

# Chapter 1

## Basics of System Identification

- 1.1 Principle of system identification
- 1.2 Linear time-invariant systems
- 1.3 System mathematical models
- 1.4 System output prediction
- 1.5 Description and analysis of random signals
- 1.6 White noises and pseudorandom codes
- 1.7 Experiment design of system identification

1

### 1.1 Principle of System Identification

#### ■ System concept

**An informal definition:** A system is a part of the world with a well-defined interface, which is acted by input and disturbance signals, and produces output signals in response.

The input can be controlled, but not the disturbance; often the disturbance cannot be measured, either.

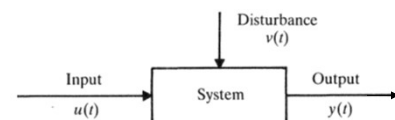
Note: the signals are functions of time, so the system evolves in time (it is dynamical).

**Example:** Consider the forward motion of the car.

Inputs: Acceleration and brake pedal positions

Output: Velocity.

Disturbance: Friction with varying road surfaces



2

### ◆ Model concept

An informal definition: A model is a description of a system that captures its essential behavior.

Crucial feature: the model is always an approximation of a real system.

- This feature is necessary and also desirable
- Exact models are unfeasible
- Simpler models are easier to understand and use

Non-math example: Mental/verbal model of a car

The model consists of verbal rules such as:

- Turning the wheel causes the car to turn.
- Pressing the gas pedal makes the car accelerate.
- Pressing the brake pedal makes the car slow down.

...



3

### ◆ Using the model

The model of a system is useful for many purposes. Some of the more important ones are:

Analyse the system (e.g. to find features such as stability, time constants etc.)

Simulate the system's response in new scenarios (e.g. how the car patient will respond to drugs). Allow studying scenarios that might be dangerous or expensive in the real system.

Predict the system's future output (e.g. weather prediction).

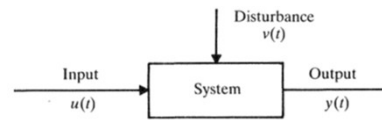
Design a controller for the system to achieve good behavior (e.g. fast response, small overshoot).

Design the system itself, by obtaining insight into how the yet-unbuilt system will behave. (Since the system is not available, requires first-principles modeling.)

4

## ■ About system identification

- ◆ System identification is an important branch of control theory.
- ◆ According to the inputs and outputs of a system with respect to time, system identification determines its mathematical model describing the system behavior.
- ◆ The basis of system identification is input and output data, and the data come from the experiments and observations of a system. So system identification is the process of extracting relevant system information from the data.
- ◆ The basic structure of an identified system includes: input, output, disturbance signals
- ◆ The goal of system identification is to obtain input and output data, and identify the mathematical model of a system through designed experiments.



5

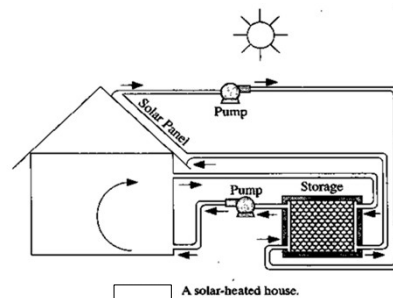
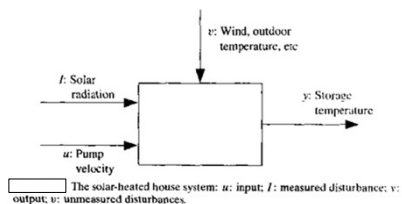
### Example:

#### A solar heated house

- The sun heats the air in the solar panels
- The air is pumped into a heat energy storage
- The stored energy is transferred to the house



**Goal:** Establish the relationship between the solar radiation, air pump velocity and storage temperature



6

#### ◆ Structure identification

- Select the specific expression form of the mathematical model in a model class.
- The structure of a linear system can be identified by input and output data.
- The general model structure is mainly obtained by prior knowledge.

#### ◆ Parameter estimation

- After knowing the model structure, use input and output data to determine the unknown parameters of the model.
- The parameter estimation is mainly based on statistical methods because actual measurements have random errors.

7

#### Example: DC motors

The DC motor is an electro-mechanical system.

