The normal distribution

QUANTITATIVE RISK MANAGEMENT IN R

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Definition of normal

- If risk factors follow GBM, then log-returns should be independent normal
- Is this the case?
- A variable x is normal if it has density:

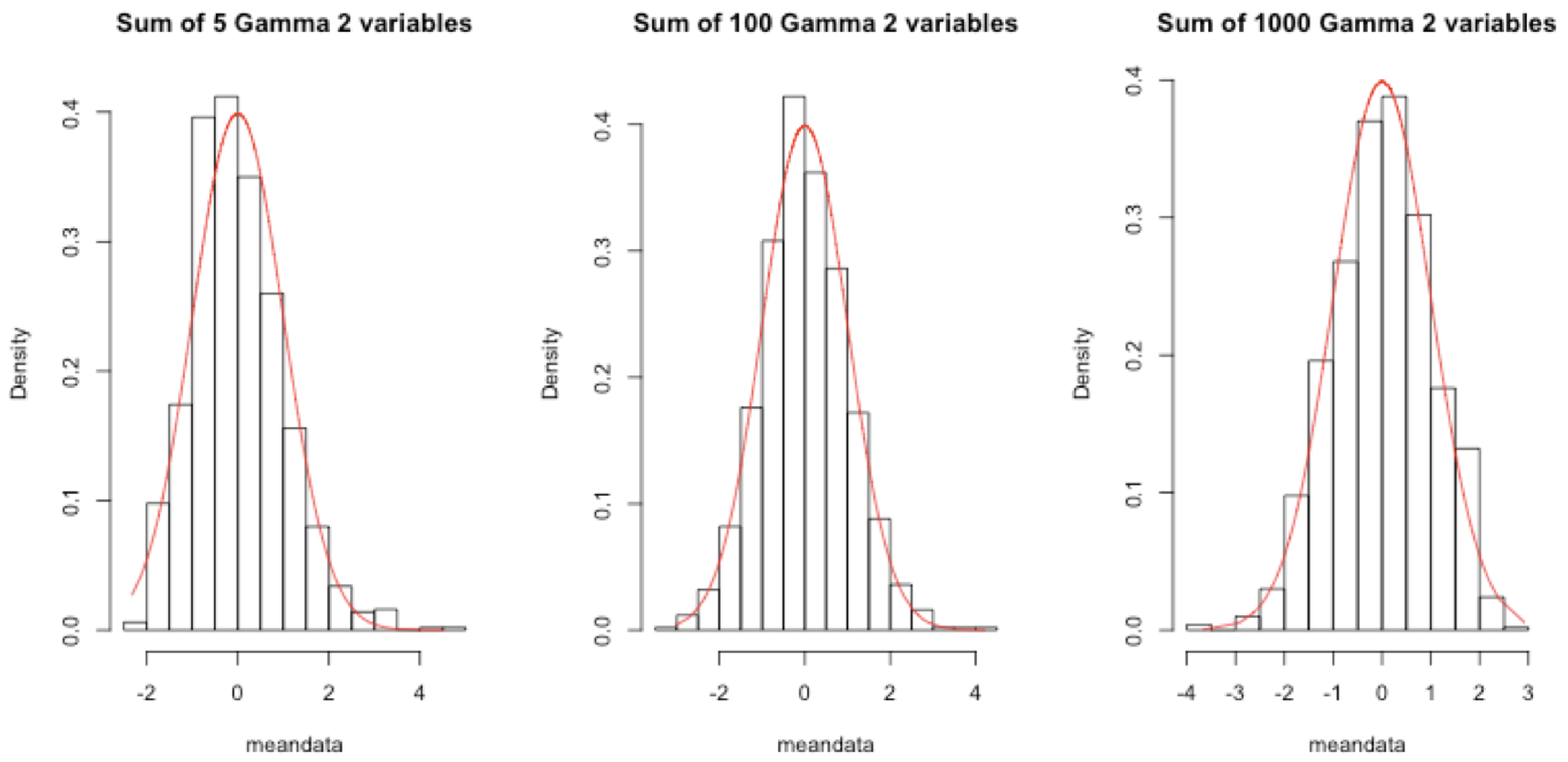
$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{\sigma^2}}$$

