Risk Adjustment in Private Equity Returns

ARTHUR KORTEWEG *

ABSTRACT

This article reviews empirical methods to assess risk and return in private equity. I discuss data and econometric issues for deal-level, fund-level, and publicly traded partnerships data. Risk-adjusted return estimates vary substantially by method, time period, and data source. The weight of evidence suggests that relative to a similarly risky investment in the stock market, the average venture capital (VC) fund earned positive risk-adjusted returns before the turn of the millennium, but net-of-fee returns have been zero or even negative since. Average leveraged buyout (BO) investments have generally earned positive risk-adjusted returns both before and after fees, relative to a levered stock portfolio. I also consider additional risk factors proposed in the literature. VC looks like a small-growth investment, while BO loads mostly on value. Liquidity and idiosyncratic risks are also discussed.

*University of Southern California Marshall School of Business.

A sizable academic literature analyzes the average returns to private equity (PE) investments (see Kaplan and Sensoy, 2015, for a thorough review).[1] Historically, institutional investors' allocations to PE were very low, and investors were effectively risk neutral to such small bets. In recent years, PE investments have grown dramatically, reaching double-digit portfolio weights for many U.S. pension funds and endowments.[2] Consequently, assessing the risks borne by PE investors has become a first-order concern.

The main goal of this review is to organize, compare, and contrast the many empirical methodologies in the growing literature on risk-adjustment of private equity returns. I separate methods developed for portfolio company returns (e.g., start-up companies in the case of venture capital) from techniques for fund-level data, because they face distinct econometric issues. These methods are broadly applicable to all asset classes under the private equity umbrella, but research to date has focused mainly on venture capital (VC) and leveraged buyout (BO). I discuss results for these two assets in detail, and briefly touch on applications to other asset classes where relevant.

Apart from evaluating manager skill, proper risk measurement is also important in the study of managerial style, persistence, the role of PE in portfolio allocations, and agency issues such as pay-for-performance and the risk-taking incentives of contractual features. These are not the main questions of interest for this review, but I will touch upon some of them where appropriate. Many questions are yet to be addressed by the literature, and I conclude with thoughts for future work.

Table I gives an overview of risk and return estimates from the literature. The table is not intended to be comprehensive, and it ignores important subsample results in both the time-series and cross-section, as well as risk estimates with respect to market indices other than the S&P 500 (the Nasdaq and Russell 2000 are popular choices for VC), and factor

[1]For the purpose of this review, the term "private equity" encompasses all types of private equity, including but not limited to leveraged buyout, venture capital, real estate, distressed debt, and natural resources.

[2]For example, in 2016, the 200 largest U.S. defined benefit public (corporate) pension funds invested an average of 9.0% (5.8%) of their assets in venture capital and leveraged buyout, and another 8.3% (5%) in real estate. University endowments allocated 17% of total assets to venture capital and buyout, and another 6% to (non-campus) private equity real estate (source: the 2016 Pensions and Investments annual survey of pension funds and the 2016 NACUBO-Commonfund Study of Endowments).

models other than the CAPM and the Fama and French (1993) three factor models. The review will highlight many of these additional results.

A quick glance at Table I reveals a substantial degree of heterogeneity in risk-adjusted return estimates, depending on the time period, empirical method, and data source used. Performance evaluation in PE is inherently complicated because returns cannot be observed on a regular basis, and payoff distributions are highly skewed, resembling option payoffs. It is therefore not possible to apply standard techniques developed for public equities, mutual, and hedge funds. Moreover, data sources have been incomplete and noisy, but more and better data sets have become available in recent years.

There has not been a consensus as to what should be the main empirical approach towards estimating risk and return in PE, or how to construct relevant benchmark returns. This lack of agreement has likely contributed to the lack of formal quantitative risk adjustment in practice. For example, in a survey of 79 leveraged buyout investors, Gompers, Kaplan, and Mukharlyamov (2016) conclude that managers rely primarily on internal rates of return (IRR) and cash multiples to evaluate investments, and that fund investors focus more on absolute performance rather than risk-adjusted returns. Gompers et al. (2016) find that of 546 institutional VCs, 64% say that they adjust their investments' target metrics for risk, though it is not quite clear how those benchmarks are determined. Recently the literature has begun to gravitate towards stochastic discount factor (SDF) approaches, in particular the public market equivalent (PME) metric and its generalizations. This metric has also started to catch on among investors in PE funds.

The review is organized as follows. Section 1 describes methods for deal-level data, section 2 considers fund-level data, and section 3 discusses publicly traded private equity. Section 4 concludes with thoughts for future research.

[INSERT TABLE 1 AROUND HERE]

1. Deal-level Data

The main advantage to analyzing individual deal data is that returns are computed before investor fees (management fee and a profit share called carried interest) are taken out, affording a direct assessment of a PE firm's skill. In addition, one can fairly cleanly estimate risk and return by industry, stage of investment (e.g., seed, early or late rounds), strategy (e.g., healthcare versus internet technology), or geography, which are often mixed inside PE funds. It is even possible in principle to consider individual managing partners' performance, though this has not yet been explored using a formal risk and return model (but see Ewens and Rhodes-Kropf, 2015).

Panel A of Table I gives an overview of the risk-adjusted returns literature that uses deal-level data. Historically, most papers consider VC only, as start-up company returns have historically been more readily available. VC returns are also easier to construct as start-ups rarely pay dividends and VCs do not charge fees to their portfolio companies, unlike buyout firms (see Phalippou, Rauch, and Umber, 2018), and data on dividends and fees are difficult to collect. However, in recent years researchers have gained access to large data sets of buyout deal returns.

The literature on deal-level data uses two main approaches. The earlier approach is to build an index from the individual deals, and then regress the index returns on a set of risk factors. Later work estimates factor models from the individual company data directly. I will discuss each approach in turn.

A. Index Method

Peng (2001) constructs an index based on valuations of 5,643 start-up companies that were funded by venture capitalists between 1987 and 1999. A key challenge that permeates the literature is that market values are only observed when companies raise money from investors, or when there is an exit event such as an initial public offering or an acquisition by another firm. In the intermediate periods between financing events, which can last several months to several years, no arms-length market prices are observed.

Peng bases his approach on the repeat-sales regression (RSR) approach originally developed in the real estate literature (Bailey, Muth, and Nourse, 1963, and Case and Shiller, 1987, 1989), to fill in the missing values and compute the index. The RSR method is used, for example, to compute the well-known S&P/Case-Shiller home price index. The main assumption is that all firms have the same expected return (and thus the same risk factor loadings) in the same period.