The electric equations:

$$V_a(t) - E_a(t) = R_a I_a(t) + L_a \frac{dI_a(t)}{dt}$$

$$E_a(t) = K_e \Phi \omega(t), \quad T(t) = K_T \Phi I_a(t)$$

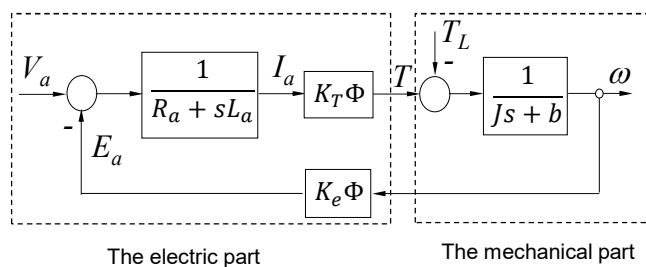
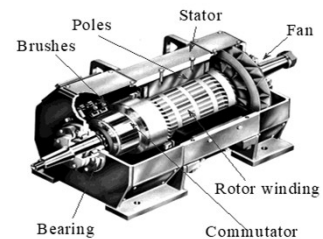
where  $V_a$ : voltage,  $\omega$ : speed,  $I_a$ : current,  $E_a$ : back-EMF,  $T$ : torque,  $R_a$ : resistance,  $L_a$ : inductance,  
 $\Phi$ : magnetic flux,  
 $K_e$  and  $K_T$ : coefficients.

The mechanical equation:

$$T = T_L + J \frac{d\omega}{dt} + b\omega$$

where

$T_L$ : load torque,  
 $J$ : inertia,  $b$ : friction



8

### Self Assessment Question

Which of the following statements are true:

- A. Parameter estimation refers to determining the specific form of the model with input and output data.
- B. Structure identification refers to the selection of specific values of mathematical models.
- C. The system identification only considers the establishment of the mathematical model of a system, and has nothing to do with the system input and output data.
- D. The model obtained from the system identification can be used to predict the future output changes of the system.

9

### ■ Functions of system identification

- ◆ The purpose of establishing a mathematical model through identification is to estimate the important parameters representing the system behavior
- ◆ Establish a model that can simulate the real system behavior
- ◆ The identification technology is used in occasions where the mathematical model and parameter estimation need to be determined by experimental data
- ◆ The identification technology has been applied to many engineering and non-engineering fields, such as, chemical processes, nuclear reactors, power systems, aerospace vehicles, biomedical systems, socio-economic systems, environmental systems, and ecosystems.

10

## ■ Concepts of system identification

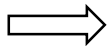
### ◆ Model

A model is the expression form of a system, process, thing or concept under study.

The model can also refer to a sample made according to experiments and drawings, which is generally used for exhibition, experiment or mold for casting machine parts.

### ◆ Type of models

- Intuitionistic model
- Physical model
- Chart model
- Mathematical model



According to the observed phenomena and practical experience of an object, a set of mathematical formulas, logic rules and specific algorithms reflecting the quantitative relationship of its internal factors are formed.

It is used to describe and study the laws of motion of objective phenomena.

11

### ◆ Modelling methods

#### 1) Theoretical analysis method

- ✓ By analysing the laws of motion of a system, using known laws, theorems and principles, such as Newton's theorem, energy balance equations, etc., deduce and establish a mathematical model of the system in a mathematical way.
- ✓ The theoretical analysis method can only be used for the modeling of relatively simple systems.
- ✓ A clear understanding of the mechanism of a system is required.
- ✓ For more complex actual systems, this modeling method has great limitations.

12

## 2) Practical test method

- ✓ Use the information provided by practical input and output data to establish a mathematical model of a system or find the parameters of a model.
- ✓ This modeling approach is system identification.
- ✓ Compared with the theoretical analysis method, the advantage of the practical test method is that it does not require a deep understanding of the mechanism of a system.
- ✓ The disadvantage is that it is often difficult to obtain effective system input and output data.

## 3) Hybrid method

- ✓ In practice, the theoretical analysis method is often combined with the practical test method.
- ✓ The part of the known mechanism (nominal model) adopts the theoretical analysis method, and the part of the unknown mechanism uses the practical test method.

13

### ◆ Definition of system identification

**Lotif Zadeh** (in 1962): System identification is to determine an equivalent model to the measured system from a class of models on the basis of input and output data.

**Lennart Ljung** (in 1978): System identification has three elements: data, model class and equivalence criterion. System identification is to select a model that fits the data best among the model classes according to certain criteria.

System identification is the process of creating a mathematical model to describe the behavior of a dynamical system, based on input and output data.

### ◆ Objectives of system identification

- System simulation
- System prediction
- System analysis
- System design and control
- Fault diagnosis and prediction
- Verification of mechanism models

14

### Self Assessment Question

Which of the following is true:

- A. It is not a good practice to combine the theoretical analysis method with the practical test method.
- B. System identification is the process of creating a model to describe system dynamics using input and output data.
- C. Only a mathematical model can describe a system.
- D. The system identification technology is employed only for engineering fields.

