and obtain the explicit adaptive controller structure and parameters.

■ Example 5.3.1

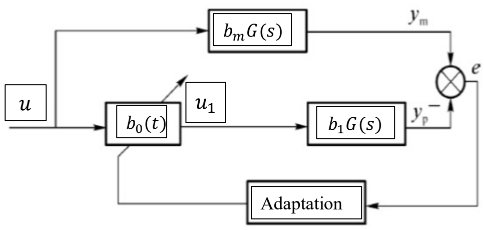
Reference model: $(1 + a_1 p + a_2 p^2) y_m(t) = b_m u(t)$

Controlled plant: $(1 + a_1 p + a_2 p^2) y_P(t) = b_1 u_1(t)$

where b_1 is an unknown constant, a_1, a_2 and b_m are given.

Output error:

$$e(t) = y_m(t) - y_P(t)$$



Introduction of a controller:

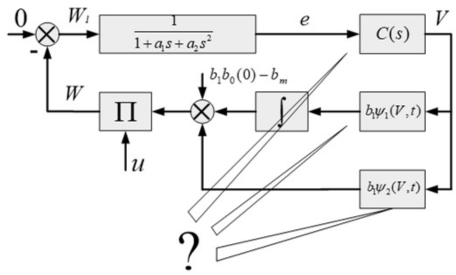
$$u_1(t) = b_0(t)\, u(t)$$

where $b_0(t)$ is the gain.

Question: How to design $b_0(t)$ using Popov hyperstability theory?

Obtain an error system: $(1 + a_1 p + a_2 p^2) e(t) = (b_m - b_1 b_0(t)) u(t)$

Let $W_1 = (b_m - b_1 b_0(t)) u(t)$



Refinement of the output error

$V(t) = C(p) e(t)$

where $C(p)$ is a polynomial.

Design the adaptive law of parameter $b_0(t)$

$$b_0(t) = \int_0^t \psi_1(V,\tau) d\tau + \psi_2(V,t) + b_0(0)$$

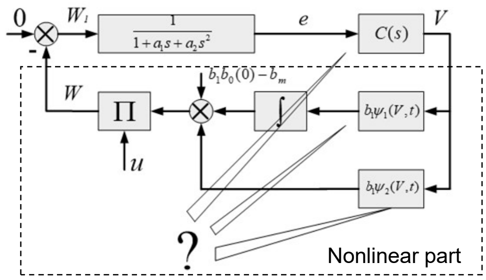
Three items to be designed:
$C(s)$, $\psi_1(V,t)$, $\psi_2(V,t)$

Let $W = -W_1$

$= (b_1 b_0(t) - b_m) u(t)$

$= \left(\int_0^t b_1 \psi_1(V,\tau) d\tau + b_1 \psi_2(V,t) + b_1 b_0(0) - b_m\right) u(t)$

Step 1: Transform the system to an equivalent feedback system with two parts (linear and nonlinear parts).



Step 2: The feedback channel satisfies Popov integral inequality.

$$\int_0^T VWdt \geq -r_0^2$$

$$\int_0^T VWdt = \int_0^T Vu\left[\int_0^t b_1\psi_1(V,\tau)d\tau + b_1\psi_2(V,t) + b_1b_0(0) - b_m\right]dt \geq -r_0^2$$

$$\int_0^T VWdt = \int_0^T Vu\left[\int_0^t b_1\psi_1(V,\tau)d\tau + b_1b_0(0) - b_m\right]dt + \int_0^T b_1\psi_2(V,t)Vudt \geq -r_0^2$$

which means if

$$\int_0^T Vu\left[\int_0^t b_1\psi_1(V,\tau)d\tau + b_1b_0(0) - b_m\right]dt \geq -r_1^2$$

$$\int_0^T b_1\psi_2(V,t)Vudt \geq -r_2^2, \quad \text{and} \quad (r_1^2 + r_2^2 \leq r_0^2)$$ then $$\int_0^T VWdt \geq -r_0^2$$

An integral equation:

$$\int_0^T \dot{f}(t)k_1f(t)dt = \frac{1}{2}k_1f^2(t)\Big|_0^T = \frac{1}{2}k_1[f^2(T) - f^2(0)] \geq -\frac{1}{2}k_1f^2(0)$$

Let $\dot{f}(t) = Vu$ and $\boxed{\psi_1(V,t) = \frac{k_1}{b_1}Vu, \qquad k_1 > 0}$

