

Probabilistic logic and statistical inference

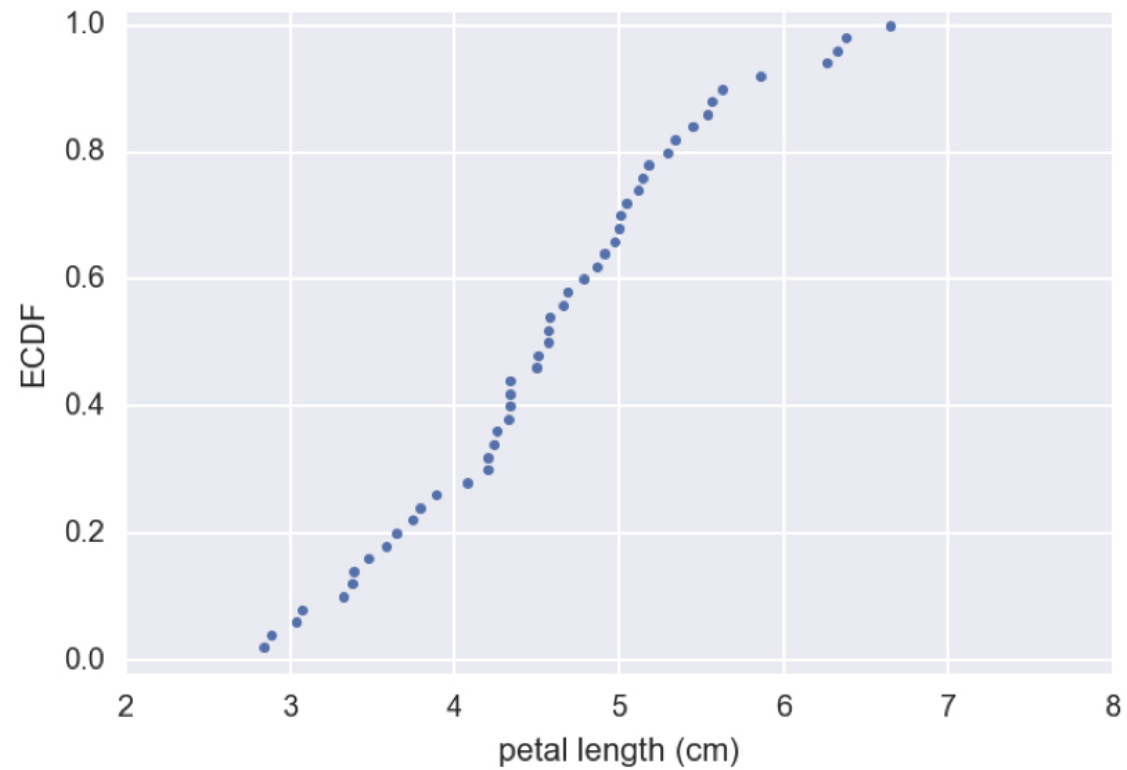
STATISTICAL THINKING IN PYTHON (PART 1)



