Chapter 6

Self-Tuning Control

6.1 Introduction to Self-Tuning Control (STC)

Self-tuning control (STC) is another kind of adaptive control.

It is different from model reference adaptive control, and also the most widely used adaptive control method.

Its basic idea is to combine the recursive algorithm of parameter estimation with different types of control algorithms to form a real-time computer control system that can automatically correct controller parameters.

Six types of self-tuning control will be introduced :

- minimum variance self-tuning control
- generalized minimum variance self-tuning control
- pole assignment self-tuning control
- self-tuning PID control
- generalized predictive control
- networked learning predictive control

■ Solution of Diophantine equation

When implementing the first two kinds of self-tuning control algorithms, the single-step Diophantine equation (Diophantine equation) needs to be solved online.

When implementing generalized predictive control, the multi-step Diophantine equation needs to be solved online.

The plant to be considered is

$$A(q^{-1})y(k) = B(q^{-1})u(k-d) + C(q^{-1})\xi(k)$$

where

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ ... + a_n\ q^{-n_a}$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + \ ... + c_{n_c} q^{-n_c}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \ ... + b_{n_b} q^{-n_b}$$

♦ Solution of single-step Diophantine equations

The equations in the form below are called Diophantine equations

$$C(q^{-1}) = A(q^{-1})E(q^{-1}) + q^{-d}G(q^{-1}) \qquad (6.1.1)$$

$$F(q^{-1}) = B(q^{-1})E(q^{-1}) \qquad (6.1.2)$$

where polynomials $A(q^{-1})$ and $C(q^{-1})$ are given,

$$F(q^{-1}) = f_0 + f_1 q^{-1} + \ ... + f_{n_f} q^{-n_f} \quad (n_f = n_b + d - 1)$$

$$G(q^{-1}) = g_0 + g_1 q^{-1} + \ ... + g_{n_g} q^{-n_g} \quad (n_g = n_a - 1)$$

$$E(q^{-1}) = 1 + e_1 q^{-1} + \ ... + e_{n_e} q^{-n_e} \quad (n_e = d - 1, \qquad a\ monic\ polynomial\ )$$

Strictly speaking, the first equation in equation (6.1.1) is usually called Diophantine equation, while the two equations are used at the same time each time, so equations (6.1.1) and (6.1.2) are collectively called Diophantine equations.

The first Diophantine equation (6.1.1) can be written as

$$1 + c_q q^{-1} + \ \dots + c_{n_c} q^{-n_c} = (1 + a_1 q^{-1} + \ \dots + \ a_n \ q^{-n_a})\left(1 + e_1 q^{-1} + \ \dots + e_{n_e} q^{-n_e}\right) +$$

$$+ g_0 q^{-d} + g_1 q^{-d-1} + \ \dots + g_{n_g} q^{-d-n_g}$$

Let the coefficients of the same power term of q^{-1} on both sides of the above equation be equal, and the recurrence formula of the parameters of Diophantine equation is

$$e_i = c_i - \sum_{j=1}^{i} e_{i-j} a_j , \qquad i = 1,2, \dots , n_e \tag{6.1.3}$$

$$g_i = c_{i+d} - \sum_{j=0}^{n_e} e_{n_e - j} a_{i+j+1} , \qquad i = 0,1, \dots , n_g \tag{6.1.4}$$

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The second equation (6.1.2) of the Diophantine equation can be written as

$$f_0 + f_1 q^{-1} + \ \dots + f_{n_f} q^{-n_f} = \ \left(b_0 + b_1 q^{-1} + \ \dots + b_{n_b} q^{-n_b}\right)\left(1 + e_1 q^{-1} + \ \dots + e_{n_e} q^{-n_e}\right)$$

Letting the coefficients of the same power term of q^{-1} on both sides of the above equation be equal gives

$$f_i = \sum_{j=0}^{i} b_{i-j} e_j , \qquad i = 0,1, \dots , n_f \tag{6.1.5}$$

In the calculation process, if a_i, b_i, c_i and e_i do not actually exist, they are replaced by 0. For example, if $i > n_a,\ a_i = 0$.