Success bias poses an additional key challenge in private equity deal data. While it is relatively easy to backfill data for startups that successfully went public (from IPO filings), and large acquisitions are more likely to have details disclosed, failures and minor acquisitions (which are often disguised failures) are not well publicized and it is difficult to find the date and value of acquisitions or liquidations. This means that returns for good outcomes are more likely to be known, and it also leaves a substantial number of "zombie" firms by the end of the sample period that appear to be alive in the sense that they have not been confirmed bankrupt or sold but are probably dead since they have not raised a financing round in many years.[3] To deal with the selection bias, Peng first constructs separate repeat sales indices for known successful and unsuccessful investments (making assumptions about the returns to the unsuccessful deals). For the zombie companies with unknown outcomes at the end of the sample period, he assigns each company a probability of success in each period using a nonparametric model, and uses that probability to distribute their value into the successful and unsuccessful indices. Finally, he combines the two indices with weights proportional to the net asset values of each index.[4]

With the final index in hand, it is straightforward to compute returns at regular intervals (typically monthly or quarterly), and regress these returns on a set of risk factors, following standard practice in the asset pricing literature. Using the S&P 500 as the market index, Peng finds a beta of 1.3. The alpha is -0.2% per month and statistically insignificant.[5] The

[3]Apart from firms that were not observed to have gone bankrupt, some of the zombie firms have only recently started operations and have not had enough time to exit (a right censoring problem), and some may be "lifestyle" companies that are still in operation even though they haven't raised a round in years. These firms provide a living for the entrepreneur but are not attracting interest from potential buyers and are too small for a public offering.

[4]The sample selection issue in repeat-sales estimators was first explored in the real estate literature. Haurin and Hendershott (1991) hypothesized its existence, and Wallace and Meese (1997) show empirical evidence. Real estate studies that apply standard selection models to this problem include Jud and Seaks (1994), Gatzlaff and Haurin (1997, 1998), Munneke and Slade (2000, 2001), Hwang and Quigley (2004), and Goetzmann and Peng (2006).

[5]For convenience I will refer to the intercept of factor regressions as alpha, though it is not clear in all cases

alpha and beta both magnify with the frequency of returns: using annual returns, the beta is 2.4 and the alpha is -0.9% per year. Using the Nasdaq as an alternative market index that is more weighted towards smaller technology companies, Peng finds a monthly (annual) beta of 0.8 (4.7), and an alpha of 0.3% per month (-3.8% per year). Unfortunately, there is no clear explanation why the annual alpha is not a properly scaled counterpart to the monthly alpha, or why the beta changes with measurement frequency.

Hwang, Quigley, and Woodward (2005) use an updated version of the data set used in Peng (2001) to construct a VC index between 1987 and 2003. Following Heckman (1979) they treat selection bias as an omitted variables problem and use a two-step estimation to correct the repeat sales index regression. The first stage is an ordered probit that models the probability of observing various outcomes for a given start-up in a given period, most importantly whether or not a valuation is observed. The second stage adds the inverse Mills ratios from the first stage to the repeat sales regression to account for the truncated error distribution that stems from high-value firms being more likely to be observed. Regressing index returns on S&P 500 (Nasdaq) returns yields a beta of 0.6 (0.4) with a quarterly alpha of 0.9% (1.0%), which is not statistically different from zero. Returns are higher in the pre-2000 period, which spans the internet boom of the late 1990s but not the subsequent bust of the early 2000s, with statistically significant alphas of around 3.5% per quarter. However, like other papers that use the index method, standard errors are not adjusted for the fact that the index itself is estimated, overstating statistical significance.

B. Individual Deal Returns

More recent VC papers bypass the index construction step and directly use the returns to the individual start-up companies, computed between financing rounds or from financing round until exit, to estimate risk and return. These returns are irregularly spaced. This is not a problem in itself. If valuations were observed randomly, then one can simply regress these returns on risk factors measured over the corresponding periods, and compute standard errors

that this is a true risk-adjusted return. The intercept is not a true alpha if one or more risk factor(s) is not a traded asset. It also contains the average return to any omitted risk factors.

to account for any cross-correlation in residuals due to the overlap in return horizons. But as discussed above, successful companies are more likely to be observed than failed ones. To tackle this issue, Cochrane (2005) specifies a selection correction that models the probability of a valuation being observed in a given period for a given firm as a function of the firm's value, and estimates the model using maximum likelihood. He assumes the underlying process for firm values follows a CAPM in logs (that is, the natural logarithm of returns follows the CAPM model) with normally distributed residuals. The log-CAPM is convenient because it compounds very nicely over time, as long-horizon returns are simple summations of short-horizon returns. The compounding of the regular (arithmetic return) CAPM to compute multi-period discount rates becomes problematic, especially when factor returns are serially correlated, or with time-varying interest rates or factor loadings (see Constantinides, 1980, Brennan, 1997, and Ang and Liu, 2004).[6] Moreover, arithmetic returns from Cochrane's model follow a lognormal distribution, whose right skew appears to fit the data better, since VC deal returns look somewhat like long call option returns.[7] Since portfolio theory applies to arithmetic returns, Cochrane shows how to adjust the parameters from the log-CAPM model to recover arithmetic return alphas and betas. From 16,638 round-to-exit returns (accounting for dilution due to intermediate financing rounds) with the S&P 500 (Nasdaq) as the market factor, he estimates a beta of 1.9 (1.4) and an annual alpha of 32% (39%). He also finds that both the alpha and beta decline for later-stage financing rounds. In the round-to-round returns data, he estimates a beta of 0.6 and an alpha of 45% per year.

Korteweg and Sorensen (2010) extend Cochrane's approach. They point out that the selection problem does not conform to the standard Heckman model assumption that unobserved outcomes (here, values) are uncorrelated with observed outcomes. Since values accumulate over time, a start-up's unobserved valuations are strongly related to its observed valuations, resulting in a more general dynamic selection problem. They formulate the problem as a state space model, where the underlying state is the value of the start-up. Returns are assumed to follow a factor model in logs, and values are observed according to a generic

[6]One drawback of the log-CAPM is that standard portfolio theory does not apply to log returns.

[7]Note that the repeat-sales regressions in the index method are usually specified on log returns as well. This impacts the index calculation (see Goetzmann, 1992).

selection equation. The time since the last financing round serves as an instrument, as it changes the probability of observing a new round (or an exit), while it should not predict future unexpected returns. Using Bayesian Markov chain Monte Carlo methods to estimate their model, they find a beta of 2.8, and a monthly alpha of 3.3%. Though the beta estimate is higher, the alpha is roughly in the ballpark of Cochrane's estimate. Splitting the sample by time period, they find an alpha of 1.6% before 1993, 5.8% during the internet boom of 1994-2000, and -2.6% from 2000 to 2005. They also estimate the Fama and French (1993) three factor model, and find that start-ups behave like small growth firms, with an SMB loading of 1.1 and an HML beta of -1.6.

The Korteweg and Sorensen model has been applied to other asset classes. Korteweg and Sorensen (2016) use it to estimate real estate indices and distributions of loan-to-value ratios, and Korteweg, Kraussl, and Verwijmeren (2016) estimate a more general version of the model in the art market.

Korteweg and Nagel (2016) estimate risk-adjusted returns from individual start-up data using a stochastic discount factor model, allowing for arbitrary return distributions. They report that a $1 investment in VC generates a net present value of about $0.50-$0.60. With the average time between rounds of roughly one year, this corresponds to about a 50-60% annual return, which is in the ballpark of Cochrane's round-to-round estimates. Because they mainly focus on fund data, their approach is discussed in more detail in section 2 below.

