Risk Adjustment of Private Equity Cash Flows

Preliminary

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Abstract

Existing stochastic discount factor methods for the valuation of private equity funds result in unrealistic time discounting. We propose and evaluate a modified method. Valuation has a risk-neutral component plus a risk adjustment, and we fix the risk-neutral component by constraining the subjective term structure of interest rates with market data. We show that (i) our approach allows for economically meaningful measurement and comparison of risk across models, (ii) existing methods estimate implausible performance when time discounting is particularly degenerate, and (iii) our approach results in lower variation of performance across funds.

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Asset allocation to buyout, venture capital, and other private equity (PE) funds has increased consistently over the past decade. It remains challenging, however, to estimate risk and performance of these funds, especially due to their illiquid secondary market and the consequential absence of reliable return data. PE returns can be extrapolated from cash flow data as in Ang, Chen, Goetzmann, and Phalippou (2018), but it requires restrictive assumptions on the return-generating process. To avoid those assumptions, Korteweg and Nagel (2016, KN) develops a stochastic discount factor (SDF) valuation framework that uses cash flows instead of returns, and that benchmarks PE against publicly traded assets.

Central to the SDF framework is a requirement for proper benchmarking: the SDF must price benchmark assets during the sample period. To satisfy this requirement, KN use a heuristic implementation. They build artificial funds invested in the benchmark assets, and they estimate SDF parameters pricing the artificial funds.

In this paper, we propose an alternative implementation which estimates a set of SDF parameters so that the subjective term structure of interest rates is determined by market data. Theoretically, our approach is based on a decomposition of PE performance in a risk-neutral part and a risk adjustment. By construction, the risk-neutral part does not vary as we add or remove risk factors from the SDF, so we can meaningfully measure the economic cost of PE risk and compare it across SDFs. Empirically, we evaluate our approach against the KN implementation, and we find that the KN method results in unrealistic time discounting which can generate implausible performance estimates. For example, a zero-coupon bond paying \$1 at 3 years maturity can have discounted value up to \$9, and a zero-coupon bond paying \$1 at 10 years maturity can have discounted value up to \$7. Our approach avoids this problem. As a result, we obtain more robust estimation of performance across SDFs and lower variation of performance across funds.

We use our method to risk-adjust PE cash flows for two types of investor: a CAPM investor, and a long-term investor who distinguishes between permanent and transitory wealth shocks. We discount net-of-fees cash flows of 1,866 PE funds started in the U.S. between 1978 and 2009, and divided in three categories: buyout, venture capital, and generalist.[1] As benchmark assets, we use the S&P 500 total return index and quarterly T-bills. For the CAPM investor, buyout has generated 30 cents of NPV per dollar of commitment, as opposed to

[1]PE data is maintained by Burgiss, and it is one of the most comprehensive PE dataset available to date.

the 7 cents of venture capital and 21 cents of generalist. Unsurprisingly, venture capital has the highest (absolute value of) risk adjustment, equal to 65 cents per dollar of commitment and about twice as large compared to 31 cents of buyout and 35 cents of generalist. We find only modest differences between the CAPM and the long-term investor. Relative to CAPM, the long-term investor assigns 3 to 8 cents lower risk adjustment across the three categories, resulting in 3 to 8 cents higher NPV.

Comparing our method with the KN method, we find large differences in the buyout category. With CAPM, the risk-adjusted performance of buyout is similar across the two methods, but performance components differ substantially. The KN method results in larger risk-neutral value which is then compensated by higher risk adjustment. Further, the standard deviation of performance across funds is 142 cents using KN, while it is 98 cents with our method. For the long-term investor, the KN method estimates very high buyout performance, up to 80 cents of NPV per dollar of commitment, in contrast to 35 cents with our method. That very high NPV, however, is not driven by lower risk adjustment; instead, it is driven by a large increase in the risk-neutral value, which goes from 80 cents with CAPM up to 250 cents with the long-term model. Standard deviation of performance in the long-term model goes up to 925 cents with the KN method, while it remains stable at 105 cents with our method.

We find differences between the two methods also in the venture capital and generalist categories. Relative to the KN method, our method assigns 20 to 30 cents higher NPV to venture capital and 10 to 20 cents higher NPV to generalist, depending on investor's type. For these categories, performance estimates using the KN method are not as implausible as for buyout, but our method consistently results in more realistic time discounting and lower variation of performance across funds.

With our implementation, we further decompose risk adjustment based on the timing of cash flows during a fund's life. For all three fund categories, cash flows have marginally negative risk exposure in the first 3 years of operations, indicating weak pro-cyclicality of contributions. After the first 3 years, risk exposure of cash flows becomes positive, and we find differences in the timing of risk across the three categories. Much of the risk adjustment is due to cash flows from year 9 to 11 for buyout and year 4 to 7 for venture capital. For generalist funds, risk adjustment is spread more homogeneously between year 4 and 10.

A potential concern remaining in our approach is that our performance decomposition does not provide clear guidance on how to estimate risk prices for proper benchmarking. In practice, we restrict the SDF to price S&P 500 returns in the sample period at 10-year horizon. This condition is heuristic, however, based on the typical horizon of PE funds. To address this concern, we study the robustness of our results by changing the price of risk exogenously. We find only weak effects on risk adjustment and NPVs of buyout and generalist funds. Their NPV remains positive over a wide range of risk prices. Venture capital, on the other hand, has higher risk exposure, and its valuation is more sensitive to the price of risk.

This paper fits into a large literature studying the risk and return of PE investments. Korteweg (2019) surveys that literature, and we build on a series of studies benchmarking PE cash flows against publicly traded assets. In this context, a popular measure of risk-adjusted performance is Kaplan and Schoar (2005)'s Public Market Equivalent (PME). The PME discounts cash flows using the realized return on a portfolio of benchmark assets. Sorensen and Jagannathan (2015) show that the PME can fit into the SDF framework as a special case of Rubinstein (1976)'s log-utility model. But the log-utility model does not necessarily price benchmark assets, and in that case the PME applies the wrong risk adjustment. To address that issue, Korteweg and Nagel (2016) propose a generalized PME, and we build on their work.[2]

Starting with Ljungqvist and Richardson (2003), several authors study the performance of PE funds adjusting for different risk factors. Franzoni, Nowak, and Phalippou (2012) along with Ang, Chen, Goetzmann, and Phalippou (2018) estimate some of the most inclusive models considering Fama-French three factors, the liquidity factor of Pástor and Stambaugh (2003), and in some cases also profitability and investment factors. With our long-term investor, we introduce a new risk factor representing shocks to investment opportunities, or discount rate news, as in the intertemporal CAPM of Campbell (1993). Closest to the spirit of our long-term investor is the work of Gredil, Sorensen, and Waller (2020), who study PE performance using SDFs of leading consumption-based asset pricing models.

Beyond the PE literature, Farnsworth, Ferson, Jackson, and Todd (2002) provide an early

[2]Parallel effort by Gupta and Van Nieuwerburgh (2021) takes a different approach to benchmark PE cash flows. They try to replicate cash flows with a portfolio of synthetic dividend strips which is then priced with standard asset pricing techniques.

study of the SDF framework for the valuation of mutual funds, and Li, Xu, and Zhang (2016) apply the framework to the case of hedge funds. A main observation in those studies is that SDF models perform better when the SDF is required to price the risk-free asset, as this helps identify the mean of the SDF. We extend that observation to the case of PE funds, which require multiperiod discounting. With multiperiod discounting, we obtain more robust performance estimates across SDFs and lower variation of performance across funds when the SDF is required to price the risk-free asset at all relevant horizons, as this helps identify the mean of the SDF at those horizons.

1 Risk Adjustment of Private Equity Cash Flows

We measure the risk-adjusted performance of PE funds using the Generalized Public Market Equivalent (GPME). In its most general form, the GPME of fund i is the sum of fund's cash flows, $C_{i,t}$, discounted with realized SDF:

$$\text{GPME}_i \equiv \sum_{h=0}^{H} M_{t,t+h} C_{i,t+h} \tag{1}$$

The term $M_{t,t+h}$ denotes a multi-period SDF discounting cash flows from $t+h$ to the start of the fund. Time t is the date of the first cash flow of the fund, and it depends on i despite the simplified notation. The letter H indicates the number of periods (quarters in our case) from the first to last cash flow of the fund. As a convention, we let H be the same across funds, and funds that are active for a lower number of periods have a series of zero cash flows in the last part of their life.

Functional forms of the SDF are discussed in Section 2. They typically include at least one risk factor and depend on a vector of parameters. Those parameters must be estimated such that the SDF reflects realized returns on benchmark assets during the sample period. This intuitive condition is necessary for proper benchmarking, but it is unclear how it should be translated into formal statements. Korteweg and Nagel (2016) propose a heuristic approach based on the construction of artificial funds that invest in the benchmark assets. They estimate parameters setting the NPV of those artificial funds to zero. In the rest of this section, we propose an alternative approach based on the GPME decomposition which we are about to describe.[3]

Investing in a random fund gives NPV equal to $E[\text{GPME}_i]$. It is useful to decompose this quantity in a typical asset pricing way:

$$E[\text{GPME}_i] = \underbrace{\sum_{h=0}^{H} E[M_{t,t+h}]E[C_{i,t+h}]}_{\text{risk-neutral value}} + \underbrace{\sum_{h=0}^{H} \text{cov}(M_{t,t+h}, C_{i,t+h})}_{\text{risk adjustment}} \tag{2}$$

As illustrated on the right-hand side of this expression, NPV is the sum of a risk-neutral value with a risk adjustment. This decomposition suggests at least one consideration: by definition, the risk-neutral value should be determined by cash flows and risk-free rates, and it should not change as we add or remove risk factors from the SDF.

Further, a main objective of a benchmarking exercise like ours is to assess risk exposure of PE to different risk factors. In general, the GPME does not allow direct measurement of risk quantities, and we are left with indirect evidence based on the behavior of risk adjustment (Jeffers, Lyu, and Posenau (2021)). As we add or remove risk factors from the SDF, it is tempting to attribute differences in GPME to changes in risk adjustment, but that interpretation is robust only when the risk-neutral value is fixed. These considerations guide our choice of restrictions for the SDF.

1.1 Moment Conditions

Restrictions are imposed to the SDF in the form of moment conditions, which we introduce in two steps. First, we describe a set of moment conditions anchoring the mean of the SDF to risk-free rates. Those conditions are a main determinant of the risk-neutral value and represent the central innovation of this paper. Second, we describe the remaining moment conditions linking the SDF to realized returns on risky benchmark assets. This second set of moment conditions is a main determinant of the risk adjustment.

We identify the mean of the SDF at different horizons with standard asset pricing conditions on risk-free rates. To illustrate, consider \$1 invested at time t in a risk-free asset that pays $R^{\text{f}}_{t,t+h}$ at time $t+h$. This investment is priced by the SDF if $E_t[M_{t,t+h}] = 1/R^{\text{f}}_{t,t+h}$. Taking

[3] A related but different decomposition is discussed by Boyer, Nadauld, Vorkink, and Weisbach (2021).

unconditional expectations on both sides, we get the following moment condition:

$$E[M_{t,t+h}] = E\left[\frac{1}{R^{\mathrm{f}}_{t,t+h}}\right] \tag{3}$$

Imposing this restriction for all horizons h from 1 to H, the risk-neutral value can be rewritten exclusively in terms of cash flows and risk-free rates, and it becomes independent of the functional form of the SDF. Importantly, the risk-neutral value does not change as we add or remove risk factors from the SDF.

Empirically, we wish to impose condition (3) to the SDF. However, the practical meaning of the expectation operator inside that condition can be elusive. How does the population condition translate into a sample condition?

To address this question, it is useful to consider the sample version of $E[\mathrm{GPME}_i]$. We call it simply GPME, and we compute it as the mean of GPME_i across N funds in a sample:

$$\mathrm{GPME} \equiv \frac{1}{N}\sum_{i=1}^{N}\sum_{h=0}^{H} M_{t,t+h}C_{i,t+h} \tag{4}$$

It is possible to decompose this quantity similarly to its population counterpart. For each horizon h, we define $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$ as the average SDF and $\overline{C}_h = \frac{1}{N}\sum_i C_{i,t+h}$ as the average cash flow across funds. With these definitions, we can write

$$\mathrm{GPME} = \sum_{h=0}^{H}\overline{M}_h\overline{C}_h + \sum_{h=0}^{H}\overline{M}_h A_h \tag{5}$$

where $A_h = \frac{1}{N}\sum_i(M_{t,t+h}/\overline{M}_h - 1)(C_{i,t+h} - \overline{C}_h)$ is the covariance between normalized SDF and cash flows. In this decomposition, the risk-neutral value is $\sum_{h=0}^{H}\overline{M}_h\overline{C}_h$ and the risk adjustment is $\sum_{h=0}^{H}\overline{M}_h A_h$. Fixing the risk-neutral value requires restrictions on $\overline{M}_h$, and the expectation operator inside condition (3) must be implemented as a cross-sectional mean. As a result, we impose the following sample condition on the SDF:

$$\frac{1}{N}\sum_{i=1}^{N} M_{t,t+h} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{R^{\mathrm{f}}_{t,t+h}} \tag{6}$$

In our applications, $R^{\mathrm{f}}_{t,t+h}$ represents the cumulative return on quarterly T-bills between t

and $t+h$, and this expression represents H moment conditions as it must hold for horizons 1 to H. For large h, however, we do not observe all returns on benchmark assets.[4] In that case, we scale N down to account for the missing observations.