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■ Example 6.1.1

Solve the single-step Diophantine equation of the following system.

$$y(k) - 1.7y(k-1) + 0.7y(k-2) = u(k-4) + 0.5u(k-5) + \xi(k) + 0.2\xi(k-1)$$

From the system difference equation

$$a_1 = -1.7,\quad a_2 = 0.7,\quad b_0 = 1,\quad b_1 = 0.5,\quad c_0 = 1,\quad c_1 = 0.2$$
$$n_a = 2,\quad n_b = 1,\quad n_c = 1,\quad d = 4$$

According to eq (6.1.1),

$$n_e = d - 1 = 3,\quad n_g = n_a - 1 = 1,\quad n_f = n_b + d - 1 = 4$$

Then it is obtained from the recurrence formula (6.1.3) - (6.1.5) ($e_0 = 1$ is known)

$$e_1 = c_1 - \sum_{j=1}^{1} e_{1-j}a_j = c_1 - e_0a_1 = 0.2 - (-1.7) = 1.9$$

$$e_2 = c_2 - \sum_{j=1}^{2} e_{2-j}a_j = c_2 - e_1a_1 - e_0a_2 = 0 - 1.9(-1.7) - 0.7 = 2.53$$

$$e_3 = c_3 - \sum_{j=1}^{i} e_{3-j}a_j = c_3 - e_2a_1 - e_1a_2 - e_0a_3 = 0 - 2.53(-1.7) - 1.9\times 0.7 - 0 = 2.971$$

$$g_0 = c_{0+4} - \sum_{j=0}^{3} e_{3-j}a_{0+j+1} = c_4 - e_3a_1 - e_2a_2 - e_1a_3 - e_0a_4 = -2.971(-1.7) - 2.53\times 0.7$$
$$= 3.2797$$

$$g_1 = c_{1+4} - \sum_{j=0}^{3} e_{3-j}a_{1+j+1} = c_5 - e_3a_2 - e_2a_3 - e_1a_4 - e_0a_5 = 0 - 2.971\times 0.7 = -2.0797$$

$$f_0 = \sum_{j=0}^{0} b_{0-j}e_j = b_0e_0 = 1$$
$$f_1 = \sum_{j=0}^{1} b_{1-j}e_j = b_1e_0 + b_0e_1 = 0.5 + 1.9 = 2.4$$
$$f_2 = \sum_{j=0}^{2} b_{2-j}e_j = b_2e_0 + b_1e_1 + b_0e_2 = 0 + 0.5 \times 1.9 + 2.53 = 3.48$$
$$f_3 = \sum_{j=0}^{3} b_{3-j}e_j = b_3e_0 + b_2e_1 + b_1e_2 + b_0e_3 = 0 + 0 + 0.5 \times 2.53 + 2.971 = 4.236$$
$$f_4 = \sum_{j=0}^{4} b_{4-j}e_j = b_4e_0 + b_3e_1 + b_2e_2 + b_1e_3 + b_0e_4 = 0.5 \times 2.971 = 1.4855$$

Therefore, the polynomial $E(q^{-1})$, $G(q^{-1})$ and $F(q^{-1})$ are

$$E(q^{-1}) = 1 + 1.9q^{-1} + 2.53q^{-2} + 2.971q^{-3}$$
$$G(q^{-1}) = 3.2797 - 2.0797q^{-1}$$
$$F(q^{-1}) = 1 + 2.4q^{-1} + 3.48q^{-2} + 4.236q^{-3} + 1.4855q^{-4}$$

♦ **Simulation codes**

example611.m

6.2 Minimum Variance Self-tuning Control

■ Basics of minimum variance self-tuning control

The minimum variance self-tuning regulator (MVSTR) was proposed by K.J. Astrom and B. Wittenmark in 1973, and the term "self-tuning" was also created by them.

This regulator belongs to direct self-tuning control.

For a SISO discrete-time system with constant but unknown parameters, the self-tuning control law is designed with the minimum output variance as the goal.

The controller whose parameters are directly estimated by the recursive least squares algorithm is the simplest self-tuning controller.

♦ Features of minimum variance control

- ✓ Minimum variance control (MVC) has the advantages of a simple algorithm, easy to understand, easy to implement.
- ✓ MVC is the basis of other self-tuning control algorithms.
- ✓ Because there is a pure delay of d in general industrial processes, the current control effect needs to delay d steps to affect the output.
- ✓ To minimize the output variance, the output must be predicted d steps in advance, and then the required control law is designed according to the predicted value.
- ✓ In this way, through continuous prediction and control, the steady-state output variance can be guaranteed to be minimum.
- ✓ It can be seen that the key to achieving minimum variance control is output prediction.

