Stan for the people

Two day introductory workshop
on Bayesian modeling

McGill University
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mc-stan.org

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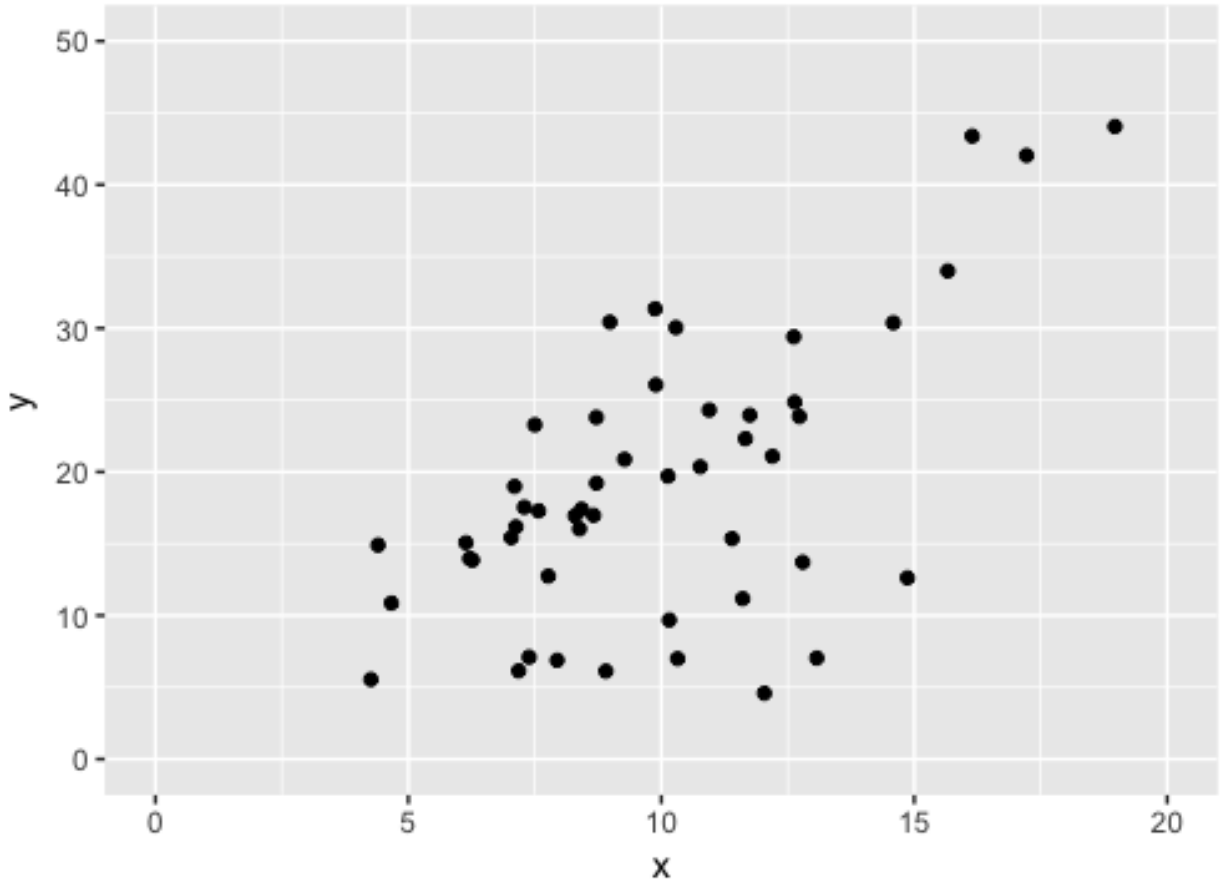
Hierarchical models

Suppose our data can be pooled into groups.

