# Cross Currency Swap Trading & Pricing Formulae - A PowerPoint Overview with Excel Pricing Examples

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# **Executive Summary**

#### **Swap Preliminaries**

- Interest Rate Swaps
- Yield Curves
- Rates Trading, Pricing & Risk

#### **Xccy Swaps**

- Xccy Swap Theory Formulae
- Xccy Swap Practice Pricing Demo

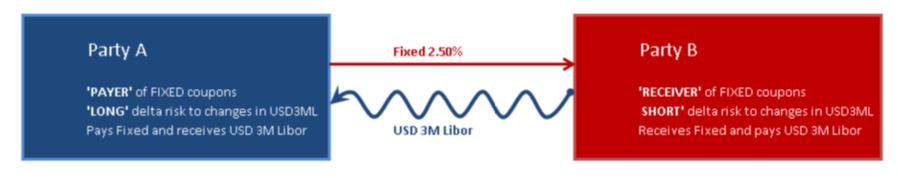
#### **Detailed Notes**

https://ssrn.com/abstract=3278907



# Interest Rate Swaps Market

- Notional Outstanding \$200tn: Traded in large volume
- **Very Liquid** : Bid-Offer 1/10th Basis Point i.e. 0.001%
- Swap Cashflows: Each set of cashflows called the trade 'leg'
- Typically Fixed-Float: or Float-Float for Basis Swaps
- Payer/Receiver: of Fixed cashflows, but long/short the risk
- Quoted at Par: Zero upfront cost



USD 1MM 5Y PAYER 2.5% VS USD3ML FLAT

USD 1MM 5Y RECEIVER 2.5% VS USD3ML FLAT

# Swap Pricing Terminology

#### **Pricing Terminology**

- **PV**: Present Value or Price
- Basis Points (bps): 1/100th of a percent i.e. 1bps = 0.01%
- Par Rate: The fixed rate in % to make the trade PV zero
- Par Spread: The float spread in bps to make trade PV zero
- **PV01**: PV sensitivity to forward rates, also called Annuity
- **DV01**: PV sensitivity to forward rates and discount factors

#### Quotation

- New Deals: Quoted as a par rate or a spread over Treasuries
- Bespoke Deals: Quoted as a PV, since not trading at par
- Basis Trades: Float-Float deals are quoted as a par spread



# Bloomberg Trading Venue

# Bloomberg Trading Portal, BBTI - Interest Rate Swaps

		2\ T	,	3) 5
Interest Ra	te Swaps	2) Too	ols 🕶	3) Settings
Venue BGL	Currency			
5) Outright		) Butterflies	8) Rolls	
10) S/A v 3M	11) S/A v 1M	12) S/A v 6M	13) Ann	√ 3M 14) MAC :
	Semi-annı	ual v 3 Month		
Tenor		Bid		Change
30)6 Months		2.668 /		-0.006 ≣
31) 12 Months		2.643 /	2.647	-0.010 ■
32) 18 Months		2.605 /	2.610	-0.015 ≣
33) 2 Year		2.552 /	2.555	-0.019
34)3 Year		2.481 /	2.484	-0.026 ≣
35)4 Year		2.453 /	2.456	-0.027 ≣
36) 5 Year		2.453 /	2.456	-0.027 ≣
37)6 Year		2.472 /	2.475	-0.028 ■
38) 7 Year		2.497 /	2.500	-0.028
39)8 Year		2.527 /	2.530	-0.028 ≣
40)9 Year		2.559 /	2.562	-0.028 ≣
41) 10 Year		2.591 /	2.594	-0.028 ■
42) 12 Year		2.648 /	2.651	-0.027 ≣
43) 15 Year		2.705 /	2.708	-0.026 ▮
44) 20 Year		2.750 /	2.754	-0.025 ≣
45) 25 Year		2.762 /	2.766	-0.024 ▮
46)30 Year		2.765 /	2.769	-0.023 I
47) 40 Year		2.743 /		-0.023 ▮
48)50 Year		2.707 /	2.714	-0.026 I

# Swaps as a Spread over US Treasuries

Par Rate = US Treasury Yield + Spread (Bps)