A puzzling result is that the alphas estimated directly from the individual deal data are substantially higher than the alphas from the index methods. It is unlikely that data is the driver of this difference, as Korteweg and Sorensen (2010) use the same data source (though updated and extended) as Peng (2001) and Hwang, Quigley, and Woodward (2005).

The VC deal return literature assumes that VCs own common equity claims, as it has been difficult to collect large samples of contract data. In practice, VCs usually own convertible preferred equity with bells and whistles such as liquidation preferences, participation rights, and board representation, amongst others. The standard argument for ignoring non-common equity features is that results are mainly driven by the large successes rather than the liquidation values of failures, and for reasonably large IPOs, VCs are automatically converted to common equity. However, IPOs have become less common since the turn of the millennium, so an increasingly large share of the payoffs have come from mergers and acquisitions. Moreover, recent studies find that reported (post-money) valuations are biased when non-common equity features are ignored (e.g., Gornall and Strebulaev, 2018, and Ewens, Gorbenko, and Korteweg, 2018), which could affect the calculation of round-to-round returns. How contracts affect returns is still an open question. With more detailed contract data becoming available, this may be a fruitful avenue of future research.

Turning to buyout data, survivorship is somewhat less of a concern compared to VC, though selection bias remains an issue as data is often sourced from one or a few LPs. Kaplan (1989) finds an average market-adjusted return of 42% (median 28%) to 25 public-to-private BO deals from the early 1980s. This is the return to all the buyout capital, debt plus equity, from the initial deal until final exit, which takes 2.7 years on average. The equity investors' return (i.e., the GPs' return) is considerably higher. BO returns have come down since the 1980s as the industry has grown and become more competitive. Groh and Gottschalg (2011) collect data from private placement memoranda (PPM), which GPs provide to potential investors when fundraising. They contain the full history of deals of a given GP, but there may be some bias due to the identity of LPs who are willing to share data, and because more successful GPs are more likely to raise a follow-on fund in the first place. The average (median) IRR to the GP across 133 U.S. buyouts is 50.1% (35.7%) per year. In comparison, the average return on a mimicking investment in the levered market portfolio is 9 to 12.6%, depending on assumptions. Franzoni, Nowak, and Phalippou (2012) collect a large data set of 4,403 individual BO investments. They regress the log modified internal rate of return (MIRR) to the GP of monthly portfolios of BO deals on log factor returns over the same period.[8] The log-CAPM beta is 0.9, and they find a positive loading on value and an insignificant loading on size in the Fama-French 3-factor model (in logs). Annualized alpha is 9.3% in the log-CAPM and 3.1% in the Fama-French model. Axelson, Sorensen, and Stromberg (2014) have data on all 2,075 deals of a single BO fund-of-funds, and find an annual

[8]Modified IRR allows for an exogenously chosen reinvestment rate. I discuss internal rate of return in more detail in the fund data section below.

alpha of 8.6% at a log-CAPM beta of 2.4. They also estimate a CAPM with jumps, motivated by the illiquid nature of buyouts. This does not alter the beta estimate significantly, but raises the alpha to 16.3% per year. Their beta estimate is considerably higher than most other BO papers. They rationalize this number from a Modigliani-Miller calculation (similar to Groh and Gottschalg, 2011), assuming the underlying company is representative of traded equities, has an (unlevered) asset beta of 0.66, and a typical leverage ratio for buyouts. Buchner and Stucke (2014) also find a high BO beta in deal-level data. Acharya et al. (2013) find a sector-adjusted PME of 1.9 for 395 buyout deals sourced from McKinsey (a consulting firm that serves large PE firms) and a large LP, and Braun, Jenkinson, and Stoff (2017) report a median PME of 1.3 for 12,541 buyout deals from three large fund-of-fund managers. The PME method is most commonly applied to fund data, and I discuss it in that context below. A PME above 1 indicates that the investment beat the market index. They find that buyout returns have been consistently high over time, although persistence of individual GPs has declined as the asset class has matured and competition has increased.

2. Fund-level Data

Private equity funds are portfolios of individual deals, typically organized as 10-year limited partnerships. The fund manager (general partner, or GP) makes investment and exit decisions on behalf of investors (limited partners, or LPs). Fund data consists of a series of cash inflows and outflows, and quarterly reported net asset values. Investors are not required to put up their committed capital at the start of the fund. Rather the GP calls capital as investment opportunities arise. As such, cash contributions by LPs are high early in the fund's life, followed by large distributions in later years when portfolio companies are sold or go public.

Contrary to individual portfolio company data, data on fully liquidated funds do not suffer from the success bias, since all investments are accounted for. Using fund data also largely avoids the problem of computing the return to the investor from a complicated contract, because the actual cash flows from the investment's payoff are observed. Risk-adjusted return estimates that are based on fund cash flows should be interpreted from the LPs' perspective, because the cash flows are reported net of fees to the GP.[9] Considering net-of-fee returns is important for answering questions such as LPs' optimal portfolio allocation to PE, but less can be said about managerial skill because rents may be absorbed by GP fees (including profit share, known as carried interest) in the spirit of Berk and Green (2004).[10] The interpretation is not as clearcut for studies that use both cash flows and net asset values (NAVs), as the NAVs do not adjust for GP performance fees (carried interest), so that return measures are a somewhere in between net and gross of fees depending on how heavily NAVs influence the results.

In one of the earliest academic papers on risk and return in PE, Gompers and Lerner (1997) use data from one PE firm, Warburg, Pincus, & Co., between 1972 and 1997.[11] It would appear straightforward to compute a quarterly return series for this GP from its net cash disbursements (distributions minus capital calls) and the change in reported quarterly NAVs of portfolio companies. However, reported NAVs have a number of drawbacks. Fund managers typically do not (aggressively) update portfolios company valuations, often leaving them at cost or, for VC, at the most recent valuation from a financing round.[12] This results in a stale index. Standard factor model regressions will yield downward biased risk estimates, since risk is determined by the covariance of the portfolio's value with the risk factors (see

[9]Net-of-fee beta estimates may differ from gross-of-fee estimates because the performance fee (carried interest) is essentially a call option contract between the GP and LPs.

[10]There is evidence that suggests that in practice, LPs may share in those rents (e.g., Hochberg, Ljungqvist, and Vissing-Jorgensen, 2014, Harris et al., 2014, and Korteweg and Sorensen, 2017, and the sizeable literature on return persistence in private equity fund returns more generally). GP skill is a necessary condition for any persistence in LP performance.

[11]Although Gompers and Lerner (1997) use firm-level rather than fund-level data, the issues are the same as in other fund data papers.

[12]Leaving portfolio companies at cost or at the most recent round valuation used to be common practice. In 2007 the Financial Accounting Standard 157 (FAS 157), now known as Accounting Standards Code (ASC) Topic 820, came into effect, which requires the fair valuation of portfolio companies. However, there is no clear market to which to mark illiquid assets such as buyout or VC portfolio companies (so-called level 3 assets), so GPs, their consultants and auditors must rely on the pricing of recent deals and valuations of comparable firms. This is a subjective exercise and many GPs maintain a conservative policy of marking assets up slowly while being quicker to mark them down if they believe their value has dropped (Anson, 2002, 2007).