- people's voting intention can be grouped by states, social status, etc.
- medical measurements are grouped by patients
- sport measurements are grouped by players

Three approaches:

- *complete pooling*: treat all the groups as identical.
- *no pooling*: treat the groups as if they had nothing in common.
- *partial pooling:* treat the groups as different, but sharing common features.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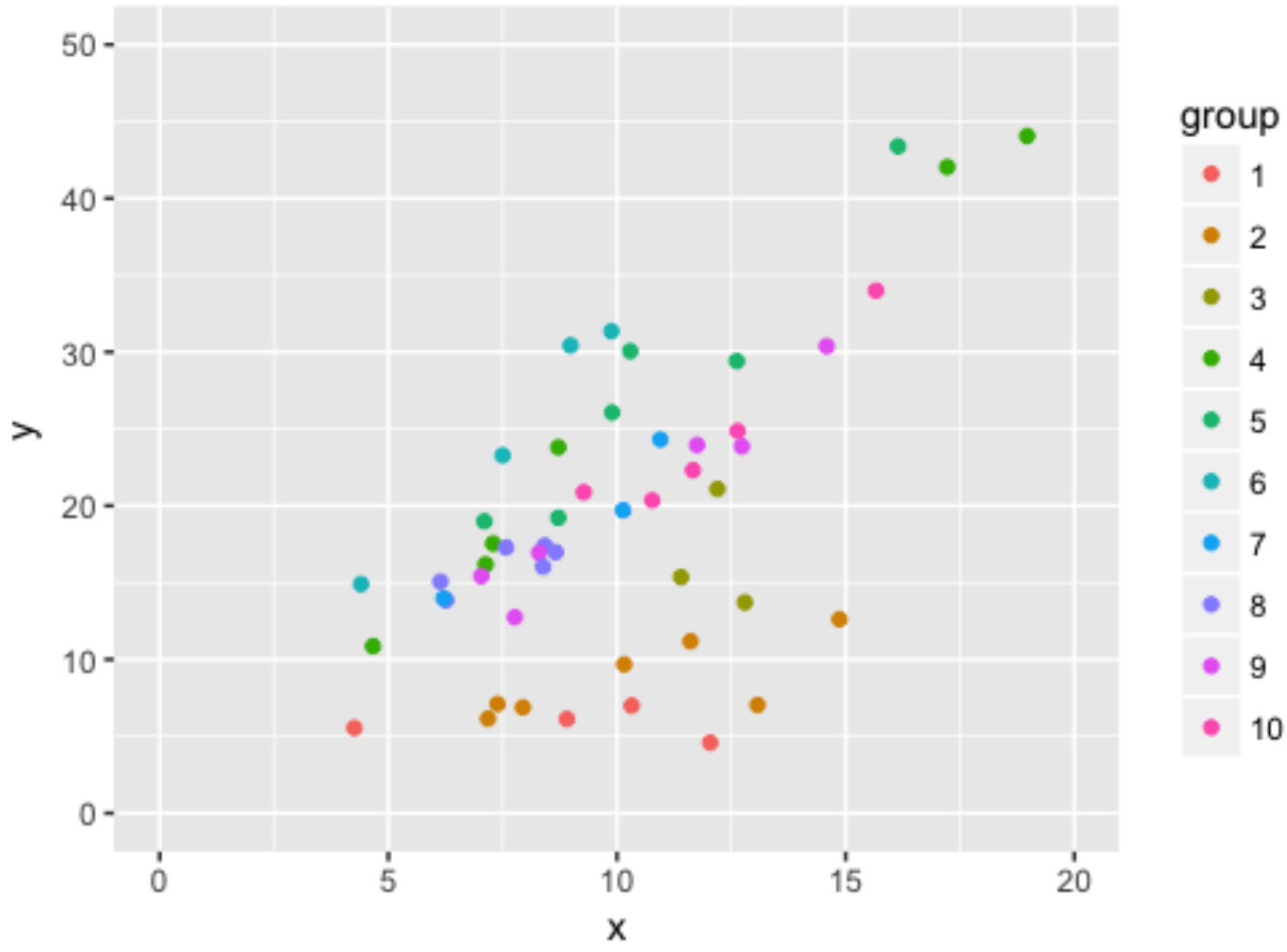
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With a hierarchical model, we can:

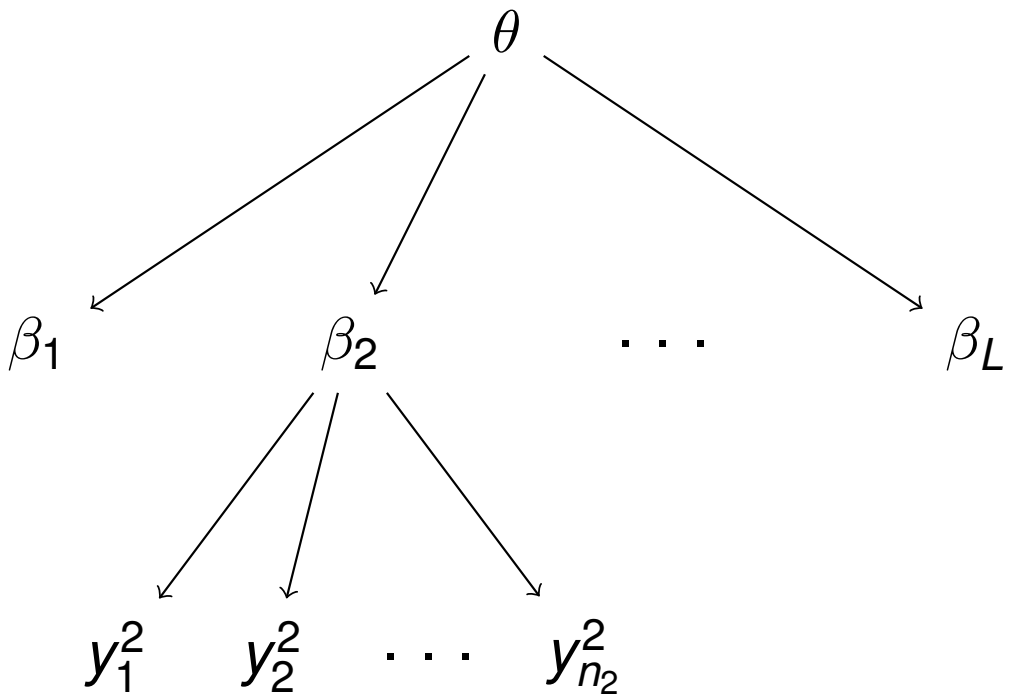
- do partial pooling.
- estimate how similar the groups are to one another.

Hierarchical model

$$\beta = (\beta_1, ..., \beta_L) \sim p(\beta|\theta)$$

$$y = (y_1, ..., y_N) \sim p(y|\beta, x)$$

Hierarchical model



Example 3: Hierarchical linear regression

Likelihood function:

$$\beta_1, \beta_2, ..., \beta_L \sim \text{Normal}(\mu, \tau^2)$$

$$y_i^1 \sim \text{Normal}(x_i \beta_1, \sigma^2)$$
$$y_j^2 \sim \text{Normal}(x_j \beta_1, \sigma^2)$$

...

Example 3: Hierarchical linear regression

Prior functions:

$$\mu \sim \text{Normal}(0.2, 1.0)$$

$$\tau \sim \text{Gamma}(2, 0.1)$$

- What happens when we do not specify a prior on σ?

The default prior in Stan is the uniform distribution over the parameter space.

```
real<lower = 0> sigma;
```

Then $\sigma \sim \text{uniform}(0, \infty)$.

Remark: improper priors are fine, as long as the posterior is proper.

Example 3: hierarchical linear regression model

Exercise: Write and fit a hierarchical linear model.

$$\mu \sim \text{Normal}(0.2, 1.0)$$

$$\tau \sim \text{Gamma}(2, 0.1)$$

$$\beta_i \sim \text{Normal}(0, \tau^2)$$

$$y_j^i \sim \text{Normal}(x_j, \sigma^2)$$

- Use `hierarchical_linear.data.r`.
- Specify an appropriate `init` function.
- Diagnose the your inference results.