15

#### 1) System simulation

To study the output of a system under different input conditions, the most direct method is to conduct experiments on the system itself, but it is difficult to achieve in practice.

- ✓ The cost of experimenting with an actual system may be too high.
- ✓ The system may be unstable during the experiment, which leads to certain dangers in the experiment process.
- ✓ The time constant of the system may be very large, so that the experiment period is too long.

For the reasons above, it is necessary to use system identification modeling, and employ a model to simulate the characteristics or behavior of the system, so as to conduct simulation research on the system indirectly.

For example, missiles, aircraft, nuclear reactors, large chemical and power plants, and large transmission machinery.

16



## 2) System prediction

Whether in the field of natural science or social science, it is often necessary to study the laws and changing trends of the future development of a system to make decisions and take measures in advance.

Most scientific quantitative predictions require the use of model methods, that is, firstly establish a mathematical model of the predicted system, and predict the future states of some variables in the system according to the model.

## 3) System analysis

After the mathematical model of a system is established according to experimental data,

- the main characteristics and changes of the system under study can be described,
- the relationship between the main variables in the system to be studied can be revealed.

The system model provides a basis for analysis, which gives clues and evidence.

17

## 4) System design and control

In engineering design, the characteristics of all components or subsystems included in a system must be mastered.

A perfect design must make the characteristics of each component of the system consistent with the overall design requirements of a system (such as output indicators, errors, stability, safety and reliability, etc.)

So, it is necessary to establish a mathematical model for analysing and examining the characteristics of each part of a system and the interaction between each part and their influence on the overall system characteristics in the design.

18

### 5) Fault diagnosis

Many complex systems, such as missiles, aircraft, nuclear reactors, large chemical and power plants, and large transmission machinery, etc., need to monitor and detect possible faults frequently in order to eliminate them in time.

This requires that the information must be continuously collected during the operation of a system, and the changes in the dynamic characteristics of the system can be deduced by establishing a mathematical model.

Then, according to the change of dynamic characteristics, it is judged whether a fault has occurred. When it occurs, determine the size of the fault, the location of the fault, etc.

19

### 6) Verification of mechanism models

After the mathematical model of a system is established according to test data, it will be very beneficial to understand the obtained test data, so that

- the influence of different input conditions on the output variables of the system can be explored and analyzed,
- the proposed mechanism model can be tested more comprehensively,
- the dynamic behavior of the system can be understood better.

20

### Self Assessment Question

Which statements are false:

- A. Some faults of nuclear reactors can be deduced by establishing a mathematical model.
- B. A system model can be used to examine the interaction between the subsystems of a system.
- C. System simulation is not good because it is not exactly the same as the real system.
- D. System identification can hardly do verification of mechanism models.

21

#### ♦ Classification of system models

- Deterministic model / stochastic model
- Static model / dynamic model
- Continuous model / discrete model
- Time-invariant model / time-varying model
- Linear system model / nonlinear model
- Parametric model / non-parametric model
- Centralized parameter model / distributed parameter model
- Single input single output model / multiple input multiple output model

22

1) The perspective of probability: deterministic and stochastic

In a system described by the deterministic model, when its state is determined, its output response is uniquely determined. For example,

$$y(k + 1) = ay(k) + bu(k)$$

where  $y(k)$  is the output,  $u(k)$  is the input,  $a$  and  $b$  are constants, and  $k$  is the discrete time.

However, in a system described by the stochastic model, when its state is determined, its output response is uncertain. For example,

$$y(k + 1) = ay(k) + bu(k) + e(k)$$

where  $e(k)$  is a random noise.

23

2) The relationship between the model and time: static and dynamic

The static model is used to describe the relationship between the state variables when the system is in a steady state (the derivative of each state variable is zero), and it is generally not a function of time. For example,

$$y(k) = au(k)$$

The dynamic model is used to describe the relationship between the various state variables when a system is in the transition process, and it is generally a function of time. For example,

$$y(k + 1) = ay(k) + bu(k)$$

24

### 3) The time scale: continuous and discrete

The models used to describe continuous systems include differential equations, s-transfer functions, etc., and the models used to describe discrete systems include difference equations, z-transfer functions, etc. For example,

$$\dot{y}(t) = ay(t) + bu(t), \quad (\text{continuous model})$$

$$y(k+1) = ay(k) + bu(k), \quad (\text{discrete model})$$

### 4) The relationship between a parameter and time: time-invariant and time-varying

The model parameters of a steady system do not change with time, while the model parameters of a time-varying system change with time. For example,

$$y(k+1) = ay(k) + bu(k), \quad (\text{time-invariant model})$$

$$y(k+1) = a(k)y(k) + b(k)u(k), \quad (\text{time-varying model})$$

25

### 5) The relationship between a parameter and input-output: linear and nonlinear

The linear model is used to describe a linear system, and its notable feature is to satisfy the principle of superposition and uniformity. For example,

$$\ddot{y}(t) = a\dot{y}(t) + bu(t)$$

$$y(k+1) = ay(k-1) + bu(k)$$

The nonlinear model is used to describe a nonlinear system, which generally does not satisfy the principle of superposition. For example,

$$\dot{y}(t) = ay(t) + by(t)u(t)$$

$$y(k+1) = ay^2(k) + bu(k)$$

26

### Self Assessment Question

What type of models is the following?

