The Systematic Risk of Private Equity*

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Abstract

This paper introduces a novel econometric approach to estimate the systematic risk and abnormal returns of illiquid assets based only on their observable cash flows. Our model assumes that the returns of a private equity investment are generated by the standard market model, and that the dividends from the investment occur at a stochastic, yet increasing rate from its unobservable interim values until the investment finally liquidates. Using the Generalized Method of Moments (GMM), we then estimate the systematic risk and abnormal returns of private equity by minimizing the distance between the model expected dividends and the cross-section of observed dividends over time. Our methodology generates asymptotically consistent estimates and we confirm its validity via a detailed Monte Carlo simulation in which all model parameters are randomly distributed, and the individual dividend rates of each investment are further correlated with the market returns and dependent on the individual risk and return profile of each investment. We apply our model to two research-quality datasets containing the net cash flows of over 1,000 mature buyout and venture capital funds, and the gross cash flows of almost 15,000 portfolio company investments. This allows us to study the risk and return characteristics at each level individually. Our estimations show that that the exposure to systematic risk is notably higher than previously estimated and widely assumed, with beta coefficients ranging from 2.5 to 3.1. While carried interest provisions reduce the exposure to systematic risk, we find that management fees effectively offset this effect. Our estimations also suggest that buyout deals have generated gross alphas of around 5% per annum, relative to the total returns of the S&P 500, whereas venture capital investments have generated alphas of more than 10%. As expected, both management fees and carried interests have a negative effect on the abnormal returns delivered to institutional investors. For buyout funds, annual alphas are slightly negative but statistically insignificantly different from zero. Net alphas of venture capital funds remain positive and range from 2% to 5%. Relative to the total returns of the Fama-French all U.S. stock market index, we find slightly lower market betas around 2.4 and consistently positive net alphas for both buyout and venture capital funds. In all estimations, our model shows excellent goodness-of-fit properties, explaining up to 95% of the cross-sectional variation in the observed monthly dividends.

Keywords: private equity, venture capital, systematic risk, abnormal return

JEL Classification: C51, G12, G23, G24

Since inception in the early 1980s, institutionalized private equity has grown to a multi-trillion dollar asset class spanning globally. Institutional investors such as pension funds and endowments commit up to 20% of their capital to private equity funds. Investments by financial sponsors account for up to 40% of M&A activity in the U.S.[1]

Despite three decades of operations and a material economic impact, the evidence on the systematic risk of private equity funds and their underlying company investments is still inconclusive.[2] Beside the general challenge of obtaining large scale sample data on private equity, the main reason is the illiquid nature of the asset class - neither the interests in a private equity fund, nor its portfolio companies are typically traded. Absent regular valuations in an efficient market, however, the application of standard econometric approaches is infeasible.

This paper aims to overcome the challenge of both unobservable market prices and data constraints. We develop a novel econometric approach to estimate the systematic risk and abnormal returns of illiquid assets based only on their observable cash flows. Our model assumes that the returns of a private equity investment are generated by the standard market model, and that the dividends from the investment occur at a stochastic, yet increasing rate from its unobservable interim values until the investment finally liquidates.[3] Using the Generalized Method of Moments (GMM), we then estimate the systematic risk and abnormal returns of private equity by minimizing the distance between the model expected dividends and the cross-section of observed dividends over time.

Our methodology generates asymptotically consistent estimates and we confirm its validity via a detailed Monte Carlo simulation in which all model parameters are ran-

[1]Estimations based on data from Preqin (on fundraising and asset allocation) and Thomson (on M&A activity).

[2]For example, estimations for the beta of buyout funds range from 0.7 to 1.3 - given the high levels of debt employed in leveraged buyouts, these numbers appear low. Estimations for the beta of venture capital vary within a range from 0.4 to 4.7. See Section 1 for a review of the literature.

[3]As we show, this assumption fits very well with the typical bounded lifecycle of private equity investments, for which dividends are low in the beginning and increase over time, with deals and funds eventually liquidating.

domly distributed, and the individual dividend rates of each investment are further correlated with the market returns and dependent on the individual risk and return profile of each investment. Even under more conservative assumptions, simulating widely dispersed parameters and highly correlated dividends, the model closely estimates the original risk and return characteristics of the investments.

We apply our model to two research-quality datasets containing the net cash flows of over 1,000 mature private equity funds with vintage years from 1980 to 2001, and the gross cash flows of almost 15,000 portfolio company investments by funds from this period. This allows us to study the risk and return characteristics at each level individually.

The estimations show that leveraged buyout deals and funds have an exposure to systematic risk that is notably higher than widely assumed. At both levels we find beta coefficients between 2.7 and 3.1, relative to the S&P 500, that are highly statistically significant. These numbers are in contrast to those in the established literature which estimate buyout betas in a range of 0.7 to 1.3. For venture capital investments we find betas between 2.5 and 2.9 that are also highly significant. These numbers are similar to some of the more recent studies that explicitly correct for selection bias in round-to-round valuations of venture-backed companies.[4]

We also find that the beta coefficient hardly changes between the level of company investments by the fund manager and the level of fund investments by limited partners. While carried interest provisions reduce the exposure to systematic risk, management fees partly offset this effect. Since only about half of all private equity funds clear the hurdle rate and share a part of their gross profits with the fund manager, we observe almost the same betas at the deal and the fund level.

The estimations also show that the abnormal returns of both buyout and venture

[4] See Cochrane (2005), Ewens (2009), and Korteweg and Sorensen (2010) for selection bias adjustments at the deal level. Driessen et al. (2012) find a similar beta as Korteweg and Sorensen (2010) at the fund level, yet an annual alpha that is substantially lower.

capital investments are positive and statistically significant. Buyouts have an annual alpha of around 5% relative to the S&P 500. Venture capital investments have an alpha of more than 10% per annum. These are meaningful excess returns, in particular compared to those observed in regular M&A transactions, that must be the result, among others, of the following sources: Fund manager skill (i.e., advising the management, exploiting market inefficiencies, or receiving credit as a preferred investor in case of venture capital), a compensation for illiquidity which already starts at the level of privately-held companies, or tax savings due to the high levels of debt in case of buyouts.

At the fund level, both management fees and carried interest have a negative effect on excess returns. For buyout funds we find an average alpha that is slightly negative but statistically insignificantly different from zero. This observation is in line with the theoretical literature predicting that excess returns are widely captured by the fund manager, while fund investors are left with returns similar to the overall market. Notable is the fact that the average buyout fund does not seem to compensate limited partners for illiquidity, though. In contrast to buyout funds, the alpha of venture capital funds is 2.1% per annum for all funds, and 4.6% for the sub-sample of U.S. funds. These positive excess returns are driven by the success of venture capital throughout the 1990s, in particular in the U.S.

In all estimations, our model shows excellent goodness-of-fit properties, explaining up to 95% of the cross-sectional variation in the observed monthly dividends. The coefficient of determination is higher for buyout investments due to the lower level of idiosyncratic risk compared to the venture-segment. Similarly, we observe a higher R^2 at the level of funds due to their diversification across a number of companies.

In the next section, we review the existing literature on the systematic risk and returns of private equity. Section 2 introduces our econometric approach and examines its accuracy in Monte Carlo simulations. Section 3 describes the two datasets of private equity cash flows that we use in the empirical section. Section 4 provides the empirical results of our estimations. The final section concludes.

1 Related Literature

This paper's distinctive contribution is to develop a new econometric approach to estimate the systematic risk and abnormal returns of private equity based only on the observable investment cash flows. After confirming the validity of the model in a number of Monte Carlo simulations, we apply the approach to two private equity datasets, containing the detailed cash flows of over 1,000 mature buyout and venture capital funds and almost 15,000 buyout and venture capital investments.

The related academic literature comprises studies investigating the risk and return profile of private equity at the level of individual company investments, and at the level of institutional investments in private equity funds. In the following, we present the main (one-factor) results of those studies. Table 1 further provides a structured overview.

At the level of individual portfolio companies, Gompers and Lerner (1997) mark-to-market a sample of 97 venture capital investments by a single fund manager and estimate betas of 1.1 to 1.4 and an annual alpha of 8.0%. Peng (2001), Woodward and Hall (2003), Hwang et al. (2003), Cochrane (2005), Ewens (2009), and Korteweg and Sorensen (2010) all use disclosed valuation data from multiple rounds of venture capital investments into the same companies and at final exit. Round-to-round valuation data provide the opportunity to construct short time series of returns and apply standard regression methods. On the flip side, this approach requires adjustments for missing investment rounds and an assessment of selection bias, as only companies that perform well appreciate multiple funding rounds and are eventually exited.[5]

Peng (2001), Woodward and Hall (2003), and Hwang et al. (2003) use these round-to-round returns to construct a venture capital index in the first place. Peng (2001) regresses monthly, quarterly and annual returns of his venture capital index against the S&P 500, and estimates betas of 1.3, 1.9 and 2.4, as well as annual alphas of around

[5]Consequently, explicit assumptions on the shape of the probability distribution of returns and on the selection process have to be made. Cochrane (2005), for example, assumes returns of venture capital investments to follow a log-normal distribution.

-1.0%. Against the Nasdaq index, the author estimates betas of 0.8, 1.1 and 4.7, as well as alphas ranging from -3.8% to 15.8%. Woodward and Hall (2003) estimate a beta of 0.9 relative to the Nasdaq index using monthly returns, as well as an alpha of 8.9% per annum. Hwang et al. (2003) find betas from 0.4 to 0.6 against both the S&P 500 and the Nasdaq index using monthly returns, as well as alphas from 0.9% to 3.5%.

Cochrane (2005) focuses extensively on the correction of selection bias and estimates betas between 0.6 and 1.2 against the S&P 500 and the Nasdaq index, and corresponding alphas ranging from 35% to 45%. Korteweg and Sorensen (2010) additionally focus on the valuation and return path between observations and find betas that average to 2.8 against the combined NYSE, Amex and Nasdaq (NAN) index, and alphas of around 47% per annum. Ewens (2009) also accounts for non-normal features of the returns distribution and finds a beta of 1.7 and an annual alpha of 40%.

Based on the sample of deal-level cash flow data, we find betas for venture capital investments that are substantially higher than those of public equities. With 2.5 to 2.7 our betas range at the upper end of previous estimates, yet in line with those of the more recent studies that explicitly correct for selection bias. Our corresponding alphas are between 11.3% and 15.1% per annum, relative to the S&P 500, which is lower than the 35% to 50% found in more recent studies.

In contrast to venture capital investments, buyout-backed companies rarely go through multiple valuation rounds. Franzoni et al. (2012) regress M-IRRs of a sample of realized buyout investments against the NAN index.[6] The authors find a beta of 1.0 and an alpha of 9.3% per annum. In early work, Axelson et al. (2013) regress regular IRRs of buyout investments against the NAN index and find betas between 2.2 and 2.4, as well as alphas ranging from 8.3% to 8.6% per annum.

For buyout investments, we find betas that are substantially higher than those of public equities. Our beta estimates range from 2.7 to 3.1, relative to the S&P 500, with

[6]The M-IRR is a smoothed version of an investment's original IRR, with interim cash flows being capitalized by the market returns into the investment's initial or final cash flow.

corresponding alphas between 4.8% and 5.1% per annum.