Then $$\int_0^t b_1\psi_1(V,\tau)d\tau + b_1b_0(0) - b_m = \int_0^t k_1\dot{f}(\tau)d\tau + b_1b_0(0) - b_m = k_1f(t)$$

where $k_1f(0) = b_1b_0(0) - b_m$

So,

$$\int_0^T Vu\left[\int_0^t b_1\psi_1(V,\tau)d\tau + b_1b_0(0) - b_m\right]dt = \int_0^T \dot{f}(t)k_1f(t)\,dt \geq -r_1^2 \qquad (5.3.3)$$

Note: Let $r_1^2 = \frac{1}{2}k_1f^2(0)$

Let $\boxed{\psi_2(V,t) = k_2(t)Vu, \qquad k_2(t) \geq 0, \qquad \forall t \geq 0}$

Possible choices for $k_2(t)$:

① Proportional adaptive law: $k_2(t) = K_2\ , K_2 \geq 0$ $\qquad \psi_2(V,t) = K_2 Vu$

② Relay adaptive law a: $k_2(t) = \frac{K_2}{|V|}, K_2 \geq 0$ $\qquad \psi_2(V,t) = K_2 sign(V)u$

③ Relay adaptive law b: $k_2(t) = \frac{K_2}{|u|}, K_2 \geq 0$ $\qquad \psi_2(V,t) = K_2 sign(u)V$

④ Relay adaptive law c: $k_2(t) = \frac{K_2}{|uV|}, K_2 \geq 0$ $\qquad \psi_2(V,t) = K_2 sign(uV)$

Choose $k_2(t) = K_2$. Thus

$$\int_0^T b_1\psi_2(V,t)Vudt = \int_0^T b_1 K_2 VuVudt \geq \int_0^T b_1 K_2 (Vu)^2 dt \geq -r_2^2 \tag{5.3.4}$$

Therefore, eqs (5.3.3) and (5.3.4) conclude:

The Popov integral inequality is satisfied, *i.e.*, $\int_0^T VWdt \geq -r_0^2$.

Step 3: Ensure that the channel of the preceding item is strict positive real.

$$h(s) = \frac{C(s)}{1 + a_1 s + a_2 s^2} \qquad\qquad C(s) = c_1 s + c_0$$

To have the zero and two poles of h(s) in the open-left s-plane,

$$a_1 > 0,\ \ a_2 > 0,\ \ c_0 c_1 > 0,$$

For $s = j\omega$,

$$h(j\omega) = \frac{c_1 j\omega + c_0}{1 + a_1 j\omega + a_2 (j\omega)^2} = \frac{(c_0 + jc_1\omega)(1 - a_2\omega^2 - ja_1\omega)}{(1 - a_2\omega^2 + ja_1\omega)(1 - a_2\omega^2 - ja_1\omega)}$$

$$= \frac{c_0\left(1 - a_2\omega^2\right) + c_1 a_1 \omega^2 + j\left(c_1\left(1 - a_2\omega^2\right) + c_0 a_1 \omega\right)}{(1 - a_2\omega^2)^2 + a_1^2\omega^2}$$

which leads to $\mathrm{Re}\left(h(j\omega)\right) > 0$ if $c_0 > 0$ and $c_1 a_1 \geq c_0 a_2$.

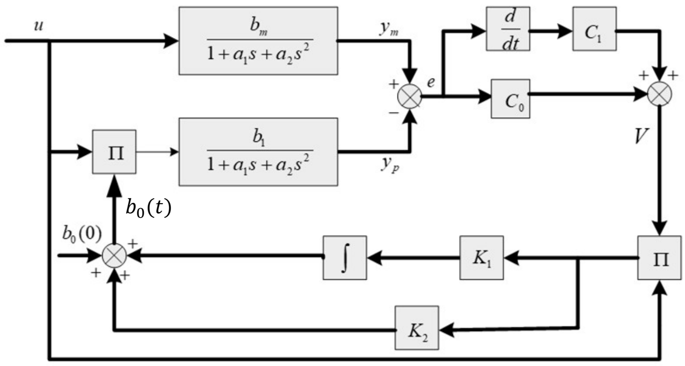
So, the SPR conditions of $h(s)$: $a_1 > 0,\ \ a_2 > 0,\ \ c_0 > 0,\ \ c_1 > 0,\ \ c_1 a_1 \geq c_0 a_2$

Step 4: Restore to a normal block diagram

The following control laws are adopted:

$$\psi_1(V,t) = K_1 V u, \quad K_1 > 0$$
$$\psi_2(V,t) = K_2 V u, \quad K_2 > 0$$