$$y(t+1) = ay^2(t) + bu(t) + e(t)$$

where  $y(t)$ ,  $u(t)$  and  $e(t)$  are the output, input and random noise, respectively,  $a$  and  $b$  are constants, and  $t$  is the discrete time.

27

6) The expression form of the model: parametric and non-parametric

The non-parametric model refers to a model established through the response obtained directly or indirectly from the experimental process of an actual system, such as a model established through a step response, impulse response, and frequency response, all of which reflect the characteristics of the system. For example,

$$y(k) = \left( \sum_{i=0}^{\infty} g(i)q^{-i} \right) u(k) \quad \begin{array}{l} q^{-1} \text{ is the delay factor, i.e.,} \\ q^{-1}u(k) = u(k-1) \end{array}$$

where  $g(i)$  is the unit impulse sequence.

Nonparametric models cannot be described by a fixed, small number of parameters, often represented as graphs or tables

The model established by the reasoning method is called the parametric model. For example,

$$y(k+1) = ay(k) + bu(k)$$

Parametric models have a fixed form (mathematical formula), with a known, often small number of parameters

28

7) The nature of the parameters: centralized parameters and distributed parameters

When the parameters of a system are only a function of time, the equations describing the system characteristics are ordinary differential equations, and the system is called a centralized (or lumped) parameter system. For example,

$$\frac{dy(t)}{dt} = ay(t) + bu(t)$$

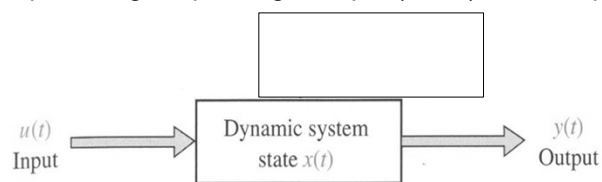
When the state parameters of a system are functions of time and space, and the state equations describing the system characteristics are partial differential equations, the system is called a distributed parameter system. For example,

$$\frac{dy(t)}{dt} = \frac{\partial y(t)}{\partial u(t)} y(t) + bu(t)$$

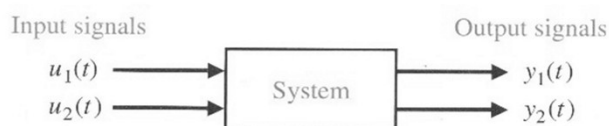
29

8) The number of inputs and outputs: single input single output (SISO) and multiple input multiple output (MIMO).

SISO system:



MIMO system:



30

### ◆ Classification of system identification

- White box modelling / black box identification / grey box identification
- Offline identification / online identification
- Open-loop system identification / closed-loop system identification

1) The experimental information provided: white box, black box, gray box

White box: A system can be called a "white box" if its structure, composition, and laws of motion are known, which is suitable for modelling through mechanism analysis.

Black box: If the objective law of a system is not clear, the response data of the system can only be measured from a system test, and the mathematical model of the system is established using the identification method, the system is called a "black box".

Gray box: If some basic laws of a system are known, but some mechanisms are still unclear, the system is called a "gray box".

White-box models: fully known in advance; Black-box models: entirely unknown; Gray-box models: partially known.

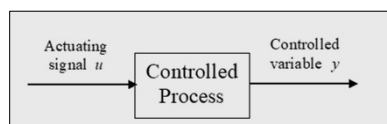
31

2) The application form of the model: offline and online.

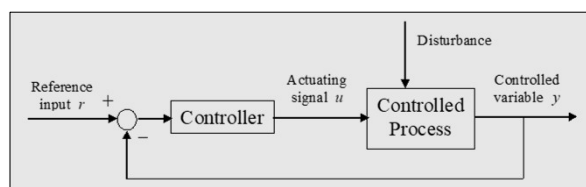
Offline identification: A system is tested, and after obtaining all the data, the identification algorithm is used to process the data intensively to obtain the estimated value of the model parameters.

On-line identification: It needs to know the structure and order of the model. After obtaining the current input and output data, the recursive identification method is used to correct the estimated value of the parameter to obtain a new estimated value of the parameter.