At the fund level, Jones and Rhodes-Kropf (2003) construct time series of returns based on quarterly net asset values reported by the fund managers. For a sample of venture capital and buyout funds, the authors estimate a beta of 0.6 and 0.2, respectively, relative to the NAN index. Adding four lagged quarters of market returns, betas increase to 1.8 for venture and 0.7 for buyout funds. In an updated version of the study, Ewens et al. (2013) find about the same beta for buyout funds, but a somewhat lower beta for venture capital funds (0.4 in the base case, and 1.2 incl. lagged market returns). Estimations for the annual alpha of buyout and venture funds are 1.2% and -0.2%, respectively. Woodward (2009) uses the same approach for a series of time-weighted returns of buyout and venture capital funds, and further controls for auto-correlation. The author estimates a beta for buyout funds of 1.0 and an annual alpha of 5.8%. The beta for venture capital funds is, again, higher at 2.2, with a corresponding alpha of 2.1% per annum.

Driessen et al. (2012) modify the standard IRR equation to accomodate a market model, and use GMM to estimate the unknowns. For buyout funds from 1980 to 1993, the authors find a beta of 1.3, as well as an annual alpha of -4.8%. Venture capital funds show a beta of 2.7, as well as an alpha of -12.3%. Jegadeesh et al. (2009) study the risk and returns of publicly listed private equity funds and publicly listed funds of (unlisted) private equity funds. Relative to the S&P 500 index these public vehicles have a beta of 1.0 and 0.7, respectively, as well as alphas of -2.3% and -3.9% per annum. Results against the MSCI World index are very similar.

At the fund level, which is the one of particular importance to institutional investors, our results differ notably. Based on our sample of cash flow data, we estimate a beta of 2.8 for all buyout funds. For the sub-sample of U.S. buyout funds, we find a beta of 2.7. These valules are higher than the 0.2 to 1.3 found in earlier studies which suggested that the market risk of buyout funds is, at most, slightly higher than the risk of public equities. Our corresponding alphas are slightly negative with -1.0% and

-0.7%, respectively. For venture capital funds, we estimate market betas of 2.9 and 2.7 for the sample of all venture capital funds and the sub-sample of U.S. venture funds, respectively. These numbers range at the upper end of previous estimates. Yet, we find consistently positive and significant alphas at 2.1% and 4.6%.

In a broader context, our paper is also related to prior work comparing the absolute returns of private equity funds to those of public markets. Studies by Ljungqvist and Richardson (2003), Robinson and Sensoy (2011), Harris et al. (2013), and Higson and Stucke (2013) all find that buyout and venture capital funds have delivered higher absolute returns than the S&P 500 performance index.[7] Although the studies do sensitivity tests for amplified market returns, they cannot explicitly estimate beta and alpha via the standard PME approach.

2 Estimation Methodology

Estimating the systematic risk and abnormal performance of private equity is inherently difficult due to the illiquid nature of the asset class. Neither private equity funds, nor their portfolio companies are typically traded in an efficient market. Hence, in the absent of regular valuations and periodic returns, standard econometric approaches to estimate the risk and return characteristics cannot be applied.

In this section we develop a novel estimation methodology that overcomes the problem of missing valuations as it relies only on the observable cash flows from private equity investments. In the first part, we define the assumptions underlying the estimation framework. Next, we present the estimation function and outline its derivation. We then provide evidence for the consistency of the derived methodology and test for potential biases of the estimates using a detailed Monte Carlo simulation experiment.

Note that our estimation methodology can be applied to both, investments in private

[7] Earlier studies by Kaplan and Schoar (2005), and Phalippou and Gottschalg (2009) do not find excess returns against the S&P 500 (see Stucke (2011) for an explanation).

equity funds by institutional investors, as well as investments in portfolio companies by the funds themselves. In the following, we use the term *investment* referring to the investment cash flows at either level.

2.1 Assumptions

The estimation framework is based of four main assumptions that are outlined and discussed in the following.

Assumption 2.1 *(Value Dynamics) The dynamics of the value $V_{i,t}$ of an investment i are given by*

$$V_{i,t} = V_{i,t-1}(1 + R_{i,t}) + \Delta T_{i,t} - \Delta D_{i,t}, \tag{2.1}$$

where $R_{i,t}$ is the period t return of investment i, $\Delta T_{i,t}$ denotes capital inflows (i.e., investments) that occur in period t, and $\Delta D_{i,t}$ denotes capital outflows (i.e., dividends) that occur in period t.[8]

Specification (2.1) is straightforward. The first term $V_{i,t-1}(1 + R_{i,t})$ states that the change in value of a private equity investment is, at first, the result of the performance of the investment already in place. In addition, the second and third term state that the value is increased by capital inflows (i.e., investments) $\Delta T_{i,t}$ and decreased by capital outflows (i.e., dividends) $\Delta D_{i,t}$. Including $\Delta T_{i,t}$ and $\Delta D_{i,t}$ into equation (2.1) takes into account that private equity investments typically involve several investment rounds and generate substantial intermediate dividends during their bounded lifecycle. As an investment is gradually exited, the dividends (whether in the form of cash or marketable securities) are directly distributed to the investors. Therefore, dividends simply decrease the investment value and there is no need to impose any assumption about the reinvestment of cash flows.

[8] Note that by period t we refer to the time period ranging from $t - 1$ to t.

Assumption 2.2 *(Dividends) Dividends or cash outflows $\Delta D_{i,t}$ of investment i in period t occur at a non-negative rate from the value of the investment at time t, i.e*

$$\Delta D_{i,t} = \delta_{i,t} V_{i,t-1}, \tag{2.2}$$

where $\delta_{i,t}$ is the period t dividend rate of investment i. We assume that the dividend rate $\delta_{i,t}$ is a stochastic function given by

$$\delta_{i,t} = 1 - \exp[-(\delta - \frac{1}{2}\,\sigma_{\delta,i}^2 + \sigma_{\delta,i}\, z_{i,t})t], \tag{2.3}$$

where $\delta > 0$ is a common factor for all investments, $\sigma_{\delta,i}$ is the volatility of the dividend rate of investment i, and $z_{i,t}$ is an i.i.d. standard normal random variable, i.e., $z_{i,t} \sim N(0,1)$.

Equation (2.2) represents the standard approach in the literature to model the dynamics of dividend paying assets.[9] It states that a fraction $\delta_{i,t}$ of the asset value of investment i at the beginning of the period t (i.e., $V_{i,t-1}$) is paid out in period t. Equation (2.3) adds the assumption that the rate $\delta_{i,t}$ is a stochastic function of time t. Taking expectations of (2.3) yields

$$E[\delta_{i,t}] = 1 - \exp(-\delta t), \tag{2.4}$$

which shows that the dividend rate increases in a non-linear way from zero to one over time.[10] From an economic perspective, this specification is reasonable as it reflects well the typical lifecycle of private equity investments, where dividends are low in the beginning and increase over the bounded life of an investment as it gets gradually realized and eventually liquidated. In addition, equation (2.3) further accounts for the fact that the dividend rate of an investment can change over time by including a stochastic

[9] See, for example, Björk (1998), Chapter 11.

[10] Note that we have tested various linear and non-linear functions of time for the dividend rate in our empirical implementation. The function given in equation (2.3) provided the best fit with the empirical data. The excellent fit of this specification is also illustrated in Section 4 by the measures of the goodness-of-fit.

component to the specification.

Assumption 2.2 is important for the estimation methodology. It enables us to overcome the problem of missing market valuations of private equity investments. This holds because the equation gives a direct connection between the unobservable market values $V_{i,t-1}$ and the observable dividends $\Delta D_{i,t}$ of the investment. Moreover, the dividend rate δ provides information on the average speed at which investments are liquidated. In this context, the parameter δ can be seen as a measure of cash flow liquidity of private equity investments.

After substituting equation (2.2) into (2.1), the value dynamics can be expressed by

$$V_{i,t} = V_{i,t-1}(1 + R_{i,t} - \delta_{i,t}) + \Delta T_{i,t}. \tag{2.5}$$

Assumption 2.3 *(Return Dynamics) The return $R_{i,t}$ of investment i in period t is generated by a single-factor market model of the form*

$$R_{i,t} = r_{f,t} + \alpha_i + \beta_i(R_{M,t} - r_{f,t}) + \epsilon_{i,t}, \tag{2.6}$$

for which $r_{f,t}$ and $R_{M,t}$ are the period t returns on the risk-free asset and on the market portfolio, respectively. Variable $\epsilon_{i,t}$ is an i.i.d. disturbance term with zero mean that is uncorrelated with the market returns for all t. In addition, it holds that disturbance terms of different investments are also uncorrelated.

This single-factor market model is the standard specification used in the performance measurement literature.[11] For a constant β_i over time the intercept α_i equals Jensen's alpha, which directly measures a manager's selection or micro-forecasting ability (see Jensen (1968) and Fama (1972)).

[11] In the private equity literature, e.g., Cochrane (2005) or Korteweg and Sorensen (2010) use similar specifications.

Assumption 2.4 *(Cross-Sectional Restrictions) Investments can be categorized according to their strategy (i.e., the sub-asset class like venture capital or buyout), their geographic focus, etc. It is assumed that investments from a certain category have a similar exposure to systematic risk β and abnormal returns α.*

The economic rationale behind this assumption is that the performance of a given investment type is subject to the same systematic risk together with an idiosyncratic component. The assumption of a similar systematic risk β and abnormal returns α is also used, for example, by Cochrane (2005).

2.2 Estimation Function

In the following, we present the estimation function. Given is a sample of N investments for which Assumptions 2.1 to 2.4 hold. For each of the $i = 1, \ldots, N$ investments we can observe a stream of periodic cash flows, i.e., periodic capital inflows $\Delta T_{i,1}, \ldots, \Delta T_{i,K}$ and dividends $\Delta D_{i,1}, \ldots, \Delta D_{i,K}$ over the total observation period of length K. In order to make investments of different sizes comparable, capital inflows and dividends are scaled on the basis of total invested capital. In addition, we can also observe the market return $R_{M,k}$ and the risk-free rate $r_{f,k}$ in each period $k = 1, \ldots, K$. Under these specifications, the parameters α, β, and δ can be estimated by an *non-linear least squares* approach.

Theorem 2.1 *Given a sample of N investments and a total observation period of length K, model parameters α, β, and δ can be estimated by*

$$\min_{\alpha,\beta,\delta} \sum_{k=1}^{K} \left(\Delta D_k - E[\Delta D_k]\right)^2 , \tag{2.7}$$

where ΔD_k are the average dividends of the N sample investments in period k, i.e.,

$$\Delta D_k = \frac{1}{N} \sum_{i=1}^{N} \Delta D_{i,k}, \tag{2.8}$$

and $E[\Delta D_k]$ are the expected dividends of the sample investments in period k, given by

$$E[\Delta D_k] = \delta_k \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{k-1} \Delta T_{i,j} \prod_{s=j+1}^{k-1} [1 + r_{f,s} + \alpha + \beta(R_{M,s} - r_{f,s}) - \delta_s], \tag{2.9}$$

for the expected dividend rate $\delta_k = 1 - \exp(-\delta k)$.