♦ **Single-step output prediction**

The system adopts the following mathematical model:

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + C(q^{-1})\xi(k) \quad (6.2.1)$$

where $C(q^{-1})$ is Hurwitz polynomial, that is, its zero point is completely located within the unit circle of the q-plane, and $u(k)$ and $y(k)$ represent the input and output of the system, $\xi(k)$ is a white noise with the variance of σ^2, $\mathrm{d} \geq 1$ is pure delay, and

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_{n_a} q^{-n_a}$$
$$B(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + \cdots + b_{n_b} q^{-n_b}$$
$$C(q^{-1}) = 1 + c_1 q^{-1} + c_2 q^{-2} + \cdots + c_{n_c} q^{-n_c}$$

Input/output data of plant (6.2.1) at time k and previous time are expressed as

$$\{Y^k, U^k\} = \{y(k), y(k-1), \ldots, u(k), u(k-1), \ldots,\}$$

Based on $\{Y^k, U^k\}$, the d-step ahead prediction of the output is denoted as $\hat{y}(k+d|k)$

The output prediction error is defined as $\tilde{y}(k+d|k) = y(k+d) - \hat{y}(k+d|k)$

Theorem 6.2.1 (optimal d-step output prediction)

To minimise the performance index (variance of prediction error)

$$\mathcal{E}\{\tilde{y}^2(k+d|k)\}$$

the d-step optimal output prediction $y^*(k+d|k)$ must satisfy the equation

$$C(q^{-1})y^*(k+d|k) = G(q^{-1})y(k) + F(q^{-1})u(k) \quad (6.2.2)$$

where
$$C(q^{-1}) = A(q^{-1})E(q^{-1}) + q^{-d}G(q^{-1})$$
$$F(q^{-1}) = B(q^{-1})E(q^{-1})$$

$$F(q^{-1}) = f_0 + f_1 q^{-1} + \ldots + f_{n_f} q^{-n_f} \qquad (n_f = n_b + d - 1)$$
$$G(q^{-1}) = g_0 + g_1 q^{-1} + \ldots + g_{n_g} q^{-n_g} \qquad (n_g = n_a - 1)$$
$$E(q^{-1}) = 1 + e_1 q^{-1} + \ldots + e_{n_e} q^{-n_e} \qquad (n_e = d - 1)$$

The variance of the optimal prediction error is $\mathcal{E}\left\{\tilde{y}^{*^2}(k+d|k)\right\} = \left(1 + \sum_{i=1}^{d-1} e_i^2\right)\sigma^2$

Proof: It can be obtained from eq (6.2.1) and eq (6.1.1) that

$$y(k) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(k) + \frac{A(q^{-1})E(q^{-1}) + q^{-d}G(q^{-1})}{A(q^{-1})}\xi(k)$$

$$y(k) = E(q^{-1})\xi(k) + \frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(k) + \frac{q^{-d}G(q^{-1})}{A(q^{-1})}\xi(k)$$

$$y(k+d) = E(q^{-1})\xi(k+d) + \frac{B(q^{-1})}{A(q^{-1})}u(k) + \frac{G(q^{-1})}{A(q^{-1})}\xi(k) \qquad (6.2.3)$$

It can be obtained from eq (6.2.1)

$$\xi(k) = \frac{A(q^{-1})}{C(q^{-1})}y(k) - \frac{q^{-d}B(q^{-1})}{C(q^{-1})}u(k)$$

Substituting the above equation into (6.2.3) and then using eqs (6.1.1) and (6.1.2) result in

$$y(k+d) = E(q^{-1})\xi(k+d) + \frac{F(q^{-1})}{C(q^{-1})}u(k) + \frac{G(q^{-1})}{C(q^{-1})}y(k)$$

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Based on the performance cost function

$$J(k) = \mathcal{E}\{\tilde{y}^2(k+d|k)\} = \mathcal{E}\{(y(k+d) - \hat{y}\,(k+d|k))^2\}$$

$$= \mathcal{E}\left\{\left(E(q^{-1})\xi(k+d) + \frac{F(q^{-1})}{C(q^{-1})}u(k) + \frac{G(q^{-1})}{C(q^{-1})}y(k) - \hat{y}\,(k+d|k)\right)^2\right\}$$

$$= \mathcal{E}\left\{\left(E(q^{-1})\xi(k+d)\right)^2\right\} + \mathcal{E}\left\{2E(q^{-1})\xi(k+d)\left(\frac{F(q^{-1})}{C(q^{-1})}u(k) + \frac{G(q^{-1})}{C(q^{-1})}y(k) - \hat{y}\,(k+d|k)\right)\right\}$$

$$+\mathcal{E}\left\{\left(\frac{F(q^{-1})}{C(q^{-1})}u(k) + \frac{G(q^{-1})}{C(q^{-1})}y(k) - \hat{y}\,(k+d|k)\right)^2\right\} \qquad (6.2.4)$$

Due to $E(q^{-1})\xi(k+d)$ is independent of $\{Y^k, U^k\}$, then the second item on the right of eq (6.2.4) is 0.