3) The system identification means: open-loop and closed-loop



The open-loop system



The closed-loop system

32



### Self Assessment Question

Which is false?

- A. A mathematical model mainly reflects the quantitative relationship of the external factors of the research object.
- B. The system identification will not discuss the input signal of the system.
- C. System identification can be used in model-based controller design.
- D. White box modeling does not require system identification.

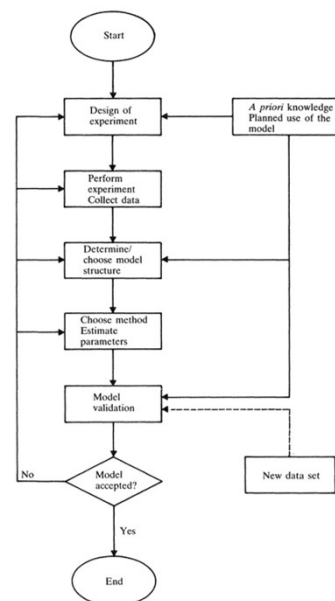
33

### ■ Workflow of system identification

#### ◆ Main steps of system identification

- Obtain prior information on a system
- Design an experiment
- Collect data
- Choose the model structure
- Determine the parameters
- Validate the model
- Accept the model

#### ◆ Block diagram of system identification



34

#### ◆ Prior information of system identification

1) Two different meanings of prior information:

- First, the information about model structure, parameters or data, which is known or collected from data.
- Second, there are assumptions made before identification work, which are some restrictions on model structure, parameters or data and are often to ensure identifiability requirements.

35

2) Prior information about model structure

- Obtained by analysing the motion law of a system (physics, chemistry, life science)
- Obtained by data analysis (linear, time-varying, noise correlation, etc.)

3) Prior information about data

- Restrictions on data
- Specified distribution of noises

4) Prior information about parameters

- Given range of parameters
- Priori estimation of parameter (can be used as the initial value of the estimation algorithm)
- A priori distribution density of parameters (improved estimation accuracy)

36

### ◆ Identification test design

The existing prior knowledge, preliminary test results and identification purpose determine how to design the test.

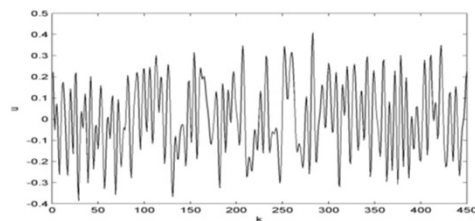
- Preliminary test (step response method, frequency response method, etc.)
- Main time constants of a system
- Allowable input signal amplitude
- Nonlinearity and time variation of a process
- Noise level
- Delay between variables, etc
- When arranging the test, first determine the variables to be observed.
- Determination of test duration (identification accuracy, test cost)

37

### A main part of experiment design consists of selecting the input signal.

- How to select the input signal (if the identified system allows to add a test signal).
- The input signal should be sufficiently rich to bring out the interesting behavior in the system.
- Different identification methods can be adopted according to the characteristics of different input signals.
- The determination of the sampling period of the observation signal is generally based on the dynamic characteristics of the system and the characteristics of the input signal.

**Example:** Input signal  $u(k)$ ,  
 $k = 0, 1, 2, \dots, N$



38

### Self Assessment Question

To implement system identification,

- A. determine the parameters after choosing the model structure
- B. the larger the input signal, the better the identification results
- C. the prior information on a identified system is unnecessary
- D. any identification test can do the job

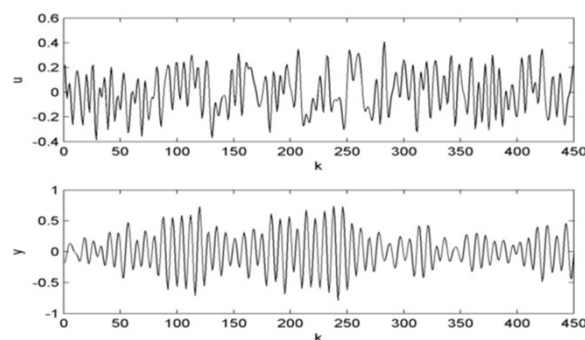
Which of the above is correct?

