Bond price volatility and the price value of a basis point

BOND VALUATION AND ANALYSIS IN R

Clifford Ang
Senior Vice President, Compass Lexecon

Bond price volatility

- Bond price volatility depends on many factors
- Some examples:
  - Size of yield change
  - Coupon rate
  - Time to maturity

Small change, symmetric effect

- Small changes in yield: % change for most bonds are similar whether yield goes up or down
- Example:
  - 100 USD par value, 10% coupon rate, 20 years, 10% yield

```
bondprc(100, 0.10, 20, 0.101) / bondprc(100, 0.10, 20, 0.10) - 1
```

```
-0.008455776
```

```
bondprc(100, 0.10, 20, 0.099) / bondprc(100, 0.10, 20, 0.10) - 1
```

```
0.008571998
```

Large change, asymmetric effect

- For large changes in yield, the percentage change is higher when the yield decreases
- Example:
  - 100 USD par value, 10% coupon rate, 20 years, 10% yield

```
bondprc(100, 0.10, 20, 0.14) / bondprc(100, 0.10, 20, 0.10) - 1
```

```
-0.2649252
```

```
bondprc(100, 0.10, 20, 0.06) / bondprc(100, 0.10, 20, 0.10) - 1
```

```
0.4587968
```

Lower coupon, more volatile

- Fixing the time to maturity and yield, bond price volatility is higher if the coupon rate is lower
- Example:
  - 100 USD par value, 20 years, 10% initial yield, 8% new yield

```
bondprc(100, 0.10, 20, 0.08) / bondprc(100, 0.10, 20, 0.10) - 1
```

```
0.1963629
```

```
bondprc(100, 0.05, 20, 0.08) / bondprc(100, 0.05, 20, 0.10) - 1
```

```
0.228328
```

Lower coupon, more volatile

```
bondprc(100, 0.00, 20, 0.08) / bondprc(100, 0.00, 20, 0.10) - 1
```

```
0.4433731
```

Longer maturity, more volatile

- Fixing the coupon rate and yield, bond price volatility is higher if the time to maturity is longer
- Example:
  - 100 USD par value, 10% coupon rate, 10% initial yield, 8% new yield

```
bondprc(100, 0.10, 20, 0.08) / bondprc(100, 0.10, 20, 0.10) - 1
```

```
0.1963629
```

```
bondprc(100, 0.10, 10, 0.08) / bondprc(100, 0.10, 10, 0.10) - 1
```

```
0.1342016
```

Shorter maturity, more volatile

```
bondprc(100, 0.10, 5, 0.08) / bondprc(100, 0.10, 5, 0.10) - 1
```

```
0.0798542
```

Price value of a basis point

- Or "dollar value of an 01" = measure of bond price volatility
  - = price of the bond if the required yield changes by 0.01%
- Example:

```
bondprc(100, 0.05, 20, 0.05)
```

```
100
```

```
bondprc(100, 0.05, 20, 0.0501)
```

```
99.87548
```

Price value of a basis point

```
abs(bondprc(100, 0.05, 20, 0.0501) - bondprc(100, 0.05, 20, 0.05))
```

```
0.1245165
```

- `abs()` to make sure the change is positive

Let's practice!

BOND VALUATION AND ANALYSIS IN R

Duration

BOND VALUATION AND ANALYSIS IN R

Clifford Ang
Senior Vice President, Compass Lexecon

What is duration?

- Estimated price change for a 100 basis point change in yield
  - Two bonds with the same duration will have same estimated price change
- A way to manage the risk of interest rate sensitive liabilities

Calculating duration

$$D = \frac{P(\text{down}) - P(\text{up})}{(2 \cdot P \cdot \Delta y)}$$

- D = Duration
- $P(\text{down})$ = Price when yield goes down
- $P(\text{up})$ = Price when yield goes up
- P = Current price
- Δy = Change in yield

Estimating price change

$$\frac{\Delta P}{P} = -D \cdot \Delta y$$

$$\Delta P = -D \cdot \Delta y \cdot P$$

$\frac{\Delta P}{P}$ = Percent change

D = duration

Δy = Change in yield

ΔP = Dollar change

P = Current price

How do you use these formulas?

- Example: $100 par value, 5% coupon rate, 10 years to maturity, initial yield = 4%, expected increase in yield = 1%

```
(p <- bondprc(100, .05, 10, .04))
```

```
108.1109
```

```
(p_down <- bondprc(100, .05, 10, .03))
```

```
117.0604
```

```
(p_up <- bondprc(100, .05, 10, .05))
```

```
100
```

How do you use these formulas?

```
(duration <- (p_down - p_up) / (2 * p * 0.01))
```