In our GPME decomposition, risk adjustment is determined by risk prices inside the SDF, and the decomposition does not provide clear guidance on how to identify the appropriate risk prices. In this case, we use heuristic rules. For each benchmark asset, b, with risky return $R^{\mathrm{b}}_{t,t+h}$, we impose the following condition:

$$\frac{1}{N}\sum_{i=1}^{N} M_{t,t+h} R^{\mathrm{b}}_{t,t+h} = 1 \tag{7}$$

This expression is the risky counterpart of the risk-free rate condition above. In our empirical applications, we have the S&P500 as the only risky benchmark. We impose this moment condition for S&P500 returns at horizon $h = 40$ quarters, or 10 years, which represents the standard horizon of a PE fund. It is also possible to impose this condition for every h between 1 and H, and we verify in unreported analysis that our empirical results are robust to that choice.

In summary, we restrict the SDF with conditions (6)-(7) in order to price benchmark assets. With the restricted SDF, we use expression (4) to estimate the NPV of investing in a random PE fund, and decomposition (5) to measure the two sources of value, risk-neutral vs. risk adjustment. This procedure fits into the GMM framework with the complication that sample size varies across moments.

1.2 Statistical Inference

For statistical inference, a main problem is that PE funds of similar vintages are likely to have positively correlated cash flows. This correlation can originate from exposure to the same factor shocks, and some of it could remain also after controlling for public factors.[5] To address this problem, Korteweg and Nagel (2016) integrate methods from spatial econometrics in their GMM framework. Below, we illustrate our inference, which is closely related to

[4] Some funds in our data operate longer than 15 years, so that $H > 60$ quarters. However, we cannot observe returns on benchmark assets at horizon $h = 60$ for funds started in 2009, for example, because that would require knowing returns realized in 2024.

[5] Ang, Chen, Goetzmann, and Phalippou (2018), for example, find a factor in PE returns which is not spanned by publicly traded factors.

their method.

To compute standard error of GPME, we ignore uncertainty about SDF parameters, but we allow for correlation between overlapping PE funds. As a start, we measure the economic distance between funds i and k by their degree of overlap. Defining $T(i)$ and $T(k)$ as the last non-zero cash flow dates of fund i and j, we compute their economic distance as follows:

$$d(i,k) \equiv 1 - \frac{\min\{T(i),T(k)\} - \max\{t(i),t(k)\}}{\max\{T(i),T(k)\} - \min\{t(i),t(k)\}} \tag{8}$$

The distance is zero if the overlap is exact, and it is 1 or greater if there is no overlap. This distance is used to construct weights that account for cross-sectional correlation in the sample estimate of the asymptotic variance. Specifically, we estimate the variance of $\sqrt{N}$ GPME as

$$v \equiv \frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{N} \max\{1 - d(i,k)/\bar{d}, 0\}\; u_i u_k \tag{9}$$

where $u_i \equiv \text{GPME}_i - \text{GPME}$. In the sum, each product $u_i u_k$ is assigned a weight between 0 and 1, and weights decrease with the distance between two funds. In our empirical work, we set $\bar{d} = 2$, and some non-overlapping pairs of funds still get positive weight. The standard error of GPME is estimated as $\sqrt{v/N}$.

The resulting standard error ignores parameters uncertainty and can be interpreted conservatively as a lower bound. Our primary objective remains obtaining point estimates of GPME that are as economically robust as possible.[6]

2 Stochastic Discount Factor

We focus on applications with exponentially affine SDFs. To illustrate, consider the case with a generic single factor f. The SDF can be written as follows:

$$M_{t,t+h} = \exp\left(a_h - \gamma_h f_{t,t+h}\right) \tag{10}$$

[6] Other than our focus on the economics of point estimates, an additional reason for ignoring parameters uncertainty is that moments with different sample sizes can complicate the derivation of GMM standard errors. To the best of our knowledge, there is only limited asymptotic GMM theory allowing for moments constructed with samples of different size. Some of that theory is developed by Lynch and Wachter (2013).

In this expression, a_h and γ_h indicate a pair of parameters per horizon, and γ_h can be interpreted as the risk price of f at horizon h. In absence of other restrictions, this SDF has a total of $2H$ parameters. Korteweg and Nagel (2016) restrict $a_h = ah$ and $\gamma_h = \gamma$, working with only 2 parameters. We do not impose any functional form on a_h, as this additional flexibility is necessary to satisfy moment conditions (6) and fix the subjective term structure of interest rates with market data. We maintain the restriction on risk prices, $\gamma_h = \gamma$, for two reasons. First, our main argument is about fixing the risk-neutral value of GPME, which is not determined by risk prices, so we maintain this part of the model as simple as possible. Second, we exploit this simplicity to study the robustness of our empirical results with respect to risk prices.

The single-factor form of the SDF can easily be extended with additional factors and corresponding risk prices. Below, we describe the form used in our empirical work.

2.1 CAPM and Long-Term Investors

We consider two risk factors. One factor is the log-return on the market, $r^{\mathrm{m}}_{t,t+h} = \ln(R^{\mathrm{m}}_{t,t+h})$. The other factor is news about future expected returns on the market, often called discount rate (DR) news in the literature. DR news arriving between t and $t+h$ is denoted $N^{\mathrm{DR}}_{t,t+h}$, and is defined as follows:

$$N^{\mathrm{DR}}_{t,t+h} \equiv (E_{t+h} - E_t) \sum_{j=1}^{\infty} \rho^j r^{\mathrm{m}}_{t+h+j} \tag{11}$$

In this expression, ρ is an approximation constant just below 1, and the right-hand side measures cumulative news between t and $t+h$ about market returns from $t+h$ onwards.[7] In a simple model with homoscedastic returns, this factor summarizes variation in investment opportunities, and positive news corresponds to better opportunities (Campbell (1993)).

With the two risk factors, we construct the following SDF:

$$M_{t,t+h} = \exp\left(a_h - \omega\gamma\, r^{\mathrm{m}}_{t,t+h} - \omega(\gamma-1) N^{\mathrm{DR}}_{t,t+h}\right) \tag{12}$$

[7] Formally, $\rho \equiv 1 - \exp(x)$ where x is the mean of the investor's log consumption-wealth ratio. In our empirical applications, one period corresponds to one quarter, and we set $\rho = 0.95^{1/4}$ corresponding to a mean consumption-wealth ratio of approximately 5% per year.

This is a two-factor version of (10) with risk price $\omega\gamma$ for market return and $\omega(\gamma-1)$ for DR news. Appendix A connects this SDF with theory, and shows that the parameter γ can be interpreted as the investor's relative risk aversion, while ω is the portfolio weight in the market, with $1-\omega$ being invested in the risk-free asset. Throughout our main analysis and unless otherwise specified, we assume $\omega = 1$ representing an investor fully allocated to the market.

This SDF recognizes that the same realized market return implies different marginal utility depending on expected returns. If expected returns are constant, $N^{\mathrm{DR}}_{t,t+h}$ is zero, and the SDF simplifies to a single-factor CAPM model. If expected returns vary over time, $N^{\mathrm{DR}}_{t,t+h}$ appears as an additional risk factor with positive risk price for investors with $\gamma > 1$. These investors are particularly averse to portfolio losses arriving jointly with negative news about expected returns. These losses are permanent in the sense that they are not compensated by higher expected returns, and a risk-averse, long-term investor fears them in particular (Campbell and Vuolteenaho (2004)).

We compare GPME estimates obtained with different restrictions on (12). In one case, we impose $a_h = 0$ and $\gamma = 1$. These restrictions correspond to a log-utility investor and result in the same SDF of Kaplan and Schoar (2005)'s PME. In addition to the log-utility investor, we consider two other types, CAPM and long-term investors, differentiated only by $N^{\mathrm{DR}}_{t,t+h}$, which is zero with CAPM and estimated below for long-term investors. The SDFs of these two investors require estimation of a_h and γ, and we compare our method with that of Korteweg and Nagel (2016). Since Korteweg and Nagel (2016) impose $a_h = ah$, we refer to their method as 'single intercept' and we refer to ours as 'multiple intercepts'.

2.2 A Model of Discount Rate News

To estimate DR news, we follow a large literature starting with Campbell (1991) that models expected market returns using vector autoregression (VAR).[8] We assume that the data are generated by a first-order VAR:

$$x_{t+1} = \mu + \Theta x_t + \varepsilon_{t+1} \tag{13}$$

[8]See for example Campbell (1991, 1993, 1996), Campbell and Vuolteenaho (2004), Lustig and Van Nieuwerburgh (2006), Cochrane (2011).

In this expression, μ is a $K \times 1$ vector and Θ is a $K \times K$ matrix of parameters. Furthermore, x_{t+1} is $K \times 1$ vector of state variables with $r^{\mathrm{m}}_{t+1} - r^{\mathrm{f}}_{t+1}$ as first element, and $\boldsymbol{\varepsilon}_{t+1}$ is a i.i.d $K \times 1$ vector of shocks with variance Σ_{ε}.

In this model, DR news is a linear function of the shocks:

$$N^{\mathrm{DR}}_{t,t+1} = \lambda \varepsilon_{t+1} \tag{14}$$

The vector of coefficients for DR news is defined as $\lambda = \rho e' \Theta (I - \rho\Theta)^{-1}$, where I is the identity matrix and $\boldsymbol{e}' = (1, 0, 0, \ldots, 0)$. Those coefficients measure the long-run sensitivity of expected returns to each element of x_t.

Combining the VAR model with the definition of multi-period DR news from (11), we obtain the following result:

$$N^{\mathrm{DR}}_{t,t+h} = \lambda \varepsilon_{t+h} + (\lambda - e'\rho\Theta)\, \varepsilon_{t+h-1} + \left(\lambda - e'\rho\Theta - e'\rho^2\Theta^2\right) \varepsilon_{t+h-2} + \cdots + (\lambda - e' \textstyle\sum_{i=1}^{h-1} \rho^i \Theta^i) \varepsilon_{t+1} \tag{15}$$

This equation expresses multi-period DR news in terms of observables, and it constitutes the empirical specification of the risk factor. In section 3, we obtain two versions of this factor by estimating two VAR models that differ in the choice of state variables.

3 Expected Returns and Discount Rate News

3.1 Public Market Data

The VAR vector x_t contains data about publicly traded assets at a quarterly frequency from 1950 to 2018. The first element of x_t is the difference between the log-return on the value-weighted S&P 500 and the log-return on quarterly T-bills. For this element, data is taken from the Center of Research in Security Prices (CRSP). The remaining elements of x_t are candidate predictors of expected returns and DR news. We consider (1) the log dividend-price ratio, (2) the term premium, (3) a credit spread of corporate bond yields and (4) the value spread. The log dividend-price ratio, term premium, and credit spread are constructed using data from Amit Goyal's website. The log dividend-price ratio is defined as the sum of the last 12 months dividends divided by the current price of the S&P 500.

Term premium is the difference between the annualized yield on 10-year constant maturity Treasuries and the annualized quarterly T-bill yield. Credit spread is the difference between the annualized yield on BAA-rated corporate bonds and AAA-rated corporate bonds. For the value spread, we rely on data from Kenneth French's data library. We construct the value spread as the difference in log book-to-market ratio of small-value and small-growth stock portfolios. These portfolios are generated from a double sort on market capitalization and book-to-market ratio.

Table 1 reports summary statistics of public market data in our sample period. From Panel A, the quarterly log equity premium is 1.6%, corresponding to 6.4% annualy, with a quarterly standard deviation of 8%. Further, our candidate return predictors are highly persistent, especially the log dividend-price ratio with autocorrelation coefficient of 0.982. Panel B reports correlations between contemporaneous and lagged state variables. The first column reports univariate correlations between one-period ahead excess market return ($r_t^{\mathrm{m}} - r_t^{\mathrm{f}}$) and lagged predictors. Market return is positively correlated with lagged dividend-price ratio, credit spread and term premium, and negatively correlated with lagged value spread.

3.2 VAR Estimation

We estimate the VAR model using OLS at a quarterly frequency in the post-war period from 1950 to 2018. We consider two different specifications: (1) a parsimonious specification including only the log dividend-price ratio as predictor and (2) a specification including the full set of predictors.

Table 2 reports the two VAR estimations. Panel A reports the parsimonious DP specification including only the dividend-price ratio, and Panel B reports the full specification. Each row corresponds to an equation in the VAR. The first row of each panel corresponds to the market return prediction equation. Standard errors are reported in brackets and the last two columns report R^2 and F-statistic for each forecasting equation. Panel A shows that dividend-price ratio significantly predicts excess market returns with a coefficient of 0.025. The R^2 is 2.8 percent and the F-statistic is statistically different from zero, consistent with the dividend-price ratio and lagged market return jointly predicting excess market returns. Panel B also includes the value spread, credit spread, and term premium in the

VAR. The first row shows that the lagged market return, dividend-price ratio and term premium positively predict excess returns. The coefficients on the dividend-price ratio and term premium are statistically significant at the five percent level. The value spread and credit spread negatively predict market return, although the corresponding coefficients are statistically insignificant.

Table 3 shows properties of one-period DR news, $N_{t,t+1}^{\mathrm{DR}}$, implied by the two VAR estimations. Panel A reports the vector of coefficients, λ, measuring the sensitivity of DR news to each element of ε_t. The "DP only" column shows that shocks to the dividend-price ratio are a significant determinant of DR news. The "Full VAR" column shows that shocks to the dividend-price ratio and term premium are significant determinants of DR news in the full VAR specification. These coefficients, however, do not represent a complete picture of how much each variable affects DR news; they do not account for the fact that elements of ε_t have different variances. We therefore decompose the unconditional variance of DR news to compare the importance of shocks to different variables.

Panel B of Table 3 decomposes the variance of DR news, $\lambda \Sigma_{\varepsilon} \lambda'$, into variance contributions from each variable's shock. The column "DP only" reports the decomposition for the parsimonious VAR. In this specifications, 105% of DR news variance originates from shocks to the dividend-price ratio and negative 5% stems from the lagged market return. Shocks to DR news are almost exclusively determined by shocks to the dividend-price ratio. The "Full VAR" column shows that the dividend-price ratio is the largest contributor to DR news variance also in the full specification. Even in this specification, the dividend-price ratio contributes nearly 100% percent of the variance. The value spread and term premium contribute only 3% and 4%, respectively, while the credit spread's contribution is essentially zero. These results suggest that both specifications rely almost exclusively on shocks to the dividend-price ratio to determine DR news.