39

#### ◆ Collect data

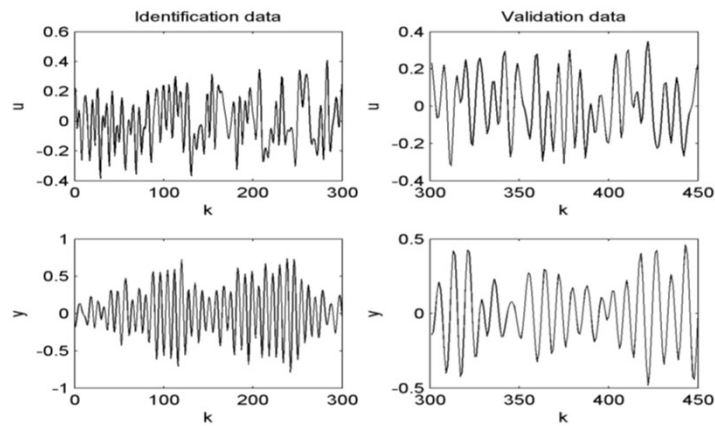
- Obtain the input-output data from experiments or practical operations
- The test data are treated with statistical methods to filter out noise pollution and random interference

**Example:** Input-output signals  $[u(k), y(k)]$ ,  $k = 0, 1, 2, \dots, N$



40

- Split the data into an identification set and a validation set (important later).



41

#### ◆ Determination of a model structure

- The definition of the model structure refers to the determination of the order and pure delay time in centralized linear time-invariant systems.
- The application purpose and accuracy requirements of the model need to be considered, which is often necessary to balance its accuracy and complexity.
- In the process of determining the structure, the understanding of the physical meaning of the identified process should be used as much as possible, rather than just using statistical methods.
- Selection of model types
  - ✓ Impulse response function
  - ✓ Linear difference equation
  - ✓ State space model
  - ✓ ...

42

### 1) Impulse response function

The discrete impulse response  $g(i)$  of a SISO system refers to the output response of the linear system to the unit impulse sequence when the initial condition is zero.

Under the action of any input, the output of the system is expressed as

$$y(k) = \left( \sum_{i=0}^{\infty} g(i)q^{-i} \right) u(k)$$

For a stable system, there is

$$y(k) = \left( \sum_{i=0}^{N_s} g(i)q^{-i} \right) u(k)$$

43

Let

$$B(q^{-1}) = \sum_{i=0}^{N_s} g(i)q^{-i}$$

For a stochastic system, considering the influence of the noise term, then

$$y(k) = B(q^{-1})u(k) + e(k)$$

where  $e(k)$  is a noise.

### 2) Linear difference equation

The difference equation is the most basic model of a discrete system.

The relationship between the input and output sequences of a dynamic discrete system can be expressed as the following  $n^{\text{th}}$ -order linear difference equation:

$$y(k) + a_1y(k-1) + \cdots + a_ny(k-n) = b_1u(k-1) + b_2u(k-2) + \cdots + b_mu(k-m)$$

44

### 3) State space model

The state space of a linear time-invariant continuous system is described as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$$

where  $x(t)$  is the state variable of the system,  $y(t)$  and  $u(t)$  represent the output and input, respectively,  $A$ ,  $B$  and  $C$  are matrices with appropriate dimensions, which are called the system matrix, input matrix and output matrix, respectively.

The state-space model of a discrete system is

$$\begin{cases} x(k+1) = A_d x(k) + B_d u(k) \\ y(k) = C_d x(k) \end{cases}$$

45

### Self Assessment Question

Which is true?

- A. A continuous-time model is not exactly the same as its discrete-time model.
- B. A validation data set includes both input and output data.
- C. A linear difference equation can not be expressed in a state space model.
- D. The discrete impulse response of a system refers to the output response of the linear system to any impulse input.

46

#### ◆ Parameter and state estimation

- Parameter estimation criteria
- Parameter estimation method, which depends on the structure chosen
- For the state space model, the state can be considered as one of the parameters, and it is also necessary to estimate the system state
- Parameter and state estimation requires numerical analysis methods

47

Commonly used error criteria for system identification

The error criterion used in the identification is one of the three elements of an identification problem and is used to measure the model's closeness to the actual system.

The error criterion is often expressed as a functional function of the error, that is,

$$J(\theta) = \sum_{k=1}^N f(\varepsilon(k), \theta)$$

where  $J(\cdot)$  is a function of the parameter  $\theta$ , and  $f(\cdot)$  is the error function defined on the interval, which generally refers to the error between the model and the actual system:

$$f(\varepsilon(k), \theta) = \varepsilon^2(k)$$

$$\varepsilon(k) = y(k) - y_m(k)$$

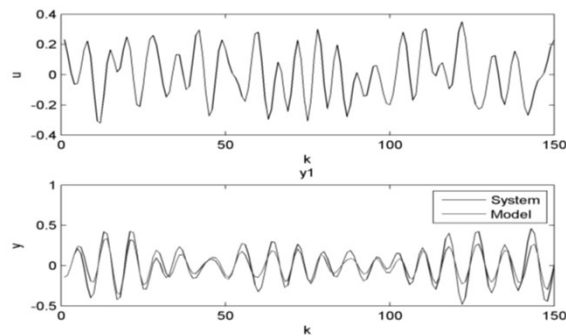
48



#### ◆ Model verification

- For multiple tests, the model identified with one set of data (obtained under a certain working condition) needs to be verified or modified with another set of data (obtained under another working condition) and cross-checked to ensure the model quality.