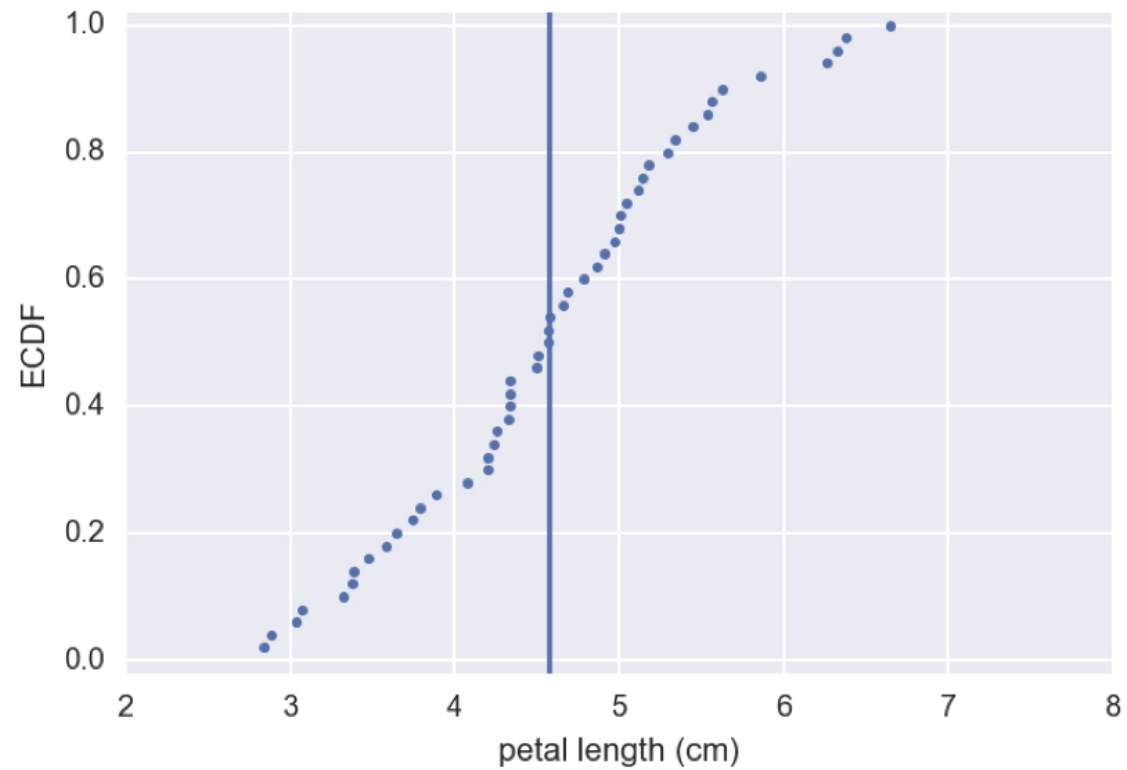
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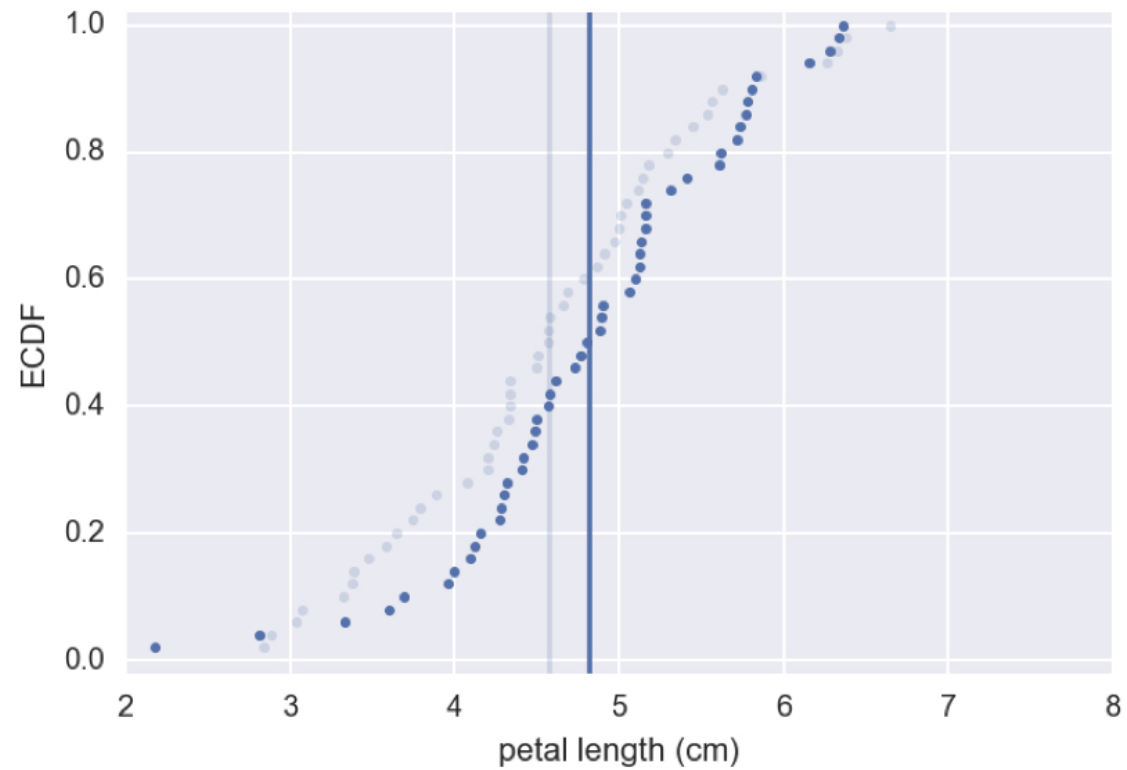
50 measurements of petal length