Stafford, 2017, for a clear illustration). Other issues with NAV are that some GPs appear to strategically manipulate their reported NAVs, especially underperforming managers with low reputations that are trying to raise a next fund (Jenkinson, Sousa, and Stucke, 2013, Barber and Yasuda, 2017, Brown, Gredil, and Kaplan, 2017). Finally, specifically to VC, to the extent that NAVs are updated using new financing round valuations, the common-equity assumption in the post-money valuation calculation means that these valuations may not reflect their true market values, as discussed above.

Instead of relying on reported NAVs, Gompers and Lerner (1997) value each portfolio company in each quarter by taking a firm's most recently observed value and updating it to the present period using the return to an equal-weighted index of publicly-traded firms in the same three-digit SIC industry. They find a CAPM quarterly alpha of 2% with a beta of 1.4. The alpha for the Fama-French three factor model is similar.

The Gompers and Lerner method is difficult to apply in general, because portfolio company data has not been available in larger data sets. Emery (2003) advocates the use of longer horizon returns to mitigate the stale NAV problem, and shows that the correlation between PE index returns and proxies for the market portfolio increase markedly when using annual rather than quarterly returns. He suggests using lagged market returns in regressions but does not report the regression coefficients. This is essentially the Dimson (1979) correction for infrequent trading, omitting the leading stock market returns, which should be negligible since the public stock market is very frequently traded. Anson (2002, 2007) also suggests the use of lagged market returns. Using index returns from VentureEconomics and including 3 lagged quarters of S&P 500 returns, Anson (2007) reports a VC (BO) alpha of 0.2% (0.8%) per quarter, and a beta of 1.4 (0.7). Woodward (2009) includes 5 lagged quarters and finds a market beta of 2.2 for VC and 1.0 for BO, with quarterly alphas of 0.5% and 1.4% in Cambridge Associate index data.

Boyer et al. (2018) take a different approach and use secondary market transactions instead of NAVs to construct private equity indices. They find a high BO beta of 2.4, similar to Axelson, Sorensen, and Stromberg (2014), and a negative CAPM alpha of 2% per year, which is surprising given that the literature consistently finds positive buyout alphas. For

VC they estimate a low beta of 1.0, and an alpha of -6%.

A different way around the myriad problems with NAVs is to simply avoid their use altogether. For fully liquidated funds, a fund's IRR is computed from cash flows only.[13] Fund IRRs are commonly reported but suffer from a number of drawbacks. Most importantly, there is no adjustment for risk.[14]

Many papers use IRR as a return metric, and typically compare IRRs to the return on the market portfolio over the fund's life (see the review by Kaplan and Sensoy, 2015). This comparison is not as straightforward as it may appear, since IRRs are a money-weighted rate of return. With cash going in and out of a fund at various times during the life of the fund, relating a fund IRR to a simple (time-weighted) market return over the fund's life is not an apples-to-apples comparison, even if PE were equally risky as the market. Recognizing this issue, Ljungqvist and Richardson (2003) compare IRRs to a matched investment strategy that invests in the market portfolio using the drawdown schedule of the fund (or, alternatively, of an average fund). They hold the portfolio until year 10, despite the fact that many exits (and hence cash distributions) occur before the final year, and it is not uncommon for exits to occur later. They also consider an NPV-type measure, discounting capital calls at the risk-free rate and the distributions at the market return (or, alternatively, a cost of capital derived from public firms from the same industries as the fund's portfolio firms). This is closely related to the public market equivalent (PME) measure that is discussed in detail in the next section. Using data on 19 VC and 54 BO funds from a large LP, Ljungqvist and Richardson find that private equity funds (including VC funds) outperform the market by 5% to 8% per year across their various measures.

Kaplan and Schoar (2005) estimate market betas by regressing fund-level IRRs on the

[13]For funds that have not yet been liquidated, it is common practice to include the latest observed NAV as final pseudo-distribution. This muddies the interpretation as NAVs are not adjusted for fees, although the issue is not as prominent as in metrics that use all quarterly NAVs.

[14]Other problems with IRR are that it may be manipulated by GPs through their choice of the size and timing of investments, it may not exist, it may not be unique, the IRR of a portfolio of investments is not a weighted average of the component IRRs, and there is an implicit reinvestment assumption (Modified IRR addresses the latter issue, but there is no guidance on the appropriate reinvestment rate). See also Kocis et al. (2009) for a detailed discussion of IRR and its shortcomings.

realized market return over the first 5 years of the fund, and report a coefficient of 1.2 for VC and 0.4 for BO. The relation between factor models and IRRs, and the econometric properties of such regressions are largely unknown. Axelson, Sorensen, and Stromberg (2014) show that in simulated data, beta estimates from such regressions tend to be downward biased. It may be more accurate to regress fund IRRs on the IRRs of a matched factor-invested portfolios, but the econometrics of such a procedure are also unknown, and this is for future work to explore. Kaplan and Schoar also propose the PME, a return metric that has become very popular. I discuss this measure in the next section.

A. Stochastic Discount Factors

Kaplan and Schoar (2005) introduce the Public Market Equivalent (PME) performance metric, building on Long and Nickels (1996). The PME takes a fund's cash distributions to LPs (and any residual NAV if the fund has not yet been liquidated) and discounts them back to fund inception at the realized public market's rate of return, and divides this by the similarly discounted value of all cash contributions made by LPs to the fund ("capital calls"). If the PME is greater than one then this is interpreted as the fund having outperformed the public stock market. As such, PME is commonly interpreted as assuming that PE has a market beta equal to one. This is not quite accurate, as I will discuss below. Sorensen and Jagannathan (2015) and Korteweg and Nagel (2016) point out that the PME is in fact an application of a stochastic discount factor (SDF) valuation, which states that the time-t price of a cash flow that realizes at time $t+h$ equals the expectation of the cash flow multiplied by a stochastic discount factor, $M_{t,t+h}$,

$$P_t = E_t[M_{t,t+h} \cdot C_{t+h}]. \tag{1}$$

The SDF can roughly be thought of as a state price, the time-t value of a \$1 payoff in each state of nature that may realize at time $t+h$, and is therefore a random variable at time t. The expected PME is thus the present value of fund distributions divided by the present value of the capital calls, using the reciprocal of the market return as the SDF. This is exactly the SDF of an investor with log-utility preferences. Note that the SDF approach takes the expectation of realized cash flows discounted at realized discount factors, as opposed to discounting expected cash flows at expected rates of return, but there is a well-known equivalence between SDF pricing and expected returns from factor models (see, for example, Cochrane's (2005) textbook). Notwithstanding this equivalence, the PME makes no distributional assumptions on the cash flows, unlike most factor models. This is important as PE payoffs are highly skewed, resembling option payoffs, and standard factor models such as the CAPM do not work well for options (unlike log-utility, see Rubinstein, 1976) as betas are highly time-varying. Note also that the endogeneity of cash flows is not important, as their riskiness is properly accounted for by the SDF.