Because the first item has nothing to do with the control sequence, it is uncontrollable. Therefore, to minimize $J(k)$, the third item on the right of eq (6.2.6) must be 0, namely

$$\hat{y}\,(k+d|k) = \frac{F(q^{-1})}{C(q^{-1})}u(k) + \frac{G(q^{-1})}{C(q^{-1})}y(k) = y^*(k+d|k)$$

So,

$$J_{min} = \mathcal{E}\left\{\left(E(q^{-1})\xi(k+d)\right)^2\right\} = \left(1 + e_1^2 + \cdots + e_{d-1}^2\right)\sigma^2$$

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♦ **Minimum variance control law**

Assuming that $B(q^{-1})$ is a Hurwitz polynomial, that is, the plant is minimum phase or inversely stable, then there is the following theorem.

Theorem 6.2.2 (minimum variance control): Let the control objective be to make the actual output $y(k+d)$ track the expected output $y_r(k+d)$ so that the performance cost function

$$J(k) = \mathcal{E}\{(y(k+d) - y_r(k+d))^2\}$$

is minimized. Then, the minimum variance control law is

$$F(q^{-1})u(k) = C(q^{-1})y_r(k+d) - G(q^{-1})y(k) \qquad (6.2.5)$$

Proof: From Theorem 6.2.1

$$y(k+d) = E(q^{-1})\xi(k+d) + y^*(k+d|k) \qquad (6.2.6)$$

Substituting eq (4.2.6) into the cost function gives

$$J(k) = \mathcal{E}\{(E(q^{-1})\xi(k+d) + \ y^*(k+d|k) - y_r(k+d))^2\}$$
$$= \mathcal{E}\left\{\left(E(q^{-1})\xi(k+d)\right)^2\right\} + \mathcal{E}\{(y^*(k+d|k) - y_r(k+d))^2\}$$

The first item on the right of the above equation is uncontrollable.

If $J(k)$ is the smallest, the second item on the right must be 0, that is

$$y^*(k+d|k) = y_r(k+d)$$

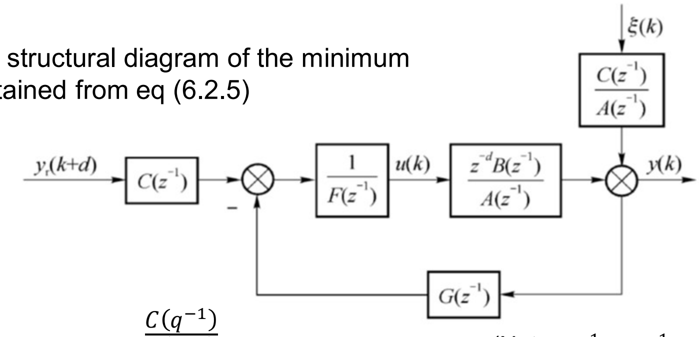
Substitute the above equation into the optimal prediction output equation (6.2.2)

$$C(q^{-1})y_r(k+d) = G(q^{-1})y(k) + F(q^{-1})u(k)$$

Then theorem 6.2.2 is proved.

♦ **Closed-loop system analysis**

For the controlled plant (6.2.1), the structural diagram of the minimum variance control system can be obtained from eq (6.2.5)



It is easy to obtain the closed-loop system equation from the diagram:

(Note: $z^{-1} = q^{-1}$)

$$y(k) = \frac{\frac{C(q^{-1})}{F(q^{-1})}\frac{q^{-d}B(q^{-1})}{A(q^{-1})}}{1+\frac{q^{-d}B(q^{-1})}{A(q^{-1})}\frac{G(q^{-1})}{F(q^{-1})}} y_r(k+d) + \frac{\frac{C(q^{-1})}{A(q^{-1})}}{1+\frac{q^{-d}B(q^{-1})}{A(q^{-1})}\frac{G(q^{-1})}{F(q^{-1})}}\xi(k)$$

$$= \frac{q^{-d}C(q^{-1})B(q^{-1})y_r(k+d) + C(q^{-1})F(q^{-1})\xi(k)}{A(q^{-1})F(q^{-1}) + q^{-d}B(q^{-1})G(q^{-1})}$$

$$= \frac{C(q^{-1})B(q^{-1})\left(y_r(k) + E(q^{-1})\xi(k)\right)}{C(q^{-1})B(q^{-1})} = y_r(k) + E(q^{-1})\xi(k) \qquad (6.2.7)$$