- Depends on two parameters: μ and σ

Properties of the normal

- μ is the mean and σ^2 is the variance
- Usual notation: $X \sim N(\mu, \sigma^2)$
- Parameters easily estimated from data
- Sum of 2+ independent normal variables is also normal

Central limit theorem (CLT)



How to estimate a normal distribution

- Data: $X_1, ..., X_n$
- **Method of moments:**

$$\hat{\mu} = \frac{1}{n}\sum_{t=1}^{n} X_t$$

$$\hat{\sigma}_u^2 = \frac{1}{n-1}\sum_{t=1}^{n}(X_t - \hat{\mu})^2$$

- Application to FTSE log-returns from 2008-09

FTSE example

```r
head(ftse)
```

```
-0.09264548 -0.08178433 -0.07428657 -0.05870079 -0.05637430 -0.05496918
```

```r
tail(ftse)
```

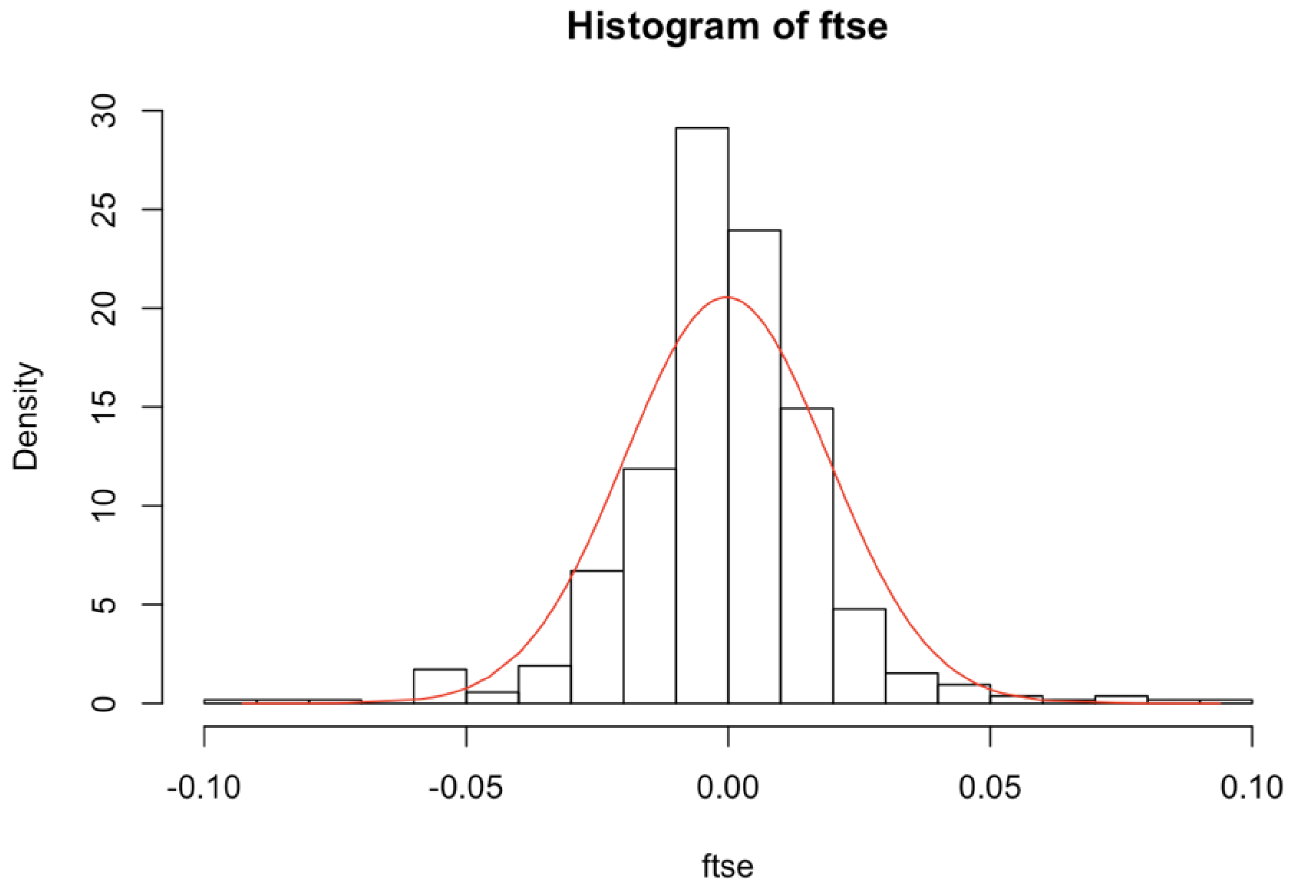
```
0.05266208 0.06006960 0.07742977 0.07936751 0.08469137 0.09384244
```

```r
mu <- mean(ftse)
sigma <- sd(ftse)
c(mu, sigma)
```

```
-0.0003378627  0.0194090385
```