PME estimates vary across periods and data sources. In a sample of 577 VC funds and 169 BO funds from Venture Economics, Kaplan and Schoar (2005) find average PMEs close to 1 for both asset classes. Size-weighted PMEs are higher at 1.21 and 0.93 for VC and BO, respectively, indicating that larger funds tend to perform better. Phalippou and Gottschalg (2009) use a slightly updated version of the same data set, and set final NAVs of funds for which no cash flows are observed for many quarters to zero. They find a lower average PME of 0.92. Stucke (2011) documents that Venture Economics fund data has an updating problem with reported NAVs and cash flows, and researchers have turned to different data sets since. McKenzie and Janeway (2008) have data on 387 VC funds from two large LPs, and estimate an average PME of 2. Higson and Stucke (2012) have a large merged dataset of 1,169 BO funds and find a PME of 1.12. Axelson et al. (2013) find a PME of 1.36, based on 706 BO funds in Preqin. Harris, Jenkinson, and Kaplan (2014) use data from Burgiss, arguably the most comprehensive high-quality fund cash flow data set currently available, and report average PMEs of 1.36 and 1.22 for VC and BO, respectively. BO performance has been consistently high over time (Braun, Jenkinson, and Stoff, 2017, also find this result in deal-level data), but VC performance has been markedly worse after the turn of the millennium, with a PME of 0.91 for post-2000 vintages.

Venturing outside the realm of traditional direct PE fund investments, Harris et al. (2018) consider the performance of PE fund-of-funds. They find average PMEs of 1.16 for VC and 1.14 for BO. This is on par with VC performance of direct fund investments, but below the PME of direct BO funds. Lerner et al. (2018) consider alternative PE investment vehicles that are offered to select LPs. The excess PME of discretionary and GP-directed buyout funds relative to the main fund vehicles of the same GP are 0.51 and 0.91, respectively. Performance for alternative VC vehicles is close to the PME of the GP's main funds. There is heterogeneity, however, and performance depends on GP quality and the relative bargaining position of LPs. Fang, Ivashina, and Lerner (2015) estimate PMEs for LPs that are invited to co-invest with a GP in a specific deal, as well as for solo deals by LPs. Although the PMEs are above 1, only solo transactions appear to outperform on average when compared to the regular PE fund investments, and outperformance is concentrated in BO.

Gredil, Griffiths, and Stucke (2014) propose a measure that effectively annualizes PME, called direct alpha. Annualization matters because there is material variation in the effective duration of funds. Direct alpha forges a closer connection between SDF-type performance measures and the intercept in factor models.

A few papers (e.g., Ljungqvist and Richardson, 2003, Phalippou and Gottschalg, 2009, Harris, Jenkinson, and Kaplan, 2014, and Phalippou, 2014) recognize that the market return may not accurately reflect the riskiness of PE. They employ alternate discount factors such as the post-IPO cost of capital for similar firms, or growth (for VC) or value portfolio returns (for BO). Harris, Jenkinson, and Kaplan (2014), Phalippou (2014) and Robinson and Sensoy (2016) use a levered market return to compute PMEs. The resulting levered PMEs tend to be lower than standard unlevered PMEs. Instead of assuming a leverage number, Driessen, Lin, and Phalippou et al. (2012) estimate the loading on the market return. They also add an alpha term to the discount rate, and estimate the parameters so as to most closely force all fund NPVs equal to zero.[15] They find large negative CAPM alphas of -1.1% for VC and -0.4% for BO, with similar results for the Fama-French 3-factor model. These alphas are more negative compared to other papers that span the same period. This could be due to their use of VentureEconomics, whose fund data is now known to be downward biased (Stucke, 2011).

Korteweg and Nagel (2016) generalize the PME by allowing for more flexibility in the SDF. Specifically, they compute a Generalized PME (GPME) using an exponential-affine

[15]Another way to view the Driessen et al. (2012) approach is that they introduce a factor model into the fund IRR calculation. Buchner and Stucke (2014) propose a similar approach that aims to match the observed distributions, under an additional assumption regarding the dividend process.

SDF, $M_{t+h} = \exp(a - b \cdot \log(R^M_{t,t+h}))$, where $R^M_{t,t+h}$ is the market return from time t to $t+h$. This is the SDF for an agent with constant relative risk aversion equal to b.[16] With the additional assumption of jointly log-normal payoffs, this model is equivalent to the CAPM in log-returns, although this assumption is not necessary. The PME is the special case when $a = 0$ and $b = 1$. Korteweg and Nagel define the (G)PME over a fund's net cash flows (scaled by the total commitment), which has better statistical properties than the ratio of discounted distributions to contributions of prior work. An additional benefit is that the (G)PME of a portfolio is a weighted average of the constituent (G)PMEs, if the same scaling is used.

Contrary to the popular claim that PME assumes a market beta of one, there is no assumption regarding betas, since the riskiness of cash flows is accounted for by the SDF. Rather, the constraints are on the risk-free rate and the equity premium implied by the SDF. For example, the risk-free rate for a log-utility investor equals $1/E[R_M^{-1}]$. This is important, as Table II shows. The table reports summary statistics of the annualized risk-free rate and market return, by decade. For example, in the 1990s – a heavily populated decade in most current private equity fund data sets – the natural logarithm of the risk-free rate and average market return was 4.8% and 17.5%, respectively, with a market volatility of 13.9% per year. Based on these numbers, the PME-implied risk-free rate over the decade was 15.6%, and the excess market return 1.9% (both in logs). It is true that with $\beta = 1$, the PME does recover the correct benchmark return. With higher betas, the PME benchmark can be above or below the log-CAPM benchmark, depending on the relative size of the market's average return and its volatility, which illustrates that the interpretation of PME as assuming a beta of one may lead to incorrect inference.

[INSERT TABLE 2 AROUND HERE]

There are two ways to interpret the (G)PME. The first is the utility interpretation described above. A drawback is that this interpretation requires knowledge of LPs' preferences, and these may vary by investor and may depend on variables that are difficult to measure. The second interpretation, favored by Korteweg and Nagel, is as a pure benchmarking exer-

[16] In this utility interpretation, the leverage factor in the levered PME is in actuality the degree of risk aversion of the agent.

cise. They fit the SDF to price public stocks and bonds, and ask whether that same SDF can also price VC funds. A GPME above zero indicates that there is a component of VC that cannot be replicated by investing in stocks and bonds. They reject the PME restrictions for their sample of VC funds, and find an overall negative GPME of -0.103. This means that relative to investing in public assets, a $1 commitment to VC led to a risk-adjusted loss of $0.103 over a fund's lifetime in present value terms. GPME was positive but insignificant for pre-1998 vintages.

The GPME method can accommodate additional benchmark factors. Korteweg and Nagel (2016) find little role for a small-growth stock portfolio in addition to the market factor. Gredil, Sorensen, and Waller (2018) use SDFs implied by habit formation and long-run risks models, and find a small positive outperformance for VC (though insignificant in some specifications), and insigificant results for BO.