The idea underlying this estimation is straightforward. We estimate the parameters such that the distance between the empirically observed dividends (2.8) and the expected dividends by the model (2.9) is minimized. Based on this approach, the systematic risk β and the abnormal returns α of private equity investments can be estimated, even though the intermediate market values of the investments are unobservable.[12] In addition, this approach allows us to estimate the dividend rate δ, which can be interpreted as a measure of cash flow liquidity of the investments.

The estimator (2.7) can be rewritten as a Generalized Method of Moments (GMM) estimator with moment condition

$$E[\Delta D_k - E[\Delta D_k]] = 0. \tag{2.10}$$

Given that the sample size N and the observation period K are sufficiently large, this generates asymptotically consistent estimates.[13]

Note that the estimation approach in Theorem 2.1 does not require any assumption on how dividends are reinvested by the investor, i.e., the estimation results are not affected by whether an investor reinvests dividends (e.g., in the stock market index or in government bonds). This assures that we only measure the systematic risk and abnormal returns of a pure private equity investment, and not a combined investment into private equity and other asset classes.

[12]Note that there is no closed-form solution to the non-linear least squares problem defined in (2.7). Instead, numerical algorithms are used to find the value of the parameters α, β and δ which minimize the objective.

[13]See, for example, Singleton (2006) for a detailed discussion of the Generalized Method of Moments.

2.3 Derivation

To derive the estimation approach presented in Theorem 2.1, first note that under equation (2.5) (**Assumption 2.2**) expected dividends of an investment i in period k are given by

$$E[\Delta D_{i,k}] = E[\delta_{i,k} V_{i,k-1}] = \delta_k E[V_{i,k-1}], \tag{2.11}$$

with $\delta_k \equiv E[\delta_{i,k}] = [1 - \exp(-\delta k)]$. The second equality in equation (2.11) holds because we assume that the dividend rate and value dynamics are uncorrelated. As outlined above, for each investment i one can only observe its stream of cash flows (i.e., capital inflows and dividends). Therefore, the value $V_{i,k-1}$ that enters the expectation on the right hand side of (2.11) is unobservable. However, it can be expressed in terms of observable variables. A starting point for this is the specification of the value dynamics of a private equity investment (**Assumption 2.1 and 2.2**) given by

$$V_{i,k} = V_{i,k-1}(1 + R_{i,k} - \delta_{i,k}) + \Delta T_{i,k}. \tag{2.12}$$

While the value still enters the right hand side of the equation, it can be resolved as follows. First, note that $V_{i,0} = 0$ holds for all investments, i.e., the value of all investments is zero when they are set-up and no capital inflow has yet occurred. Based on this condition, we can recursively solve equation (2.12) for the value at any point in time. It turns out

$$
\begin{aligned}
V_{i,0} =& 0, && (2.13)\\
V_{i,1} =& \Delta T_{i,1}, && (2.14)\\
V_{i,2} =& V_{i,1}(1 + R_{i,2} - \delta_{i,2}) + \Delta T_{i,2} &&\\
=& \Delta T_{i,1}(1 + R_{i,2} - \delta_{i,2}) + \Delta T_{i,2}, && (2.15)\\
V_{i,3} =& V_{i,2}(1 + R_{i,3} - \delta_{i,3}) + \Delta T_{i,3} &&\\
=& \Delta T_{i,1}(1 + R_{i,2} - \delta_{i,2})(1 + R_{i,3} - \delta_{i,3}) &&\\
&+ \Delta T_{i,2}(1 + R_{i,3} - \delta_{i,3}) + \Delta T_{i,3} && (2.16)\\
\vdots &&&\\
V_{i,k} =& \sum_{j=1}^{k} \Delta T_{i,j} \prod_{s=j+1}^{k} (1 + R_{i,s} - \delta_{i,s}). && (2.17)
\end{aligned}
$$

Substituting the return dynamics (**Assumption 2.3**) from equation (2.6) into (2.17) yields

$$V_{i,k} = \sum_{j=1}^{k} \Delta T_{i,j} \prod_{s=j+1}^{k} [1 + r_{f,s} + \alpha + \beta(R_{M,s} - r_{f,s}) + \epsilon_{i,s} - \delta_{i,s}]. \tag{2.18}$$

This equation states that the value of an investment i in period t is the sum of its compounded capital inflows, with compounding being carried out by the single-factor market model that is corrected for the periodical dividends. Taking expectations on both sides of equation (2.18) gives

$$E[V_{i,k}] = E\left\{\sum_{j=1}^{k} \Delta T_{i,j} \prod_{s=j+1}^{k} [1 + r_{f,s} + \alpha + \beta(R_{M,s} - r_{f,s}) - \delta_{s}]\right\}. \tag{2.19}$$

Note that the error term $\epsilon_{i,s}$ does no longer appear in the expectation on the right hand side of equation (2.19). This follows as $\epsilon_{i,s}$ has zero expectation. Moreover, the error terms $\epsilon_{i,s}$ and the market returns $R_{M,t}$ are uncorrelated for all t and s, and the expectations of cross-products of the form $\epsilon_{i,s}\epsilon_{i,t}$ (as well as higher-order cross-products) are equal to zero for $s \neq t$. Additionally, we have replaced $\delta_{i,s}$ by its expectation $\delta_s \equiv E[\delta_{i,s}] = 1 - \exp(-\delta s)$, assuming that the dividend rate is independently distributed.[14]

Inserting (2.19) into equation (2.11), the expected dividends of investment i in period k are represented by

$$E[\Delta D_{i,k}] = \delta_k E\left\{\sum_{j=1}^{k-1} \Delta T_{i,j} \prod_{s=j+1}^{k-1} [1 + r_{f,s} + \alpha + \beta(R_{M,s} - r_{f,s}) - \delta_s]\right\}. \tag{2.20}$$

The expectation on the right hand side of the equation cannot be evaluated directly, as the *expected* capital inflows of a specific investment i are not observable. However, we assume a sample of N investments that satisfy the cross-sectional restrictions given in **Assumption 2.4**. Under this assumption, and given that the sample size N is sufficiently large, the expectation in (2.20) can be approximated by averaging across all N investments, i.e.,

$$E[\Delta D_k] = \delta_k \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{k-1} \Delta T_{i,j} \prod_{s=j+1}^{k-1} [1 + r_{f,s} + \alpha + \beta(R_{M,s} - r_{f,s}) - \delta_s]. \tag{2.21}$$

$E[\Delta D_k]$ gives the expected dividends of the sample investments predicted by the model for period k. In addition, as the dividends of the sample investments can be observed directly, we can calculate the average sample dividends in each period k, i.e.,

$$\Delta D_k = \frac{1}{N} \sum_{i=1}^{N} \Delta D_{i,k}. \tag{2.22}$$

$E[\Delta D_k]$ in equation (2.21) gives the expected model dividends for period k, whereas ΔD_k in equation (2.22) gives the average of the empirically observed dividends of the sample investments for the same period. Given these two types of information, the idea is to estimate the parameters α, β and δ by minimizing the distance between the

[14]In the Monte Carlo Simulation we show that under more general specifications, assuming various cross-dependencies of the dividend rate, the precision of the estimates remains at a very high level.

observed dividends and the model expected dividends over time. This can be done by a *non-linear least squares* estimation. For a total observation period of length K, the goal function for the estimation is then given by the optimization problem

$$\min_{\alpha,\beta,\delta} \sum_{k=1}^{K} \left(\Delta D_k - E[\Delta D_k]\right)^2, \tag{2.23}$$

where $E[\Delta D_k]$ and ΔD_k are given by Equations (2.21) and (2.22), respectively.

2.4 Monte Carlo Simulation

As argued above, our estimation methodology generates asymptotically consistent estimates of the model parameters when the sample size N is sufficiently large. To validate the consistency of the derived methodology and to test for potential biases of the estimation, we conduct a detailed Monte Carlo simulation experiment in the following.

2.4.1 Implementation

To implement the Monte Carlo simulation, we start by modelling the cash flow dynamics of private equity investments. We assume that each investment has a total size given by C. Capital inflows of investment i in period t are $\Delta T_{i,t}$, and cumulated capital inflows up to time t are denoted by $T_{i,t}$. We assume that capital inflows in each period t occur at some constant, non-negative rate from the remaining capital at the beginning of the period, $C - T_{i,t-1}$. Hence, the dynamics of the capital inflows can be described by

$$\Delta T_{i,t} = \gamma(C - T_{i,t-1}), \tag{2.24}$$

for which $\gamma > 0$ denotes the constant investment rate that is the same for all investments.[15]

[15] This deterministic modeling framework for the capital inflows of private equity investments is similar to Takahashi and Alexander (2002). It can easily be extended to a stochastic setting, assuming that γ

In the following simulations, we choose model parameters that closely match the characteristics of venture capital investments at the *deal level*. Since deal level returns have a higher idiosyncratic risk than fund level returns, model parameters are more difficult to estimate at the deal level. Hence, if the estimation methodology produces consistent estimates at the deal level, it should do so at the fund level as well. Motivated by the same argument, we choose the venture segment for our experiment, since venture capital investments typically involve higher levels of idiosyncratic risk compared to buyout deals.

Consistent with the sample of venture capital investments used in the empirical analysis, we set $\gamma = 0.8$ per annum. This implies that 80% of the total investment size C is invested within the first year.

Dividends of the investments occur according to the specification (see **Assumption 2.2**)

$$\Delta D_{i,t} = \delta_{i,t} V_{i,t-1}, \tag{2.25}$$

for which the dividend rate $\delta_{i,t}$ is a stochastic function given by

$$\delta_{i,t} = 1 - \exp[-(\delta - \frac{1}{2}\,\sigma_\delta^2 + \sigma_\delta\, z_{i,t})t], \tag{2.26}$$

where $\delta > 0$ and the volatility σ_δ are the same for all investments. Consistent with the estimation results for venture capital deals, we set $\delta = 0.18$ per annum. In the base case, we assume $\sigma_\delta = 0$ and extend this to a stochastic setting later on.

The unobservable value dynamics of the investments are given by (see **Assumptions 2.1 and 2.3**)

$$V_{i,t} = V_{i,t-1}[1 + r_f + \alpha + \beta(R_{M,t} - r_f) + \epsilon_{i,t}] + \Delta T_{i,t} - \Delta D_{i,t}, \tag{2.27}$$

for which the returns are generated by the standard market model, in which $R_{M,t}$ is the

follows some stochastic process. See, for example, Malherbe (2004).

market return in period t and r_f is the risk-free rate.

Given the large idiosyncratic volatility of the returns of venture capital investments, assuming a normal distribution for $R_{M,t}$ and $\epsilon_{i,t}$ would generate returns that could fall below -100% with non-negligible probability. We therefore use shifted log-normal distributions for both variables (see Driessen et al. (2012)). For the market return $R_{M,t}$ we assume an i.i.d. shifted log-normal distribution over time, i.e., $R_{M,t} = \exp(X) + c_M$, where X is normally distributed with mean μ_M and variance σ_M^2, and c_M is a constant. Similarly, we assume $\epsilon_{i,t}$ is i.i.d. shifted log-normal across investments and over time. Finally, we assume that the risk-free rate is constant at $r_f = 0.05$ per year.