Displaying the fitted normal

```
hist(ftse, nclass = 20, probability = TRUE)
lines(ftse, dnorm(ftse, mean = mu, sd = sigma), col = "red")
```



Let's practice!

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Testing for normality

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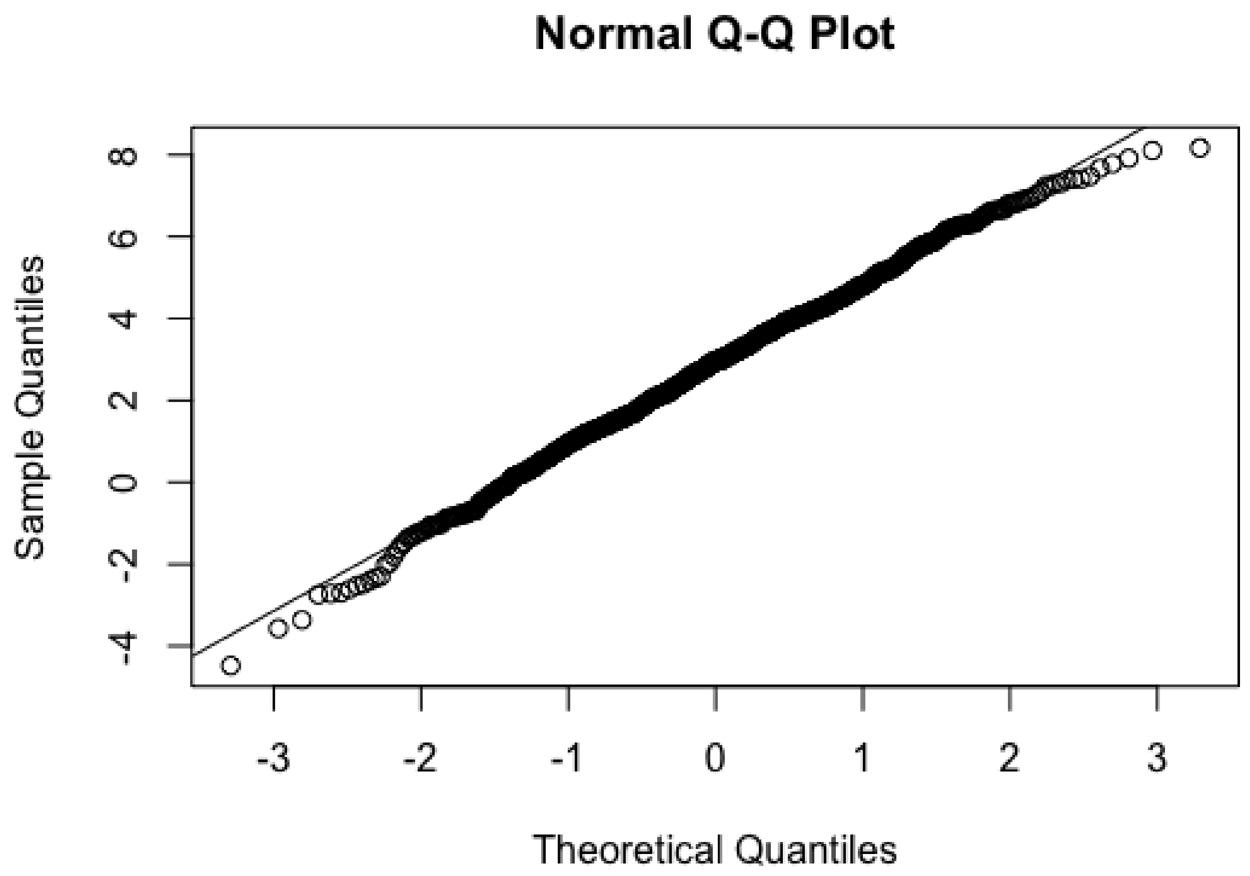
How to test for normality

