QUANTUM HACKATHON

ECONOMIC CAPITAL ESTIMATION

Grenoble

01 – 02 Octobre 2022

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CORPORATE & INVESTMENT BANK

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Introduction

Within banks, quantitative risk management issues are generally solved by Monte Carlo simulations. For some years now, a consensus around this technique has emerged for various reasons:

- Complexity of the problems and a strong need for precision in the measures calculated: banks seek to estimate the risk of their portfolio as a function of different parameters (risk horizon, confidence level) and are interested in increasingly accurate measures (e.g. the contribution of each of their transactions to the portfolio)
- High inertia of financing portfolios: It is not necessary to refresh the calculations frequently (a monthly or even quarterly calculation is often sufficient). Time consuming computations can be tolerated without real impact on risk management.
- Conclusive research work on numerical methods and in particular on simulation: banks have acquired considerable experience in the use of numerical simulation techniques, particularly within the capital markets, benefiting from both academic and internal research

For risk-related applications such as dynamic portfolio management (optimisation of risk-return indicators) or the calculation of profitability indicator for credit lending, however, computation time is a major issue, in particular for economic capital.

Definition of Economic Capital

Economic Capital, a key tool of risk management, is computed by banks to determine the amount of risk capital that they require to remain solvent in the face of adverse yet realistic conditions. Banks are exposed to many forms of risk such as credit risk which is the risk of a monetary loss resulting from a counterparty failing to meet a financial obligation. For instance, a payment may not be

made in due time or at all. Risk metrics such as Value at Risk and the Economic Capital Requirement (ECR) are often calculated for many different scenarios. Monte Carlo (MC) simulations are thus the method of choice for this task. In a MC simulation a parameter is estimated by building a distribution obtained by taking M samples from the model input distributions. The error on the resulting

estimation scales as O(M). Evaluating credit risk with MC is a rare-event simulation problem which requires many samples thereby making MC computationally costly.

Modelling Economic Capital

Loss Function

ECR summarizes in a single figure the amount of capital (or own funds) required to remain solvent at a given confidence level (usually linked to the risk appetite or target solvency rating) and a time horizon (usually one year). It is a complementary metric to the regulatory capital requirements that refers to the amount of own funds required following regulatory criteria and rules. For this use case, we consider only the ECR related to default risk, which is the loss that occurs when an obligor does not fulfill the repayment of a loan. The main components of an ECR model for a portfolio of assets are the single asset default probabilities (PD), the loss given default (LGD), and the correlation (r) among the single-asset default events. In the following, we first introduce a general form of the credit risk analysis problem considered in this use case and then define concrete models in detail.

We consider without loss of generality a portfolio of M loans granted to M different clients, with :

D_i default indicator of client i (equals to 1 in case of default, 0 otherwise)

EAD_i exposure at default of the client i

LGD_i loss given default of the loan i, often considered as a beta random variable (mean = E_i, standard deviation = σ_i) independent from the default indicator.

Using these notations, the loss function L of the portfolio can be written as follows :

$$L = \sum_{i=1}^{M} EAD_i . LGD_i . D_i$$

The Merton Model links the default indicator of the client i to the asset return X_i via the probability of default PD_i :

$$P(D_i = 1) = P\left(X_i < N^{-1}[PD_i]\right)$$

$X_i = r_i . Y_i + \sqrt{1 - r_i^2} \, . \varepsilon_i$ where Y_i represents a systematic variable driving the asset return of the client i and ε_i the idiosyncratic variable, both are independent and follow a normal standard distribution. r_i is the correlation between X_i and Y_i.

Y_i depends on a set of macroeconomic factors representing the global economy, countries or industries noted $Z_{k,(k=1,\dots,N)}$ and following a normal standard distribution.

Thus for each client i, $Y_i = \sum_{i=1}^{N} \alpha_{ik} Z_k \ avec \ \sum_{k=1}^{N} \alpha_{ik}^2 = 1$

The Value at Risk (VaR) for a given confidence level $\alpha \in [0,1]$ is defined as the smallest total loss that still has a probability greater than or equal to α, i.e.,

$$VaR_\alpha[L] = \inf_{x \geq 0} \{x | P[L \leq x] > \alpha\}$$

The ECR at confidence level α is thus defined as:

$$ECR_\alpha[L] = VaR_\alpha[L] - \mathrm{E}[L]$$

Some practitioners replace VaR by Expected Shortfall (ES) to calculate the ECR.

$$ES_\alpha[L] = \mathrm{E}\big[L | L > VaR_\alpha[L]\big]$$

Common values of α for ECR are around 99.9%

ECR allocation

For pricing and optimization purposes, one might need to determine the contribution of each loan to the portfolio-level VaR. This contribution is called Tail Risk Contribution (TRC). The loss function of a client or a loan i is written as follows: $L_i = EAD_i LGD_i D_i$

Thus: $TRC_i = \mathrm{E}[L_i | L = VaR_\alpha(L)]$

For ES, $TRC_i = \mathrm{E}[L_i | L > VaR_\alpha(L)]$

Application

Portfolio Description

We will now use a two-factor setup. The loans are grouped into two buckets: A and B. Bucket u (index u can take values 1 or 2) contains M_u identical loans characterized by the total exposure EAD_u, a single probability of default PD_u, expected LGD_u, systematic factor Y_u and factor correlation r_u.