For the market return, we match the S&P 500 total return index over the sample period from 1980 to 2008. We set $c_m = -0.2$ which is close to the minimum monthly return of -21.76% during this period. The mean μ_M and variance σ_M^2 of the shifted log-normal distributed market returns are calculated by

$$\mu_M = \ln\left[\frac{(E(R_{M,t}) - c_M)^2}{\sqrt{Var(R_{M,t}) + (E(R_{M,t}) - c_M)^2}}\right], \tag{2.28}$$

$$\sigma_M^2 = \ln\left[\frac{Var(R_{M,t})}{(E(R_{M,t}) - c_M)^2} + 1\right], \tag{2.29}$$

for which $E(R_{M,t})$ and $Var(R_{M,t})$ are the average and the variance of the arithmetic monthly S&P 500 returns, respectively. It turns out that $\mu_M = -1.58$ and $\sigma_M = 0.18$ per month.

For the idiosyncratic error $\epsilon_{i,t}$, parameter c_ϵ is fixed such that returns cannot fall below -100%, i.e., $r_f + \alpha + \beta(c_M - r_f) + c_\epsilon = -1$ holds. In the base case the 'true' alpha is set to zero and the 'true' beta is set to 2.5. This gives $c_\epsilon = -0.43$. Similar to above, the mean μ_ϵ and variance σ_ϵ^2 of the shifted log-normal distributed error terms can be calculated by [16]

$$\mu_\epsilon = \ln\left[\frac{c_\epsilon^2}{\sqrt{Var(\epsilon_{i,t}) + c_\epsilon^2}}\right], \tag{2.30}$$

$$\sigma_M^2 = \ln\left[\frac{Var(\epsilon_{i,t})}{c_\epsilon^2} + 1\right], \tag{2.31}$$

for which $Var(\epsilon_{i,t})$ is the idiosyncratic variance of the private equity investments. We set $Var(\epsilon_{i,t}) = 0.16$ per month, which closely matches the monthly idiosyncratic volatility of 41% as reported by Korteweg and Sorensen (2010) for venture capital deals. Using this result, we get $\mu_\epsilon = -1.17$ and $\sigma_\epsilon^2 = 0.80$.

Table 2 summarizes all model parameters for the base case of our Monte Carlo simulation experiment.

2.4.2 Simulation Results

The simulation is carried out using 5,000 iterations, i.e., parameters are estimated from a sample of 5,000 investments.[17] The simulation is repeated 1,000 times and the mean, standard deviation, median, and interquartile range of the 1,000 estimated pairs of parameters is calculated.

Table 3 presents the estimation results from the Monte Carlo simulation. The results show that all parameters can be estimated with very high precision. In the Base Case in Panel A, the methodology estimates an average alpha of -0.01% per month, an average beta of 2.51, and an average delta of 0.18 across simulations. These numbers are very close or equal to the true parameters of alpha=0%, beta=2.5, and delta=0.18. Note that we have repeated the simulations using different pairs of the true parameters alpha, beta, and delta. In all cases, the precision of the resulting estimates remained at the same high level.

[16] Note that $E(\epsilon_{i,t}) = 0$ holds by definition.

[17] This number is similar in magnitude to the sample sizes at the deal level used in the empirical analysis.

Moreover, Figure 1 indicates that our estimation approach does not suffer from any numerical problems.[18] The figure shows the objective function space of the parameters alpha and beta for the optimal value of delta. We can observe that the objective function exhibits a well defined and easy to determine minimum at the true values of alpha and beta.

The estimation methodology relies on the fact that idiosyncratic shocks in investment returns average out over large samples. Consequently, the precision of the estimation depends on the idiosyncratic volatility of the investments and on the size of the sample. Panel A further shows how a change in the idiosyncratic volatility affects the accuracy of the estimates. We assume two different cases, Low and High, with an idiosyncratic volatility of 20% per month (i.e., about 69% p.a.) and 60% per month (i.e., about 208% p.a.), respectively. The results show that the level of precision further increases when the idiosyncratic volatility is relatively lower, whereas the precision slightly decreases in case of larger idiosyncratic shocks. Nevertheless, given the very large idiosyncratic shocks in the high volatility case, the accuracy of the resulting estimates is still at a high level for all model parameters.

The last two columns in Panel A show how a change in the sample size affects the precision of the parameter estimates. As we would expect, the estimators converge towards the true values with further precision as the sample size increases, whereas the precision slightly decreases as the sample size is reduced. This behavior provides further evidence of the statistical consistency of our estimation methodology.

In Panel B, we relax the assumption that the dividend rate is deterministic, and examine how different cross-dependencies between the dividend rate and other model variables affect the precision of the estimates. First of all, we make the dividend rate stochastic by setting $\sigma_\delta = 0.05$. In the Base Case, the dividend rate is independently

[18]Note that this is unfortunately the case for the earlier cash flow-based approach by Driessen et al. (2012), since their objective function is not globally convex. As acknowledged by the authors "Using this method is therefore problematic in practice, especially if one does not have good starting values for the optimization algorithm.". We contrast the corresponding objective function space of their approach in Figure 2.

distributed. In the next two columns, we add the property that the dividend rate is correlated with lagged market returns over the last six months. The results show that a positive correlation, i.e., higher proportional dividends following strong public markets, leads to slightly upward biased estimates of beta, whereas alpha and delta can still be estimated with very high precision. However, with less than 3%, the bias of the estimated beta is rather small given the relatively large correlation of 0.5.

In the last two columns, we analyze to what extent cross-dependencies between the dividend rate and the parameters alpha and beta can affect the precision of the estimation. We first create a cross-sectional variation in alpha among the simulated sample deals by assuming that alpha is normally distributed with a zero mean and a standard deviation of 8%. This specification generates a variation of alpha among the sample such that about 99% of the deals have alphas being in the interval between -20% and +20%. To incorporate a dependency between alpha and the dividend rate, we make the constant δ in equation (2.26) investment specific by the function $\delta_i = \exp(\alpha_i)\delta$. In this case, investments with a low (negative) abnormal returns α_i will distribute capital slower, which seems reasonable as private equity fund managers typically hold on to their less successful investments. The estimation results show that this specification does not have any material effect on the estimation precision. The average beta is slightly too low, but this bias seems negligible.

In the last column we examine the effect a cross-dependency between beta and the dividend rate. To incorporate a dependency between beta and the dividend rate, we replace the constant δ in equation (2.26) by the function $\delta_i = (\beta/\beta_i)\,\delta$, for which β is the average beta of all sample deals and β_i is the beta of investment i. This specification results in a lower dividend rate of high beta investments, which seems reasonable due to the longer time needed to reestablish a sustainable capital structure and exit the investment. The beta of the sample investments is assumed to be normally distributed with a mean of 2.5 and a standard deviation of 0.4. This results in a variation of beta among the sample such that about 99% of the deals have a beta between 1.5 and 3.5.

Again, the results show only a very small effect on the estimation precision and all estimates are close to the true values.

Overall, the simulations in Panel B confirm that, even though we assume independently distributed dividend rates in our derivation of the estimation methodology, the results do not exhibit any meaningful bias under more general specifications where the dividend rate is correlated with (lagged) market returns and further depends on the parameters alpha and beta of the investments.

3 The Data

We apply our methodology to two research-quality cash flow datasets at the level of private equity investments into companies, and at the level of institutional investments into private equity funds. In the following, we describe the nature of the data and outline the sample distribution.

3.1 Sources of Data

To study the systematic risk and abnormal returns of private equity backed companies, we use cash flow data at the individual deal level. These investment cash flows are by definition gross of management fees and carried interest provisions, and represent the sole investments and realizations between a private equity fund and its portfolio companies. The data has been provided by the Centre of Private Equity Research (Cepres), which maintains one of the largest databases of precisely timed deal level cash flows covering over 30,000 companies. Earlier versions of this dataset have been used by Krohmer et al. (2009), and Cumming et al. (2010) for venture capital investments, as well as Franzoni et al. (2012) for buyout investments. An advantage of this data is that we do not have to adjust for selection bias as the data covers the complete history of company investments by each fund and manager.

To explore the risk and return characteristics from the perspective of investors in private equity funds, we use cash flow data at the individual fund level, which is net of management fees and carried interest payments. This data has been provided by Burgiss, a leading provider of portfolio management software, services, and analytics to limited partners investing in private capital. Burgiss maintains one of the largest databases of precisely timed fund level cash flows, containing over 5,300 private capital funds sourced directly from around 300 limited partners. This fund cash flow data has been used in recent studies by Harris et al. (2013), Brown et al. (2013), and Harris et al. (2013) who confirm its high level of quality. An advantage of this data is that it is not subject to backfilling bias, as it contains the complete portfolios of the institutional investors since program inception.

3.2 Sample Description

For our empirical analysis we use the buyout and venture capital parts of each dataset with a geographic focus on both all funds and deals worldwide as well as the sub-sample of U.S. funds and deals only. To minimize a possible impact by estimated net asset values of unrealized investments, we only use funds with vintage years from 1980 to 2001. Similarly, we only include deals into our analysis that are conducted by funds with vintage years up to 2001.

Table 4 shows the distribution of funds and deals over time. Overall, our analysis includes 1,109 private equity funds and 14,918 company investments. The 1,109 funds split into 413 buyout funds and 696 venture capital funds. The number of buyout funds with a primary focus on the U.S. market is 307 (74%). The number of venture capital funds with a primary focus on the U.S. market is 620 (89%).

As suggested, the geographic focus by a certain fraction of the private equity funds is not limited to a single market. For example, some U.S. buyout funds also invest, to a minor extent, outside the U.S., and vice versa. Similarly, a certain number of buyout fund managers have historically invested a small fraction of their funds into venture or growth capital deals. To maintain comparability between our estimations at the net and the gross level, we categorize all company investments by the primary geographic and sub-asset class focus of their underlying funds.

Out of the 14,918 company investments, 5,715 deals are conducted by funds, raised until 2001, that have a primary focus on buyouts. 9,203 investments are conducted by funds with a primary focus on venture capital. Since the investment period of private equity funds typically stretches over 5 to 6 years, their company investments reach into the mid-2000s. The number of investments by buyout funds with a main focus on the U.S. market is 1,737 (30%). Due to the European background of Cepres, the fraction of international buyout fund managers is disproportionately larger. As we see in the next section, the risk characteristics by U.S. and non-U.S. deals and funds is, however, very similar since buyout investors typically seek to maximize the fraction of debt in their deal structures in either region. The number of investments by venture capital funds with a main focus on the U.S. is 6,374 (69%).

4 Estimation Results

In this section we estimate the exposure of private equity to systematic risk and the corresponding abnormal returns. We use the S&P 500 Total Return Index to proxy the market portfolio and the one-month U.S. Treasury Bill rate as the return of the risk-free asset. In the first part, we estimate beta factors and alpha residuals at the level of private equity investments into companies. In the second part, we follow the perspective of investors in private equity funds and further examine the impact of management fees and carried interest provisions on beta and alpha.