Gupta and Van Nieuwerburgh (2018) include a total of five priced risk factors in their SDF. Together, they pin down the level and slope of the yield curve and the unconditional risk premium on publicly traded stocks, real estate, and infrastructure. Their model allows for richer risk price dynamics by pricing the benchmark assets period by period. Average risk-adjusted returns to VC, BO, real estate, and infrastructure funds are close to zero (higher in the pre-2000 vintages, and lower afterwards).

Korteweg and Nagel (2018) argue that while GPME works well when averaged across many funds, the performance metric is noisy for individual funds. They propose a benchmark return that is consistent with aggregate GPME but reduces the noise in individual fund returns. Giacoletti and Westrup (2018) apply this method to real estate investments.

Ang et al. (2018) also consider the benchmarking question. Using Bayesian estimation, they filter a quarterly time series of realized PE returns. Their main goal is to explore whether PE is spanned by publicly traded assets, rather than estimating expected returns, but they do show risk loadings from regressing their realized PE returns on factor returns. These risk estimates are in line with other studies. They find evidence of a PE-specific factor, such that PE returns cannot be perfectly replicated by simple passive strategies of traded assets.

Stafford (2017) takes a somewhat different approach to constructing a benchmark portfolio that matches the riskiness of PE investments. He shows that a portfolio of publicly traded small firms with low EBITDA multiples can produce returns that are consistent with pre-fee BO index returns. A replicating portfolio of public equities earns a monthly alpha of 0.7% at a CAPM beta of 1.8, which is in the range of pre-fee BO estimates in the literature.

Given the disagreement in the literature, especially on BO, it is clear that more work is still needed to determine whether PE returns can be replicated with publicly traded assets. The weight of evidence at this stage is that there appears to be something special about BO, though its magnitude has shrunk over time, while for VC it may have disappeared altogether (at least for the average fund). There are still many questions to explore, most importantly regarding the set of risk factors in PE. I will discuss some further results along these lines in the next section.

B. Other Risk Factors

The bulk of the literature estimates CAPM or Fama-French 3-factor models.[17] Recent work has started to push beyond these standard models to explore other potential sources of risk. The two main risk factors that have been considered are liquidity and idiosyncratic volatility.

B.1. Liquidity

With LPs committing to PE funds that last 10 years or longer, it would seem obvious that there should be a large liquidity risk premium in PE returns. However, since not all capital is called immediately and many exits occur before the 10 years are up, effective duration is well under 10 years. Moreover, many investors select into PE because they are less prone to liquidity shocks, and PE allocations used to be quite small. This may lower the required illiquidity premium. Franzoni, Nowak, and Phalippou (2012) add the PS liquidity factor (Pastor and Stambaugh, 2003) to the Fama-French 3-factor model, and find a positive and statistically significant loading for their BO data. They compute a liquidity premium of about 3% per year. Ang et al. (2018) also include the PS factor and find a quantitatively similar

[17] Ang et al. (2018) also use the 5-factor Fama-French model

positive and significant loading for BO, but the loading for VC is insignificant. Buchner (2016) also finds a negligible effect of including the PS factor in VC.

Notwithstanding some supportive results, the PS factor was developed for public equities and is geared toward liquidity risk stemming from order flow fluctuations. This is relevant for public equity markets, but PE investors are more concerned with illiquidity due to search and information frictions from locating a counterparty in an over-the-counter market. These liquidity costs are substantial: Nadauld et al. (2017) find that the LPs who sell their PE fund stakes in the secondary market accept an average discount to NAV of 13.8%, though this fluctuates with fund age and market conditions. For the most common transactions (which occur outside of the financial crisis) the discount was 9%. The buyers realize a higher return than the sellers, about 5% per year in both IRR and PME terms.

Sorensen, Wang, and Yang (2014) specify a contingent-claims model of PE valuations. If PE is spanned by traded assets, there is no illiquidity cost because investors can hedge their PE exposure. Their model quantifies the cost of illiquidity due to unspanned risk. Calibrating the model to a typical PE fund, they find that the cost of illiquidity is high, on the same order of magnitude as GP fees (management fee plus carried interest). Bollen and Sensoy (2016) build a valuation model that explicitly allows for a secondary market (as well as stochastic capital flows). Calibrations suggest that a moderately risk-averse LP should be indifferent between a 10 to 20% portfolio allocation to PE and an 80-20 portfolio of public stocks and Treasuries.

An interesting, as of yet unanswered question is whether the liquidity premium has changed over time. On one hand, the financial crisis may have heightened awareness of liquidity concerns, given that some LPs defaulted on their commitments or sold out at large discounts. On the other hand, the growth of a (currently still small) intermediated secondary market has increased overall liquidity. Having a more model-free liquidity factor that is pertinent to PE would help to answer these and other important questions.

B.2. Idiosyncratic volatility

Ang et al. (2006) uncover evidence that idiosyncratic risk is negatively priced in the public market (that is, stocks with high exposure to idiosyncratic risk shocks have low average returns). In PE, idiosyncratic volatility may instead be positively priced due to the underdiversification of GPs, who demand compensation by pricing their exposure into contracts when negotiating with entrepreneurs. Ewens, Jones, and Rhodes-Kropf (2013) find supporting evidence for this hypothesis in VC and BO returns. Peters (2018) documents a positive loading on aggregate idiosyncratic risk shocks in VC, and attributes this to the option-like nature of VC payoffs. Gompers et al. (2016), in their survey of VCs, find that 42% of VCs treat systematic and idiosyncratic risk the same, 5% discount systematic risk more, and 14% discount idiosyncratic risk more.

Entrepreneurs are also underdiversified, even more so than GPs. They do not appear to be compensated for idiosyncratic risk, as average returns to entrepreneurs are quite poor (Moskowitz and Vissing-Jorgensen, 2002, and Hall and Woodward, 2010). Whether this is due to a preference for skewness, overoptimism, overconfidence, or some other channel, is as of yet an open question.

B.3. Summary

PE research has traditionally confined itself to the CAPM and Fama-French 3 factor models, but a recent movement has started to consider other risk factors that may play a role in PE. Some of these risks are also present to some degree in public markets (e.g., liquidity, idiosyncratic volatility), but the nature of many of these risks in PE tends to be different from publicly traded assets. Term structure features must be important for PE returns measured over long horizons, and though there has been some exploration along these lines (Gupta et al., 2018), more remains to be done. Finally, there is some (mixed) evidence suggesting a PE-specific return component that is not spanned by public assets (e.g., Korteweg and Sorensen, 2010, and Ang et al., 2018), the nature of which has not yet been explored in the literature.