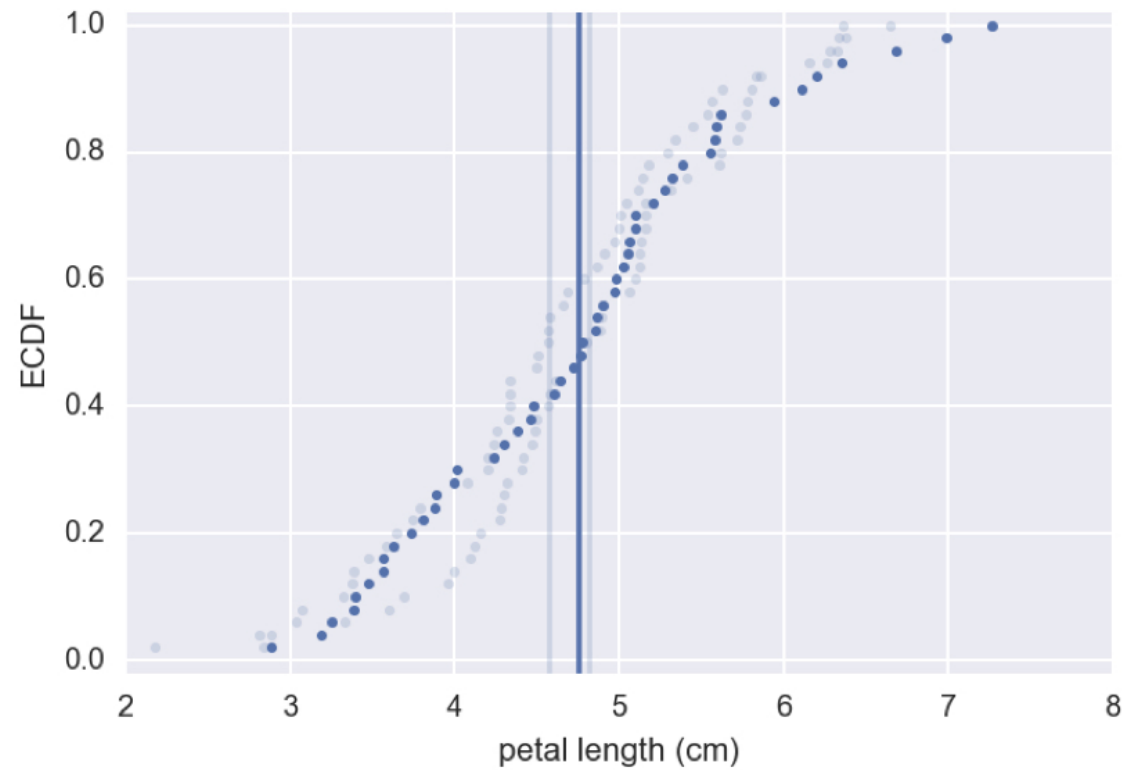
50 measurements of petal length



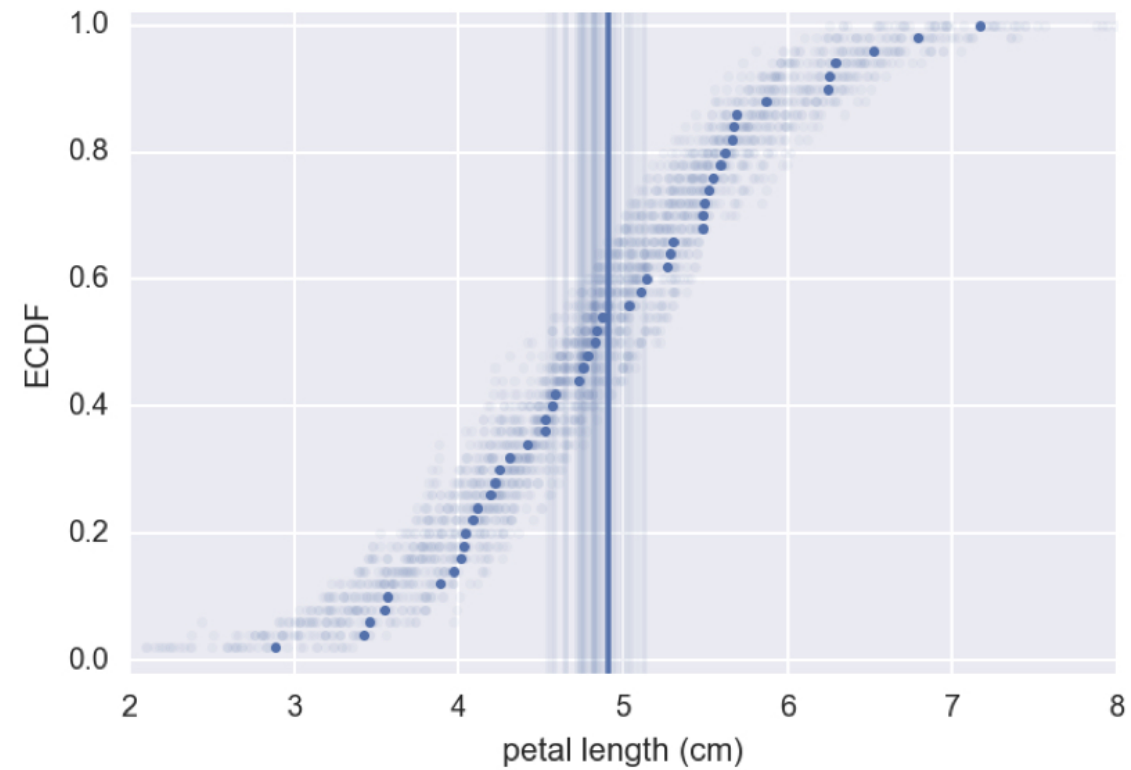
50 measurements of petal length



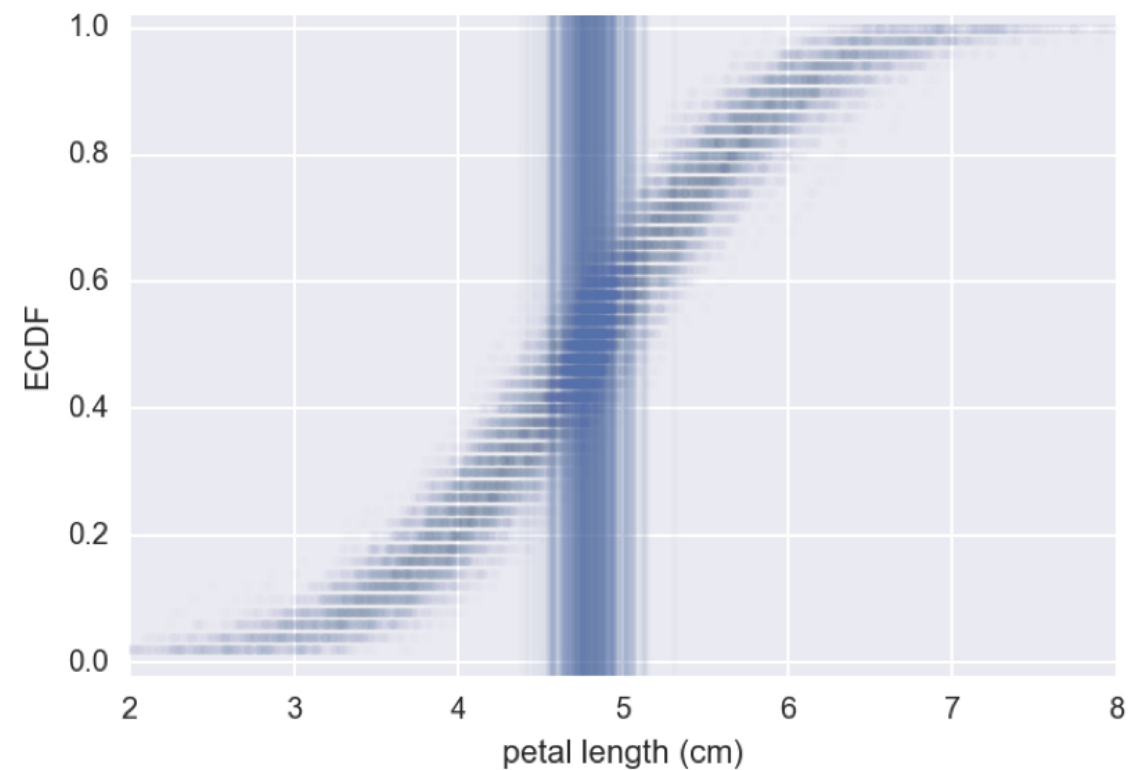
50 measurements of petal length



50 measurements of petal length



Repeats of 50 measurements of petal length

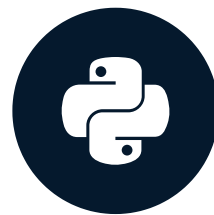


Let's practice!