4 Private Equity Performance

4.1 Funds Data

We analyze PE data maintained by Burgiss. Our final sample contains net-of-fees cash flows of 1866 PE funds started in the U.S. between 1978 and 2009, and divided in three mutually exclusive categories: buyout, venture capital, and generalist funds.

Burgiss provides at least two levels of classification for each fund. At the most general level (Tier 1), funds are primarily classified as 'equity', 'debt', or 'real assets'. We focus exclusively on equity. At a more detailed level (Tier 2), we distinguish between equity funds classified as buyout, venture capital, and generalist. We define buyout and venture capital funds using the homonymous Tier 2 classes. In our generalist category, we include funds with Tier 2 classification of 'generalist', 'expansion capital', 'unknown', and 'not elsewhere classified'.

To obtain our final sample, we exclude funds with less than 5 million USD of commitment from the raw data. We also exclude funds whose majority of investments are not liquidated by 2019. For that, we impose two conditions. First, we only include funds of vintage year 2009 or earlier. Second, among those funds, we exclude those with a ratio of residual NAV over cumulative distributions larger than 50%. Finally, we normalize cash flows and residual NAV by each fund's commitment.

Figure 1 plots the aggregate sum of normalized contributions, distributions, and net cash flows for the three fund categories over time. For all the categories, our sample constitutes mostly of cash flows observed between 1995 and 2019. For venture capital, we see uniquely large distributions in year 2000, corresponding to the dot-com bubble; those distributions are almost 10 times larger than distributions and contributions observed at any other time.

Table 4 summarizes our PE data. Panel A reports descriptive statistics and shows that the sample consists of 652 buyout funds, 971 venture capital funds and 243 generalist funds. The median (average) fund size is \$421 (\$1,099) million for buyout, \$126 (\$222) million for venture capital and \$225 (\$558) million for generalist funds. The average number of years between the first and last buyout fund cash flow is 14.16 years, 15.52 years for venture capital, and 14.42 years for generalist funds. The average number of cash flows per fund is approximately 36 for buyout, 28 for venture capital and 33 for generalists. The sample includes 311 unresolved buyout funds with an average NAV-to-Distributions ratio of 0.10, 353 unresolved venture capital funds with NAV-to-Distributions ratio of 0.14, and 88 unresolved generalist funds with a NAV-to-Distributions ratio of 0.12.

Panel B of Table 4 reports Total Value to Paid-In ratios (TVPI) across vintage years.[9] Across the three categories, TVPIs fluctuates over time and are typically higher for earlier vintages. Furthermore, venture capital shows peculiarly high TVPIs between 1993 and 1996. These large multiples can be ascribed, at least in part, to funds of these vintages deploying capital in the period leading up to the 2001 dot-com bubble and exiting investments before 2001.

4.2 Buyout

Table 5 reports GPME estimation for buyout funds. The first estimation corresponds to "Log-Utility" and uses the inverse of the market return as SDF. The resulting GPME is a re-formulation of Kaplan and Schoar (2005)'s PME defined as the sum of discounted cash flows rather than the ratio of discounted distributions over contributions. The log-utility GPME is 0.20 and significantly different from zero at the one percent level; buyout funds provide log-utility investors with 20 cents of abnormal profits per dollar of committed capital.

Other than the log-utility model, Table 5 reports GPME estimation for CAPM and long-term (LT) investors assuming they are fully invested in the market ($\omega = 100\%$). The "Single Intercept" columns estimate only one intercept parameter a, with $a_h = ah$, and the estimations use the moment conditions of Korteweg and Nagel (2016). The "Multiple Intercepts" columns use our method as described in Section 1, which does not impose a functional restriction on a_h and estimates multiple intercepts linking the subjective term-structure of interest rates to market data.

4.2.1 Buyout Value for CAPM Investors

Table 5 shows a GPME of 0.28 for the CAPM SDF using a single intercept. The estimate is statistically different from zero at the ten percent level. With multiple intercepts, The GPME is 0.30 and statistically significant at the one percent level. These numbers are close, and they imply that buyout funds provide CAPM investors with 28-30 cents of NPV per dollar of committed capital. CAPM investors thus derive 8-10 cents more value than log-utility investors from a marginal allocation to buyout funds.

Even though GPME estimates for CAPM investor are similar across the two methods, we

[9] If there are less than five funds per vintage year, figures are omitted due to confidentiality.

show below that the different restrictions placed on the single and multiple intercepts specifications imply markedly different SDF properties. To further explore differences between the two methods, Table 5 also reports the two GPME components from the decomposition of Section 1. The first component is $\sum_h \overline{M}_h \overline{C}_h$ and corresponds to the risk-neutral part of GPME. For each horizon, we take the average cash flow across funds and discount it with the average SDF across funds. We then sum over horizons. The second component is $\sum_h \overline{M}_h A_h$ and corresponds to the total risk adjustment inside GPME.

For the CAPM investor, the risk-neutral component is 0.61 with multiple intercepts and 0.79 with a single intercept. Risk adjustment, instead, is -0.31 with multiple intercepts and -0.51 with a single intercept. In this case, the single intercept specification achieves similar GPME estimate by assigning higher risk-neutral value, but also more negative risk adjustment, relative to the multiple intercepts case. Differences between the two methods become more evident considering long-term investors below.

4.2.2 Buyout Value for Long-Term Investors

In Table 5, the "LT" columns report GPME estimations for long-term investors. In particular, the "LT (DP)" columns use the VAR specification with only the dividend-price ratio as return predictor to measure DR news. With a single intercept, the LT (DP) investor assigns a GPME of 0.80 to buyout. This point estimate is considerably higher relative to the CAPM investor, but it is not statistically significant due to the large standard error. Further, a NPV of 80 cents per dollar of commitment is large enough to appear economically implausible, and our decomposition shows that this GPME results from summing a risk-neutral value of 2.50 with a risk adjustment of -1.69. Compared to the CAPM, this large value is not generated by a change in risk adjustment. Instead, it comes from a large increase of the risk-neutral component. With multiple intercepts, we estimate a GPME of 0.35 for the LT (DP) investor. This point estimate is statistically significant, and it is only marginally higher compared to CAPM. By construction, the difference relative to CAPM is entirely due to risk adjustment.

The "LT (Full)" columns of Table 5 use DR news computed with the full VAR specification which includes the dividend-price ratio, term premium, credit spread, and value spread as return predictors. With single intercept, the resulting GPME is 0.41, although not statistically significant, and substantially lower than 0.80 obtained in the LT (DP) case.

With multiple intercepts, GPME is 0.34, statistically significant at the one percent level, and close to the 0.35 estimated in the LT (DP) case. While the single intercept methodology suggests that the two VAR specifications result in DR news which generate large GPME differences, the multiple intercepts method suggests similar GPME implications of the two VAR specifications. In Section 3, we show that both VAR estimations generate DR news that vary almost exclusively from shocks to the dividend-price ratio, suggesting that the full VAR might have similar dynamics to the parsimonious VAR. Consistent with this interpretation, the multiple intercepts method estimates virtually identical GPMEs for LT (DP) and LT (Full) investors.

4.2.3 Time Discounting of Buyout Funds

Figure 2 plots the average SDF across funds as a function of horizon. At each horizon h, the average SDF measures the present value of one dollar paid for certain at that horizon by all funds in the sample. The figure compares single intercept and multiple intercepts specifications from Table 5. By construction, multiple intercepts estimations using the same sample imply the same average SDF as a function of horizon. Single intercept estimations impose less structure to the SDF, and the average SDF at each horizon varies depending on the risk factors considered.

The figure shows unrealistic time discounting for the single intercept estimation with a peculiar pattern of negative time discounting in the first 3 years, positive discounting from year 4 to 8, and negative discounting again from year 8 to 11. This pattern is qualitatively consistent across investors and it is quantitatively strongest for the LT (DP) model, suggesting that the very large GPME estimate obtained with this model might be due to this implausible time discounting pattern resulting from the single intercept method.

With multiple intercepts, Figure 2 shows that time discounting is consistently positive and stable across horizons, and this result corresponds to more stable GPME estimates across models, as shown in Table 5. It also corresponds to lower variation of GPME_i across funds, as we show below.

4.2.4 Cross-Sectional Variation of Performance

Table 6 summarizes the cross-sectional distribution of GPME_i resulting from the different estimations. The table contains results for all three fund categories. We focus primarily on buyout, and a similar discussion applies for venture capital and generalist funds studied below. For each estimation, we report the mean of GPME_i, which corresponds to the GPME estimates of Table 5. Below the mean, we report the standard deviation and selected percentiles of the GPME_i distribution.

Differences in the distribution of GPME_i are interesting because the multiple intercepts estimations, just like the single intercept ones, restrict the SDF using exclusively public market data, and ignoring any information about PE cash flows. Thus, differences in the GPME_i distribution are a result which is not imposed by construction. We find that the multiple intercepts estimations imply consistently lower variation of GPME_i across funds, relative to single intercept.

For buyout, the log-utility model generates the lowest standard deviation of GPME_i, equal to 0.64. The single intercept CAPM model implies a standard deviation of 1.42, while the multiple intercepts CAPM model implies a standard deviation of 0.98. Thus, the multiple intercepts model generates substantially lower standard deviation with CAPM, even though the two models have similar mean (0.28 vs. 0.30). Further, the lower standard deviation of the multiple intercepts CAPM model comes with less extreme tail observations as indicated by the reported percentiles. For long-term investors, we see qualitatively similar differences between the single intercept and multiple intercepts methods, with more extreme magnitudes. Especially for the LT (DP) investor, the GPME_i standard deviation of the single intercept model is extremely high, 9.25, relative to 1.05 obtained with multiple intercepts model.

4.2.5 Components of Buyout Performance across Horizons

With multiple intercepts, we take the GPME decomposition one step further. Not only do we decompose GPME in a risk-neutral part and a risk adjustment, but we also decompose the risk-neutral part and the risk adjustment based on the contribution of each horizon. To illustrate, we decompose the risk-neutral part as follows:

$$\sum_{h=0}^{H}\overline{M}_h\overline{C}_h = \overline{M}_0\overline{C}_0 + \sum_{h=1}^{4}\overline{M}_h\overline{C}_h + \sum_{h=5}^{8}\overline{M}_h\overline{C}_h + \cdots + \sum_{h=53}^{56}\overline{M}_h\overline{C}_h + \sum_{h=57}^{H}\overline{M}_h\overline{C}_h \tag{16}$$

These components correspond to values coming from year 0, year 1, year 2, . . . , year 14, and year 15 or higher. A similar decomposition is done for risk adjustment.

Figure 3 plots the resulting GPME components against horizon for selected models with multiple intercepts. The figure focuses on CAPM and LT (DP) models. By construction, the risk-neutral component from each horizon (grey bars) is identical across models, and differences originate exclusively from components of risk adjustment plotted as black bars for CAPM and white bars for LT (DP).

Decomposing the risk-neutral part, the grey bars in Figure 3 show the "J-curve" typical of PE cash flows.[10] Investors contribute capital primarily in the first 4 years, corresponding to negative average cash flows at short horizons. Average cash flows turn positive from year 5, as funds distribute capital.

Decomposing risk adjustment, Figure 3 shows that CAPM and LT (DP) models are similar not only on the overall risk adjustment but also on its components across horizons. Surprisingly perhaps, risk adjustment is moderately positive in the first years of fund operations and turn negative only after the third year. At short horizons, net cash flows are dominated by contributions, and a positive risk adjustment suggest that buyout funds tend to call less capital in bad times with high SDF realizations. This tendency decreases risk and has small but positive effect on GPME. Further, this result is consistent with Robinson and Sensoy (2016), who also find pro-cyclicality in contributions.

The components of risk adjustment turn negative at longer horizons, after year 3, and they are most negative between year 9 and 11. Interestingly, risk adjustment is small for years 6 to 8, even though average cash flows are high during those years. This result appears consistent with Gupta and Van Nieuwerburgh (2021) finding that buyout funds generate cash flows that appear to be risk-free in part of their harvesting period.

4.3 Venture Capital

Table 7 reports GPME estimations for venture capital funds. As a starting point, we estimate a log-utility GPME of 0.14 and statistically indistinguishable from zero. For comparison,

[10] Grey bars represent the risk-neutral present value of average cash flows at each horizon. Instead, the J-curve is typically plotted as the average cash flows at each horizon without discounting. Nonetheless, the two quantities are close, especially at horizons shorter than 10 years.

Korteweg and Nagel (2016) find a marginally positive log-utility GPME of 0.05 for venture capital funds. The higher GPME in our sample might come from a larger number of funds in pre-1998 vintages. Historically, those vintages have high risk-adjusted performance for venture capital.

Considering the GPME decomposition for log-utility, venture capital has risk-neutral value of 0.48 and risk adjustment of -0.35. Compared to buyout, this risk adjustment is considerably larger (-0.35 vs. -0.09). The log-utility model has constant risk price of $\gamma = 1$ across samples, and differences in risk adjustment are entirely due to different covariance between cash flows and market returns. Thus, higher risk adjustment for venture capital suggests higher market exposure of venture capital's cash flows relative to buyout. Larger risk adjustment for venture capital is consistent with Driessen, Lin, and Phalippou (2012), who estimate a market beta of 2.4 for venture capital and 1.3 for buyout, and with Ang et al. (2018), who estimate a market beta 1.8 for venture capital and 1.2 for buyout.

4.3.1 Venture Capital for CAPM and Long-Term Investors

In Table 7, the GPME estimate for the CAPM investor is -0.15 with single intercept and 0.07 with multiple intercepts. Both methods indicate that CAPM implies lower GPME relative to log-utility, and this qualitative difference is consistent with the findings of Korteweg and Nagel (2016) in a smaller sample. The single intercept and multiple intercepts methods disagree on the GPME sign and magnitude, however, and multiple intercepts result in marginally positive GPME for venture capital.