Example 3: hierarchical linear regression model

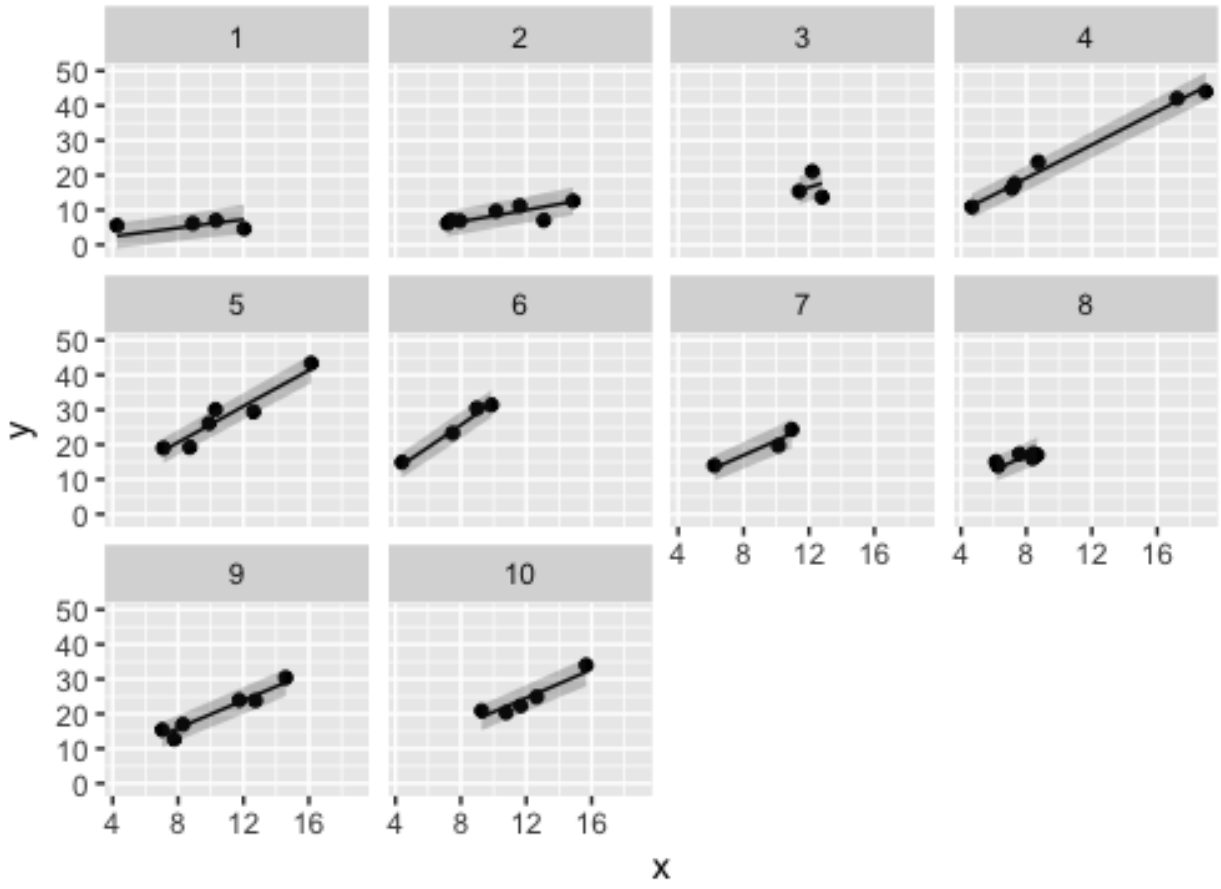
- What posterior predictive checks should we do?