4.1 Deal Level Estimations

To estimate the systematic risk and abnormal returns of buyout and venture capital investments, we use the individual deal level cash flows. These cash flows are gross of management fees and carried interest payments to the fund manager. Panel A of Table 5 presents the estimation results, corresponding standard errors, and measures of the goodness-of-fit of the estimations.

For leveraged buyout investments we find an exposure to systematic risk that is statistically and economically significant. For the full sample of buyout investments we estimate a beta factor of 2.70. For the sub-sample of U.S. buyouts we estimate a beta of 3.15. Both coefficients are statistically significant at the 1% level.

These beta values are in line with what corporate finance theory would predict. For example, between 1990 and 2007, the average fraction of net debt to enterprise value in the S&P 500, representing the market portfolio with a beta of 1.0, has been 13.4%.[19] Over the same period, leveraged buyouts have been funded, on average, with approximately 70% in net debt.[20] Given that the overall purchase price in a leveraged buyout contains certain acquisition related costs (i.e., a takeover premium and transaction fees), these 70% correspond to a hypothetical fraction of net debt in the S&P 500 of at least 85%. Depending on the assumption for the market risk of buyout debt, the resulting equity beta should be above 3.5 immediately following a leveraged buyout. For a successful investment that does not undergo a subsequent recapitalization, this high initial beta then decreases towards 1.6 as the debt-to-equity ratio is improved.[21]

Our estimations for the systematic risk of buyout investments are substantially higher than previously estimated by Franzoni et al. (2012). A possible explanation for their

[19]S&P CapitalIQ and own calculations. Percentage numbers for the Russell 2000, the Russell 3000, and the S&P 1500 are very similar.

[20]S&P Loan Commentary and Data. Axelson et al. (2013) report a fraction of (gross) debt in leveraged buyouts of 75%.

[21]For LBO-backed IPOs, Cao and Lerner (2009) find a post-IPO beta of 1.3. Since most or all of the IPO proceeds are typically used to pay down outstanding debt principal, which is then further reduced in the months and years following the IPO, their beta 'at IPO' would be similar to our calculations.

(one-factor) market beta of 0.95 might be the use of the M-IRR as the dependent variable. In early work, Axelson et al. (2013) use regressions against the regular IRR of buyout investments and estimate betas between 2.2 and 2.4.

With respect to venture capital, we estimate a beta factor of 2.72 for the full sample containing all venture capital investments worldwide. For the sub-sample of U.S. investments we estimate a beta of 2.53. As for buyouts, both coefficients are statistically significant at the 1% level.

From a theoretical perspective, the high systematic risk of venture capital can be explained by the fact that growth opportunities play an important role in the valuation of young companies. Growth opportunities typically include embedded options, which add implicit leverage to this type of, usually, all-equity investments (see Berk et al. (2004), and Bernardo et al. (2007)). As a result, venture capital carries a material market risk and its returns are particularly dependent on the general economic conditions.

Both beta factors are in line with more recent estimations based on round-to-round valuation data that explicitly correct for selection bias (see Korteweg and Sorensen (2010)), but higher than in any previous study.

Our estimations also suggest that both buyout and venture capital investments earn positive abnormal returns. The average alpha of all buyout investments worldwide is 5.1% per annum. For U.S. buyouts we find an annual alpha of 4.8%. With respect to venture capital investments we find an alpha of 11.3% per annum in the full sample, and 15.1% in the sub-sample of U.S. investments. All excess returns are statistically significant at the 1% level. Table 1 shows how our alphas relate to, and differ from, those in earlier deal-level studies.

These positive alphas result from one or more of the following sources: Fund manager skills - these include advising and supporting a company's management, identifying and exploiting market inefficiencies, or receiving credit as a preferred investor in case of venture capital; a compensation for illiquidity due to the private nature of the company investments; tax savings following substantially increased debt tax shields in the case of leveraged buyouts.[22]

Panel B of Table 5 reports the cost of capital and the market model implied expected returns of the investments. Using weighted averages of the monthly market returns and the monthly risk-free rates over the individual periods of the sample investments, the cost of capital according to the CAPM are 18.0% per annum for the full sample of buyout investments, and 20.0% for U.S. buyouts. The cost of capital for all venture capital investments and the sub-sample of U.S. deals are, respectively, 11.4% and 11.1%. The resulting expected returns implied by the market model are 23.2% and 24.8% per annum for buyout investments, and 22.7% and 26.2% for venture capital investments. These estimated expected returns are close to the average gross IRRs of each sample (23.8% and 25.3% for buyouts; 21.2% and 21.9% for venture capital deals).

The estimation methodology also allows drawing inference on the expected cash flows of private equity investments via the estimated dividend rates δ (not shown in the table). The delta factor is 0.17 per annum for both the full sample of buyout investments and the sub-sample of U.S. buyouts. For venture capital, the factors are 0.18 and 0.19, respectively. Figure 3 illustrates the expected dividend flows that follow equation (2.9). For the full sample of buyout investments, the expected dividends are around 16% in the first year, relative to the overall invested capital, and the expected time to break-even is around 3.2 years. The corresponding values for U.S. buyouts are 18% and 3.1 years, respectively. Results for the venture capital segment are similar, with an expected time to break-even of 3.5 years for the full sample of venture investments, and 3.3 years for U.S. venture capital deals.

Figure 3 also shows an excellent fit between the model expected dividends and the empirical observations. This close relationship is quantified by the two measures of the goodness-of-fit in Table 5. With a coefficient of determination, R^2, of 87.2% our model explains a very high degree of the variation in monthly dividends of all buyout

[22] Note that this list is not necessarily exhaustive.

investments. In addition, the root mean squared error, $RMSE$, is as low as 0.0043 per month. For venture capital investments, the measured R^2 is slightly lower at 82.1%. The reason is the higher level of idiosyncratic risk in the venture segment.

4.2 Fund Level Estimations

We now use the fund level cash flow data of institutional investors who are limited partners in private equity partnerships. All cash flows are net of management fees and carried interest payments to the fund manager.

Before presenting estimations for the systematic risk and abnormal returns from the perspective of fund investors, however, we first use these net cash flows to re-engineer the approximate gross cash flows of each fund *before* management fees and carried interests. Our objective is twofold. On the one hand, we want to draw a comparison with the deal level estimations from the previous section to test for consistency.[23] On the other hand, we want to learn about the impact of fees and carried interests on the systematic risk and the remaining alpha for investors.

4.2.1 Results Based on the (Approximated) Gross Cash Flows

To approximate the gross cash flows of the sample of buyout and venture capital funds we adjust their original net cash flows by management fees and carried interest payments. During the investment period (i.e., the first 5 years of a fund's operations), we assume fund management fees to be 2% of committed capital. From year 6 to 10 we assume management fees to step down to 1.5%. We also change the fee basis from committed capital to a fund's remaining net invested capital which we estimate via the ratios of cumulative distributions to total values (see Metrick and Yasuda (2010)). With respect to carried interest payments, we assume a hurdle rate of 8% and a profit sharing agreement of 20%. Table 6 presents the estimation results, corresponding standard errors,

[23]Note that we do not have any information on the overlap between both anonymized dataset.

and measures of the goodness-of-fit of the estimation.

For the gross cash flows of both buyout and venture capital funds we find beta factors that are similar to those estimated at the individual deal level. For the full sample of buyout funds we estimate a beta of 2.76, while U.S. buyout funds have a beta of 2.65. For the full sample of venture capital funds and the sub-sample of U.S. funds we estimate betas of 2.83 and 2.66, respectively. All coefficients are statistically significant at the 1% level.

The similar beta factors are notable. Without information on the overlap between the Burgiss and the Cepres datasets the level of coincidence remains unclear. Nonetheless, the fact that the estimated gross betas are at the same high levels using both original deal level cash flows and approximated gross fund cash flows provides further reassurance in the accuracy of the estimations. We further observe that the measured R^2 has increased in all cases compared to the deal level estimations. The reason is that the idiosyncratic risk has decreased due to the diversified nature of investments within each fund.

The resulting excess returns of the approximated gross fund cash flows are also similar to those estimated at the deal level. The annual alpha is 4.7% for all buyout funds, and 4.5% for U.S. buyout funds. The full sample of venture capital funds and its sub-sample of U.S. funds have an alpha of, respectively, 11.8% and 13.2% per annum.

4.2.2 Results Based on the Net Cash Flows to Fund Investors

We now continue with the original net cash flows of the sample of private equity funds, i.e., we measure the exposure to systematic risk and abnormal returns that limited partners in these funds have experienced. In this context, we first gauge the impact of management fees and carried interest payments on the risk and return profile.

As for carried interests, the impact on the market beta is straightforward. By their nature, carried interests represent call-option like positions on the gross returns of the company investments. Due to the implicit leverage in call options, the beta of the expected carry provision must be higher than the beta of the underlying gross cash flows. Consequently, we would expect the net beta of private equity funds after carry payments to be lower than the corresponding gross beta. Management fees, in turn, are contractually fixed payments and, as such, essentially unrelated to the market. With a market beta of (close to) zero, their systematic risk is lower than that of gross returns. Consequently, we would expect the net beta after management fees to be higher than the corresponding gross beta. The combined effect of managements fees and carry provisions on beta is a priori unspecified and depends on the specific terms of a fund and its probability of receiving carry payments. Appendix A formalized these considerations. Panel A of Table 7 presents the empirical results.

For the full sample of buyout funds we estimate a beta factor of 2.80. For U.S. buyout funds we estimate a beta of 2.67. Both coefficients are statistically significant at the 1% level, and similar to their corresponding values at the gross level, indicating that the impact of management fees and carried interests effectively balance each other.

Our estimations for the systematic risk of buyout funds is substantially higher than estimated in previous fund-level studies and widely assumed by industry participants. Jones and Rhodes-Kropf (2003) find a beta factor of 0.66. Driessen et al. (2012) arrive at a (one-factor) market beta of 1.31. Ewens et al. (2013) estimate a beta of 0.72.

For venture capital funds we estimate a beta factor of 2.85 for the full sample, and 2.70 for the sub-sample of U.S. funds. Again, both coefficients are statistically significant at the 1% level, and similar to their corresponding values at the gross level. For venture capital funds, Jones and Rhodes-Kropf (2003) estimate a beta factor of 1.79. Driessen et al. (2012) estimate a (one-factor) market beta of 2.73.[24] Ewens et al. (2013) estimate a beta of 1.23.

With respect to abnormal returns, both management fees and carried interest pay-

[24]Note that, despite a similar beta to Driessen et al. (2012), our venture capital alphas are positive and (weakly) significant as shown below, whereas Driessen et al. (2012) find an alpha of -12.3% that is highly statistically significant.

ments have an obvious negative effect. The gross alpha of buyout funds drops by around 5.5%, resulting in a net alpha to fund investors of -1.0% per annum for the full sample, and -0.7% for the sub-sample of U.S. buyout funds. However, both residual values are statistically insignificantly different from zero. For venture capital funds we find a decrease in the gross alpha of around 9.0%. The net alpha of all funds is 2.1% per annum, while the sub-sample of U.S. venture capital funds has a net alpha of 4.6% which is statistically significant at the 8% level.