♦ **The control system block diagram**

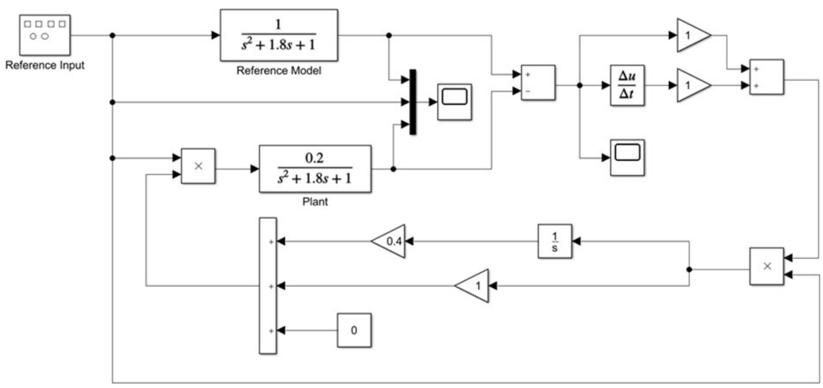


♦ **Simulation**

The reference input is a square wave and choose the following parameters:

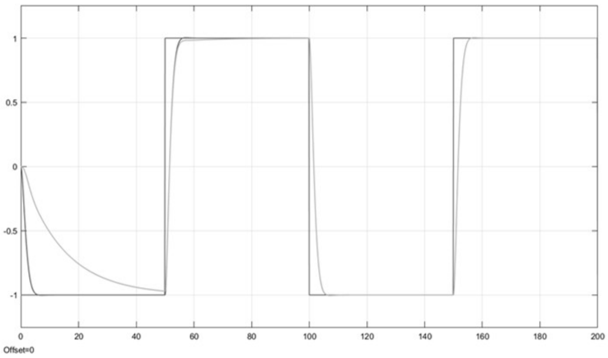
$a_1 = 1.8, a_2 = 1,\ b_m = 1, b_1 = 0.2,\ c_0 = 1, c_1 = 1,\ K_1 = 0.4,\ K_2 = 1,\ b(0) = 0$

Simulation using Simulink:

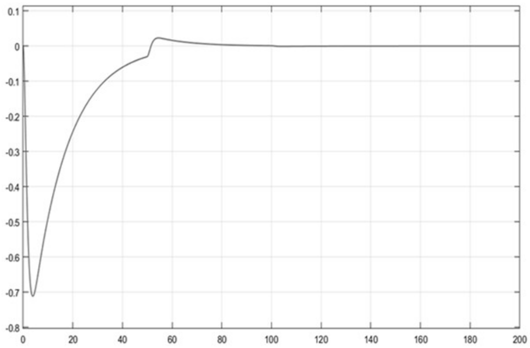


♦ **Results:**

The reference input, reference model output and plant output



The error between the reference model output and plant output



♦ **Simulation codes** example531.slx

5.4 Stable Adaptive Control Method

■ Introduction

The adaptive control law introduced in section 5.2 requires that all state variables can be measured, which is difficult to meet in the actual system, so its application is greatly limited.

To overcome these difficulties, two solutions have been proposed: indirect method and direct method.

- ✓ In the indirect method, an adaptive observer needs to be designed to estimate the unmeasurable states of the object in real time using the input and output data of the system, and then the adaptive law is realized by these state estimates.
- ✓ The direct method does not need to estimate the states of the plant, but directly uses the input/output data to construct the adaptive law. It can be seen that the direct method has obvious advantages.

A direct method called the stable adaptive controller scheme was proposed by S. Narendra[1].

[1] K.S. Narendra and L.S. Valavani, Stable Adaptive Controller Design - Direct Control, *IEEE Transactions on Automatic Control*, 23 (4) , pp.570-583, 1978.

■ Stable adaptive control scheme

♦ The plant

Let the state equation and output equation of SISO systems be

$$\dot{x}_p(t) = A_p x_p(t) + b_p u(t)$$
$$y_p(t) = h^T x_p(t)$$

where $x_p(t)$ is the n-dimensional state vector, $u(t)$ is the control input, $y_p(t)$ is the output, A_p is n × n matrix, B_p and h are n × 1 vectors.

Assumptions: m and n are known, the parameters A_p and b_p of the controlled plant are unknown or slowly time-varying.

The transfer function corresponding to the controlled plant is

$$G_p(s) = h^T\left(sI - A_p\right)^{-1} b_p = k_p \frac{N_p(s)}{D_p(s)}$$

where $N_p(s)$ is a monic Hurwitz polynomial and $D_p(s)$ is a monic polynomials, whose orders are m and n (m ≤ n-1) respectively, $k_p > 0$ is the gain of the controlled plant (its sign is assumed to be positive here).