**Example:** Verify the model using the validation data that are separated from the start.



49

- All assumptions should be checked as far as possible. If model verification is not possible due to test conditions, it is also necessary to judge whether the model is within a reasonable deviation range according to the data obtained from the test and the understanding of the process.
- The quality of the model is mainly determined by actual application effects.

If the determined system model is suitable, the identification ends.

Otherwise, change the prior model structure of the system and re-execute the identification process until a satisfactory model is obtained.

50

#### ◆ Limitations of traditional system identification

- Completely based on numerical calculation of a mathematical model, it is difficult to process complex information, such as language and image.

So, fusion and processing of complex information need to be considered.

- The identification result is a nominal system (a point), and the allowable error range of modeling is not given, which cannot meet the requirements of robust control design.

So, robust identification needs to be considered.

51

#### Self Assessment Question

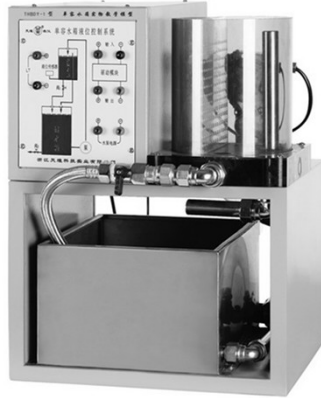
Which is true?

- A. Parameter estimation and state estimation cannot be carried out at the same time.
- B. The more complex the model, the higher the precision of the model.
- C. System identification will use the same data for model identification and verification.
- D. The prior information affects the results of system identification.

52

## ■ An example of system identification

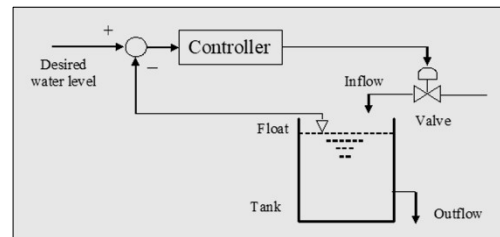
### ◆ Water tank level control system



Water tank



Controller



53

### ◆ The schematic diagram of a single water tank

$V$ : a water storage volume

$h = h_0 + \Delta h$ : the water level height

$h_0$ : the steady-state value of liquid level (m);

$\Delta h$ : the increment of liquid level (m);

$Q_i$ : the steady-state value of inflow water flow ( $\text{m}^3/\text{s}$ );

$\Delta Q_i$ : the increment of inflow water flow ( $\text{m}^3/\text{s}$ );

$Q_o$ : the steady-state value of the outflow water flow ( $\text{m}^3/\text{s}$ );

$\Delta Q_o$ : the increment of the outflow water flow ( $\text{m}^3/\text{s}$ );

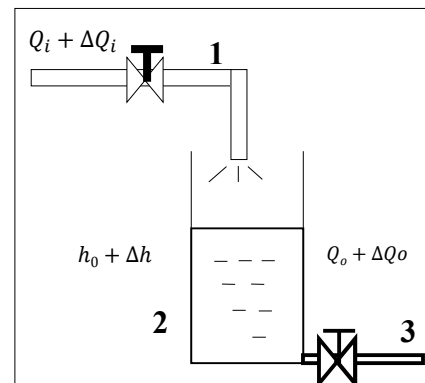
$\Delta u$ : the opening of the control valve (inlet valve) ( $\text{m}^2$ ).

$A$ : be the cross-sectional area of the tank ( $\text{m}^2$ ).

### ◆ Mechanism modeling

The difference between the inflow and outflow is equal to the rate of change of liquid storage in the tank, namely:

$$\Delta Q_i - \Delta Q_o = \frac{dV}{dt} = A \frac{d\Delta h}{dt}$$



1: Control valve

2: Water storage tank

3: Load valve

54

### ◆ Transfer function

According to the prior knowledge, the single tank is a first-order system with time delay, and its transfer function is

$$G(s) = \frac{K}{Ts + 1} e^{-\tau s}$$

Its three parameters need to be identified: gain K, time constant T and time delay  $\tau$ .