Panel B of Table 7 reports the cost of capital and the market model implied expected returns of the investments. Using weighted averages of the market returns and the risk-free rates with respect to the estimated amounts of net invested capital throughout the life of each fund, the cost of capital according to the CAPM are 14.9% per annum for the full sample of buyout funds, and 14.7% for U.S. buyout funds. The cost of capital for all venture capital funds and the sub-sample of U.S. funds are, respectively, 15.1% and 14.7%. The resulting expected returns implied by the market model are 13.9% per annum for both buyout fund samples, as well as 17.2% and 19.3% for venture capital funds. These estimated expected returns are close to the average net IRRs of each sample (13.6% and 13.8% for buyout funds; 19.0% and 18.2% for venture capital funds).

For the full sample of buyout funds we observe the best fit between the model expected dividends and the empirical observations. With an R^2 of 94.6% our model explains almost the entire variation in the observed dividends. The sub-sample of U.S. buyout funds has an R^2 of 91.1%. As observed for individual company investments, the R^2 for venture capital is slightly lower at 86.2% and 85.8%, respectively, due to the higher level of idiosyncratic risk in the venture segment.

Table 8 presents empirical results using the total returns of the combined NYSE, Amex, and Nasdaq (NAN) U.S. stock index as a proxy for the market portfolio.[25] For both buyout and venture capital funds the beta is lower with coefficients between 2.36 and 2.45. The annual alphas for buyout funds are positive and significant with values

[25]The index data has been downloaded from Kenneth French's website.

around 3.7% to 3.8%. Correspondingly, venture capital alphas are also higher at 6.6% to 6.7% and highly statistically significant. All coefficients of determination, R^2, are essentially the same as in the preceding estimations against the S&P 500.

Our observations are in line with the theoretical literature predicting that excess returns are widely captured by the fund manager, while fund investors are left with returns similar to the overall market. In the context of mutual funds, Berk et al. (2004) assume a competitive market for fund investments where fund managers are in possession of scarce skills and have certain bargaining power to adjust fund terms such that they capture most or all of the rents from these skills. Under this condition, abnormal fund returns net of fees, are close to zero in equilibrium. With respect to venture capital fund managers, Hochberg et al. (2014) argue that continuing investors are able to effectively prevent major adjustments in fund terms. Our estimations, net of fees, support these predictions. For venture capital funds we find consistently positive abnormal returns over the observation period, yet the majority of excess returns are indeed captured by the fund managers. The non-positive alpha for buyout funds relative to the total returns of the S&P 500 suggests that investors do not receive a compensation for the illiquidity of their long-term engagements in buyout funds (a possible explanation for this puzzle is given by Lerner and Schoar (2004)). With respect to the NAN index, the net-of-fees alpha is positive, though.

5 Conclusions

Estimating the systematic risk and abnormal returns of private equity has been particularly challenging due to the illiquid nature of the asset class. Since neither the interests in a private equity fund, nor its portfolio companies are typically traded in an efficient market, regular (fair) valuations are not available and, hence, the application of standard econometric approaches is infeasible. As a result, institutional investors in this asset class have a hard time with respect to constructing minimum variance portfolios and applying modern portfolio theory tools.

In this paper we introduce a novel econometric approach to estimates the systematic risk and abnormal returns of illiquid assets based only on their observable cash flows. Our methodology generates asymptotically consistent estimates and, in contrast to earlier work based, our objective function is globally convex. Using two large scale datasets of mature private equity funds and portfolio company investments, we study the exposure to systematic risk and the resulting abnormal returns individually at each level.

The estimations show that market betas are notably higher than previously estimated and typically assumed by industry participants. Using the total returns of the S&P 500 as a proxy for the market portfolio, we find beta coefficients ranging from 2.5 to 3.1, as well as corresponding deal-level alphas of around 5% per annum for buyouts and above 10% for venture capital investments. These are meaningful excess returns, in particular, compared to those observed in regular M&A transactions.

Assessing the individual impact by both, carried interest provision and fund management fees, on the change in systematic risk theoretically, we predict that both effects should widely offset each other. This is indeed what we observe in our empirical tests. Beta coefficient are very similar at the deal and the fund level. In contrast, however, the corresponding alphas drop by several percentage point, as one would expect. Annual net alphas of buyout funds are slightly negative but statistically insignificantly different from zero. Net alphas of venture capital funds, in turn, are positive. Using the total returns of the combined NYSE, Amex, Nasdaq stock index we find a slightly lower exposure to systematic risk, with beta coefficients around 2.4, and consistently positive annual net alphas.

A Appendix

In this appendix, we show how management fees and carried interest payments affect the systematic risk of a private equity investment.

A.1 Relationship Between Net and Gross Beta

Let V_t^{Gross} denote the time-t present value of the gross cash flows on a private equity investment, V_t^{Net} the corresponding present value of the net cash flows, V_t^{MF} the present value of the management fees, and V_t^{CI} the present value of the carried interest payments. Given that management fees and carry payments reduce the gross investment cash flows, the following identity must hold:

$$V_t^{Gross} = V_t^{Net} + V_t^{MF} + V_t^{CI}. \tag{A.1}$$

Using this result, it is straightforward to show that the beta of the gross investment cash flows β_t^{Gross} at some time t can be calculated by

$$\beta_t^{Gross} = \left(1 - \frac{V_t^{MF}}{V_t^{Gross}} - \frac{V_t^{CI}}{V_t^{Gross}}\right)\beta_t^{Net} + \frac{V_t^{MF}}{V_t^{Gross}}\beta_t^{MF} + \frac{V_t^{CI}}{V_t^{Gross}}\beta_t^{CI}. \tag{A.2}$$

for which β_t^{Net} is the beta of the net cash flows, β_t^{MF} is the beta of the management fees, and β_t^{CI} is the beta of the carried interest payments.

Because management fees are essentially fixed contractual payments that are uncorrelated with market returns, $\beta_t^{MF} = 0$ holds per definition: This simplifies equation (A.2) to

$$\beta_t^{Gross} = \left(1 - \frac{V_t^{MF}}{V_t^{Gross}} - \frac{V_t^{CI}}{V_t^{Gross}}\right)\beta_t^{Net} + \frac{V_t^{CI}}{V_t^{Gross}}\beta_t^{CI}. \tag{A.3}$$

Equation (A.5) allows to study the effects of management fees and carry payments on the gross beta.

A.2 Effects of Management Fees on Beta

Let's start with the simple case in which there are only management fees and no carried interest payments, i.e., $V_t^{CI} = 0$ for all t. Under this condition, the gross beta is given by

$$\beta_t^{Gross} = \left(1 - \frac{V_t^{MF}}{V_t^{Gross}}\right) \beta_t^{Net}. \tag{A.4}$$

Because $(1 - V_t^{MF}/V_t^{Gross}) < 1$, it must hold that $\beta_t^{Gross} < \beta_t^{Net}$. That is, management fees generally increase the beta of the investment cash flows. The magnitude of this effect will depend on the level of the management fees. Larger management fees lead to larger increases in beta because the fraction V_t^{MF}/V_t^{Gross} in equation (**??**) is an increasing function in the fee level.

A.3 Effects of Carried Interest Payments on Beta

We now turn to the effects of the carried interest payments on beta. In case there are only carried interest payments and no management fees, i.e., $V_t^{MF} = 0$ for all t, equation (A.2) reduces to

$$\beta_t^{Gross} = \left(1 - \frac{V_t^{CI}}{V_t^{Gross}}\right) \beta_t^{Net} + \frac{V_t^{CI}}{V_t^{Gross}} \beta_t^{CI}. \tag{A.5}$$

By construction, carried interest payments are call option-like positions on the gross investment cash flows. Because of the implicit leverage of call options, $\beta_t^{CI} > \beta_t^{Gross}$ must hold. From this relationship, it directly follows that

$$\beta_t^{Gross} > \left(1 - \frac{V_t^{CI}}{V_t^{Gross}}\right) \beta_t^{Net} + \frac{V_t^{CI}}{V_t^{Gross}} \beta_t^{Gross}, \tag{A.6}$$

which can be reduced to $\beta_t^{Gross} > \beta_t^{Net}$. That is, carried interest payments decrease the beta of the investment cash flows. The magnitude of this effect will depend on the carried interest level, on the hurdle rate, and on the probability that the investment performance will exceed the hurdle rate.

A.4 Combined Effects on Beta

Overall, we have two opposing effects on beta. Management fees increase the net beta, whereas carry payments reduce the net beta. To gain some intuition, under which circumstances gross beta will be larger or smaller than net beta, consider two limiting cases.

First, assume that the probability of getting carried interest payments is close to one, i.e., the carry option is deep in-the-money. Under this condition, it follows from standard option pricing theory that $\beta_t^{CI} \approx \beta_t^{Gross}$. Substituting this result into equation (A.2) gives

$$\beta_t^{Gross} \approx \frac{\left(1 - \frac{V_t^{MF}}{V_t^{Gross}} - \frac{V_t^{CI}}{V_t^{Gross}}\right)}{\left(1 - \frac{V_t^{CI}}{V_t^{Gross}}\right)} \beta_t^{Net} < \beta_t^{Net}. \tag{A.7}$$

That is, for samples where carried interest in paid for a large fraction of the investments, the estimated net beta should be larger than the estimated gross beta.

Second, assume that the probability of getting carry payments is close to zero. In this case, standard option pricing theory implies that $\beta_t^{CI} \gg \beta_t^{Gross}$. Under this condition, equation (A.2) shows that it will generally hold that $\beta_t^{Gross} > \beta_t^{Net}$. This means that the estimated net beta should be lower than the estimated gross beta for samples where carried interest in paid only for a small fraction of the investments.

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Table 1: Literature on the Systematic Risk of Private Equity

This table gives an overview on the established literature estimating the risk and return characteristics of private equity-backed companies and private equity funds. Panel A and B list studies that focus on venture capital-backed and buyout-backed company investments, respectively. Panel C list studies focusing on fund-level estimations. Presented numbers reflect those of model specifications most comrapable to ours.