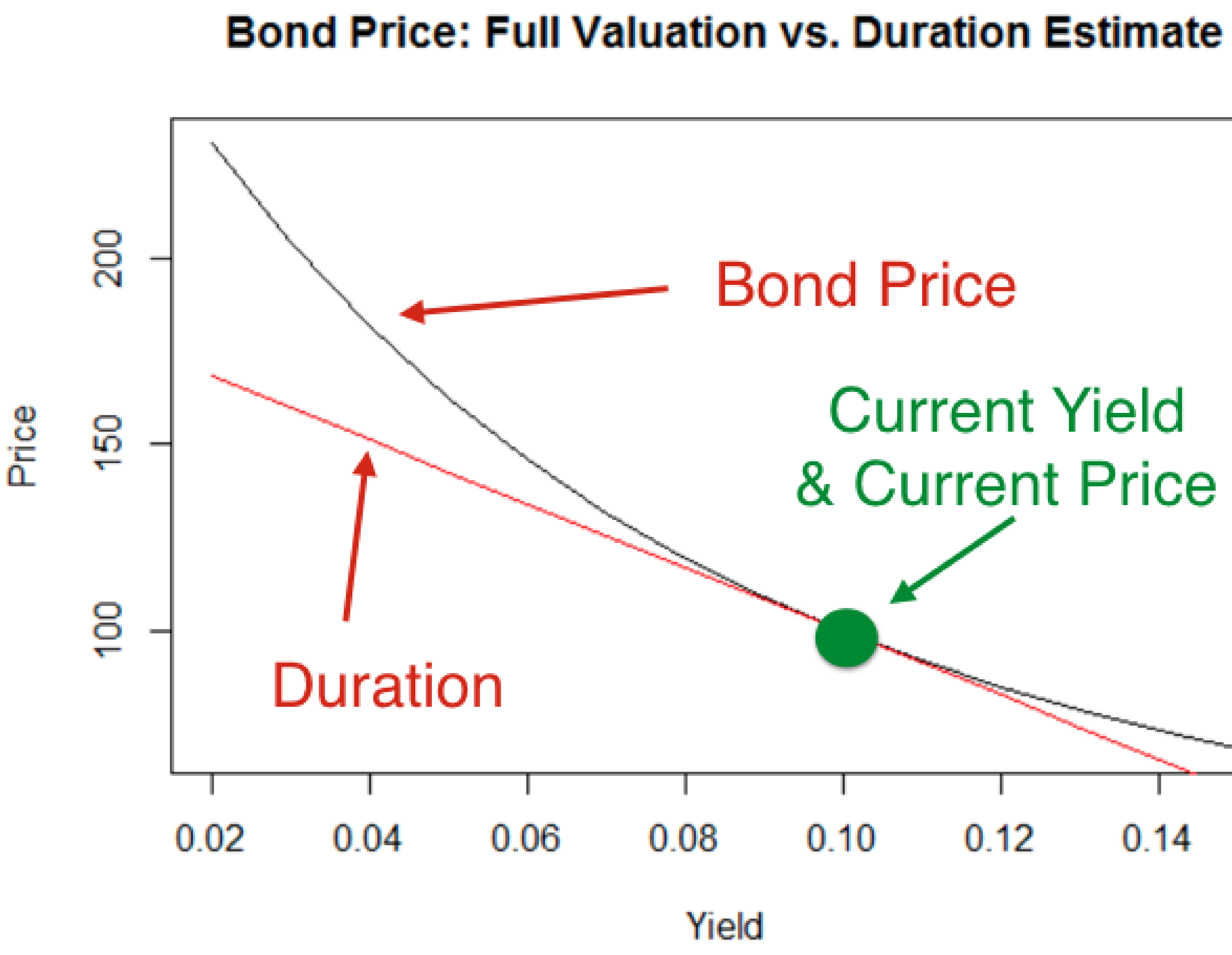
```
7.890234
```

```
(duration_pct_change <- - duration * 0.01)
```

```
-0.07890234
```

```
(duration_dollar_change <- duration_pct_change * p)
```

```
-8.530203
```


Bond Price: Full Valuation vs. Duration Estimate
Bond Price
Current Yield
& Current Price
Duration
Price
200
150
100
0.02
0.04
0.06
0.08
0.10
0.12
0.14
Yield

Let's practice!

BOND VALUATION AND ANALYSIS IN R

Convexity

BOND VALUATION AND ANALYSIS IN R

Clifford Ang
Senior Vice President, Compass Lexecon

Convexity measure

- Duration does a poor job when yield changes are large
- Convexity measure is used to adjust the duration estimate

Calculating the convexity measure

$$C = \frac{P(down) + P(up) - 2 \cdot P}{(P \cdot (\Delta y)^2}$$

- C = Convexity measure
- $P(down)$ = Price when yield goes down
- $P(up)$ = Price when yield goes up
- P = Current price
- $(\Delta y)^2$ = Change in yield squared
- $2 \cdot P$ = 2x current price

Estimating effect on price

$$\frac{\Delta P}{P} = 0.5 \cdot C \cdot (\Delta y)^2$$

- $\frac{\Delta P}{P}$ = Percent change

$$\Delta P = 0.5 \cdot C \cdot (\Delta y)^2 \cdot P$$

- ΔP = Dollar change
- C = Convexity measure
- $(\Delta y)^2$ = Change in yield squared
- P = Current price

How do you use these formulas?