Example 3: hierarchical linear regression model

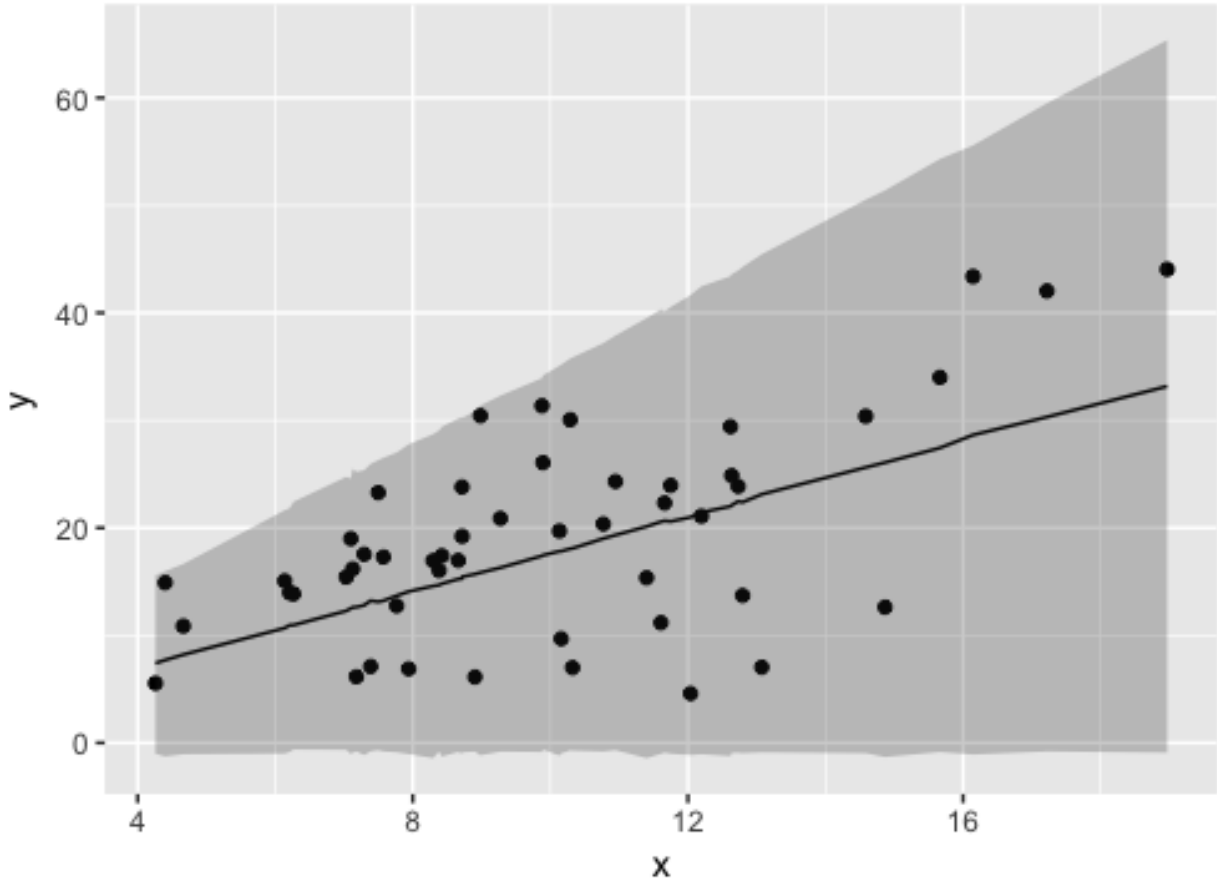
Exercise:

- Generate new data for each group.
- Generate new data for a new group.

Example 3: hierarchical linear regression model



Example 3: hierarchical linear regression model



Example 3: hierarchical linear regression model

- The shaded region is too broad.
- Are we overestimating the variance through τ and σ?
- Do we need more regularizing priors?
- Our model could be misspecified, i.e. we have not constructed the correct data generating process.

- Often, hierarchical models exhibit complex geometric pathologies.
- For an excellent discussion, see [Betancourt and Girolmi, 2015].
- We go over these in our next example.

References I

[Betancourt and Girolmi, 2015] Betancourt, M. and Girolmi, M. (2015).
Hamiltonian monte carlo for hierarchical models.
Current trends in Bayesian methodology with applications, 79.