The control input

$$u(k) = \frac{\frac{C(q^{-1})}{F(q^{-1})}}{1+\frac{q^{-d}B(q^{-1})}{A(q^{-1})}\frac{G(q^{-1})}{F(q^{-1})}} y_r(k+d) + \frac{-\frac{C(q^{-1})}{A(q^{-1})}\frac{G(q^{-1})}{F(q^{-1})}}{1+\frac{q^{-d}B(q^{-1})}{A(q^{-1})}\frac{G(q^{-1})}{F(q^{-1})}}\xi(k)$$

$$= \frac{C(q^{-1})A(q^{-1})y_r(k+d) - C(q^{-1})G(q^{-1})\xi(k)}{A(q^{-1})F(q^{-1}) + q^{-d}B(q^{-1})G(q^{-1})}$$

$$= \frac{C(q^{-1})\left(A(q^{-1})y_r(k) - G(q^{-1})\xi(k)\right)}{C(q^{-1})B(q^{-1})} = \frac{A(q^{-1})y_r(k) - G(q^{-1})\xi(k)}{B(q^{-1})} \qquad (6.2.8)$$

As can be seen from the above equation, the essence of minimum variance control is to use the poles of the controller [the zeros of $F(q^{-1})$] to cancel the zeros of the controlled plant [the zeros of $B(q^{-1})$].

Control issues of minimum variance control:

- ✓ According to eq (6.2.7), when $B(q^{-1})$ is unstable, the output $y(k)$ is bounded.
- ✓ But, according to eq (6.2.8), at this time, the input $u(k)$ of the plant will increase exponentially and reach the saturation, eventually leading to system instability.
- ✓ Therefore, when the minimum variance control is adopted, the plant must be the minimum phase system. This is the main defect of minimum variance control.

The minimum variance control has two other defects:

① The minimum variance control is very sensitive to stable zeros near the unit circle;

② The control effect of minimum variance control is not constrained.

When the interference variance is large, the variance of the control input is also large because the minimum variance control needs to complete the correction in one step, which will accelerate the wear of the actuator.

Moreover, some plants do not want or allow the adjustment process to be too drastic.

♦ **Algorithm 6.2.1** (minimum variance control, MVC)

Given the structure and parameters of a controlled plant.

Step 1: Enter the initial data.

Step 2: Solve Diophantine equations (6.1.1) –(6.1.2) and obtain the coefficients of polynomials $E(q^{-1})$, $F(q^{-1})$ and $G(q^{-1})$.

Step 3: Sample the current actual output $y(k)$ and the expected output $y_r(k+d)$ (or $y_r(k)$).

Step 4: Use the controller equation (6.2.5) to calculate and implement $u(k)$.

Step 5: Return to Step 3 (k → k+1) and continue the loop.

♦ **Example 6.2.1:**

Let the controlled plant be

$$y(k) - 1.7y(k-1) + 0.7y(k-2) = u(k-4) + 0.5u(k-5) + \xi(k) + 0.2\,\xi(k-1)$$

where $\xi(k)$ is a white noise with the variance of σ^2.

According to simulation example 6.1.1, the solution of Diophantine equation of the system is

$$F(q^{-1}) = 1 + 2.4q^{-1} + 3.48q^{-2} + 4.236q^{-3} + 1.4855q^{-4}$$

$$G(q^{-1}) = 3.2797 - 2.0797q^{-1} \qquad E(q^{-1}) = 1 + 1.9q^{-1} + 2.53q^{-2} + 2.971q^{-3}$$

Then, the minimum variance control law of the system is obtained from equation (6.2.5)

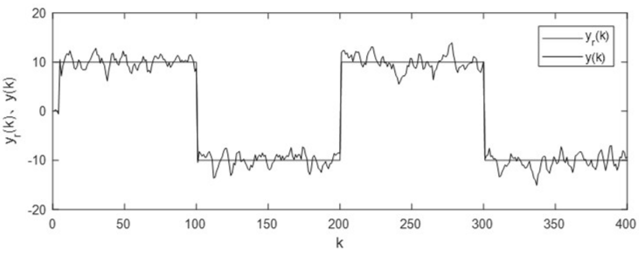
$$\left(1 + 2.4q^{-1} + 3.48q^{-2} + 4.236q^{-3} + 1.4855q^{-4}\right)u(k)$$
$$= (1 + 0.2q^{-1})y_r(k+4) - (3.2797 - 2.0797q^{-1})y(k)$$