♦ **The reference model**

Select the reference model as

$$\dot{x}_{\mathrm{m}}(t) = A_m x_m(t) + b_m y_r(t)$$
$$y_m(t) = h^T x_m(t)$$

where $x_m(t)$ is the n-dimensional state vector, $y_r(t)$ is the piecewise continuous uniformly bounded input, $y_m(t)$ is the reference model output, and A_m is n × n matrix, b_m is n × 1 vector.

The transfer function corresponding to the reference model is

$$G_m(s) = h^T(sI - A_m)^{-1}b_m = k_m \frac{N_m(s)}{D_m(s)}$$

where $G_m(s)$ is strictly positive real, $N_m(s)$ and $D_m(s)$ are monic Hurwitz polynomials, and their orders are m and n, respectively, k_m is the reference model gain.

♦ **Adaptive mechanism**

Let the generalized output error be

$$e(t) = y_m(t) - y_p(t) \qquad (5.4.1)$$

The design objective of the control system is to design an adaptive control law using Lyapunov stability theory, and generate a bounded control input from it so that the generalized output error $e(t)$ satisfies

$$\lim_{t\to\infty} e(t) = 0$$

To realize the perfect match between the adjustable system and the reference model, the adaptive controller must have adequate adjustable parameters.

When the denominator of the transfer function of the controlled plant $G_p(s)$ is n-order and the numerator is m-order, plus the amplification factor k_p, the maximum number of adjustable parameters is n+m+1.

Therefore, the adaptive mechanism should also have n+m+1 adjustable parameters corresponding to it.

Only n-m=1 are introduced here.

When n-m=1, $D_p(s)$ has n unknown coefficients, $N_p(s)$ has n-1 unknown coefficients, plus k_p, a total of 2n unknown parameters.

To ensure the complete match between the plant and reference model, there are at least 2n adjustable parameters.

For this purpose, the adjustable gain k_c and two auxiliary signal generators $F_1(s)$ and $F_2(s)$ are set, which together constitute an adaptive controller.

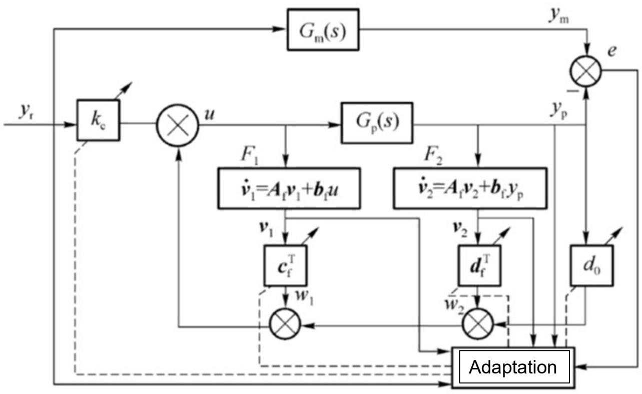
♦ **Adaptive control structure**

The input of $F_1(s)$ is the control $u(s)$ of the controlled plant, with n-1 adjustable parameters $c_i \ (i = 1, 2, \ldots, n-1)$.

The input of $F_2(s)$ is the output $y_p(s)$ of the controlled plant, with n adjustable parameters $d_i(i = 0,1,2,\ldots,n-1)$.

Plus the feedforward gain k_c.

A total of 2n adjustable parameters.



The state equations of the two auxiliary signal generators and their corresponding transfer functions are

$$\dot{v}_1(t) = A_f v_1(t) + b_f u(t)$$
$$w_1(t) = c_f^T v_1(t)$$
$$G_1(s) = c_f^T \left(sI - A_f\right)^{-1} b_f = \frac{N_c(s)}{D_f(s)}$$

$$\dot{v}_2(t) = A_f v_2(t) + b_f y_p(t)$$
$$w_2(t) = d_f^T v_2(t) + d_0 y_p(t)$$
$$G_2(s) = d_0 + d_f^T \left(sI - A_f\right)^{-1} b_f = d_0 + \frac{N_d(s)}{D_f(s)}$$

where $v_1(t)$ and $v_2(t)$ are (n-1)-dimensional column vectors,

the denominator $D_f(s)$ of transfer functions $G_1(s)$ and $G_2(s)$ is the monic Huiwitz polynomial of (n-1)-order, and

the numerators $N_c(s)$ and $N_d(s)$ are (n-2)-order polynomials, namely

$$D_f(s) = s^{n-1} + d_{f(n-1)} s^{n-2} + \cdots + d_{f2} s + d_{f1}$$
$$N_c(s) = c_{n-1} s^{n-2} + c_{n-2} s^{n-3} + \cdots + c_2 s + c_1$$
$$N_d(s) = d_{n-1} s^{n-2} + d_{n-2} s^{n-3} + \cdots + d_2 s + d_1$$