			IDC Tro	ling Dorts	1
*			iks irac	ding Porta	
	^				
S/A 15) IMM S/A	16) IMM Ann	17) OIS	18) SOFR	19) FOMC	
	Spreads \	/ Treasur	ies		
Tenor			Ask	Change	
1 Year		14.627	/ 15.614	-0.794	
70) 2 Year		9.991	/ 10.374	+0.068	■ ■
71) 3 Year		8.082	/ 8.432	-0.262	≣
4 Year		5.250	/ 5.535	-0.385	
72) 5 Year		5.053	/ 5.446	-0.360	≣
6 Year		2.500	/ 2.875	-0.253	
73) 7 Year		0.356	/ 0.671	-0.308	≣
8 Year		0.503	/ 0.809	-0.877	
9 Year		-0.125	/ 0.500	-0.377	
74) 10 Year		0.072	/ 0.441	-0.471	
12 Year		6.113	/ 6.424	-1.038	
15 Year		1.125	/ 1.375	-0.563	
20 Year		-4.875	/ -4.500	-0.565	
25 Year		-13.500	/ -13.000	-1.125	
75)30 Year		-24.171	/ -23.786	-0.715	≣

# Interest Rate Swap Pricing

#### Swap Specification & Pricing

To specify a swap many parameters are required to generate the swap cashflow schedules accurately. To price a swap we require Libor forecast rates, OIS discount rates and a Swap pricing formula.

$$PV^{Swap} = N \sum_{\forall i} r^{Fixed} \tau_i P(t_0, t_i) - N \sum_{\forall j} (L_j + s) \tau_j P(t_0, t_j)$$



# Bloomberg Swap Manager, SWPM

#### Par Swap - 5Y Receiver vs USD3ML



# IRS Pricing Formula

# Fixed Leg

$$PV(Fixed) = N \times r^{Fixed} \underbrace{\sum_{\forall i} \tau_i P(t_0, t_i)}_{Annuity}$$

Float Leg

$$PV(Float) = N \sum_{\forall j} (L_j + s) \tau_j P(t_0, t_j)$$

Swap Price

$$PV(Swap) = \phi(PV(Fixed) - PV(Float))$$

Swap Rate

$$ParRate = \frac{PV(Float)}{N \times Annuity}$$

# Other Swap Types

- 1 Interest Rate Swaps Rate Hedging
- **2 Tenor Basis Swaps** Frequency Matching
- **3 Xccy Swaps** Funding in another Currency
- 4 Asset Swaps Bond & Corporate Financing
- **6 Credit Default Swaps** Hedging Counterparty Risk
- **6 Inflation Swaps** Inflation Hedging

# Libor Benchmark Rate Reform [New Swaps]

- Risk-Free Curves (RFRs)
- Alternative Reference Rates (ARRs)
- SOFR Swaps Secured Overnight Funding Rate

# **Xccy Swap Overview**

# Xccy Basis Swaps

- Exchange a set of cashflows for an equivalent set in another currency
- Used to secure cheaper funding in a different currency, manage FX exposures and liquidity risk

#### Features

- Marked-to-Market (MTM)
- Notional Resets to Reduce Credit Exposures
- Choice of Valuation Currency
- Spot FX Required
- Multi-Currency Yield Curves Required
- OIS Discounting with Foreign CSA Collateral
- Xccy Par-Spread usually on Non-USD leg

# Bloomberg MtM Xccy Swap USD/EUR

# USD/EUR MtM Xccy Swap 1Y



# Bloomberg Xccy Swap - USD Leg

# USD/EUR MtM Xccy Swap 1Y - USD Leg



# Bloomberg Xccy Swap - EUR Leg

# USD/EUR MtM Xccy Swap 1Y - EUR Leg



# Yield Curves, Fwd Rates & Disc Factors

#### **Curve Construction & Dependencies**

- 1 First Calibrate Standard/Native CSA Curves
- 2 Then build USD CSA Curves using Xccy Swaps
- 3 Then build Non-USD CSA Curves using FX Fwd Invariance