So,

$$u(k) = -2.4u(k-1) - 3.48u(k-2) - 4.236u(k-3) - 1.4855u(k-4) +$$
$$y_r(k+4) + 0.2y_r(k+3) - 3.2797y(k) + 2.0797y(k-1)$$

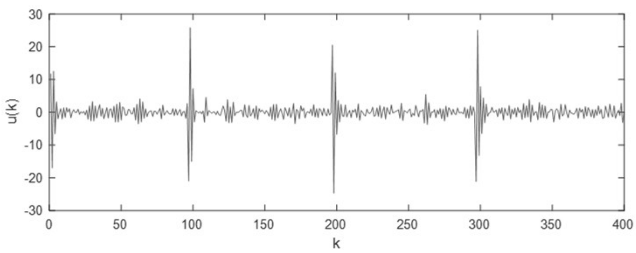
♦ **Results**

Assume that the historical input/output data $u(-4), u(-3), u(-2), u(-1)$, $u(0), y(-1)$ and $y(0)$ are all 0.



Let the expected output $y_r(k)$ be a square wave signal with amplitude of 10 and $\sigma^2 = 0.1$.

The control effect of MVC algorithm is shown on the right.



♦ **Simulation codes**

example621.m

■ Minimum variance indirect self-tuning control

When the parameters of the controlled plant (6.2.1) are unknown, the recursive least squares method can be used to estimate the parameters of the plant in real-time online, and then the minimum variance control law can be designed.

The design of the plant parameter estimator and the controller are separated, and the indirect algorithm of the minimum variance self-correction control is formed.

The algorithm is simple and easy to understand, but the calculation is heavy.

♦ **Algorithm** 6.2.2 (minimum variance indirect self-tuning control)

Given the model order n_a, n_b, n_c and delay d.

Step 1: Set initial values $\hat{\theta}(0)$ and $P(0)$, enter the initial data.

Step 2: Sample the current actual output $y(k)$ and the expected output $y_r(k+d)$.

Step 3: Use the recursive least squares method to estimate the parameters $\hat{\theta}(k)$ of the controlled plant online in real time, namely $\hat{A}(q^{-1})$, $\hat{B}(q^{-1})$ and $\hat{C}(q^{-1})$.

Step 4: Solve Diophantine equation (6.1.1) to obtain the coefficients of polynomials $E(q^{-1})$, $F(q^{-1})$ and $G(q^{-1})$.

Step 5: Use equation (6.2.5) to calculate and implement $u(k)$.

Step 6: Return to Step 2 (k → k+1) and continues the loop.

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♦ **Example:**

Consider the controlled plant

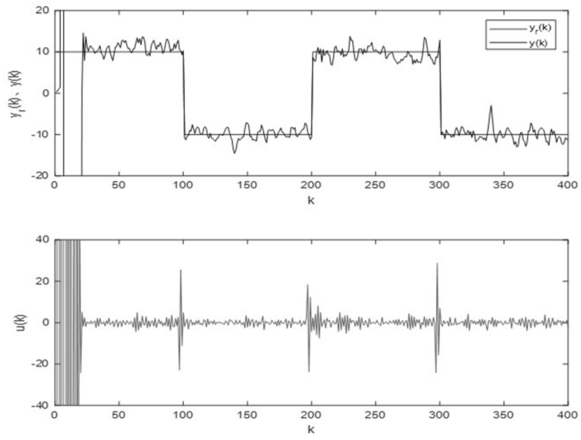
$$y(k) - 1.7y(k-1) + 0.7y(k-2) = u(k-4) + 0.5u(k-5) + \xi(k) + 0.2\,\xi(k-1)$$

where $\xi(k)$ is a white noise with variance of 0.1.

Take the initial values $\hat{\theta}(0) = 0.001$, and $P(0) = 10^6 I$.

The expected output $y_r(k)$ is a square wave signal with amplitude of 10.

The minimum variance self-tuning control indirect algorithm is adopted.



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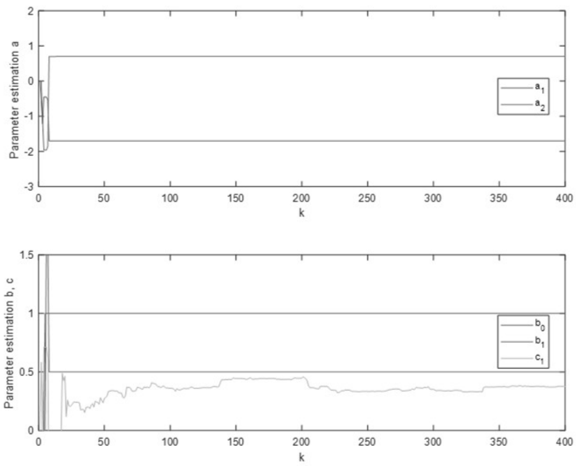
The estimation results of the parameters:



Discussions

You can't take it here $\hat{\theta}(0) = 0$, because at the time of $\mathrm{k} = 1$, $f_0(1) = \hat{b}(1) = 0$ when the control quantity is calculated, $u(1) = \infty$, the system will diverge.