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A_f is to be selected (n-1) × (n-1) asymptotically stable matrix, c_f and d_f are (n-1)-dimensional column vectors, which can be expressed as

$$A_f = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & 1 \\ -d_{f1} & -d_{f2} & \cdots & -d_{f(n-1)} \end{bmatrix}$$

$$b_f = [0 \quad 0 \quad \cdots \quad 1]^{\mathrm{T}} \in \Re^{(n-1)\times 1}$$

$$c_f^T = [c_1 \quad c_2 \quad \cdots \quad c_{n-1}]$$

$$d_f^T = [d_1 \quad d_2 \quad \cdots \quad d_{n-1}]$$

The elements of vectors c_f and d_f and d_0 are 2n-1 adjustable parameters, and matrix A_f is shared by the two auxiliary signal generators.

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♦ **The adaptive control law**

1) The adaptive controller can be designed as follows:

$$\dot{v}_1(t) = A_f v_1(t) + b_f u(t) \tag{5.4.2}$$

$$\dot{v}_2(t) = A_f v_2(t) + b_f y_p(t) \tag{5.4.3}$$

2) The adaptive law of adjustable parameters is

$$\dot{\theta}(t) = \Gamma \varphi(t) e(t) \tag{5.4.4}$$

where $\theta(t) = \left[k_c, c_f^T, d_0, d_f^T\right]^T \in \mathbb{R}^{2n\times1}$, $\varphi(t) = \left[y_r(t), v_1^T(t), y_p(t), v_2^T(t)\right]^T \in \mathbb{R}^{2n\times1}$

$\Gamma \in \mathbb{R}^{2n\times2n}$ is a positive definite symmetric matrix.

3) The adaptive control law is

$$u(t) = \theta^T(t)\varphi(t) \tag{5.4.5}$$

which can stablise the closed-loop adaptive control system by Lyapunov stablility method.

■ **Algorithm** 5.4.1 (Narendra MRAC when n-m=1)

It is known that the order of the controlled plant $G_p(s)$ is n and n-m=1.

Step 1: Select the reference model $G_m(s)$ as a strictly positive real, stable minimum phase system, which has the same order and relative order as $G_p(s)$, and has ideal dynamic performance. The auxiliary signal generator state matrix A_f is constructed by $N_m(s)$.

Step 2: Set initial value $\theta(0)$, select adaptive gain matrix Γ and input signal $y_r(t)$, and initialize relevant data.

Step 3: Sample the current reference model output $y_m(t)$ and the actual system output $y_p(t)$, and calculate $e(t)$ from equation (5.4.1).

Step 4: Uses equation (5.4.2) and (5.4.3) to calculate $v_1(t)$ and $v_2(t)$.

Step 5: Use formula (5.4.4) to calculate parameter vector $\theta(t)$.

Step 6: Establish $\varphi(t)$ and calculate $u(t)$ from equation (5.4.5).

Step 7: t → t+h, return to Step 3 and continue the loop.

Note:

Generally speaking, the initial value of the adjustable parameters in the adaptive law $\theta(0)$ can be any value, but it is worth noting that if the initial value is improperly selected, the closed-loop system may be in an unstable initial state.

- ✓ If this happens, sometimes the adaptive adjustment $\theta(t)$ makes the system out of the unstable state.
- ✓ But sometimes, it may be too divergent to make the system continue to work before the adaptive law makes the closed-loop system stable.

Therefore, the selection of the initial value of the adjustable parameter needs to be tested or selected near its ideal parameter values.