STATISTICAL THINKING IN PYTHON (PART 1)

Random number generators and hacker statistics

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Hacker statistics

- Uses simulated repeated measurements to compute probabilities.



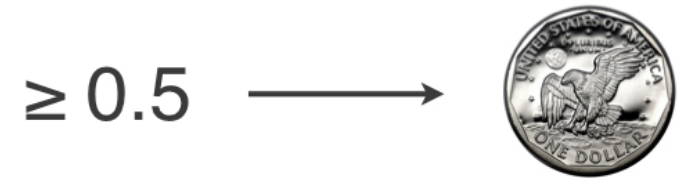
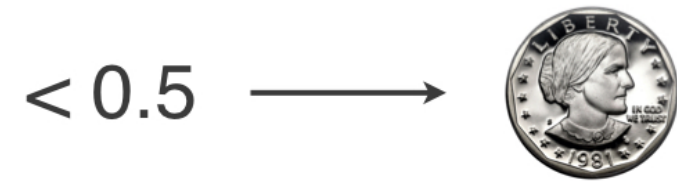
Blaise Pascal

¹ Image: artist unknown



¹ Image: Heritage Auction

Simulating coin flips



Bernoulli trials

- An experiment that has two options, "success" (True) and "failure" (False).

The np.random module

```
import numpy as np  
rng = np.random.default_rng()
```

```
rng
```

```
Generator(PCG64) at 0x7F9433D38120
```

Random number seed

- Integer fed into random number generating algorithm
- Manually seed random number generator (only) if you need reproducibility
- Specified using `rng = np.random.default_rng(seed)`

Simulating 4 coin flips

```
rng = np.random.default_rng(42)

random_numbers = rng.random(size=4)
random_numbers
```

```
array([0.77395605, 0.43887844, 0.85859792, 0.69736803])
```

```
heads = random_numbers < 0.5
heads
```

```
array([False,  True, False, False])
```

```
np.sum(heads)
```

```
1
```

Simulating 4 coin flips

```
n_all_heads = 0 # Initialize number of 4-heads trials
for _ in range(10000):
    heads = np.random.random(size=4) < 0.5
    n_heads = np.sum(heads)
    if n_heads == 4:
        n_all_heads += 1

n_all_heads / 10000
```

0.0607

Hacker stats probabilities

- Determine how to simulate data
- Simulate many many times
- Probability is approximately fraction of trials with the outcome of interest

Let's practice!

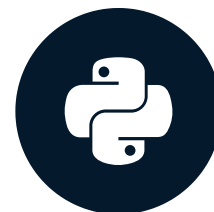
STATISTICAL THINKING IN PYTHON (PART 1)

Probability distributions and stories: The Binomial distribution

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Probability mass function (PMF)

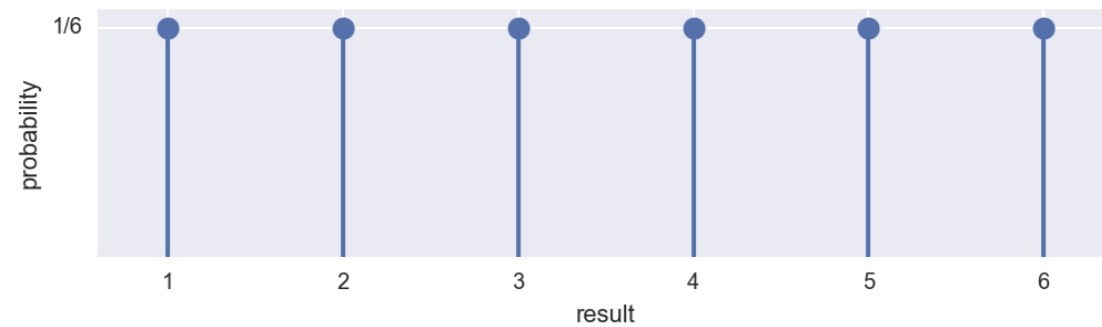
- The set of probabilities of discrete outcomes

Discrete Uniform PMF

Tabular

					
1/6	1/6	1/6	1/6	1/6	1/6

Graphical



Probability distribution

- A mathematical description of outcomes

Discrete Uniform distribution: the story

The outcome of rolling a single fair die is

- Discrete
- Uniformly distributed.