#### **CSA Discount Factors**

- Forward rates are independent of CSA
- Discount factors depend on CSA

# Standard/Native CSA Disc Factors

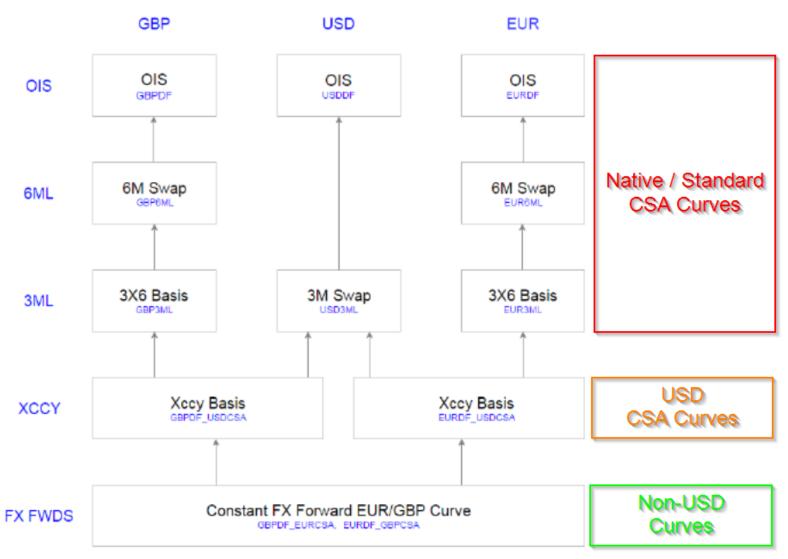
- Native CSA discount factors same as No CSA
- Example: USDOIS\_USDCSA = USDOIS



# Curve Dependency Tree

Example: EURDF with GBP Collateral

**EUROIS-GBPCSA** 



# **Xccy Curves - Discounting with Collateral**

#### **USD CSA Discount Factors**

- Implied directly from market Xccy Swap Spreads
- Typically Xccy trades have a USD leg and post USD collateral

# **Example: USD/EUR Xccy**

- We know: Par Spread, USDOIS, USD3ML and EURIBOR3M
- from which we **imply** EUROIS-USDCSA

# Xccy Trade - Curves & DF Requirements

#### SWPM -FLFL -XCCY -MTM -USD 1MM 5Y

91) Actions 🕶	ctions • 92) Products • 9		94) Info + 95) Se		ings 🕶	Swap Manager	
Solver (Premium) •		Load	Save		Trade 🕶	CCP *	
3) Main 4 Details	5) Curves 6) Cashf	low 7) Resets	9) Scenario 10) R	tisk 12) I	Matrix		
□ Deal 🗈	MTM XCCY Swa	p Counterparty	SWAP CNTRPA	RTY •	Ticker / SWAP	20) Properties	
■ Swap	*Notional Reset b		3 Month	Euribor	■ Valuation Setting	S	
Leg 1:Float	Receive	Leg 2:Float	Pay		Curve Date	03/25/2019	
Notional	1MM	Notional	883,704.4	49	Valuation	03/27/2019	
Currency	USD	Currency	EUR	▼.	CSA Coll Ccy	USD •	
Effective	OD 03/27/2019	Effective	OD 03/27/2	2019	Valuation Ccy	USD •	
Maturity	5Y 03/27/2024 t	Maturity	5Y 03/27/2	2024	FX Rate	1.131600	
Index	3M US0003M	Index	3M EURO	D3M	OIS DC Strippin	ng .	
Spread	0.000 b	p Spread	-12.750	bp			
Leverage	1.00000	Leverage	1.00000				
Latest Index	2.60988	Latest Index	-0.31000	0			
Reset Freq	Quarterly	Reset Freq	Quarterly	_			
Pay Freq	Quarterly	Pay Freq	Quarterly	•			
Day Count	ACT/360	Day Count	ACT/360	•			
■ Market	3						
Dscnt 42 • M	USD OIS (ICVS I	Dscnt 403 •	MBB USD Coll f	or EUR			
Fwd 23 • M	USD (30/360, S/A	Fwd 201 • I	M V EUR (vs. 3M E	URIE •			
■ Leg 1: NPV	1,011,849.8	8 Leg 2: NPV	-1,011	,849.88		,	
∀ Valuation Results					22) Calculators •		
Principal		0 Premium		0.00000	BR01 92:EUR vs.	-514.42	
Accrued	0.0			0.00000	DV01	0.00	
NPV	0.0	O			Gamma (1bp)	0.00	

# Non-USD CSA Curves

#### **FX Forward Invariance**

For Non-USD CSA calculations we assume FX forward invariance

# FX Forward Invariance Example for EUR\_JPYCSA $FwdFX(t,T)^{EUR/JPY} = \underbrace{S\frac{P(t,T)^{EUR\_JPYCSA}}{P(t,T)^{JPY\_JPYCSA}}}_{JPY\_JPYCSA} = \underbrace{S\frac{P(t,T)^{EUR\_USDCSA}}{P(t,T)^{JPY\_USDCSA}}}_{From Xccy Curve}$ where t denotes the valuation date, S the FX spot rate, P(t,T) the discount factor at time t for tenor T with 0 < t < T