♦ **Simulation example 5.4.1**

Consider the unstable controlled plant

$$G_p(s) = \frac{s+1}{s^2 - 5s + 6}$$

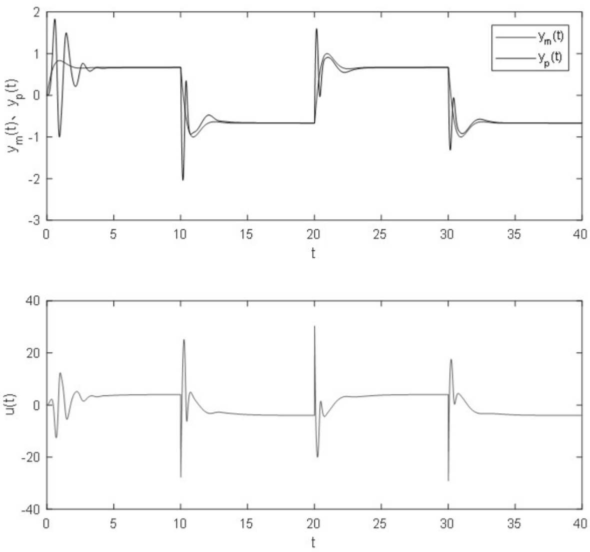
Select the reference model as

$$G_m(s) = \frac{s+2}{s^2 + 3s + 6}$$

It can be verified that $G_m(s)$ is strictly positive real.

The adaptive gain matrix $\Gamma = 10I^{4\times 4}$.

The reference input signal $y_r(t)$ is a square wave signal with amplitude $r = 2$, and the Narendra adaptive control law is adopted.



♦ **Simulation code**

example541.m

Self Assessment Question

In example 5.4.1, try different adaptive gain matrix Γ (e.g., $0.1I$, $1I$, $100I$, $500I$).

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5.5 Gain Scheduling Adaptive Control

■ Gain scheduling control systems

Examples of gain scheduling control systems

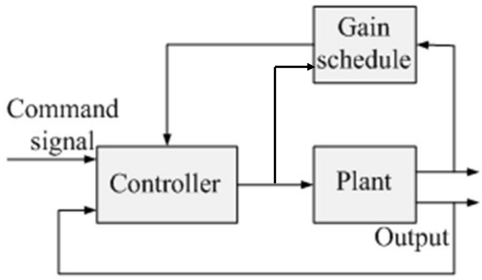
— Nonlinear valve control systems

— Water level control systems

— General nonlinear systems

The gain scheduling control is a design method of a linear variable parameter (LVP) system

Gain scheduling adaptive control system



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♦ **Gain scheduling adaptive control**

Idea: use auxiliary variables to measure the changes of the environment or the controlled plant itself, such as "gain" changes, and then use the controller to compensate for the reduction of the control system performance caused by the "gain" changes.

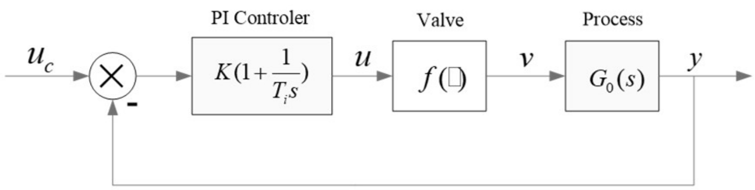
Objective: The control performance of the system is similar to the one of a desired reference model, e.g., a time-invariant linear system.

Implementation: through "function setting" or "lookup table method", also known as the gain list compensation method.

Applications: In aircraft control (x-15 fighter), flight Mach number and flight altitude are used as scheduling variables.

■ Example of a gain scheduling adaptive control system

Nonlinear valve control system (pp. 393, Adaptive Control, K. J. Astrom, B. Wittenmark, Science Press).



Linear plant:

$$G_0(s) = \frac{1}{(s+1)^3}$$

Nonlinear valve:

$$v = f(u) = u^4, u \geq 0$$

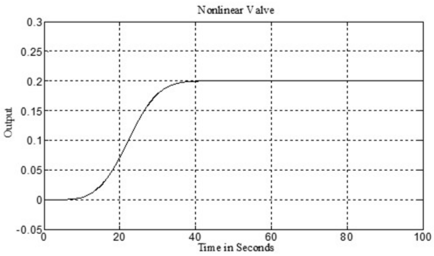
Controller:

$$G_C(s) = K\left[1 + \frac{1}{T_i s}\right]$$

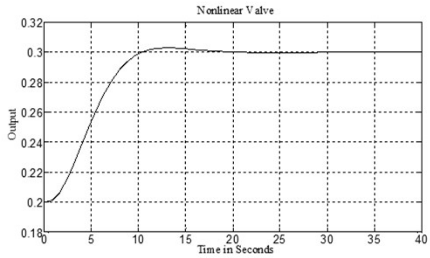
where

$$T_i = 1,\ \mathrm{K} = 0.15$$

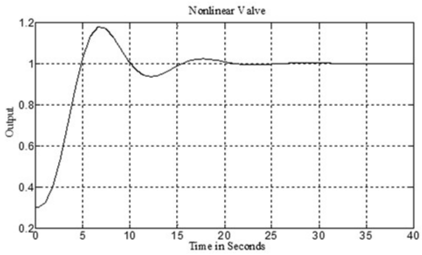
Output curve when the command is 0.2



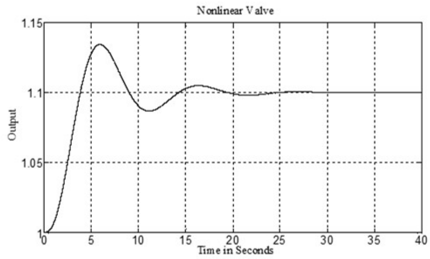
The steady state changes from 0.2 to 0.3



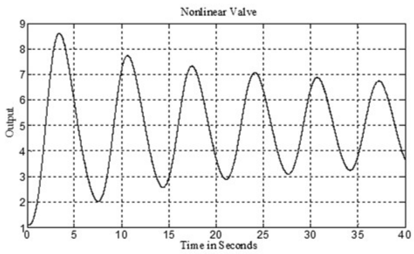
The steady state changes from 0.3 to 1.0



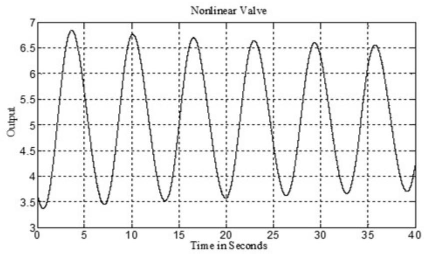
The steady state changes from 1.0 to 1.1



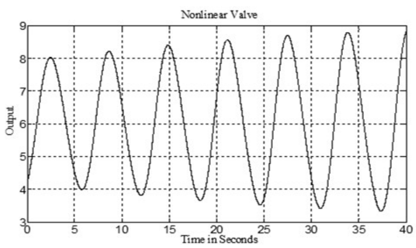
The stable state changes from 1.1 to 5.0



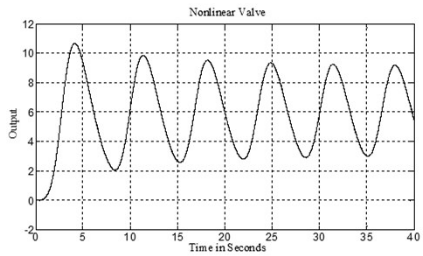
The stable state changes from 5.0 to 5.1



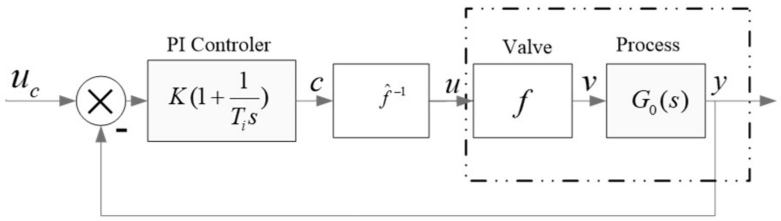
The stable state changes from 5.1 to 6.0



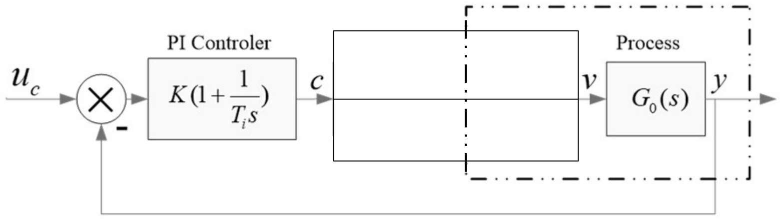
The stable state changes from 0 to 6.0



Control system with a nonlinear controller

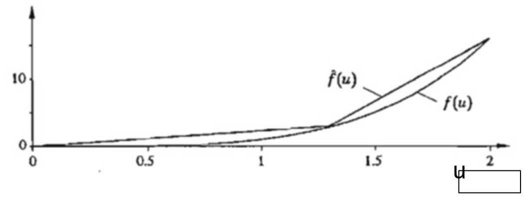


Linearized control system (the reference model)



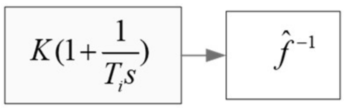
Design a nonlinear unit (an approximation of the inverse of the nonlinear function $f(u)$.

$$u = \hat{f}^{-1}(c) = \begin{cases} 0.433c, & 0 \le c \le 3 \\ 0.0538c + 1.139, & 3 \le c \le 16 \end{cases}$$