At the initial stage of parameter estimation, the estimated parameters obtained are inaccurate and may deviate far from the true value of the actual parameters, making the polynomial $B(q^{-1})$ unstable and eventually leading to system divergence.

There are two ways to solve the problem:

- ✓ Method 1: If there are a small number of input/output data, a batch processing algorithm can be used for parameter estimation before implementing self-tuning control to obtain the estimated parameter values close to the actual parameter values.

- ✓ Method 2: If there is no ready-made input/output data, the estimated value of parameter $B(q^{-1})$ can be limited to a stable range when implementing self-tuning control.

♦ **Simulation codes**

example622.m

■ Minimum variance direct self-tuning control

When the parameters of the controlled plant (6.2.1) are unknown, the recursive algorithm can also be used to estimate the parameters of the minimum variance controller directly, that is, the direct algorithm of the minimum variance self-tuning control.

In 1973, K.J. Aström and B. Wittenmark proposed this direct algorithm, which has a small amount of computation, but the physical meaning of the estimated parameters is not clear.

The direct algorithm requires direct estimation of controller parameters, so a new estimation model is needed.

According to (6.2.2),

$$y^*(k+d|k) = G(q^{-1})y(k) + F(q^{-1})u(k) + \left(1 - C(q^{-1})\right)y^*(k+d|k) = \varphi^T(k)\theta \tag{6.2.9}$$

where

$$\varphi(k) = \left[y(k), \cdots, y\left(k-n_g\right), u(k), \cdots, u\left(k-n_f\right), y^*(k+d-1|k-1), \cdots, y^*(k+d-n_c|k-n_c)\right]^T$$

$$\theta = \left[g_0 \quad \cdots \quad g_{n_g} \quad f_0 \quad \cdots \quad f_{n_f} \quad c_1 \quad \cdots \quad c_{n_c}\right]^T$$

The estimation model obtained from equation (6.2.9) is

$$y(k+d) = \varphi^T(k)\theta + E\xi(k+d)$$

Let the time be d-step back and rewrite the above as

$$y(k) = \varphi^T(k-d)\theta + E\xi(k) \qquad (6.2.10)$$

where

$$\varphi(k-d) = \left[y(k-d), \cdots, y(k-d-n_g), u(k-d), \cdots, u(k-d-n_f), y^*(k-1|k-d-1), \cdots, y^*(k-n_c|k-d-n_c)\right]^T$$

$$\varepsilon(k) = E\xi(k) = \xi(k) + e_1\xi(k-1) + \cdots + e_{d-1}\xi(k-d+1)$$

Since the plant parameters are unknown, it is also impossible to know the optimal output prediction in the data vector $\varphi(k-d)$.

One solution is to replace $y^*(k|k-d)$ with its estimated value $\hat{y}^*(k)$, that is

$$\hat{y}^*(k) = \hat{\varphi}^T(k-d)\hat{\theta}(k-1)$$

where

$$\hat{\varphi}(k-d) = \left[y(k-d), \cdots, y(k-d-n_g), u(k-d), \cdots, u(k-d-n_f), \hat{y}^*(k-1), \cdots, \hat{y}^*(k-n_c)\right]^T$$

Then, for model (6.2.10), the recursive least squares parameter estimation algorithm is

$$\left.\begin{aligned}
\hat{\theta}(k) &= \hat{\theta}(k-1) + K(k)\left(y(k) - \hat{\varphi}^T(k-d)\hat{\theta}(k-1)\right) \\
K(k) &= \frac{P(k-1)\hat{\varphi}(k-d)}{1 + \hat{\varphi}^T(k-d)P(k-1)\hat{\varphi}(k-d)} \\
P(k) &= \left(I - K(k)\hat{\varphi}^T(k-d)\right)P(k-1)
\end{aligned}\right\} \qquad (6.2.11)$$

The minimum variance control law obtained from equation (6.2.5) is

$$u(k) = \frac{1}{\hat{f}_0}\left(-\sum_{i=1}^{n_f}\hat{f}_i u(k-i) + y_r(k) + \sum_{i=1}^{n_c}\hat{c}_i y_r(k-i) - \sum_{i=0}^{n_g}\hat{g}_i y(k-i)\right) \qquad (6.2.12)$$

- ✓ According to the above equation, if $\hat{f}_0$ tends to zero during the implementation of the control algorithm, the phenomenon of division by zero will occur.
- ✓ Therefore, the minimum value of $\hat{f}_0$ should be constrained, which means that the sign and lower bound of $\hat{f}_0$ should be known in advance, or the value of $\hat{f}_0$ should be determined in advance.