Binomial distribution: the story

- The number r of successes in n Bernoulli trials with probability p of success, is Binomially distributed
- The number r of heads in 4 coin flips with probability 0.5 of heads, is Binomially distributed

Sampling from the Binomial distribution

```
rng.binomial(4, 0.5)
```

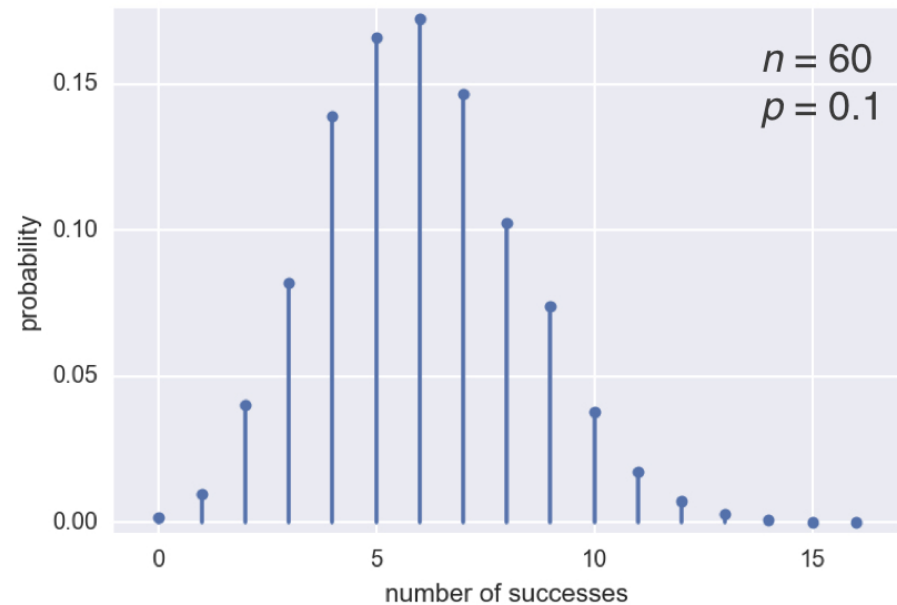
```
2
```

```
rng.binomial(4, 0.5, size=10)
```

```
array([4, 3, 2, 1, 1, 0, 3, 2, 3, 0])
```

The Binomial PMF

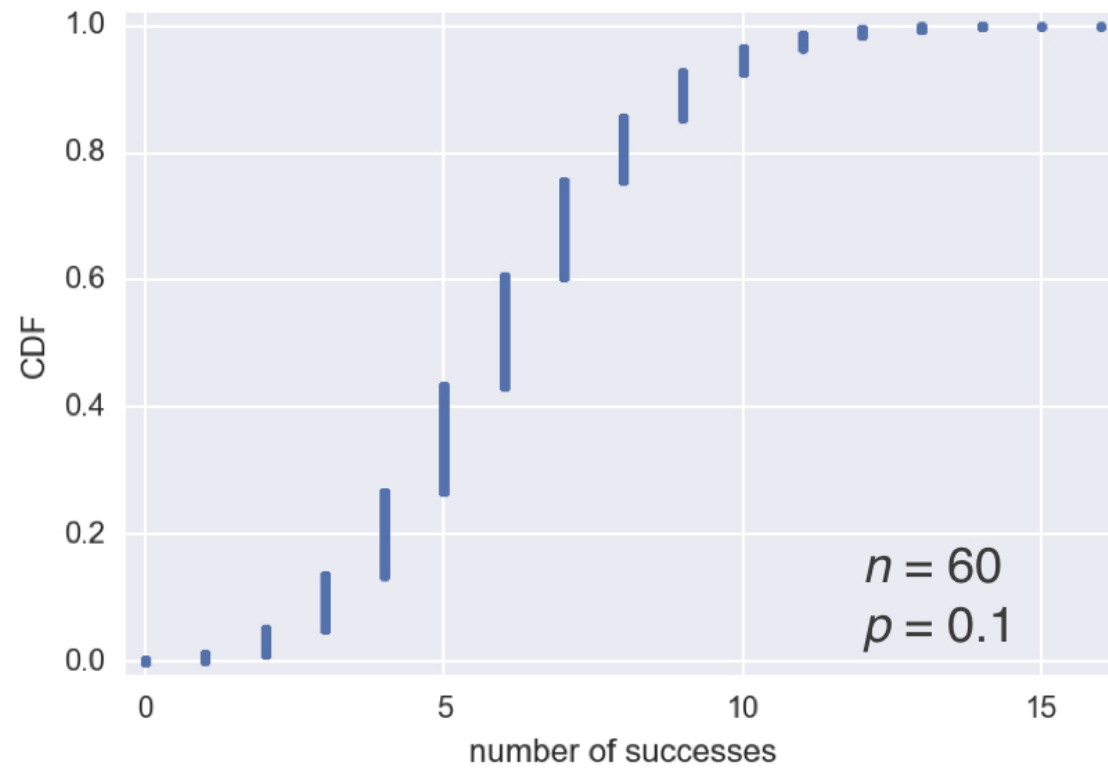
```
samples = rng.binomial(60, 0.1, size=10000)
n = 60
p = 0.1
```