Differences in GPME between single intercept and multiple intercepts can arise from differences in the SDF intercepts, a_h, but also from differences in the estimated risk price, γ. Table 7 shows that CAPM's risk price is 2.93 with single intercept and 2.03 with multiple intercepts. We show in Section 5.1 that differences in γ do not entirely explain this GPME difference between the two methods, as the GPME of the CAPM investor with multiple intercepts remain higher relative to single intercept even assuming the same risk price of 2.93 for both models.

For the long-term investor, we observe qualitatively similar differences between single and multiple intercepts. With single intercept, we estimate negative GPMEs of -0.19 for LT (DP) and -0.08 for LT (Full). With multiple intercepts, we estimate positive GPMEs of 0.13

for LT (DP) and 0.15 for LT (Full). In Section 5.1, we also show that this difference is not entirely explained by lower risk prices with multiple intercepts.

Comparing GPMEs between CAPM and long-term investors, we find that the long-term investor assigns higher value to venture capital, relative to the CAPM investor, with multiple intercepts. GPME estimates with multiple intercepts under LT (DP) and LT (Full) are almost double the GPME estimate under CAPM. Considering the single intercept method, we find more stable performance across investors for venture capital relative to buyout. To investigate this result, we plot the implied discounting of the different venture capital estimations.

For venture capital, Figure 4 plots the cross-sectional average SDF as a function of horizon. As we do for buyout, the figure distinguishes between multiple intercepts and the three models with single intercept. This figure confirms that multiple intercepts imply a more stable time discounting across horizons. The single intercept method implies qualitatively similar time discounting between buyout and venture capital estimations. With venture capital, however, time discounting of the single intercept method does not vary as widely across the three models.

4.3.2 Components of Venture Capital Performance across Horizons

Following the discussion of buyout results, we also decompose GPMEs by horizon for venture capital. Figure 5 plots the decomposition for CAPM and LT (DP) models with multiple intercepts. We focus on the decomposition of risk adjustment.

In the figure, risk adjustment varies similarly with horizon across the two models. For both models, risk adjustment is marginally positive from year 0 to 2. This result suggests that contributions tend to be slightly pro-cyclical, and it is similar to buyout funds although quantitatively smaller. Starting from year 3, risk adjustment turns negative, and it is most important between year 4 and 7. This result contrasts with buyout, whose risk adjustment tends to be small especially in year 6 and 7. Distributions of venture capital funds show substantially different risk across horizons, relative to buyout.

4.4 Generalist

Our analysis of generalist funds is similar to that of buyout and venture capital. Here we provide an overview of the results.

Table 8 shows the results of our GPME estimations for generalist funds. With log-utility, we estimate a statistically significant GPME of 0.16, which is lower relative to buyout and marginally higher relative to venture capital. For CAPM and long-term investors, we estimate consistently positive GPME with single intercept and multiple intercepts methods. GPME estimates are higher and more stable across estimations with multiple intercepts relative to single intercept.

Figure 6 shows differences in time discounting across methods plotting the average SDF by horizons implied by the different estimations. Qualitatively, the figure shows results similar to buyout and venture capital. With multiple intercepts, time discounting is positive, constant across investors, and stable across horizons. With single intercept, we observe time discounting being negative in the first 3 years, positive in the next 5 years, and negative again in the next 3 years.

Figure 7 plots the risk-neutral value and risk adjustment components by horizon. From year 0 to 2, the figure shows risk adjustment similar to the other fund categories and consistent with contributions hedging some risk for PE investors. From year 3 onwards, risk adjustment turns negative. Compared to the other categories, risk adjustment of generalist fund can be attributed more homogeneously to cash flows received from year 4 to 10.

5 Robustness

In this section, we focus exclusively on the method with multiple intercepts, and we explore the robustness of our results with respect to two parameters. First, we study the sensitivity of GPME as we exogenously change risk aversion, γ. Second, we estimate GPMEs for investors whose portfolio weight in the market is either $\omega = 50\%$ or 200%, as opposed to 100% in Section 4.

5.1 Risk Aversion

Our GPME decomposition does not provide clear guidance on how to estimate risk prices for proper benchmarking of PE cash flows. As described in Section 1, our method identifies risk prices by constraining the SDF to price risky benchmark returns at 10-year horizon. This is a heuristic approach based on the typical horizon of PE funds, and we study the sensitivity to this heuristic by changing risk prices exogenously. Specifically, we change risk aversion, γ, because risk prices are primarily determined by this parameter for our investors.

As we change risk aversion exogenously, we do not need to run new estimations. Instead, the resulting GPME with multiple intercepts can be computed as follows:

$$\text{GPME}(\gamma) = \sum_{h=1}^{H} \left(\frac{1}{N} \sum_{i=1}^{N} \frac{1}{R^{\text{f}}_{t,t+h}} \right) \left(\overline{C}_h + A_h(\gamma) \right) \tag{17}$$

To obtain this expression, we rewrite the GPME decomposition (5) using time discounting restrictions: $\frac{1}{N}\sum_{i=1}^{N} M_{t,t+h} = \frac{1}{N}\sum_{i=1}^{N} 1/R^{\text{f}}_{t,t+h}$. We use this notation to highlight that A_h is the only term of GPME affected by risk prices. Further, A_h is the only term affected by the SDF, but it does not depend on intercept parameters.[11]

Using expression (17), we compute GPMEs with risk aversion between 1 and 12 for each type of investor in each fund category. Figure 8 plots the resulting GPMEs as a function of γ. For each category, the three lines correspond to different investors. The solid line represents CAPM, the dotted line represents LT (DP), and the dash-dotted line represents LT (Full). Further, there are two circles over each line. The black circle represents the combination of GPME and γ estimated for that investor with multiple intercepts in Table 5, Table 7, or Table 8, depending on the category. For comparison, the white circle corresponds to γ estimated with single intercept and GPME computed with expression (17).

The top-left panel of Figure 8 shows results for buyout. For the CAPM investor, GPME ranges from 0.5 to 0.1, it is monotonically decreasing in risk aversion, and it remains positive even at risk aversion of 12. For long-term investors, the LT (DP) and LT (Full) models imply approximately the same GPME across all levels of risk aversion. For these investors, GPME ranges from 0.5 to 0.3, and it is non-monotonic in risk aversion. Overall, we find robustly

[11] To see why A_h does not depend on intercept parameters, recall that $A_h = \frac{1}{N}\sum_i (M_{t,t+h}/\overline{M}_h - 1)(C_{i,t+h} - \overline{C}_h)$ with $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$. The SDF enters A_h only through its normalized form, $M_{t,t+h}/\overline{M}_h$, and intercepts cancel out because of the normalization.

positive performance of buyout funds, and only moderate sensitivity to risk prices.

The top-right panel of Figure 8 shows results for venture capital. As opposed to buyout, we find high sensitivity of venture capital's GPME to risk prices. This sensitivity is highest for the CAPM investor, whose GPME estimate goes from 0.35 to -0.25 as risk aversion increases. For long-term investors, GPME shows marginally lower sensitivity, ranging from 0.35 to -0.15. Across all investors, we find that venture capital's GPME is most sensitive to risk aversion in the range of risk aversion between 1 and 3, which contains the three point estimates of γ with multiple intercepts from Table 7.

In the bottom panel of Figure 8, we report sensitivity results for generalist funds. Similarly to buyout, the GPME of generalist funds display only moderate sensitivity to risk prices, and it remains positive for all investors at all levels of risk aversion between 1 and 12. GPME ranges from 0.35 to 0.05 for the CAPM investor, and from 0.35 to 0.15 for long-term investors.

Across investors and fund categories, we find tendency for GPME to decrease in risk aversion, especially for risk aversion between 1 and 5, which is typically the most relevant range. For buyout and generalist funds, we find quantitatively modest GPME sensitivity to risk prices, and GPME remains positive across a wide range of risk aversion. For venture capital, instead, we find high sensitivity of GPME to risk prices, with positive GPME for risk aversion below 2 and negative GPME for risk aversion above 3. Because of this high sensitivity, venture capital seems the most problematic category to evaluate.

5.2 Investor Leverage

An additional way to compare CAPM and long-term investors is by looking at the effect of investor's leverage on GPME. A natural measure of leverage in our model is the portfolio weight in the market, ω, and while we assume $\omega = 100\%$ in most of the paper, here we consider two different values representing a conservative investor with low leverage ($\omega = 50\%$) and an aggressive investor with high leverage ($\omega = 200\%$).

After further inspection, SDF expression (12) presented in Section 2 suggests two considerations about leverage. First, CAPM investors with different leverage assign the same GPMEs. For those investors, ω enters the SDF only in the product $\omega\gamma$ that determines market risk price, and it is a redundant parameter. For CAPM investors with higher leverage, our GPME estimation will mechanically result in proportionally lower risk aversion. Second, long-term investors with different leverage can assign different GPMEs. For long-term investors, ω affects the importance of market risk price, $\omega\gamma$, relative DR news risk price, $\omega(\gamma - 1)$. Since $\gamma > \gamma - 1$, risk from DR news is less important for aggressive investors with large ω, and if DR news matters when evaluating PE, leverage can affect GPMEs.

In Table 9, we report GPME estimations similar to the multiple intercepts part of Table 5, Table 7, and Table 8, except that we do not assume $\omega = 100\%$. Instead, we assume $\omega = 50\%$ in first part of the table and $\omega = 200\%$ in the second part. For CAPM investors, the table confirms that leverage has no effect on GPME, and higher leverage is mechanically offset by lower risk aversion. For long-term investors, we find some differences in GPME across leverage. The conservative long-term investor assigns GPME of 0.35 to buyout, 0.25 to generalist, and in the 0.15-0.20 range to venture capital. The aggressive long-term investor, instead, assigns GPME of 0.33 to buyout, 0.22 to generalist, and 0.07 to venture capital. Thus, the conservative investor assigns higher value to PE across all three fund categories, and by construction, these differences are entirely due to different risk adjustments. Quantitatively, however, these GPME differences across leverage seem largely negligible at least for the case of buyout and generalist funds.

Overall, we estimate that all three fund categories provide positive values to both CAPM investors and long-term investors across a wide range of leverage levels.

6 Conclusion

PE funds are illiquid investments whose true fundamental return is typically unobservable. Since investment returns are unobservable, risk and performance cannot be estimated with standard approaches, and the literature has developed methods to evaluate these investments by discounting funds' cash flows with SDFs. In this paper, we show that existing SDF methods for the valuation of PE funds result in unrealistic time discounting, which can generate implausible performance estimates. We propose a modified method and compare it to existing ones.

Theoretically, our approach is based on a standard asset pricing decomposition of PE performance in a risk-neutral part and a risk adjustment. We fix the risk-neutral part by constraining the SDF such that the subjective term structure of interest rates is determined by market data. By construction, the risk-neutral part does not vary as we add or remove risk factors from the SDF, so we can meaningfully measure the economic cost of PE risk and compare it across models. Empirically, we evaluate our approach against existing methods, and find that our approach results in more stable PE performance across models and lower variation of performance across funds.

We use our method to measure PE performance and risk adjustment for two types of investors: a CAPM investor, and a long-term investor who distinguishes between permanent and transitory wealth shocks. We discount net-of-fees cash flows of 1,866 PE funds started in the U.S. between 1978 and 2009, and divided in three categories: buyout, venture capital, and generalist. We find largely negligible differences between the two investors, especially for buyout and generalist funds. Overall, we find positive performance of buyout, generalist, and venture capital funds. For venture capital, however, high risk exposure makes performance estimates particularly sensitive to estimated risk prices.

Our performance decomposition does not provide clear guidance on how to estimate risk prices for proper benchmarking of PE cash flows. Because of this, we rely on heuristic SDF restrictions, and we study the sensitivity of performance estimates with respect to risk prices. The open issue on the estimation of risk prices, among others, is left to future research.

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Figure 1
Aggregate Cash Flows

This figure plots aggregate (normalized) contributions, distributions, and net cash flows for the three fund categories. Buyout corresponds to the top-left, venture capital is in the top-right, and generalist at the bottom. The blue area represents distributions, the red area represents contributions, and the solid line represents net cash flows. The grey shaded areas correspond to NBER recessions.