- Use the **quantile-quantile** plot (Q-Q plot)
- Sample of quantiles of data versus theoretical quantiles of a normal distribution

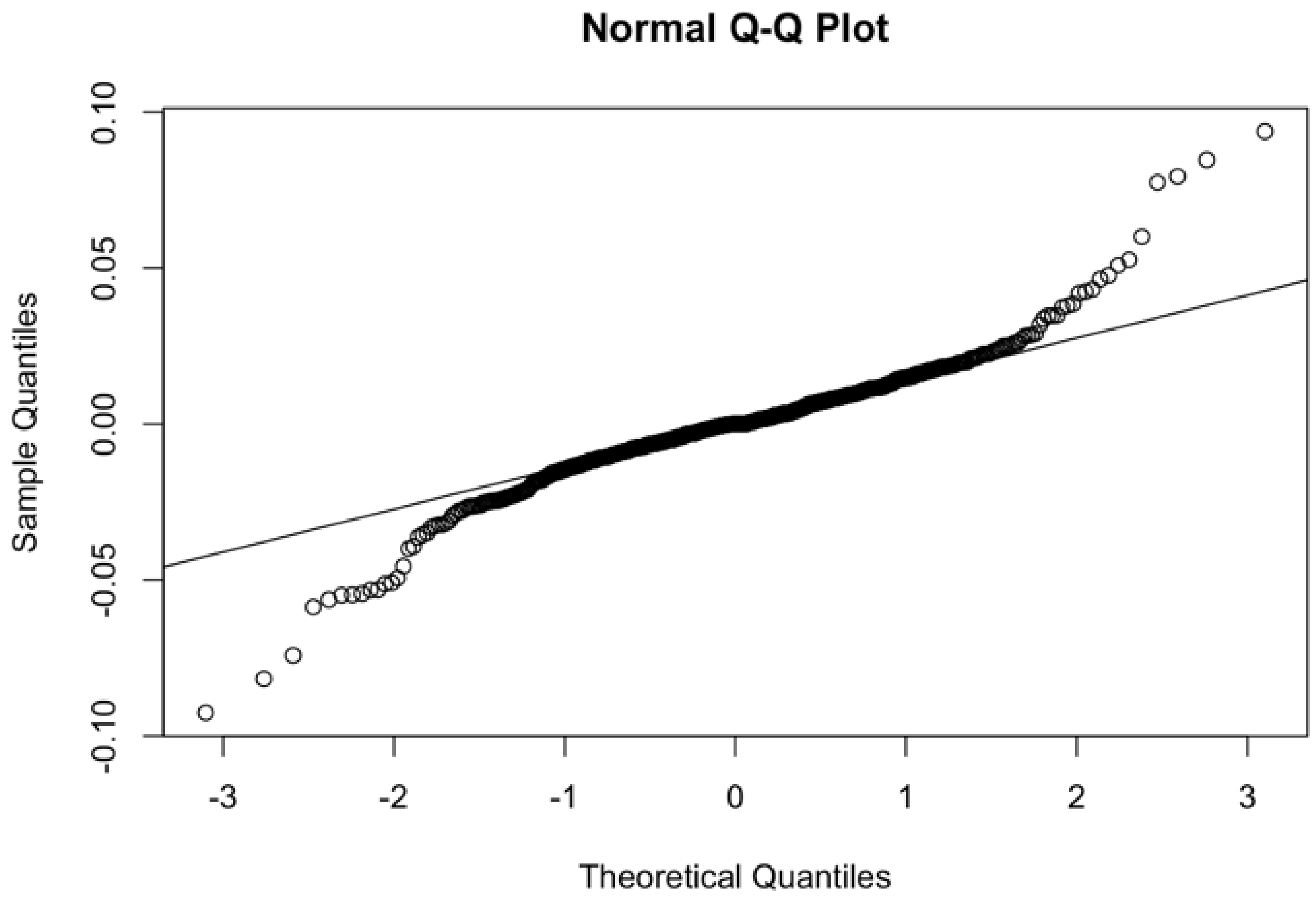
```
data <- rnorm(1000,
         mean = 3,
         sd = 2)
qqnorm(data)
qqline(data)
```