♦ **Algorithm 6.2.3** (minimum variance direct self-tuning control)

Given the model order n_a, n_b, n_c and delay d.

Step 1: Set initial values $\hat{\theta}(0)$ and $P(0)$, enter the initial data.

Step 2: Sample the current actual output $y(k)$ and the expected output $y_r(k+d)$.

Step 3: Construct observation data vector $\hat{\varphi}(k-d)$ using recursive algorithm (6.2.11) to estimate controller parameters $\hat{\theta}$ online in real time, namely, $\hat{G}(q^{-1})$, $\hat{F}(q^{-1})$ and $\hat{C}(q^{-1})$.

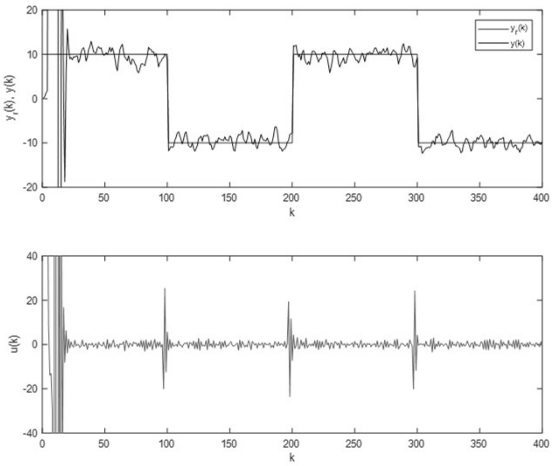
Step 4: Use eq. (6.2.12) to calculate and implement $u(k)$.

Step 5: Return to Step 2 (k → k+1) and continue the loop.

♦ **Example 6.2.3:**

Let the controlled plant be

$$y(k) - 1.7y(k-1) + 0.7y(k-2) = u(k-4) + 0.5u(k-5) + \xi(k) + 0.2\xi(k-1)$$



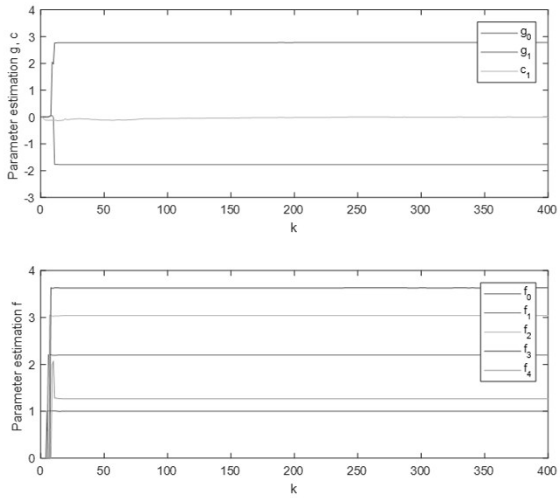
where $\xi(k)$ is a white noise with a variance of 0.1.

Take the initial values $\hat{\theta}(0) = 0$ and $P(0) = 10^6 I$, the lower bound of $\hat{f}_0 = 0.1$.

The expected output $y_r(k)$ is a square wave signal with amplitude of 10.

The minimum variance self-tuning control direct algorithm is adopted.

The estimation results of the parameters:



♦ **Simulation codes**

example623.m

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Exercise 6.1

Consider a time-varying plant

$$y(k) + a_1 y(k-1) + a_0 y(k-2) = b_1 u(k-2) + b_0 u(k-3) + \xi(k) + 0.2\xi(k-1)$$

where $a_1 = -1.7\big(1 + 0.1\sin(0.004\pi k)\big), a_0 = 0.7\big(1 - 0.2\cos(0.005\pi k)\big)$,
$b_1 = \big(1 + 0.1\sin(0.006\pi k)\big),\ b_0 = 0.5\big(1 - 0.05\cos(0.007\pi k)\big)$ and $\xi(k)$ is a white noise with variance of 0.1.

Assume the reference input $y_r(k)$ is a square wave signal with amplitude of 20 and period of 300 steps.

Using the minimum variance self-tuning control scheme (both direct and indirect methods), design self-tuning controllers and make simulations.

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