#### Non-USD CSA Discount Factors

- Use LHS to imply from using FX Forwards
- Use RHS to imply from Xccy Basis quotes

#### Replication Logic

- We can create a synthetic EUR/JPY forward FX rate
- Borrow JPY, buy spot EUR/JPY and deposit the EUR
- The forward FX rate is constant for any given CSA

# **Xccy Swap Pricing Formula**

# FOR/DOM Xccy Price

$$\begin{split} PV(\Omega_{Xccy}) &= \phi \Big[ PV(\Omega_{FOR}) - PV(\Omega_{DOM}) \Big] \\ PV\left(\Omega_{Xccy}\right) &= \phi \Big[ PV\Big( \text{Cpn}, \Omega_{FOR} \Big) + PV\Big( \text{Exch}, \Omega_{FOR} \Big) + PV\Big( \text{Resets}, \Omega_{FOR} \Big) \\ &- PV\Big( \text{Cpn}, \Omega_{DOM} \Big) - PV\Big( \text{Exch}, \Omega_{DOM} \Big) - PV\Big( \text{Resets}, \Omega_{DOM} \Big) \Big] \end{split}$$

# Example: USD/EUR MtM Xccy Swap

- Valuation Currency: USD
- Collateral Currency: USD
- Reset Currency: USD
- Quotation: EUR Par-Spread
- EUR Leg: 3M EURIBOR with EUROIS-USDCSA discounting
- USD Leg: 3M USD Libor with USDOIS discounting



# Xccy Swap Pricing Formula [Expanded]

$$PV\left(\Omega_{Xccy}\right) = \phi \bigg[ \underbrace{\sum_{j=1}^{m} N_{t_0}^{FOR} \Psi(t_j)^{FOR}(l_j + s_{FOR}) \tau_j P(0,t_j)^{FOR-CSA}}_{Foreign Float Coupons} \\ + \underbrace{\left(\underbrace{N_{t_0}^{FOR} \Psi(t_m)^{FOR} P(0,t_m)^{FOR-CSA}}_{Foreign Final Exchange} - \underbrace{N_{t_0}^{FOR} \Psi(t_0)^{FOR} P(0,t_0)^{FOR-CSA}}_{Foreign Upfront Exchange} \right)}_{Foreign Notional Resets} \\ + 1 \bigg\{ \underbrace{\Omega = MtM} \big\} 1 \bigg\{ \underbrace{FOR = C^{Reset}}_{Domestic Float Coupons} \bigg\} \underbrace{\sum_{j=1}^{m} N_{t_0}^{FOR} \left(\Psi(t_j)^{FOR} - \Psi(t_{j+1})^{FOR}\right) P(0,t_j)^{FOR-CSA}}_{Foreign Notional Resets} \\ - \underbrace{\sum_{j=1}^{m} N_{t_0}^{DOM} \Psi(t_j)^{DOM} (l_j + s_{DOM}) \tau_j P(0,t_j)^{DOM-CSA}}_{Domestic Float Coupons} \\ - \underbrace{\left(\underbrace{N_{t_0}^{DOM} \Psi(t_m)^{DOM} P(0,t_m)^{DOM-CSA}}_{Domestic Float Exchange} - \underbrace{N_{t_0}^{DOM} \Psi(t_0)^{DOM} P(0,t_0)^{DOM-CSA}}_{Domestic Upfront Exchange} \right)}_{Domestic Upfront Exchange} \\ - 1 \bigg\{ \underbrace{\Omega = MtM} \big\} 1 \bigg\{ \underbrace{POM = C^{Reset}}_{Domestic} \underbrace{\underbrace{N_{t_0}^{DOM} \Psi(t_0)^{DOM} - \Psi(t_{j+1})^{DOM}}_{DOM} P(0,t_j)^{DOM-CSA}} \right]$$

Domestic Notional Resets

# Xccy Features: MtM

#### Marked-to-Market

- Xccy swaps present users with FX risk
- The MtM feature mitigates this
- MtM Xccy Swaps track and reset the FX rate each period
- FX losses on the Notional reimbursed on each FX fixing date