The Binomial CDF

```
import matplotlib.pyplot as plt
import seaborn as sns
sns.set()
x, y = ecdf(samples)
_ = plt.plot(x, y, marker='.', linestyle='none')
plt.margins(0.02)
_ = plt.xlabel('number of successes')
_ = plt.ylabel('CDF')
plt.show()
```

The Binomial CDF

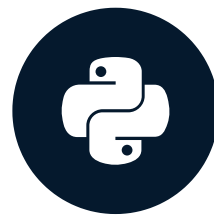


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STATISTICAL THINKING IN PYTHON (PART 1)

Poisson processes and the Poisson distribution

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Poisson process

- The timing of the next event is completely independent of when the previous event happened

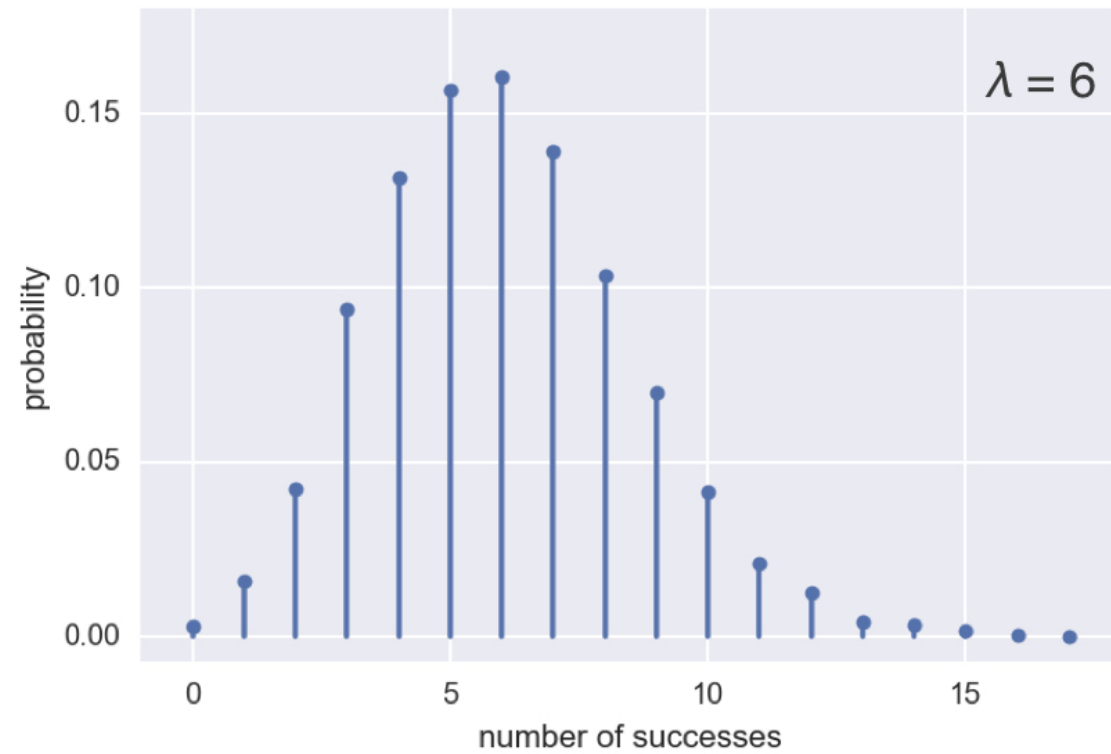
Examples of Poisson processes

- Natural births in a given hospital
- Hits on a website during a given hour
- Meteor strikes
- Molecular collisions in a gas
- Aviation incidents