The systematic factors are correlated with correlation ρ. In these notations, correlation inside bucket u is r_u^2, while the asset correlation between the buckets is $\rho r_1 r_2$. We also introduce bucket weight w_u defined as the ratio of the net exposure of all loans in bucket u to the net exposure of all loans in the portfolio.

Step 1: One-Factor Loan Portfolio

We will use a one-factor setup as a starting point with bucket 1:

- $M_1 = 100,$
- $PD_1 = 0.5\%$
- $r_1 = 0.5$
- $LGD_1 = 0.6$
- $EAD_1 = 100\ Billions\ €$

Task 1: Calculate ECR using both VaR and ES at the confidence level 99.9%

Task 2: Compute the tail risk contribution (TRC) for each loan within the portfolio

Step 2: One-Factor Loan Portfolio with CDS Hedges

A Credit Default Swap (CDS) is a financial instrument used by banks to mitigate the potential loss arising from the default of a client within the portfolio. It is a way for banks to transfer the default risk of a specific client which is the reference entity of the CDS. The contract is agreed between the seller and the buyer. The buyer makes regular premium payments to the seller. If the reference entity defaults, the protection seller pays the buyer an amount proportional to the nominal of the contract. This seller's payment represents a gain for the bank. Therefore, the CDS exposure will be recorded as a negative figure.

The loss function[1] of a CDS can be written: $L_i = -EAD_i LGD_i D_i$

As in the previous step, we are required to calculate the risk metrics on a portfolio composed of buckets 1 and 2:

- $M_2 = 20,$
- $PD_2 = 0.7\%$
- $r_2 = 0.5$
- $LGD_2 = 0.4$
- $EAD_2 = -50\ Billions\ €$

[1] Premium payments by the bank are not taken into account

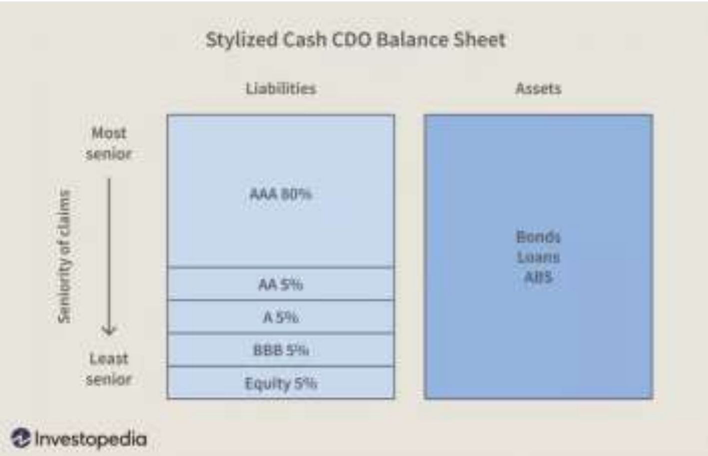
The correlation between buckets is $\rho = 1$

Task 1: Calculate ECR using both VaR and ES at the confidence level 99.9%

Task 2: Compute the tail risk contribution (TRC) for each loan within the portfolio

Step 3: One-Factor Synthetic Securitization

As for credit default swap, synthetic securitizations are also used to transfer risk. It is a financial product backed by a pool of loans and to institutional investors by tranches. A tranche is piece of the pool with a specific risk profile. A tranche is characterized by the attachment point (A) and the detachment point (D).



The loss function of a securitization tranche is:

$$L_{tr} = -min(max(L - A, 0), D - A)$$

$D - A$ is the thickness of the tranche

The loan pool is described by bucket 1.

The objective is this step:

Task 1: Calculate the expected loss of the tranches [0,3%], [3%,6%] and [6%, 100%] backed by the bucket 1

Task 2: Assuming a new portfolio as a combination of bucket 1 and the securitization tranche [3%, 6%]

Task 2.1 Calculate the ECR using VaR and ES of the new portfolio

Task 2.2 Calculate the TRC of each transaction within the portfolio including the securitization tranche

Step 4: Multifactor Portfolio

Now we assume that there are two systematic factors in the model. The entire portfolio is composed of two buckets and the securitization tranche [3%, 6%] backed by bucket 1. The correlation between buckets is $\rho = 0.2$.

Task 1: Calculate ECR using both VaR and ES at the confidence level 99.9%

Task 2: Compute the tail risk contribution (TRC) for each loan within the portfolio

Task 3: What are the conditional losses associated with each systematic factor at the confidence level 99.9%?

Step 5: (Bonus) Real World Portfolio

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