# Understanding the Notional

#### **Xccy Coupon Notional**

We always scale the **initial** notional by  $\Psi_i$ 

$$N_i = N_0 \Psi_i$$

#### **Notional Reset Factor**

$$\Psi_{i} = \alpha(t_{0}, C^{Leg}) \underline{\beta(t_{i}, C^{Leg})}$$
Valuation Adj FX Reset Adj

#### **Spot FX Valuation Adjustment,** $\alpha$

$$\alpha(t,C^{Leg}) = \begin{cases} 1 & \text{, if } C^{Leg} = C^{Val} \\ s^{FOR/DOM} & \text{, if } C^{Leg} \neq C^{Val} \text{ and } C^{Leg} = C^{FOR} \\ s^{DOM/FOR} & \text{, if } C^{Leg} \neq C^{Val} \text{ and } C^{Leg} = C^{DOM} \end{cases}$$

#### Forward FX MtM Reset Adjustment, $\beta$

$$\beta(t,C^{Leg}) = \begin{cases} 1 & \text{, if } C^{Leg} \neq C^{Reset} \\ \left(\frac{f(t)^{FOR/DOM}}{s^{FOR/DOM}}\right) & \text{, if } C^{Leg} = C^{Reset} \text{ and } C^{Leg} = C^{FOR} \\ \left(\frac{f(t)^{DOM/FOR}}{s^{DOM/FOR}}\right) & \text{, if } C^{Leg} = C^{Reset} \text{ and } C^{Leg} = C^{DOM} \end{cases}$$



# Notional Resets & Exchanges

#### Initial & Final Exchanges

Borrow funds at start and return at end

Initial Exchange = 
$$N_0 \Psi_0 P(0, t_0)$$

Final Exchange = 
$$-N_0\Psi_nP(0,t_n)$$

#### **Notional FX Resets**

FX immunized: FX losses reimbursed each FX fixing date

FX Reset = 
$$N_i P(0, t_i) - N_{i+1} P(0, t_i)$$
  
=  $N_0 \Psi_i P(0, t_i) - N_0 \Psi_{i+1} P(0, t_i)$ 

# Coupon Pricing

#### Floating Cashflows

$$PV(Coupon) = \sum_{i=1}^{n} N_{0}\Psi_{i}(I_{i} + s)\tau_{i}P(0, t_{i})$$

$$= \sum_{i=1}^{n} N_{0}\Psi_{i}I_{i}\tau_{i}P(0, t_{i}) + s\sum_{i=1}^{n} N_{0}\Psi_{i}\tau_{i}P(0, t_{i})$$

$$= \sum_{i=1}^{n} N_{0}\Psi_{i}I_{i}\tau_{i}P(0, t_{i}) + s \text{ Annuity}$$

where

Annuity = 
$$\sum_{i=1}^{n} N_0 \Psi_i \tau_i P(0, t_i)$$

# Xccy Par Spread

#### Par Spread, s

Applied to the Non-USD Leg

$$s = -\left(\frac{\text{PV(Trade with No Spread)}}{\text{Annuity(Non-USD Leg)}}\right)$$

where

$$Annuity = \sum_{i=1}^{n} N_0 \Psi_i \tau_i P(0, t_i)$$

# Bloomberg: Xccy Quotes

#### **Bloomberg Par Rates**



# Xccy Pricing Demo Workbook

#### **Demo Workbook:**

# 5Y USD/EUR MTM XCCY SWAP USD 1MM

Leg1 - FUR Cashflows

Cross Currency Swap, $\Omega_{\text{Xccy}}$								
TradeDate	Fri, 26-Oct-18							
Maturity (Years)	5Y	Wed, 25-Oct-23						
Trade Notional	1,000,000							
Trade Currency	USD							
MtM	YES							
NotionalExchanges	YES							
Reset Currency	USD	USD						
CSA Currency	USD							
Valuation Currency	USD							
SpotFX	1.14030	USD/EUR						
LegCurrency	EUR	USD						
LegNotional	876,962	1,000,000						
PayOrReceive	PAY	RECEIVE						
LegType	FLOATING	FLOATING						
RateOrSpread (%)	0.00000%	0.00000%						
FloatIndex	EUR EURIBOR 3M	USD LIBOR 3M						
Frequency	QUARTERLY	QUARTERLY						
LegResetsRequired	NO	YES						
LegSpotFX	0.87696	1.14030						
ValuationFXAdj	1.14030	1.00000						
DaycountBasis	ACT/360	ACT/360						
UseMarketSchedule	NO	NO						