- Example (same as duration)
  - \$100 par value, 5% coupon rate, 10 years to maturity, initial yield = 4%, expected increase in yield = 1%

```
p
```

```
108.1109
```

```
(convexity <- (p_down + p_up - 2 * p) / (p * (0.01^2)))
```

```
77.56981
```

How do you use these formulas?

```
(convexity_pct_change <- 0.5 * convexity * 0.01 ^ 2)
```

```
0.00387849
```

```
(convexity_dollar_change <- 0.5 * convexity * 0.01 ^ 2 * p)
```

```
0.4193071
```

Effect of duration and convexity

- Estimated change in price

```
duration_dollar_change
```

```
-8.530203
```

```
convexity_dollar_change
```

```
0.4193071
```

```
duration_dollar_change + convexity_dollar_change
```

```
-8.110896
```

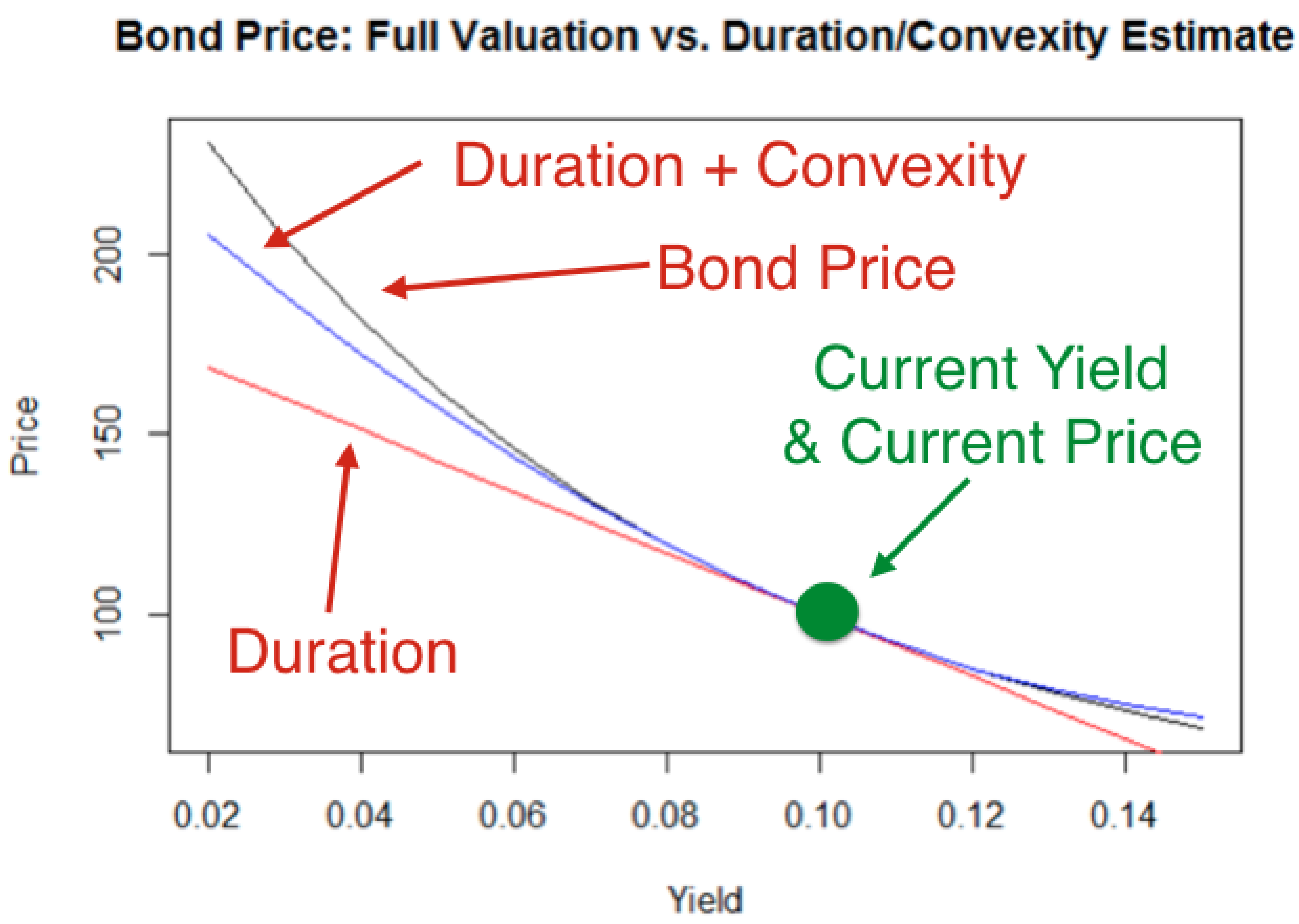
- Estimated price

```
p
```

```
108.1109
```

```
duration_dollar_change + convexity_dollar_change + p
```

```
100
```


Bond Price: Full Valuation vs. Duration/Convexity Estimate
Duration + Convexity
Bond Price
Current Yield
& Current Price
Duration
Price
200
150
100
0.02
0.04
0.06
0.08
0.10
0.12
0.14
Yield

Let's practice!

BOND VALUATION AND ANALYSIS IN R