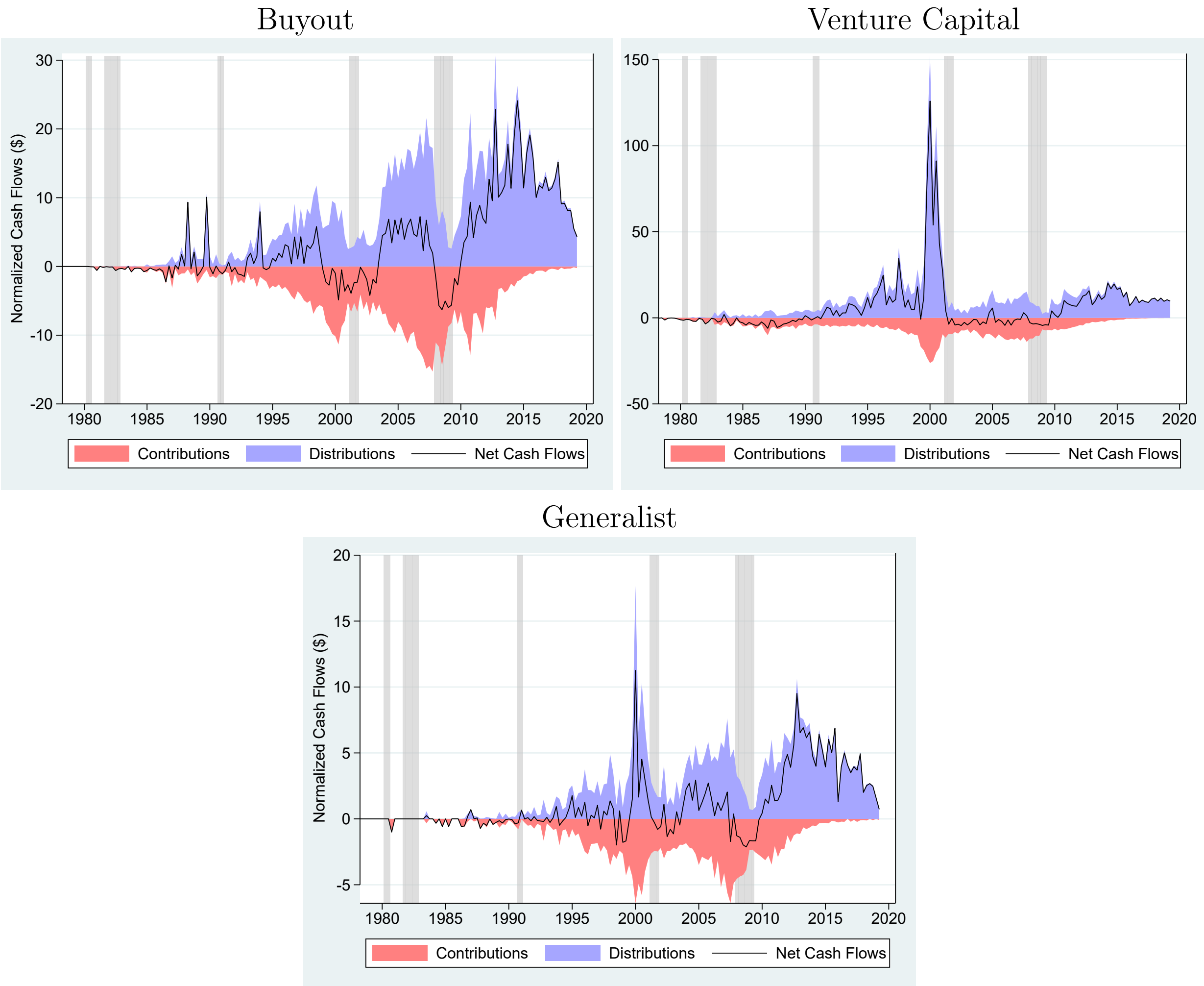

Figure 2
Time Discounting for Buyout

This figure plots average multi-period SDF, $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$, across buyout funds every quarter. We consider different SDFs resulting from the estimations of Table 5.

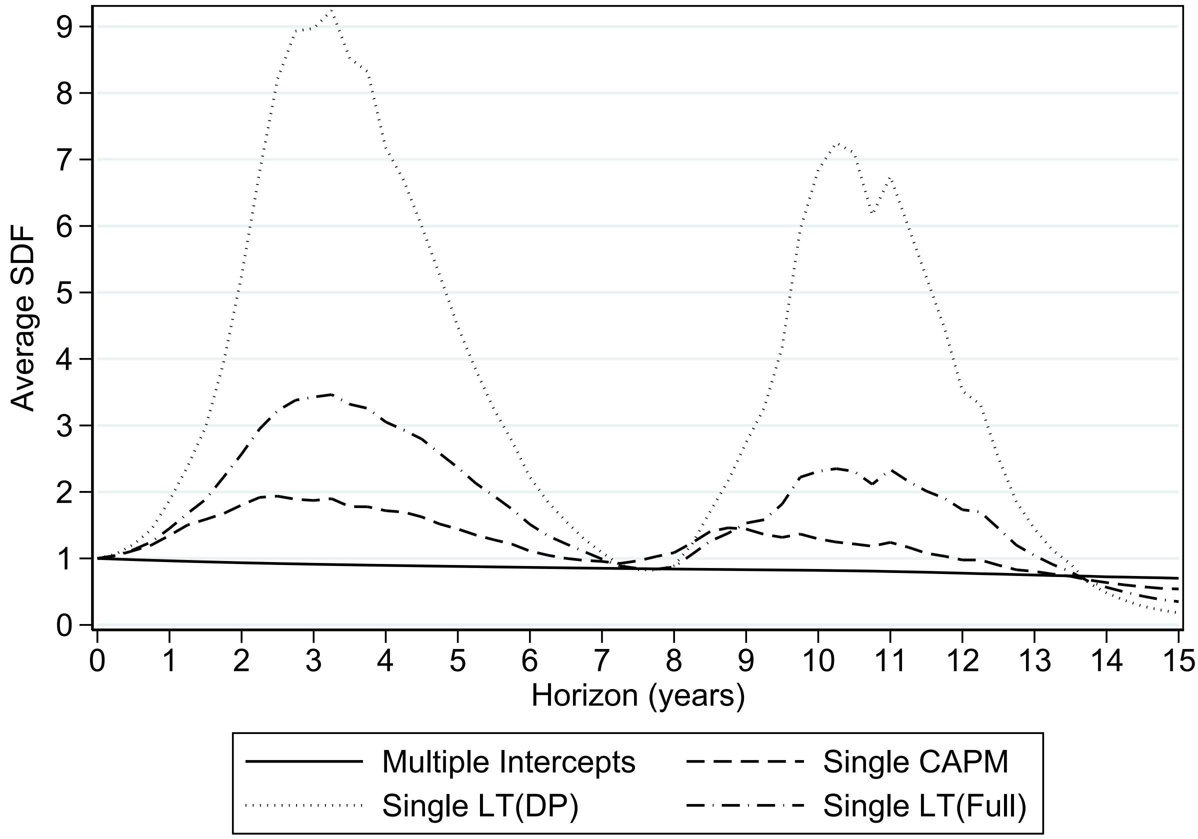

Figure 3
GPME Decomposition for Buyout

In this figure, we decompose GPMEs estimated in Table 5 for the CAPM and LT(DP) models with multiple intercepts. Discounted Value of Avg. Cash Flow is $\overline{C}_0$ in year 0, $\sum_{h=1}^{4}\overline{M}_h\overline{C}_h$ in year 1, $\sum_{h=4y-3}^{4y}\overline{M}_h\overline{C}_h$ in year y between 2 and 14, and $\sum_{h=57}^{H}\overline{M}_h\overline{C}_h$ in year 15. By construction, Discounted Value of Avg. Cash Flow is the same across the two models. Discounted Value of Risk is 0 in year 0, $\sum_{h=1}^{4}\overline{M}_h A_h$ in year 1, $\sum_{h=4y-3}^{4y}\overline{M}_h A_h$ in year y between 2 and 14, and $\sum_{h=57}^{H}\overline{M}_h A_h$ in year 15.

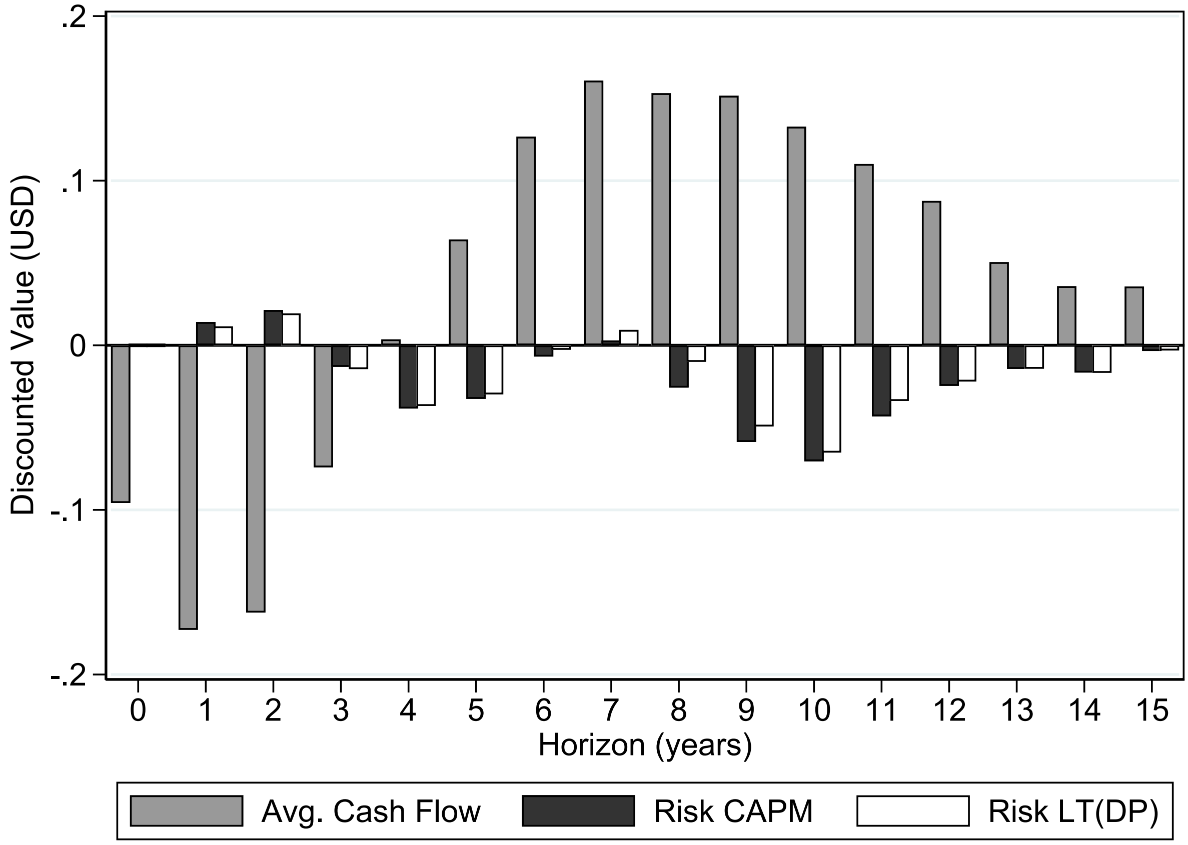

Figure 4

Time Discounting for Venture Capital

This figure plots average multi-period SDF, $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$, across venture capital funds every quarter. We consider different SDFs resulting from the estimations of Table 7.

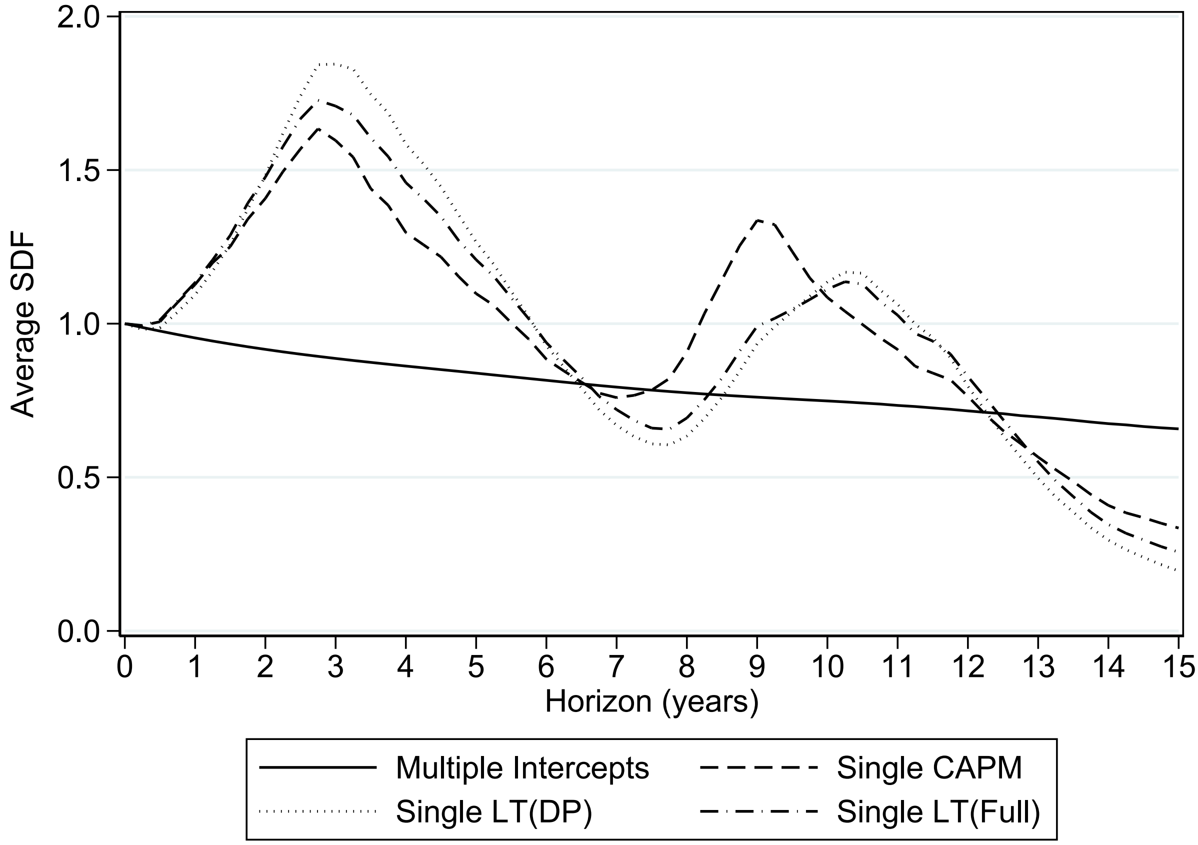

Figure 5

GPME Decomposition for Venture Capital

In this figure, we decompose GPMEs estimated in Table 7 for the Multiple Intercepts CAPM and LT(DP) models. The plot is constructed as in Figure 3.