3. Publicly Traded Partnerships

In order to avoid the problems with fund and deal-level data, researchers have turned to publicly traded PE partnership data. Jegadeesh, Kraussl, and Pollet (2015) collect data on 24 traded funds-of-funds and 129 traded PE partnerships. They find statistically insignificant VC and BO alphas for both the CAPM and a 4-factor (Fama-French 3 factors plus momentum) models.[18] These results are quite different from Harris et al. (2018), who find that fund-of-funds on average outperform the market, and the weight of evidence that the average BO fund outperforms. McCourt (2018) finds results that are more in line with the unlisted fund data: based on 134 listed PE funds, he finds positive excess returns for BO funds, and insignificant performance for VC, using the same 4-factor model as Jegadeesh et al. (2015). Using traditional tools from the mutual fund literature to separate skill and luck, he also finds evidence of skill in BO, but not in VC (unlike Hochberg, Ljungqvist, and Vissing-Jorgensen, 2014, and Korteweg and Sorensen, 2017, who find skill in both). One reason for the difference in results between the Jegadeesh et al. and the McCourt papers may be that the latter includes the period following the financial crisis.

The publicly traded partnerships are a useful alternate lens through which to view PE risk-adjusted returns. However, it is not clear how the results should be compared to the literature on unlisted funds. Because it is difficult to get a good sense of true NAVs (due to the issues discussed above), and payout policies tend to be highly smoothed, the market returns of listed funds may not capture all the underlying variation in PE returns in a timely manner, which could explain why estimated betas are at the lower end of the spectrum. Though this may all work out in the long run when funds liquidate, the traded funds data are only on the order of 20 years (and unlisted funds data not much longer). The second issue is that the sample of traded funds is small and possibly selected, although both Jegadeesh et al. and McCourt argue that they are representative. Third, the fact that the funds are publicly traded may change GPs' incentives compared to the same fund if it were unlisted, complicating direct comparisons. Finally, to the extent that results are based on publicly

[18] Jegadeesh et al. (2015) also exploit the divergence between market prices and NAVs to obtain an alternate estimate of manager skill.

traded firms such as Blackstone or KKR, these more closely reflect the GP's share of returns rather than the LPs' share, and they tend to reflect of mix of investments that is not limited to only PE.

4. Conclusion

Many methods have been proposed to evaluate risk and return in PE. The recent literature appears to be converging towards the use of stochastic discount factors and benchmarking approaches, but much work remains to be done. A key open question is to identify the set of risk factors in PE. Are PE returns spanned by publicly traded assets or is there a component to returns that cannot be captured otherwise? If the latter, is that component a risk premium unique to PE, or is it pure alpha? How much cross-sectional and time-series variation (both in calendar time and over the life of a fund or portfolio company) is there in factor loadings and, ultimately, in risk-adjusted returns? Can we reconcile pre-fee and post-fee returns, how does the fee structure affect risk taking, and what does that imply for GP compensation?

With more insight on the risk and return question, many applications remain to be explored in more depth: What is the degree of persistence in GP and LP risk-adjusted returns (and why is there persistence in LP returns in the first place?), do managers have styles, how do agency problems or measurement issues from contractual arrangements (between GPs and LPs, and between GPs and portfolio companies) affect returns, and how does PE fit into a broader portfolio of assets?

Finally, the methods developed for PE can be applied to other assets that are infrequently traded (e.g., real estate, distressed debt), as they are plagued by similar empirical problems.

With the increasing availability of PE data sets of higher quantity and quality, and with recent developments in methodology, this looks to be a promising area for future research.

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Table I.
Overview of Papers

This table shows the main results for papers discussed in the review. It is not intended to be comprehensive. Papers are shown in chronological order by asset class (venture capital or buyout). Panels A and B show results for deal-level data, and Panels C and D show results for fund-level data.The sample period column contains the period over which deal-to-deal returns are observed (Panels A and B), or fund vintages (Panels C and D). The column labeled "alpha" shows the arithmetic, equal-weighted risk-adjusted return for U.S. data, in percentages (that is, 5.0 means 5%), to the extent these numbers are reported. The superscript shows the data frequency (m for monthly, q for quarterly, a for annual, and x for some other frequency as described in the "method and notes" column). The β_M column reports the market beta estimates, using the S&P 500 as market index if multiple indices are used. If the Fama-French 3-factor model was estimated, the next two columns show the loadings on the size (SMB) and value (HML) factors. For papers that estimate both the CAPM and Fama-French 3-factor models, both are shown on consecutive lines. For stochastic discount factor (SDF) based methods in Panels B and D, risk loadings are usually not estimated, and the "alpha" column shows the risk-adjusted return metric described in the "Method and notes" column. N/R stands "for not reported". Data source abbreviations are: B for Burgiss, C for CEPRES; CA for Cambridge Associates; G for stock delistings and Grimm's Mergerstat Review; LP# means sourced from # of LPs; MK for McKinsey; LPS for LP Source; MKT for publicly listed market data; P for Preqin; PPM for private placement memoranda; R for Thomson Reuters M&A; S for secondary transactions intermediary; SHE for Sand Hill Econometrics; SS for State Street; VE for Venture Economics; VO for VentureOne (now VentureSource); WP for Warburg Pincus. See the main text for detailed descriptions of papers and methods used.

Paper	Sample period	α (%)	β_M	β_{SMB}	β_{HML}	Data source	Method and notes
Panel A: Portfolio Company Returns (gross of fees)							
Venture Capital:							
Peng (2001)	'87-'99	-0.2^m	1.3			VO	Index returns; Annual β_M is 2.4
Cochrane (2005)	'87-'00	32^a	1.9			VO	Selection model
Hwang, Quigley, Woodward (2005)	'87-'03	0.9^q	0.6			SHE	Index returns; Heckman
Korteweg and Sorensen (2010)	'87-'05	3.3^m	2.8			SHE	Selection model
	'87-'05	3.5^m	2.3	1.1	-1.6		
Buchner and Stucke (2014)	'80-'01	15.1^a	2.5			C	Matching fund distributions
Peters (2018)	N/R	0.8^m	1.6	0.0	-0.7	SHE	Index returns
Buyout:							
Kaplan (1989)	'80-'85	41.9^x	-			G	Market-adjusted return to all BO capital over the deal (average 2.7 years)
Groh and Gottschalg (2011)	'84-'04	N/R	1.4			PPM	Mimicking portfolios
Franzoni, Nowak, Phalippou (2012)	'75-'06	9.3^a	0.9			C	Log-MIRR regressions
	'75-'06	3.1^a	1.4	-0.1	0.7		
Axelson, Sorensen, Stromberg (2014)	'94-'07	8.6^a	2.4			LP1	Log-CAPM w/ heteroscedasticity
Buchner and Stucke (2014)	'80-'01	4.8^a	3.2			C	Matching fund distributions
Stafford (2017)	'86-'16	0.7^m	1.8			R	Mimicking portfolios