### ◆ Unit step output response

1) For the time delay  $\tau=0$ , 
$$y(t) = L^{-1} \left[ \frac{K}{Ts + 1} \times \frac{1}{s} \right] = L^{-1} \left[ \frac{K}{s} - \frac{KT}{Ts + 1} \right] = K \left( 1 - e^{-\frac{t}{T}} \right)$$

2) For the time delay  $\tau \neq 0$ , 
$$y(t) = K \left( 1 - e^{-\frac{t-\tau}{T}} \right)$$

55

### ◆ How to estimate the three parameters K, T and $\tau$ ?

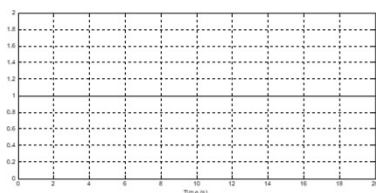
– The gain K:  $K = y(\infty)$

– Use the two-point method to estimate parameters T and  $\tau$

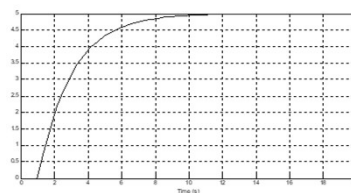
Choose two points:  $y(t_1)=0.39K$  and  $y(t_2)=0.632K$

Formula of estimating the parameters:  $T \approx 2(t_2 - t_1)$ ,  $\tau \approx 2t_1 - t_2$

### ◆ Conduct a unit step response test



Input (a unit step signal)



Output (a unit step response )

56

### ♦ Experimental results

— the measured data:

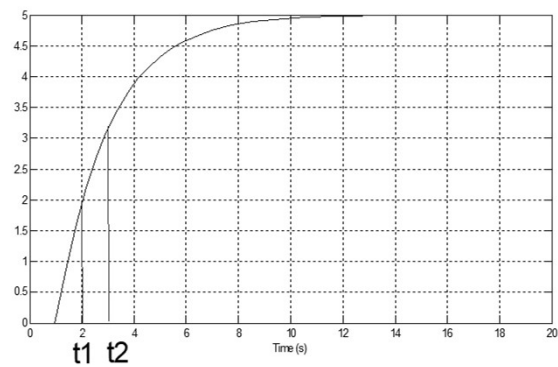
$t_1 = 2$  (reaches 39%),  
 $t_2 = 3$  (reaches 63.2%)  
 $y(\infty) = 5$  (final value)

— the model parameters:

$K = y(\infty) = 5$   
 $T = 2(t_2 - t_1) = 2$   
 $\tau = 2t_1 - t_2 = 1$

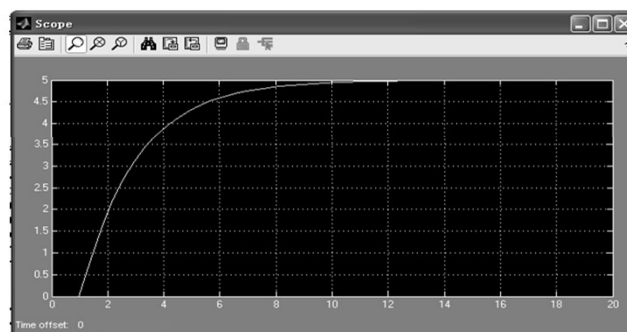
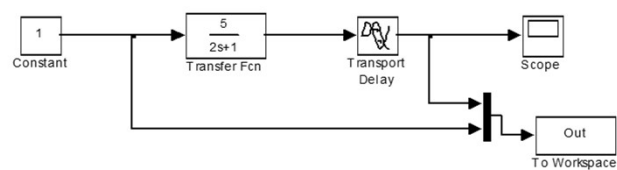
— the model:

$$G(s) = \frac{5}{2s+1} e^{-s}$$



57

### ♦ Simulation verification



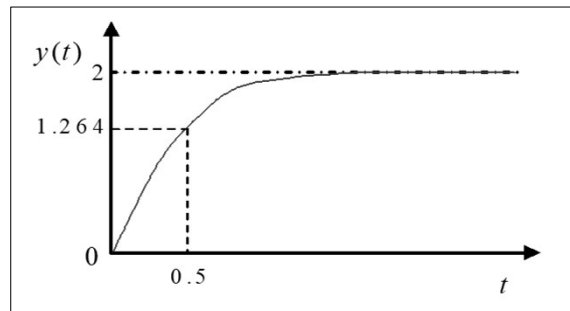
58

### Self Assessment Question

For a first-order system, its transfer function is

$$G(s) = \frac{b}{s + a}$$

If its unit step response is as shown in the figure below, estimate parameters  $a$  and  $b$ .



59

### Exercise 1.1

Explain the following:

- 1) the principle of system identification;
- 2) the importance of the three key elements in system identification;
- 3) the workflow of creating an electric kettle model.

60