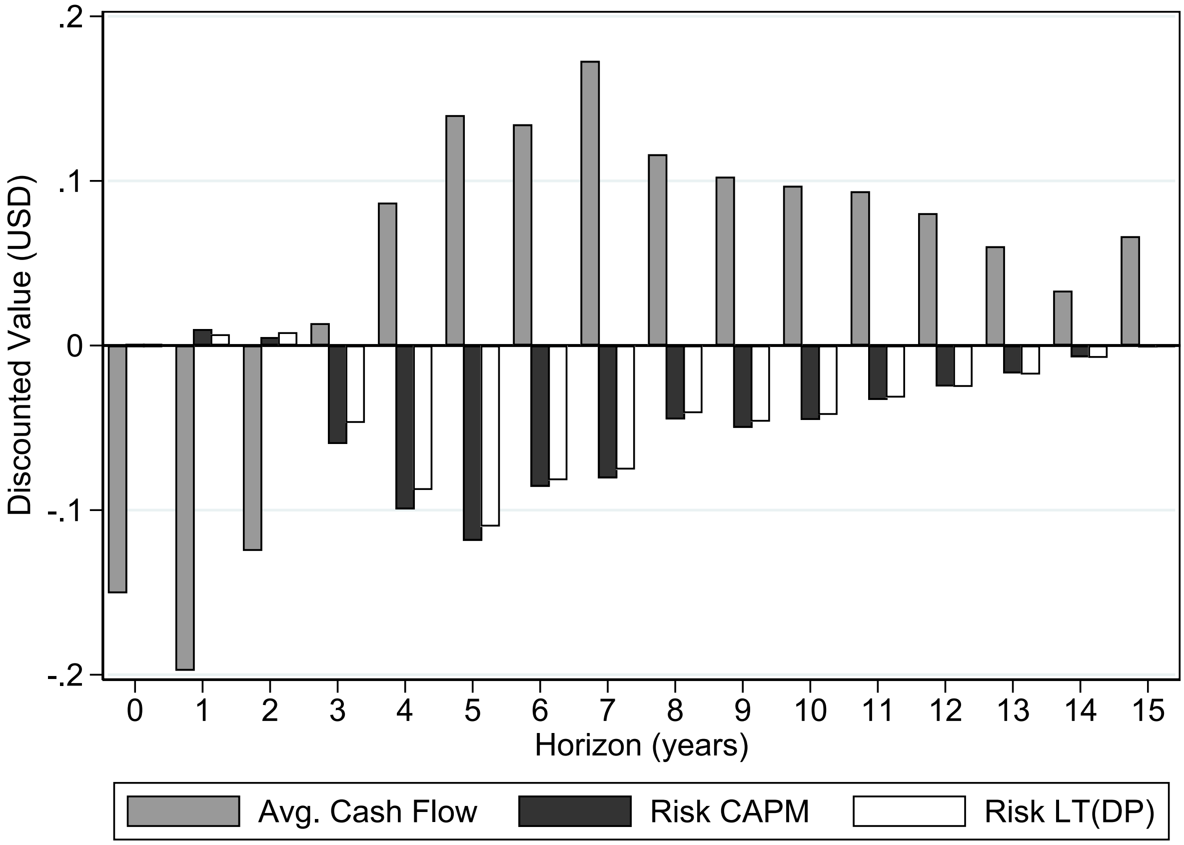

Figure 6

Time Discounting for Generalist

This figure plots average multi-period SDF, $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$, across generalist funds every quarter. We consider different SDFs resulting from the estimations of Table 8.

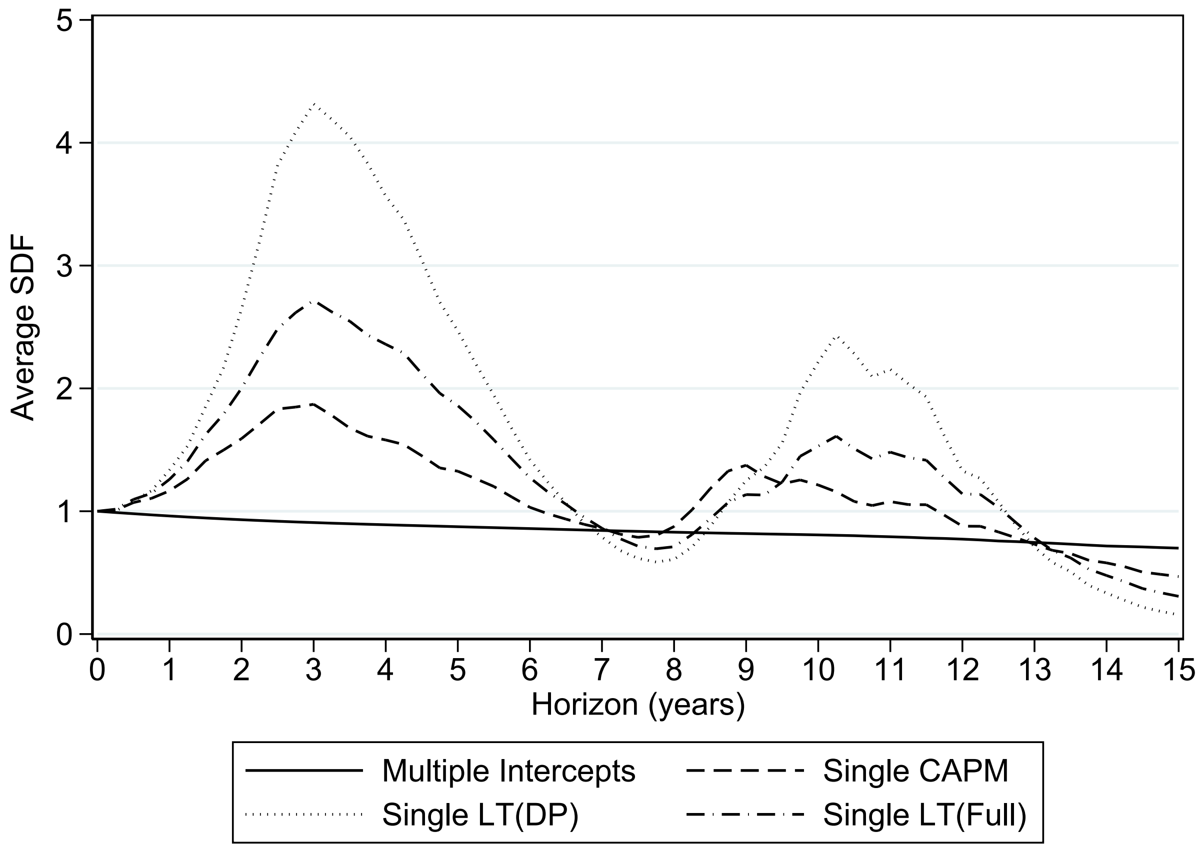

Figure 7

GPME Decomposition for Generalist

In this figure, we decompose GPMEs estimated in Table 8 for the Multiple Intercepts CAPM and LT(DP) models. The plot is constructed as in Figure 3.

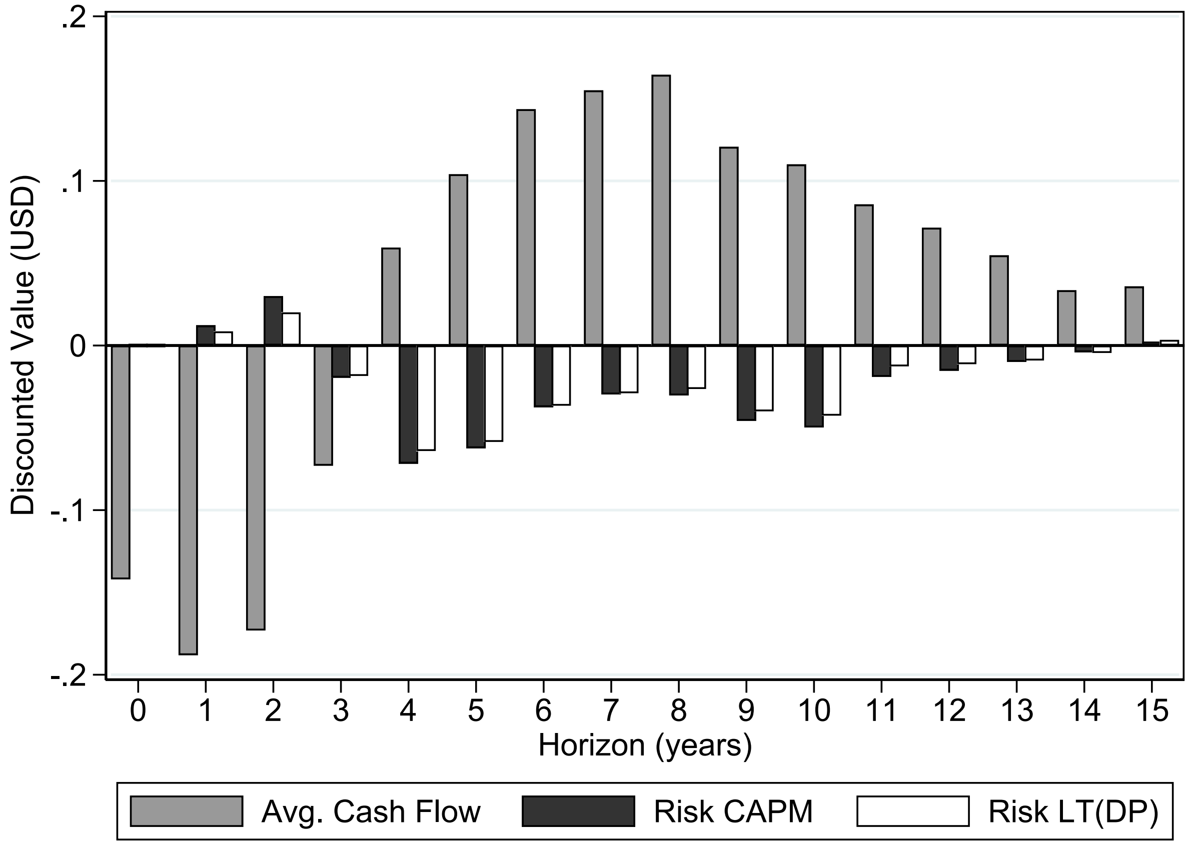

Figure 8

GPME Sensitivity to Risk Aversion Estimates

In this figure, we plot GPME for models with multiple intercepts using exogenous values of γ between 1 and 12. For each category and for each model, we compute GPME as $\sum_h(\frac{1}{N}\sum_i 1/R^{\mathrm{f}}_{t,t+h})\,(\overline{C}_h+A_h)$, where $A_h = \frac{1}{N}\sum_i(M_{t,t+h}/\overline{M}_h-1)(C_{i,t+h}-\overline{C}_h)$. The term $M_{t,t+h}/\overline{M}_h$ is computed as $\exp(-\gamma r^{\mathrm{m}}_{t,t+h})/\frac{1}{N}\sum_i \exp(-\gamma r^{\mathrm{m}}_{t,t+h})$ for CAPM and similarly with an additional risk factor for LT(DP) and LT(Full). On each line, the black circle indicates the GPME estimate using γ from the corresponding model in Table 5 for buyout, Table 7 for venture capital, and Table 8 for generalist. The white circle indicates the GPME estimate using γ from the single intercept version of the model.

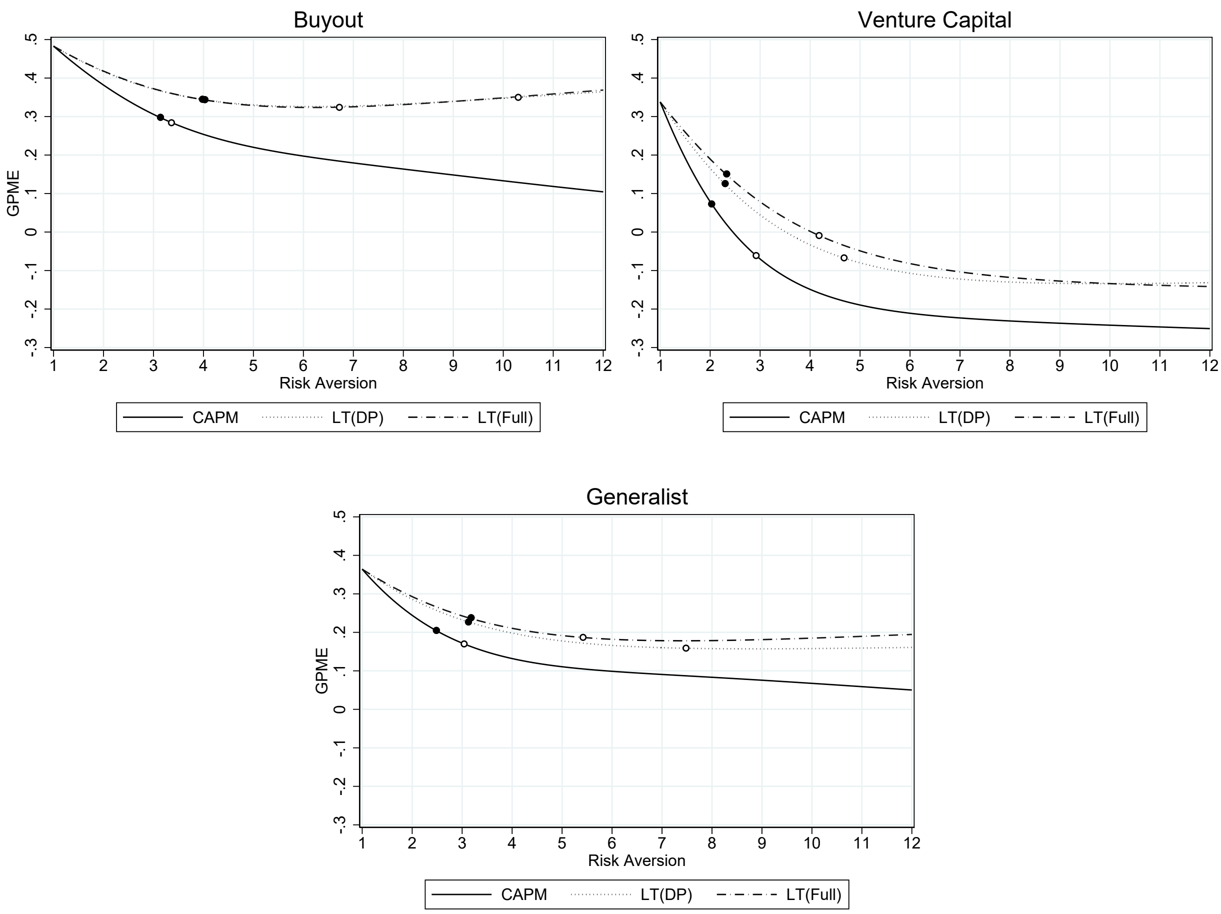

Table 1
Summary Statistics of VAR Variables

This table reports summary statistics of variables entering the full VAR model. Variables are computed quarterly from 1950 to 2018 for a total of 276 observations. The expression $r^{\mathrm{m}} - r^{\mathrm{f}}$ indicates the excess log-return on the S&P 500, DP is the logarithm of dividend yield on the S&P 500, VS is the difference in the log book-to-market ratio of small-value and small-growth stocks, CS is the yield difference between BAA and AAA rated corporate bonds, and $TERM$ is the yield difference between treasuries with 10-year and 3-month maturity. Panel A reports descriptive statistics. Panel B reports correlations between contemporaneous and lagged variables.