Author and Paper	Data Source	Sample Period	Benchmark	Beta	Alpha
Panel A: Portfolio Company Level - Venture Capital					
Gompers and Lerner, Risk and Reward in Private Equity Investments: The Challenge of Performance Assessment, 1997	GP data	1972-1997	n/a	1.08-1.40	8.00%
Peng, Building a Venture Capital Index, 2001	VentureOne	1987-1999	S&P 500 Nasdaq	1.29-2.41 0.81-4.66	-0.9% to -1.0% -3.8% to 15.8%
Woodward and Hall, Benchmarking the Returns to Venture, 2004	Sand Hill	1987-2001	Nasdaq	0.86	8.90%
Hwang et al., An Index for Venture Capital 1987-2003, 2005	Sand Hill	1987-2001/03	S&P 500 Nasdaq	0.48-0.59 0.42-0.43	0.9% to 3.5% 1.0% to 3.2%
Cochrane, The Risk and Return of Venture Capital, 2005	VentureOne	1987-2000	S&P 500 Nasdaq	0.6 1.2	45% 35%
Ewens, A New Model of Venture Capital Risk and Return, 2009	VentureOne	1987-2007	Wilshire 5k	1.7	40%
Korteweg and Sorensen, Risk and Return Characteristics of Venture Capital-Backed Entrepreneurial Companies , 2010	Sand Hill	1987-2005	NAN	2.7 to 2.8	46.8% to 47.5%
This study	Cepres	1980-2001/08	S&P 500	2.53-2.72	11.3% to 15.1%

Table 1 continued

Author and Paper	Data Source	Sample Period	Benchmark	Beta	Alpha
Panel B: Portfolio Company Level - Buyout					
Franzoni et al., Private Equity Performance and Liquidity Risk, 2012	Cepres	1975-2006	NAN	0.95	9.30%
This study	Cepres	1980-2001/08	S&P 500	2.70-3.15	4.8% to 5.1%
Panel C: Fund Level - Venture Capital and Buyout					
Jones and Rhodes-Kropf, The Price of Diversifiable Risk in Venture Capital and Private Equity, 2003	Thomson VE	1980-1999	NAN	0.36-1.05 (all) 0.63-1.79 (VC) 0.22-0.66 (BO)	n/a
Woodward, Measuring Risk for Venture Capital and Private Equity Portfolios, 2009	Cambridge Associates	1989-2008	Wilshire 5k	0.77-2.22 (VC) 0.42-1.06 (BO)	0.4% to 9.4% 5.8% to 8.6%
Jegadeesh et al., Risk and Expected Returns of Private Equity Investments: Evidence Based on Market Prices, 2010	Public data	1994-2008	S&P 500	0.98 (LPE) 0.71 (FoF)	-2.30% -3.90%
Driessen et al., A New Method to Estimate Risk and Return of Non-Traded Assets from Cash Flows: The Case of Private Equity Funds, 2008/2011	Thomson VE	1980-1993	S&P 500	1.31 (BO) 2.73 (VC)	-4.8% -12.3%
Ewens et al., The Price of Diversifiable Risk in Venture Capital and Private Equity, 2013	Thomson VE, Preqin, LP Source	1980-2007	NAN	0.32-0.93 (all) 0.37-1.23 (VC) 0.30-0.72 (BO)	0.60% -0.20% 1.20%
This study	Burgiss	1980-2001	S&P 500	2.70-2.85 (VC) 2.67-2.80 (BO)	2.1% to 4.6% -0.7% to -1.0%

Table 2: Monte Carlo Simulation Parameters

This table reports the base case parameters used in the Monte Carlo simulation experiment. Model parameters represent monthly values, if not stated otherwise.

Parameter	Symbol	Value
Riskless rate (p.a.)	r_f	0.05
Alpha	α	0.00
Beta	β	2.50
Dividend rate (p.a.)	δ	0.18
Investment rate (p.a.)	γ	0.80
Minimum market return	c_M	-0.20
Lognormal mean market	μ_M	-1.58
Lognormal volatility market	σ_M	0.18
Minimum error term	c_ϵ	-0.43
Lognormal mean error term	μ_ϵ	-1.17
Lognormal volatility error term	σ_ϵ	0.80

Table 3: Monte Carlo Simulation

This table presents the estimation results from the Monte Carlo simulation experiment. The simulation is carried out using 5,000 iterations, i.e., parameters are estimated from a sample of 5,000 investments. Investment returns are modeled by a single-factor market model, for which market returns and error terms are assumed to follow a shifted log-normal distribution. In the base case of Panel A, idiosyncratic volatility is matched to that of Korteweg and Sorensen (2010) at 40% per month. Idiosyncratic volatility is set to 20% per month and 60% per month in the lower and higher volatility case, respectively. The last two columns of Panel A show the results when the sample size is reduced to 1,000 and increased to 10,000 observations. In Panel B, we relax the assumption that the dividend rate is deterministic and analyze how different cross-dependencies between the dividend rate and other model variables affect estimation precision (see text for details). All simulations are repeated 1,000 times with the true parameters set to alpha=0, beta=2.5, and delta=0.18. The mean, median, standard deviation, and interquartile range are based on 1,000 sets of estimated parameters.

Panel A	True Model	Idiosyncratic Volatility			Sample Size	
	Parameters	Base Case	Low	High	1,000	10,000
Alpha						
mean	0.00%	-0.01%	0.00%	-0.02%	-0.02%	0.00%
median		-0.07%	0.00%	-0.28%	-0.19%	0.00%
std.		0.52%	0.08%	1.49%	1.04%	0.19%
interquartile		[-0.24%,0.11%]	[-0.05%,0.05%]	[-0.68%,0.31%]	[-0.50%,0.23%]	[-0.10%,0.08%]
Beta						
mean	2.50	2.51	2.50	2.56	2.52	2.50
median		2.52	2.50	2.51	2.51	2.50
std.		0.65	0.11	1.98	1.33	0.12
interquartile		[2.23,2.77]	[2.43,2.56]	[1.76,3.28]	[1.91,3.12]	[2.41,2.58]
Delta						
mean	0.18	0.18	0.18	0.19	0.18	0.18
median		0.18	0.18	0.19	0.18	0.18
std.		0.01	0.00	0.03	0.02	0.00
interquartile		[0.17,0.19]	[0.18,0.18]	[0.18,0.20]	[0.17,0.20]	[0.18,0.18]

Table 3 continued

Panel B	True Model	Dividend Rate Stochastic			Dividend Rate Dependent on...	
	Parameters	Base Case	Corr=0.25	Corr=0.5	Alpha	Beta
Alpha						
mean	0.00%	-0.03%	0.01%	0.03%	0.08%	0.08%
median		-0.07%	-0.07%	-0.05%	0.00%	-0.04%
std.		0.43%	0.59%	0.70%	0.60%	1.09%
interquartile		[-0.25%,0.14%]	[-0.24%,0.15%]	[-0.24%,0.17%]	[-0.18%,0.20%]	[-0.23%,0.21%]
Beta						
mean	2.50	2.53	2.55	2.57	2.43	2.49
median		2.52	2.54	2.56	2.47	2.50
std.		0.77	0.91	0.95	0.90	0.97
interquartile		[2.18,2.89]	[2.54,2.90]	[2.21,2.94]	[2.11,2.80]	[2.17,2.83]
Delta						
mean	0.18	0.18	0.18	0.18	0.18	0.18
median		0.18	0.18	0.18	0.18	0.18
std.		0.01	0.01	0.02	0.01	0.01
interquartile		[0.18,0.19]	[0.17,0.20]	[0.17,0.20]	[0.18,0.19]	[0.17,0.19]

Table 4: Sample Distribution

This table shows the distribution of buyout and venture capital funds in the Burgiss dataset (left panel) by fund vintage year, and the distribution of company investments by buyout and venture capital funds raised until 2001 in the Cepres dataset (right panel) with respect to the year of the first investment. We show numbers for all buyout and venture capital funds worldwide, as well as for the respective sub-samples of U.S. buyout and U.S. venture capital funds only. Equivalently, we group the company investments according to the geographic and strategy focus of their underlying funds.

	Burgiss Funds by Vintage Year				Cepres Deals by Entry Year			
	Buyout		Venture Capital		Buyout		Venture Capital	
	All	U.S.	All	U.S.	All	U.S.	All	U.S.
1980	2	2	9	9	0	0	0	0
1981	1	1	7	6	1	0	8	8
1982	2	2	13	11	1	0	18	18
1983	2	2	17	16	4	1	26	19
1984	2	1	20	20	11	7	41	33
1985	0	0	24	22	28	13	53	49
1986	6	6	17	15	41	21	76	64
1987	9	8	23	19	51	16	78	67
1988	8	6	22	19	77	34	105	95
1989	13	11	23	22	90	54	147	132
1990	6	3	16	14	145	49	122	104
1991	3	3	10	8	187	37	156	93
1992	10	9	18	18	217	62	151	119
1993	12	10	23	21	214	77	197	131
1994	23	17	25	22	350	86	224	148
1995	25	22	30	27	335	85	287	194
1996	20	15	25	23	401	96	457	294
1997	45	35	42	39	474	115	525	362
1998	56	43	55	52	510	157	741	540
1999	46	32	92	81	570	152	1,202	882
2000	75	52	120	101	663	207	1,814	1,122
2001	47	27	65	55	380	107	856	570
2002	-	-	-	-	282	94	573	414
2003	-	-	-	-	254	98	496	363

Table 4 continued

	Burgiss Funds by Vintage Year				Cepres Deals by Entry Year			
	Buyout		Venture Capital		Buyout		Venture Capital	
	All	U.S.	All	U.S.	All	U.S.	All	U.S.
2004	-	-	-	-	203	81	459	315
2005	-	-	-	-	146	56	238	149
2006	-	-	-	-	65	29	116	71
2007	-	-	-	-	13	3	26	11
2008	-	-	-	-	2	0	11	7
Total	413	307	696	620	5,715	1,737	9,203	6,374

Table 5: Estimation Results at the Deal Level

Panel A reports the estimated abnormal performance (Alpha, in % per annum) and the systematic risk (Beta Market) for the deals in the Cepres dataset, whose underlying funds were raised until 2001 and had a primary strategic focus on buyout or venture capital. We show estimations for the full sample of company investments, as well as the sub-samples of deals conducted by U.S. funds. The S&P 500 total return index is used as the proxy for the market returns and the one-month U.S. Treasury Bill rate is used as the return of the risk-free asset. Standard errors are derived from the Hessian matrix and shown in parentheses. ***, ** and * denotes statistical significance at the 1%, 5% and 10% level, respectively. RMSE measures the difference between the observed dividends and the model expected dividends. R^2 measures the extent to which the variation in observed dividends can be explained by the model expected dividends. Panel B calculates the corresponding cost of capital and expected returns. The market return and the risk-free rate are weighted averages with respect to the individual periods of each investment.

	Buyout		Venture Capital	
	All	U.S.	All	U.S.
Panel A: Model Estimations				
Alpha (in % p.a.)	5.14***	4.84***	11.34***	15.05***
	(1.00)	(1.03)	(2.38)	(3.36)
Beta Market	2.70***	3.15***	2.72***	2.53***
	(0.10)	(0.10)	(0.32)	(0.49)
RMSE	0.00	0.01	0.00	0.00
R^2	0.87	0.78	0.82	0.81
Observations	5,715	1,737	9,203	6,374
Panel B: Cost of Capital and Expected Returns				
Market Return (in % p.a.)	9.1	9.0	6.5	6.7
Risk-free Rate (in % p.a.)	3.8	3.8	3.7	3.8
Risk Premium (in % p.a.)	14.2	16.1	7.7	7.4
Cost of Capital (CAPM; in % p.a.)	18.0	20.0	11.4	11.1
Expected Return (Market Model; in % p.a.)	23.2	24.8	22.7	26.2

Table 6: Estimation Results at the Fund Level (Gross Cash Flows)

This table reports the estimated abnormal performance (Alpha, in % per annum) and the systematic risk (Beta Market) for the approximated gross cash flows of the buyout and venture funds in the Burgiss dataset raised until 2001. We show estimations for the full sample of buyout and venture capital funds, as well as the sub-samples of U.S. funds only. The S&P 500 total return index is used as the proxy for the market returns and the one-month U.S. Treasury Bill rate is used as the return of the risk-free asset. Standard errors are derived from the Hessian matrix and shown in parentheses. ***, ** and * denotes statistical significance at the 1%, 5% and 10% level, respectively. RMSE measures the difference between the observed dividends and the model expected dividends. R^2 measures the extent to which the variation in observed dividends can be explained by the model expected dividends.