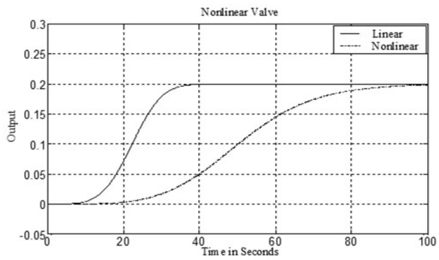
The controller structure is as follows:

$$K\left[1 + \frac{1}{T_i s}\right] \times \hat{f}^{-1}(c)$$

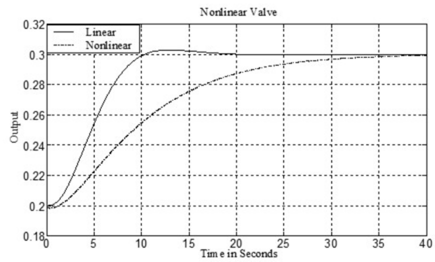


The input of the process is used as a scheduling variable to change the "open-loop gain" to compensate for the impact of valve nonlinear characteristics on the system.

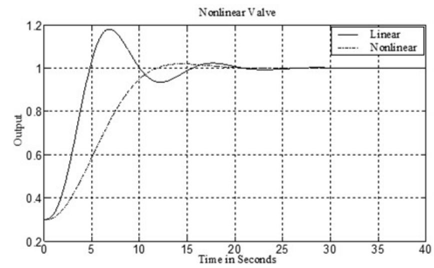
Output curve when the command is 0.2



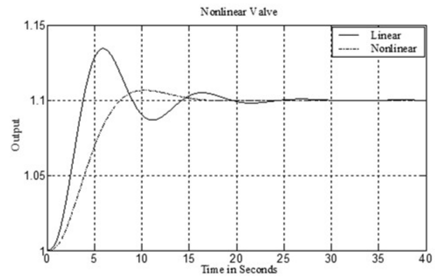
The steady state changes from 0.2 to 0.3



The steady state changes from 0.3 to 1.0



The steady state changes from 1.0 to 1.1



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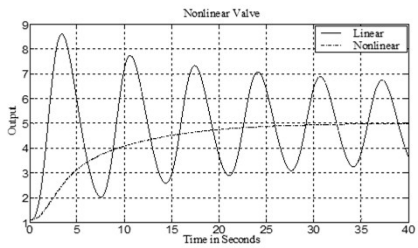
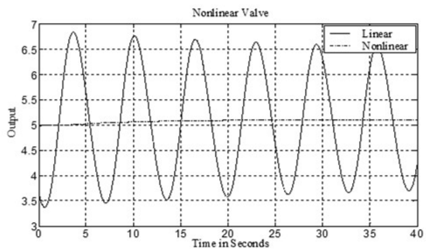
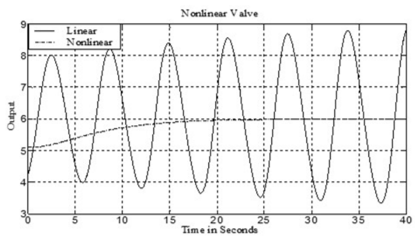
The steady state changes from 1.1 to 5.0



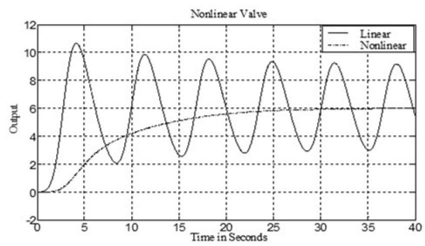
The steady state changes from 5.0 to 5.1



The steady state changes from 5.1 to 6.0



The steady state changes from 0 to 6.0



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Exercise 5.2

Consider a time-varying plant

$$\ddot{y}_p(t) + a\dot{y}_p(t) + by_p(t) = \dot{u}(t) + cu(t)$$

where $a = -5\big(1 + 0.1\sin(0.1\pi\mathrm{t})\big)$, $b = 6\big(1 - 0.2\cos(0.2\pi\mathrm{t})\big), c = \big(1 + 0.1\sin(0.3\pi\mathrm{t})\big)$.

Assume the reference model is

$$G_m(s) = \frac{s+1}{s^2 + 2.4s + 4}$$

and the reference input signal $y_r(t)$ is a square wave signal with amplitude of 4 and period of 15s.

Using the model reference stable adaptive control method (Narendra MRAC method), design an adaptive controller and make simulation.