Panel A: Descriptive Statistics

	Mean	Std	Min	Max	Autocorr.
$r_t^{\mathrm{m}} - r_t^{\mathrm{f}}$	0.016	0.078	−0.311	0.192	0.099
DP_t	−3.536	0.424	−4.497	−2.624	0.982
VS_t	1.575	0.157	1.280	2.111	0.890
CS_t	0.010	0.004	0.003	0.034	0.878
$TERM_t$	0.017	0.014	−0.035	0.045	0.841

Panel B: Correlations

	$r_t^{\mathrm{m}} - r_t^{\mathrm{f}}$	DP_t	VS_t	CS_t	$TERM_t$
$r_{t-1}^{\mathrm{m}} - r_{t-1}^{\mathrm{f}}$	0.099	−0.069	−0.017	−0.220	0.060
DP_{t-1}	0.130	0.982	−0.439	0.159	−0.221
VS_{t-1}	−0.099	−0.429	0.890	0.030	0.326
CS_{t-1}	0.022	0.144	0.060	0.878	0.321
$TERM_{t-1}$	0.103	−0.273	0.283	0.171	0.841

Table 2
VAR Estimation

This table reports the results of two VAR estimations. In Panel A, the VAR includes only the excess log-return and the logarithm of dividend yield on the S&P 500. In Panel B, the VAR includes also the value spread, credit spread, and term premium as defined in Table 1 and in the main text. Variables are computed quarterly from 1950 to 2018 for a total of 276 observations. OLS Standard errors are reported in parenthesis, and the symbols ***, **, and * indicate significance at 1%, 5%, and 10%.

Panel A: DP only							
	Constant	$r^{\mathrm{m}}_t - r^{\mathrm{f}}_t$	DP_t	VS_t	CS_t	$TERM_t$	R^2
$r^{\mathrm{m}}_{t+1} - r^{\mathrm{f}}_{t+1}$	0.102***	0.107*	0.025**				0.028
	(0.039)	(0.060)	(0.011)				
DP_{t+1}	−0.084**	−0.100	0.977***				0.965
	(0.040)	(0.062)	(0.011)				
Panel B: Full VAR							
	Constant	$r^{\mathrm{m}}_t - r^{\mathrm{f}}_t$	DP_t	VS_t	CS_t	$TERM_t$	R^2
$r^{\mathrm{m}}_{t+1} - r^{\mathrm{f}}_{t+1}$	0.162***	0.082	0.027**	−0.040	−0.622	0.929**	0.051
	(0.055)	(0.060)	(0.012)	(0.034)	(1.118)	(0.373)	
DP_{t+1}	−0.101*	−0.085	0.975***	0.019	−0.705	−0.780**	0.966
	(0.056)	(0.062)	(0.013)	(0.035)	(1.153)	(0.385)	
VS_{t+1}	0.101**	0.064	−0.027**	0.868***	1.840*	−0.240	0.798
	(0.051)	(0.056)	(0.012)	(0.031)	(1.039)	(0.347)	
CS_{t+1}	0.001	−0.009***	0.000	0.000	0.875***	−0.011	0.798
	(0.001)	(0.002)	(0.000)	(0.001)	(0.029)	(0.010)	
$TERM_{t+1}$	−0.011**	−0.004	0.000	0.006*	0.371***	0.789***	0.725
	(0.005)	(0.006)	(0.001)	(0.003)	(0.106)	(0.036)	

Table 3
Discount Rate News

This table decomposes point estimates and variance of N_t^{DR} from the two VAR estimations of Table 2. In Panel A, we report estimates of the vector λ such that $N_t^{\mathrm{DR}} = \lambda\varepsilon_t$ where ε_t is the VAR error term. The vector is $\lambda = \rho e'\Theta(I - \rho\Theta)^{-1}$, where Θ is the matrix of VAR coefficients, I is the identity matrix, e is a column vector with 1 as first element and 0 elsewhere, and $\rho = 0.95^{1/4}$. In parentheses, we report standard errors calculated with delta method. Statistical significance is computed using the normal distribution, and the symbols ***, **, and * indicate significance at 1%, 5%, and 10%. In Panel B, we decompose the variance of N_t^{DR} expressed as $\lambda\Sigma_\varepsilon\lambda'$. We report the vector of variance components $\lambda \circ \lambda\Sigma_\varepsilon$, where $\circ$ is the element-wise product. We also report components as percentages of the total variance.

Panel A: Long-Run Coefficients

	DP only	Full VAR
	λ	λ
$r^{\mathrm{m}} - r^{\mathrm{f}}$	0.039	0.046
	(0.027)	(0.041)
DP	0.713***	0.782***
	(0.112)	(0.139)
VS		−0.110
		(0.097)
CS		−4.869
		(3.843)
$TERM$		1.977***
		(0.747)

Panel B: Variance Decomposition

	DP only		Full VAR	
	$\lambda \circ \lambda\Sigma_\varepsilon$	%	$\lambda \circ \lambda\Sigma_\varepsilon$	%
$r^{\mathrm{m}} - r^{\mathrm{f}}$ shock	−0.00016	−5.4	−0.00019	−5.4
DP shock	0.00303	105.4	0.00343	99.2
VS shock			0.00011	3.0
CS shock			−0.00002	−0.4
$TERM$ shock			0.00013	3.6
$\mathrm{Var}(N_t^{\mathrm{DR}})$	0.00287	100.0	0.00345	100.0

Table 4
Summary Statistics of Funds Data

This table reports summary statistics of funds data from Burgiss. In Panel A, *#Funds* is the sample size, and for each fund, Fund Size is total commitment, Effective Years counts the years between the first and last available cash flow, #Cash flows/Fund is the number of cash flows, TVPI is the ratio of distributions over contributions. Furthermore, #Unresolved Funds counts funds that are not fully liquidated in sample, and NAV/Distributions is the ratio of their residual NAV over total distributions. Panel B reports mean and median TVPI for different vintage years. For confidentiality, we hide figures computed with 4 funds or less.

Panel A: Descriptive Statistics

	Buyout			Venture Capital			Generalist		
	Mean	Median	St.dev.	Mean	Median	St.dev.	Mean	Median	St.dev.
#Funds	652			971			243		
Fund Size ($M)	1,099.10	421.00	2,118.19	222.10	126.00	282.75	557.77	225.00	1,290.66
Effective Years	14.16	13.75	3.13	15.52	15.25	3.37	14.42	13.50	3.37
#Cash flows/Fund	35.83	36.00	10.45	27.60	27.00	9.62	32.99	33.00	10.53
TVPI	1.83	1.68	1.15	2.08	1.39	3.07	1.80	1.64	1.02
#Unresolved Funds	311			353			88		
NAV/Distributions	0.10	0.06	0.11	0.14	0.10	0.13	0.12	0.07	0.13

Panel B: TVPI by Vintage Year

	Buyout			Venture Capital			Generalist		
Vintage	# Funds	Mean	Median	# Funds	Mean	Median	# Funds	Mean	Median
1978-91	57	3.09	2.22	238	2.12	1.77	15	2.76	2.55
1992	8	1.97	1.64	17	3.19	1.76	5	3.12	2.62
1993	7	1.68	1.72	20	5.35	3.15	7	2.30	1.90
1994	18	1.73	1.49	16	6.15	4.50	8	2.60	2.18
1995	26	1.63	1.53	27	5.69	2.72	6	3.08	2.37
1996	17	1.64	1.70	18	6.68	3.31	8	2.00	1.43
1997	26	1.24	1.23	47	3.48	1.94	17	1.49	1.28
1998	40	1.45	1.45	53	1.97	1.18	17	1.48	1.39
1999	34	1.45	1.55	94	0.86	0.72	20	1.29	1.09
2000	50	1.79	1.68	119	0.96	0.85	26	1.43	1.41
2001	31	1.90	1.94	60	1.26	1.12	6	2.04	2.20
2002	21	1.89	1.85	21	1.08	1.11	7	1.73	1.63
2003	22	2.07	1.82	21	1.38	1.10	4	* * *	* * *
2004	38	1.77	1.63	34	1.49	0.91	10	1.66	1.74
2005	57	1.64	1.52	52	1.63	1.31	18	1.84	1.53
2006	61	1.67	1.64	54	1.56	1.47	27	1.53	1.39
2007	66	1.77	1.70	45	2.21	1.90	22	1.58	1.68
2008	55	1.72	1.69	28	2.06	1.71	13	1.96	1.81
2009	18	2.06	2.05	7	1.92	2.01	7	1.71	1.59

Table 5
Buyout Performance

For $N = 652$ buyout funds in our sample, we estimate expected GPME by summing discounted cash flows of each fund and averaging the result across funds. Cash flows are discounted with the following SDF:

$$M_{t,t+h} = \exp\left(a_h h - \omega\gamma\, r^{\mathrm{m}}_{t,t+h} - \omega(\gamma-1)N^{\mathrm{DR}}_{t,t+h}\right)$$

In this table, the investor has stock allocation $\omega = 100\%$, and each column corresponds to different restrictions on the SDF. With Log-Utility, $a_h = 0$ and $\gamma = 1$ as in the PME of Kaplan and Schoar (2005). With Single Intercept, $a_h = a$ for all h, and the two parameters (a and γ) are estimated following Korteweg and Nagel (2016) so that the SDF prices artificial funds invested in the S&P 500 and in quarterly T-bills. With Multiple Intercepts, we estimate γ and one a_h for every h such that the SDF prices T-bills investments at every horizon and stock investments at horizon $h = 40$ quarters. CAPM corresponds to $N^{\mathrm{DR}}_{t,t+h} = 0$, while LT(DP) and LT(Full) use $N^{\mathrm{DR}}_{t,t+h}$ from the VAR estimations of Table 2. For each specification, we report point estimates of GPME and γ. In parentheses, we report GPME standard errors that account for error dependence between overlapping funds and ignore parameters uncertainty. In brackets, we report p-values for a J-test of GPME $= 0$. In the last two rows of the table, we divide the GPME in two components, and we let $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$ be the average SDF at each horizon, $\overline{C}_h = \frac{1}{N}\sum_i C_{i,t+h}$ be the average cash flow, and $A_h = \frac{1}{N}\sum_i (M_{t,t+h}/\overline{M}_h - 1)(C_{i,t+h} - \overline{C}_h)$ be a risk adjustment.

		Single Intercept			Multiple Intercepts		
	Log-Utility	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)
GPME	0.203	0.279	0.803	0.405	0.298	0.350	0.346
	(0.027)	(0.159)	(0.706)	(0.285)	(0.113)	(0.118)	(0.114)
	[0.000]	[0.079]	[0.255]	[0.155]	[0.008]	[0.003]	[0.002]
γ	1.00	3.37	10.30	6.72	3.16	3.98	4.03
Components of GPME							
$\sum_h \overline{M}_h \overline{C}_h$	0.293	0.792	2.502	1.139	0.608	0.608	0.608
$\sum_h \overline{M}_h A_h$	−0.090	−0.513	−1.699	−0.735	−0.310	−0.258	−0.261

Table 6
GPME Distributions

The table reports mean, standard deviation, and selected percentiles of the GPME distribution for buyout, venture capital, and generalist funds across different models. The models are estimated in Table 5, 7, and 8.

		Single Intercept			Multiple Intercepts		
	Log-Utility	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)
Buyout ($N = 652$)							
Mean	0.20	0.28	0.80	0.41	0.30	0.35	0.35
St.Dev.	0.64	1.42	9.25	2.99	0.98	1.05	1.04
Min	−1.25	−4.15	−15.39	−5.86	−2.21	−1.60	−1.66
p10	−0.36	−0.83	−4.30	−1.65	−0.47	−0.48	−0.47
p25	−0.12	−0.38	−0.85	−0.56	−0.20	−0.23	−0.20
p50	0.13	−0.04	−0.17	−0.14	0.02	0.03	0.04
p75	0.44	0.59	0.61	0.69	0.51	0.61	0.58
p90	0.82	1.72	4.06	2.76	1.35	1.56	1.51
Max	10.89	13.98	165.94	42.62	8.78	8.32	8.66
Venture Capital ($N = 971$)							
Mean	0.14	−0.15	−0.19	−0.08	0.07	0.13	0.15
St.Dev.	1.40	1.18	1.43	1.68	1.07	1.23	1.32
Min	−1.14	−2.57	−3.21	−2.84	−1.50	−1.36	−1.40
p10	−0.67	−0.95	−1.12	−1.02	−0.66	−0.65	−0.65
p25	−0.44	−0.63	−0.68	−0.63	−0.42	−0.41	−0.41
p50	−0.19	−0.33	−0.37	−0.34	−0.16	−0.16	−0.15
p75	0.23	0.04	−0.04	0.03	0.26	0.30	0.31
p90	0.88	0.72	0.73	0.90	0.87	0.91	0.98
Max	15.13	16.47	20.42	24.55	15.51	17.31	17.46
Generalist ($N = 243$)							
Mean	0.16	0.13	0.05	0.15	0.21	0.24	0.25
St.Dev.	0.52	1.01	2.28	1.58	0.75	0.79	0.79
Min	−1.15	−2.72	−6.43	−3.75	−1.50	−1.47	−1.51
p10	−0.39	−0.72	−1.95	−1.08	−0.41	−0.46	−0.42
p25	−0.17	−0.44	−0.69	−0.51	−0.24	−0.24	−0.23
p50	0.06	−0.12	−0.24	−0.17	0.00	0.03	0.06
p75	0.44	0.42	0.34	0.40	0.46	0.51	0.53
p90	0.79	1.19	2.38	2.07	1.01	1.11	1.10
Max	2.63	6.11	11.52	8.94	4.15	4.01	4.20

Table 7
Venture Capital Performance

We estimate expected GPME for $N = 971$ venture capital funds in our sample. The construction of this table follows the description of Table 5 using the sample of venture capital funds.