	Buyout		Venture Capital	
	All	U.S.	All	U.S.
Alpha (in % p.a.)	4.70*	4.52*	11.84***	13.15***
	(2.49)	(2.39)	(2.38)	(2.44)
Beta Market	2.76***	2.65***	2.83***	2.66***
	(0.38)	(0.36)	(0.37)	(0.37)
RMSE	0.01	0.01	0.02	0.02
R^2	0.94	0.91	0.86	0.86
Observations	413	307	696	620

Table 7: Estimation Results at the Fund Level (Net Cash Flows)

Panel A reports the estimated abnormal performance (Alpha, in % per annum) and the systematic risk (Beta Market) for the original net cash flows of the buyout and venture funds in the Burgiss dataset raised until 2001. We show estimations for the full sample of buyout and venture capital funds, as well as the sub-samples of U.S. funds only. The S&P 500 total return index is used as the proxy for the market returns and the one-month U.S. Treasury Bill rate is used as the return of the risk-free asset. Standard errors are derived from the Hessian matrix and shown in parentheses. ***, ** and * denotes statistical significance at the 1%, 5% and 10% level, respectively. RMSE measures the difference between the observed dividends and the model expected dividends. R^2 measures the extent to which the variation in observed dividends can be explained by the model expected dividends. Panel B calculates the corresponding cost of capital and expected returns. The market return and the risk-free rate are weighted averages with respect to the estimated net invested capital over the life of each fund.

	Buyout		Venture Capital	
	All	U.S.	All	U.S.
Panel A: Model Estimations				
Alpha (in % p.a.)	-0.97	-0.74	2.05	4.58*
	(3.17)	(3.16)	(2.39)	(2.53)
Beta Market	2.80***	2.67***	2.85***	2.70***
	(0.50)	(0.50)	(0.38)	(0.40)
RMSE	0.01	0.01	0.01	0.01
R^2	0.95	0.92	0.87	0.86
Observations	413	307	696	620
Panel B: Cost of Capital and Expected Returns				
Market Return (in % p.a.)	7.4	7.6	7.5	7.6
Risk-free Rate (in % p.a.)	3.3	3.4	3.4	3.5
Risk Premium (in % p.a.)	11.6	11.3	11.7	11.3
Cost of Capital (CAPM; in % p.a.)	14.9	14.7	15.1	14.7
Expected Return (Market Model; in % p.a.)	13.9	13.9	17.2	19.3

Table 8: Estimation Results for the NAN index (Net Cash Flows)

This table reports the estimated abnormal performance (Alpha, in % per annum) and the systematic risk (Beta Market) for the approximated gross cash flows of the buyout and venture funds in the Burgiss dataset raised until 2001. We show estimations for the full sample of buyout and venture capital funds, as well as the sub-samples of U.S. funds only. The total returns of the combined NYSE, Amex, Nasdaq (NAN) index is used as the proxy for the market returns and the one-month U.S. Treasury Bill rate is used as the return of the risk-free asset. Standard errors are derived from the Hessian matrix and shown in parentheses. ***, ** and * denotes statistical significance at the 1%, 5% and 10% level, respectively. RMSE measures the difference between the observed dividends and the model expected dividends. R^2 measures the extent to which the variation in observed dividends can be explained by the model expected dividends.

	Buyout		Venture Capital	
	All	U.S.	All	U.S.
Alpha (in % p.a.)	3.78**	3.65**	6.74***	6.55***
	(1.70)	(1.74)	(1.54)	(1.52)
Beta Market	2.44***	2.36***	2.43***	2.45***
	(0.52)	(0.50)	(0.40)	(0.39)
RMSE	0.01	0.01	0.01	0.01
R^2	0.94	0.91	0.87	0.88
Observations	413	307	696	620

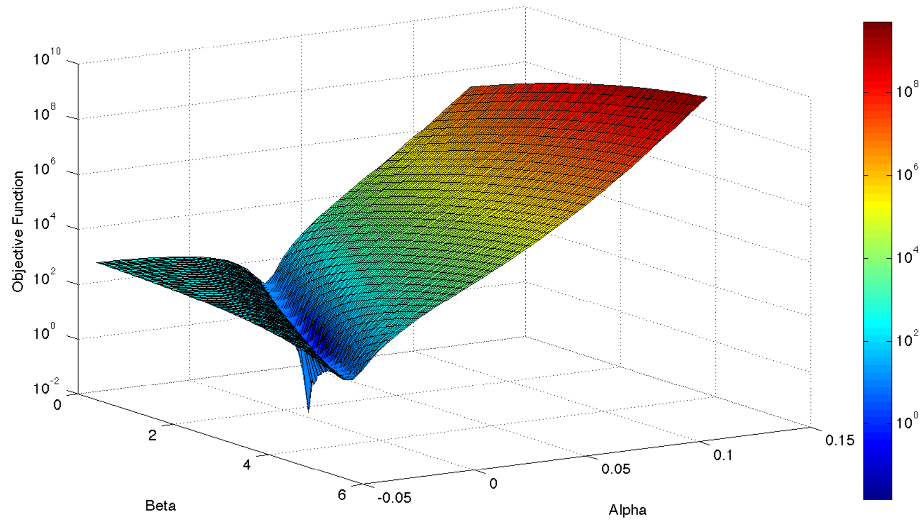


Figure 1: Objective Function Space of Parameters Alpha and Beta for the Optimal Delta

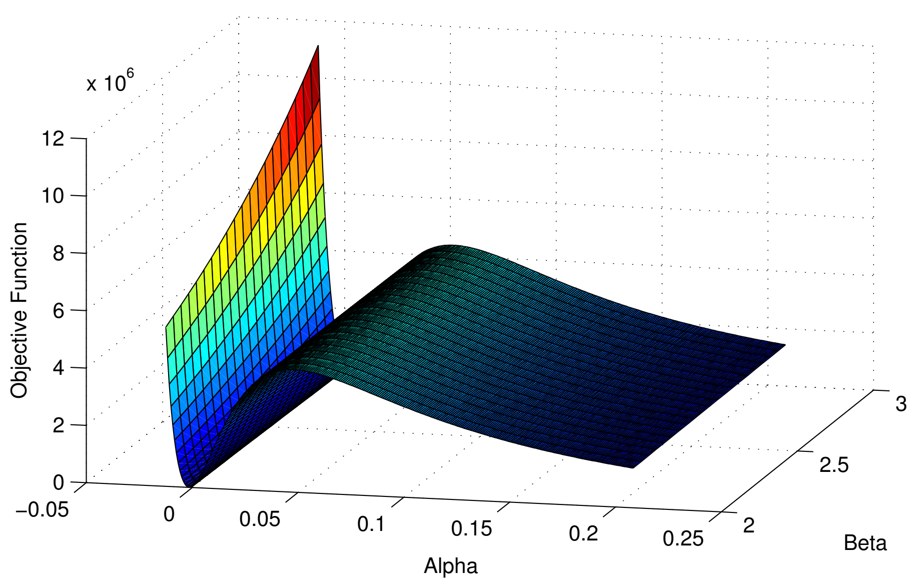
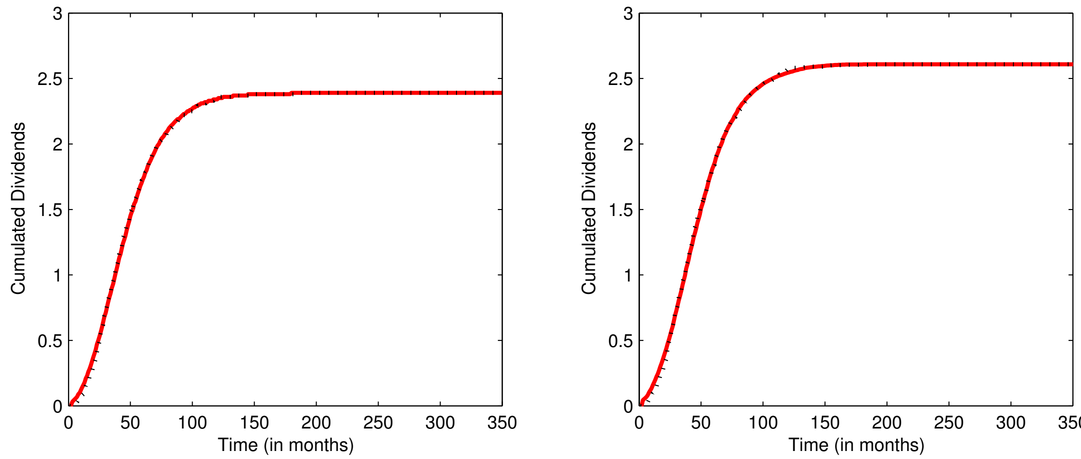
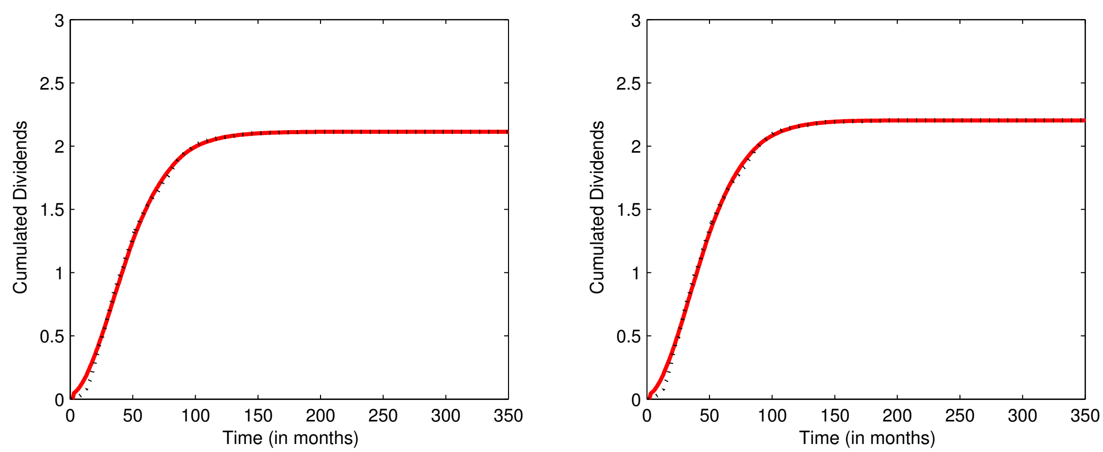


Figure 2: Objective Function Space of Parameters Alpha and Beta as per Driessen et al. (2012)



(a) Buyout Deals: Cumulated Dividends of All Investments (Left) and U.S. Investments (Right)



(b) Venture Capital Deals: Cumulated Dividends of All Investments (Left) and U.S. Investments (Right)

Figure 3: Goodness-of-Fit Model expectations of the dividends are plotted as compared to sample average dividends. Solid lines represent model expectations, dotted lines represent sample means.