- Buses in Poissonville

Poisson distribution

- The number r of arrivals of a Poisson process in a given time interval with average rate of λ arrivals per interval is Poisson distributed.
- The number r of hits on a website in one hour with an average hit rate of 6 hits per hour is Poisson distributed.

Poisson PMF



Poisson Distribution

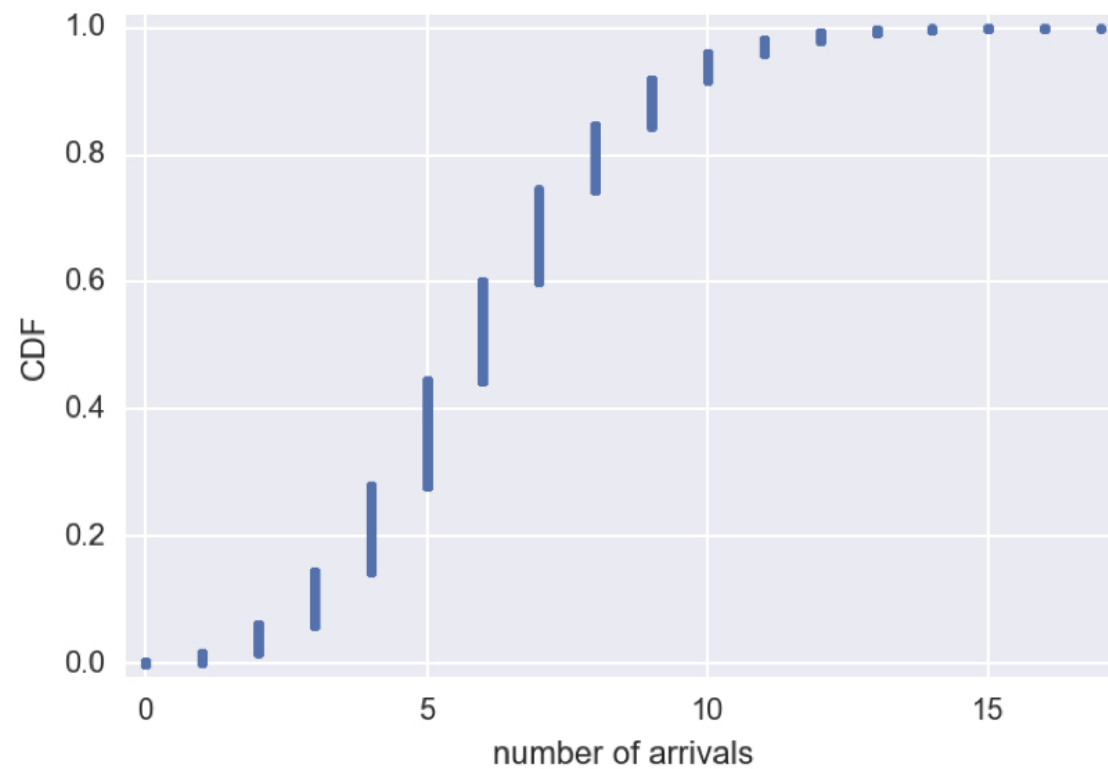
- Limit of the Binomial distribution for low probability of success and large number of trials.
- That is, for rare events.

The Poisson CDF

```
samples = rng.poisson(6, size=10000)
```

```
x, y = ecdf(samples)
_ = plt.plot(x, y, marker='.', linestyle='none')
plt.margins(0.02)
_ = plt.xlabel('number of successes')
_ = plt.ylabel('CDF')
plt.show()
```

The Poisson CDF



Let's practice!

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