		Single Intercept			Multiple Intercepts		
	Log-Utility	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)
GPME	0.135	−0.150	−0.188	−0.077	0.073	0.126	0.151
	(0.084)	(0.067)	(0.081)	(0.101)	(0.065)	(0.078)	(0.084)
	[0.109]	[0.026]	[0.020]	[0.447]	[0.261]	[0.108]	[0.072]
γ	1.00	2.93	4.68	4.19	2.03	2.37	2.40
Components of GPME							
$\sum_h \overline{M}_h \overline{C}_h$	0.484	0.910	0.903	0.892	0.725	0.725	0.725
$\sum_h \overline{M}_h A_h$	−0.349	−1.060	−1.091	−0.969	−0.652	−0.599	−0.574

Table 8
Generalist Performance

We estimate expected GPME for $N = 243$ generalist funds in our sample. The construction of this table follows the description of Table 5 using the sample of generalist funds.

		Single Intercept			Multiple Intercepts		
	Log-Utility	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)
GPME	0.156	0.127	0.046	0.153	0.213	0.241	0.249
	(0.023)	(0.094)	(0.173)	(0.129)	(0.069)	(0.074)	(0.071)
	[0.000]	[0.177]	[0.791]	[0.234]	[0.002]	[0.001]	[0.000]
γ	1.00	3.05	7.49	5.43	2.53	3.14	3.19
Components of GPME							
$\sum_h \overline{M}_h \overline{C}_h$	0.318	0.703	1.043	0.832	0.563	0.563	0.563
$\sum_h \overline{M}_h A_h$	−0.162	−0.576	−0.998	−0.679	−0.350	−0.321	−0.313

Table 9
PE Performance and Investor's Leverage

We estimate expected GPME for of a conservative investor with 50% of wealth in stocks ($\omega = 50\%$) and an aggressive investor with 200% of wealth in stocks ($\omega = 200\%$). This table reports the results separately for buyout, venture capital, and generalist funds. The estimation follows the description of Table 5 limited to the case with Multiple Intercepts.

	Buyout			Venture Capital			Generalist		
	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)
Conservative Investor ($\omega = 50\%$)									
GPME	0.298	0.358	0.354	0.073	0.155	0.198	0.212	0.250	0.262
	(0.113)	(0.119)	(0.114)	(0.065)	(0.086)	(0.096)	(0.069)	(0.076)	(0.071)
	[0.008]	[0.003]	[0.002]	[0.262]	[0.071]	[0.040]	[0.002]	[0.001]	[0.000]
γ	6.32	8.29	8.41	4.08	5.07	5.17	5.07	6.66	6.78
Components of GPME									
$\sum_h \overline{M}_h \overline{C}_h$	0.608	0.608	0.608	0.725	0.725	0.725	0.563	0.563	0.563
$\sum_h \overline{M}_h A_h$	−0.310	−0.250	−0.254	−0.652	−0.570	−0.527	−0.350	−0.313	−0.301
Aggressive Investor ($\omega = 200\%$)									
GPME	0.298	0.329	0.327	0.073	0.075	0.075	0.212	0.223	0.225
	(0.113)	(0.116)	(0.114)	(0.065)	(0.065)	(0.066)	(0.069)	(0.071)	(0.069)
	[0.008]	[0.004]	[0.004]	[0.262]	[0.253]	[0.250]	[0.002]	[0.002]	[0.001]
γ	1.58	1.81	1.83	1.02	1.02	1.03	1.27	1.38	1.39
Components of GPME									
$\sum_h \overline{M}_h \overline{C}_h$	0.608	0.608	0.608	0.725	0.725	0.725	0.563	0.563	0.563
$\sum_h \overline{M}_h A_h$	−0.310	−0.279	−0.281	−0.652	−0.650	−0.649	−0.350	−0.339	−0.337

A Theoretical Stochastic Discount Factor

In this section, we derive the theoretical version of the long-term SDF introduced in the main text. Using the setup of Campbell (1993), we extend his results to price payoffs received several periods in the future.

The investor has infinite-horizon Epstein-Zin preferences over consumption, and these preferences correspond to the following general form of SDF (Epstein and Zin (1989)):

$$M_{t,t+1}^{\text{theory}} = \exp\left(\theta \log \delta - \frac{\theta}{\psi}(c_{t+1} - c_t) - (1-\theta)\, r_{t+1}^{\mathrm{W}}\right) \tag{A.1}$$

In this expression, c_t is the natural logarithm of consumption, and r_{t+1}^{W} is the log-return on wealth. Greek letters indicate parameters: δ is the subjective discount factor, ψ is the elasticity of intertemporal substitution, and $\theta \equiv \frac{1-\gamma}{1-1/\psi}$ depends on ψ and relative risk aversion γ.

The budget constraint of the investor can be written as follows:

$$W_{t+1} = (W_t - C_t) R_{t+1}^{\mathrm{W}} \tag{A.2}$$

We indicate wealth with W_t, while $C_t = \exp(c_t)$ is consumption, and $R_{t+1}^{\mathrm{W}} = \exp(r_{t+1}^{\mathrm{W}})$ is the return on wealth. Following Campbell (1993), we represent the investor's budget constraint with the following log-linear approximation:

$$w_{t+1} - w_t = r_{t+1}^{\mathrm{W}} + k + \left(1 - \frac{1}{\rho}\right)(c_t - w_t) \tag{A.3}$$

In this expression, w_t is the logarithm of wealth, while k and ρ are constants of approximation.

A.1 Consumption Growth

Assuming that second and higher conditional moments of r_{t+1}^{W} do not vary over time, consumption can be expressed as a function of returns. For that, we use two equations derived by Campbell (1993) under the same set of assumptions. The first equation connects expected consumption growth to expected return on wealth:

$$E_t\left[c_{t+1}-c_t\right]=\psi E_t\left[r^{\mathrm{W}}_{t+1}\right]+\underbrace{\psi\log(\delta)+\frac{1}{2}\frac{1-\gamma}{1-\psi}\mathrm{Var}_t\left(c_{t+1}-c_t-\psi r^{\mathrm{W}}_{t+1}\right)}_{\text{constant}} \tag{A.4}$$

The second equation connects the unexpected component of consumption growth to the unexpected component of the return on wealth and to discount rate (DR) news about investor's wealth:

$$c_{t+1}-c_t-E_t\left[c_{t+1}-c_t\right]=r^{\mathrm{W}}_{t+1}-E_t\left[r^{\mathrm{W}}_{t+1}\right]+(1-\psi)\underbrace{\left(E_{t+1}-E_t\right)\sum_{j=1}^{\infty}\rho^j r^{\mathrm{W}}_{t+1}}_{N^{\mathrm{DR,W}}_{t+1}} \tag{A.5}$$

Combining (A.4) with (A.5), we get the following expression for the logarithm of consumption growth:

$$c_{t+1}-c_t=r^{\mathrm{W}}_{t+1}+(1-\psi)N^{\mathrm{DR,W}}_{t+1}-(1-\psi)E_t\left[r^{\mathrm{W}}_{t+1}\right]+\text{constant} \tag{A.6}$$

A.2 Return on Wealth

DR news on wealth, $N^{\mathrm{DR,W}}_{t+1}$, can be expressed in terms of DR news on underlying assets. For that, we specify a simple investment strategy for the investor, whose portfolio is a constant combination of two assets. One asset is risk-free, while the other is a market index of public equities. The return on the risk-free asset is constant and its logarithm is denoted r^{f}. The return on the market is risky and its logarithm is denoted r^{m}_t. Expected log-return on the market can vary over time, while its variance and higher conditional moments remain constant.

Following Campbell and Viceira (1999), log-returns on individual assets determine log-return on wealth through the following approximate relation:

$$r^{\mathrm{W}}_{t+1}=\omega\left(r^{\mathrm{m}}_{t+1}-r^{\mathrm{f}}\right)+r^{\mathrm{f}}+\frac{1}{2}\omega(1-\omega)\mathrm{var}\left(r^{\mathrm{m}}_{t+1}\right) \tag{A.7}$$

In this expression, ω is the constant portfolio weight in the market. Under our assumptions, this approximation implies:

$$N_{t+1}^{\mathrm{DR,W}} = \omega N_{t+1}^{\mathrm{DR}} \tag{A.8}$$

where N_{t+1}^{DR} is one-period DR news on the market as defined in the main text.

A.3 Theoretical Long-Term SDF

To obtain the theoretical form of the long-term SDF, we substitute (A.6) and (A.7) inside the general SDF expression (A.1). As a result, we obtain the one-period theoretical version of the long-term SDF:

$$M_{t+1}^{\mathrm{theory}} = \exp\left(a_t - \omega\gamma\, r_{t+1}^{\mathrm{m}} - \omega(\gamma-1)\, N_{t+1}^{\mathrm{DR}}\right) \tag{A.9}$$

with $a_t = \omega(\gamma-1)E_t\left[r_{\mathrm{m},t+1}\right] + \mathrm{constant}$.

A.3.1 Two Periods

For the two-period version, the product $M_{t+1}^{\mathrm{theory}} \times M_{t+2}^{\mathrm{theory}}$ can be written as follows:

$$M_{t,t+2}^{\mathrm{theory}} = \exp\left(a_{2,t} - \omega\gamma\, r_{t,t+2}^{\mathrm{m}} - \omega(\gamma-1)N_{t,t+2}^{\mathrm{DR}}\right) \tag{A.10}$$

This expression contains the following objects:

$$a_{2,t} = \omega(\gamma-1)\left(E_t[r_{t+1}^{\mathrm{m}}] + E_t[r_{t+2}^{\mathrm{m}}]\right) + 2\cdot\mathrm{constant} \tag{A.11}$$

$$\begin{aligned} N_{t,t+2}^{\mathrm{DR}} &= (E_{t+2}-E_t)\sum_{j=1}^{\infty}\rho^j r_{t+2+j}^{\mathrm{m}} - (1-\rho)(E_{t+1}-E_t)r_{t+2}^{\mathrm{m}} \\ &\approx (E_{t+2}-E_t)\sum_{j=1}^{\infty}\rho^j r_{t+2+j}^{\mathrm{m}} \end{aligned} \tag{A.12}$$

and the approximation of $N_{t,t+2}^{\mathrm{DR}}$ is accurate for ρ close 1.

A.3.2 Many Periods

For the multiperiod version, the product $M_{t+1}^{\text{theory}} \times M_{t+2}^{\text{theory}} \times \cdots \times M_{t+h}^{\text{theory}}$ can be written as:

$$M_{t,t+h}^{\text{theory}} = \exp\left(a_{h,t} - \omega\gamma\, r_{t,t+h}^{\text{m}} - \omega(\gamma-1) N_{t,t+h}^{\text{DR}}\right) \tag{A.13}$$

where

$$a_{h,t} = \omega(\gamma-1)\sum_{s=1}^{h} E_t[r_{t+s}^{\text{m}}] + h \cdot \text{constant} \tag{A.14}$$

Furthermore, multi-period DR news is written as follows:

$$\begin{aligned}
N_{t,t+h}^{\text{DR}} &= \sum_{s=1}^{h} N_{t+s}^{\text{DR}} - \sum_{s=1}^{h-1}\left[(E_{t+s}-E_{t+s-1})\sum_{j=1}^{h-s} r_{t+s+j}^{\text{m}}\right] && \text{(A.15)}\\
&= \sum_{s=1}^{h}\left[(E_{t+s}-E_{t+s-1})\sum_{j=1}^{\infty}\rho^j r_{t+s+j}^{\text{m}}\right] - \sum_{s=1}^{h-1}\left[(E_{t+s}-E_{t+s-1})\sum_{j=1}^{h-s} r_{t+s+j}^{\text{m}}\right]\\
&= \sum_{s=1}^{h}\left[(E_{t+s}-E_{t+s-1})\sum_{j=h-s+1}^{\infty}\rho^j r_{t+s+j}^{\text{m}}\right] - \sum_{s=1}^{h-1}\left[(E_{t+s}-E_{t+s-1})\sum_{j=1}^{h-s}(1-\rho^j) r_{t+s+j}^{\text{m}}\right]\\
&\approx \sum_{s=1}^{h}\left[(E_{t+s}-E_{t+s-1})\sum_{j=h-s+1}^{\infty}\rho^j r_{t+s+j}^{\text{m}}\right]\\
&= \sum_{s=1}^{h}\left[(E_{t+s}-E_{t+s-1})\sum_{j=1}^{\infty}\rho^j r_{t+h+j}^{\text{m}}\right]\\
&= (E_{t+h}-E_t)\sum_{j=1}^{\infty}\rho^j r_{t+h+j}^{\text{m}} && \text{(A.16)}
\end{aligned}$$

The approximation is accurate for ρ close to 1.