Prices / ParSpreads	LEG1: EUR	LEG2: USD
LegPV	-24,992	15,686
SwapPV	-9,306	USD

	regi - row c	asilliows									
	Notional	FXFixingDate	ForwardFX	NotionalExchange	Spread	FloatRate	Coupon	DiscountFactor	CouponPV	SpotFX	ValuationPV
0				876,962			876,962	1.000000	876,962	1.1403	1,000,000
1	-876,962	Fri, 26-Oct-18	1.00000	0	0.00000%	-0.31695%	703	1.002365	704	1.1403	803
2	-876,962	Fri, 25-Jan-19	1.00000	0	0.00000%	-0.31644%	701	1.004182	704	1.1403	803
3	-876,962	Fri, 26-Apr-19	1.00000	0	0.00000%	-0.28931%	641	1.005926	645	1.1403	736
4	-876,962	Fri, 26-Jul-19	1.00000	0	0.00000%	-0.22709%	503	1.007807	507	1.1403	579
5	-876,962	Sat, 26-Oct-19	1.00000	0	0.00000%	-0.13634%	302	1.009467	305	1.1403	348
6	-876,962	Sat, 25-Jan-20	1.00000	0	0.00000%	-0.05021%	111	1.010835	113	1.1403	128
7	-876,962	Sat, 25-Apr-20	1.00000	0	0.00000%	0.02216%	-49	1.011997	-50	1.1403	-57
8	-876,962	Sat, 25-Jul-20	1.00000	0	0.00000%	0.08249%	-183	1.013047	-185	1.1403	-211
9	-876,962	Sun, 25-Oct-20	1.00000	0	0.00000%	0.13501%	-299	1.013962	-303	1.1403	-346
10	-876,962	Sun, 24-Jan-21	1.00000	0	0.00000%	0.19845%	-440	1.014734	-446	1.1403	-509
11	-876,962	Sun, 25-Apr-21	1.00000	0	0.00000%	0.27912%	-619	1.015295	-628	1.1403	-716
12	-876,962	Sun, 25-Jul-21	1.00000	0	0.00000%	0.37754%	-837	1.015577	-850	1.1403	-969
13	-876,962	Mon, 25-Oct-21	1.00000	0	0.00000%	0.48748%	-1,081	1.015536	-1,097	1.1403	-1,251
14	-876,962	Mon, 24-Jan-22	1.00000	0	0.00000%	0.58832%	-1,304	1.015210	-1,324	1.1403	-1,510
15	-876,962	Mon, 25-Apr-22	1.00000	0	0.00000%	0.67584%	-1,498	1.014663	-1,520	1.1403	-1,733
16	-876,962	Mon, 25-Jul-22	1.00000	0	0.00000%	0.74980%	-1,662	1.013957	-1,685	1.1403	-1,922
17	-876,962	Tue, 25-Oct-22	1.00000	0	0.00000%	0.81171%	-1,799	1.013132	-1,823	1.1403	-2,079
18	-876,962	Tue, 24-Jan-23	1.00000	0	0.00000%	0.87156%	-1,932	1.012214	-1,956	1.1403	-2,230
19	-876,962	Tue, 25-Apr-23	1.00000	0	0.00000%	0.93160%	-2,065	1.011157	-2,088	1.1403	-2,381
20	-876,962	Tue, 25-Jul-23	1.00000	-876,962	0.00000%	0.99282%	-879,163	1.009939	-887,901	1.1403	-1,012,474

Leg2 - USD Cashflows										
	Notional	FXFixingDate	ForwardFX NotionalExchange	Spread	FloatRate	Coupon	DiscountFactor	CouponPV	SpotFX	ValuationPV
0			-1,000,000			-1,000,000	1.000000	-1,000,000	1.0000	-1,000,000

# Appendix - Useful Resources

#### References

- 1. Collateralization & CSA Fundamentals https://ssrn.com/abstract=3035648
- 2. Discounting with Collateral https://ssrn.com/abstract=3009281
- 3. An Interest Rate Swap Primer https://ssrn.com/abstract=2815495
- 4. Cross Currency Swaps https://ssrn.com/abstract=3278907
- 5. Interest Rate Modeling: Volume I-III
  Atlantic Financial Press Vladimir Piterbarg
- 6. Interest Rate Models Theory & Practice Springer Damiano Brigo, Fabio Mercurio