Table I - Continued

Paper	Sample period	α (%)	β_M	β_{SMB}	β_{HML}	Data source	Method and notes
Panel B: Portfolio Company Returns (gross of fees) using Stochastic Discount Factors							
Buyout:							
Acharya et al. (2018)	'91-'08	1.9				MK + LP1	PME using sector return as discount rate
Braun, Jenkinson, Stoff (2017)	'74-'13	1.3				LP3	Median PME
Panel C: Fund-level Returns (net of fees)							
Venture Capital:							
Gompers and Lerner (1997)	'72-'97	2.0^q	1.4			WP	Index method
	'72-'97	1.7^q	1.3	0.8	0.1		
Anson (2007)	'85-'05	0.2^q	1.4			VE	Dimson betas (3 lags)
Woodward (2009)	'96-'08	0.5^q	2.2			CA	Dimson betas (5 lags)
Driessen, Lin, Phalippou (2011)	'80-'93	-1.1^m	2.7			VE	NPV method
	'80-'93	-0.7^m	2.4	0.9	-0.2		
Ewens, Jones, Rhodes-Kropf (2013)	'80-'07	-0.2^q	1.2			VE+P+LPS	Dimson betas (4 lags)
	'80-'07	0.5^q	1.1	-0.1	-0.9		
Buchner and Stucke (2014)	'80-'01	13.2^a	2.7			B	Matching fund distributions
Jegadeesh, Kraussl, Pollet (2015)	'97-'08	-0.3^m	1.0			MKT	Fund-of-funds
	'97-'08	0.0^m	1.2			MKT	Partnerships
Ang et al. (2017)	'94-'08	0^a	1.8			P	Bayesian filtering of realized PE returns
	'94-'08	1^a	1.7	0.8	-0.6	P	
Boyer et al. (2018)	'06-'17	-6^a	1.0			S	Secondary transactions-based index
Peters (2018)	N/R	0.6^q	1.4			CA	Index returns, Dimson betas (5 lags)
McCourt (2018)	'95-'15	0.1^m	1.4	1.1	-1.4	MKT	Partnerships; incl. momentum factor
Buyout:							
Anson (2007)	'85-'05	0.8^q	0.7			VE	Dimson betas (3 lags)
Woodward (2009)	'96-'08	1.4^q	1.0			CA	Dimson betas (5 lags)
Driessen, Lin, Phalippou (2011)	'80-'93	-0.4^m	1.3			VE	NPV method
	'80-'93	-1.0^m	1.7	-0.9	1.4		
Ewens, Jones, Rhodes-Kropf (2013)	'80-'07	1.2^q	0.7			VE+P+LPS	Dimson betas (4 lags)
	'80-'07	0.9^q	0.8	0.1	0.2		
Buchner and Stucke (2014)	'80-'01	4.5^a	2.7			B	Matching fund distributions
Jegadeesh, Kraussl, Pollet (2015)	'97-'08	0.2^m	0.7			MKT	Fund-of-funds
	'97-'08	-0.0^m	0.9			MKT	Partnerships
Ang et al. (2017)	'94-'08	4^a	1.3			P	Bayesian filtering of realized PE returns
	'94-'08	1^a	1.2	0.5	0.6	P	
Boyer et al. (2018)	'06-'17	-2^a	2.4			S	Secondary transactions-based index
McCourt (2018)	'95-'15	0.8^m	1.1	0.6	0.3	MKT	Partnerships; incl. momentum factor
Mixed VC + Buyout:							
Ljunqvist and Richardson (2003)	'81-'93	7.2^a	-			LP1	IRR minus matched market investment

Table I - Continued

Paper	Sample period	α (%)	β_M	β_{SMB}	β_{HML}	Data source	Method and notes
Panel D: Fund-level Returns (net of fees) using Stochastic Discount Factors							
Venture Capital:							
Kaplan and Schoar (2005)	'80-'94	0.96				VE	PME
McKenzie and Janeway (2008)	'80-'02	2.0				LP2	PME
Harris, Jenksinson, Kaplan (2014)	'84-'08	1.36				B	PME
Fang, Ivashina, Lerner (2015)	'91-'10	1.01				LP7	Direct investments PME
Robinson and Sensoy (2016)	'84-'09	1.01				LP1	2x levered PME
Korteweg and Nagel (2016)	'79-'08	-0.10				P	Generalized PME (GPME)
Gredil, Sorensen, Waller (2018)	'79-'08	0.08				B	excess NPV, long-run risk model
	'79-'08	0.11				B	excess NPV, habit model
Gupta and van Nieuwerburgh (2018)	'90-'10	-0.01				P	GPME extension, SDF: 5 priced factors
Harris et al. (2018)	'87-'07	1.16				B	Fund-of-funds PME
Lerner et al. (2018)	'87-'17	0.06				SS	Discretionary funds, excess PME
	'91-'17	-0.06				SS	GP-directed funds, Excess PME
Buyout:							
Kaplan and Schoar (2005)	'80-'94	0.97				VE	PME
Higson and Stucke (2012)	'80-'08	1.12				CA+CalPERS	PME
Axelson et al. (2013)	'87-'09	1.36				P	PME
Phalippou (2014)	'93-'10	0.97				P	1.3x levered PME
Harris, Jenksinson, Kaplan (2014)	'84-'08	1.22				B	PME
Fang, Ivashina, Lerner (2015)	'91-'10	1.34				LP7	Direct investments PME
Robinson and Sensoy (2016)	'84-'09	1.15				LP1	1.3x levered PME
Gredil, Sorensen, Waller (2018)	'85-'08	0.02				B	Excess NPV, long-run risk model
	'85-'08	0.05				B	Excess NPV, habit model
Harris et al. (2018)	'87-'07	1.14				B	Fund-of-funds PME
Lerner et al. (2018)	'87-'17	0.51				SS	Discretionary funds, excess PME
	'91-'17	0.91				SS	GP-directed funds, excess PME
Gupta and van Nieuwerburgh (2018)	'90-'10	0.01				P	GPME extension, SDF: 4 priced factors
Mixed VC + Buyout:							
Phalippou and Gottschalg (2009)	'80-'93	0.92				VE	Value-weighted PME

Table II.
Market and SDF-implied Returns

Panel A shows the natural logarithm of the average risk-free rate and the average market return, and the standard deviation of log stock market returns by decade, computed from monthly data downloaded from Kenneth French's data library. Panel B shows the log risk-free rate and market premium implied by the stochastic discount factor (SDF) underlying the public market equivalent (PME) metric. Panel C shows the benchmark rates of return (in logs) for various market risk levels (β). All returns are annualized and in percentages. See Korteweg and Nagel (2016) for more details.

	1960 -1969	1970 -1979	1980 -1989	1990 -1999	2000 -2009	2010 -2017
Panel A: Market return statistics						
Risk-free rate	3.8	6.1	8.5	4.8	2.7	0.2
Market return	8.7	7.3	16.9	17.5	1.0	14.0
Market volatility	12.6	16.9	17.0	13.9	16.9	12.3
Panel B: PME-implied market returns						
Risk-free rate	7.1	4.5	14.1	15.6	-1.9	12.5
Market premium	1.6	2.9	2.9	1.9	2.9	1.5
Panel C: Benchmark returns						
β = 1:						
PME	8.7	7.3	16.9	17.5	1.0	14.0
log-CAPM	8.7	7.3	16.9	17.5	1.0	14.0
β = 2:						
PME	10.3	10.2	19.9	19.4	3.8	15.5
log-CAPM	13.6	8.5	25.4	30.1	-0.8	27.8
β = 3:						
PME	11.9	13.0	22.8	21.3	6.7	17.0
log-CAPM	18.5	9.7	33.8	42.8	-2.6	41.6