Interpreting the Q-Q plot

- Data with heavier tails than normal: inverted S shape
- Data with lighter tails than normal: S shape
- Data from a very skewed distribution: curved shape

```
qqnorm(ftse)
qqline(ftse)
```



Let's practice!

QUANTITATIVE RISK MANAGEMENT IN R

Skewness, kurtosis and the Jarque-Bera test

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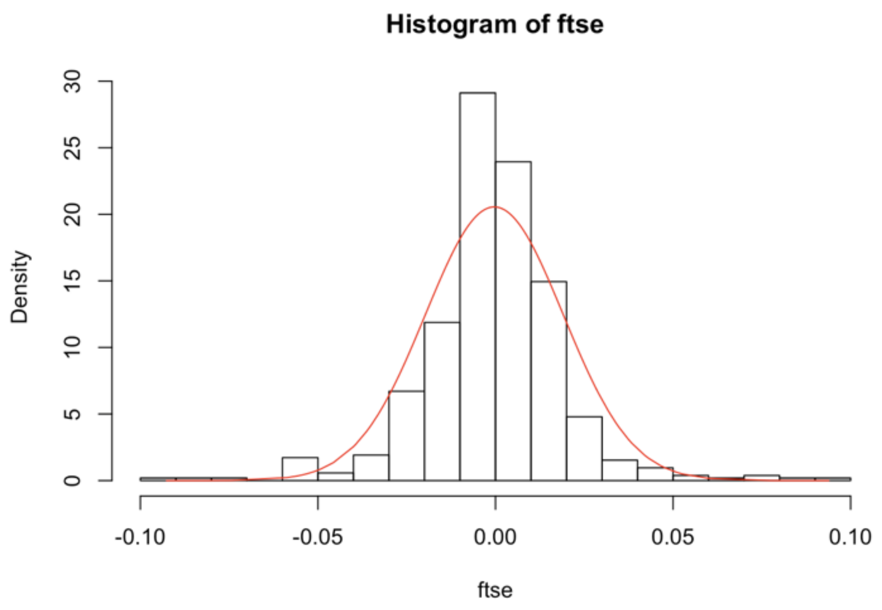
Skewness and kurtosis

- **Skewness** (b) is a measure of asymmetry
- **Kurtosis** (k) is a measure of heavy-tailedness
- Skewness and kurtosis of normal are 0 and 3, respectively



$$= \frac{\frac{1}{n}\sum_{t=1}^{n}(X_t - \hat{\mu})^3}{\hat{\sigma}^3}$$

$$= \frac{\frac{1}{n}\sum_{t=1}^{n}(X_t - \hat{\mu})^4}{\hat{\sigma}^4}$$

Skewness and kurtosis (II)

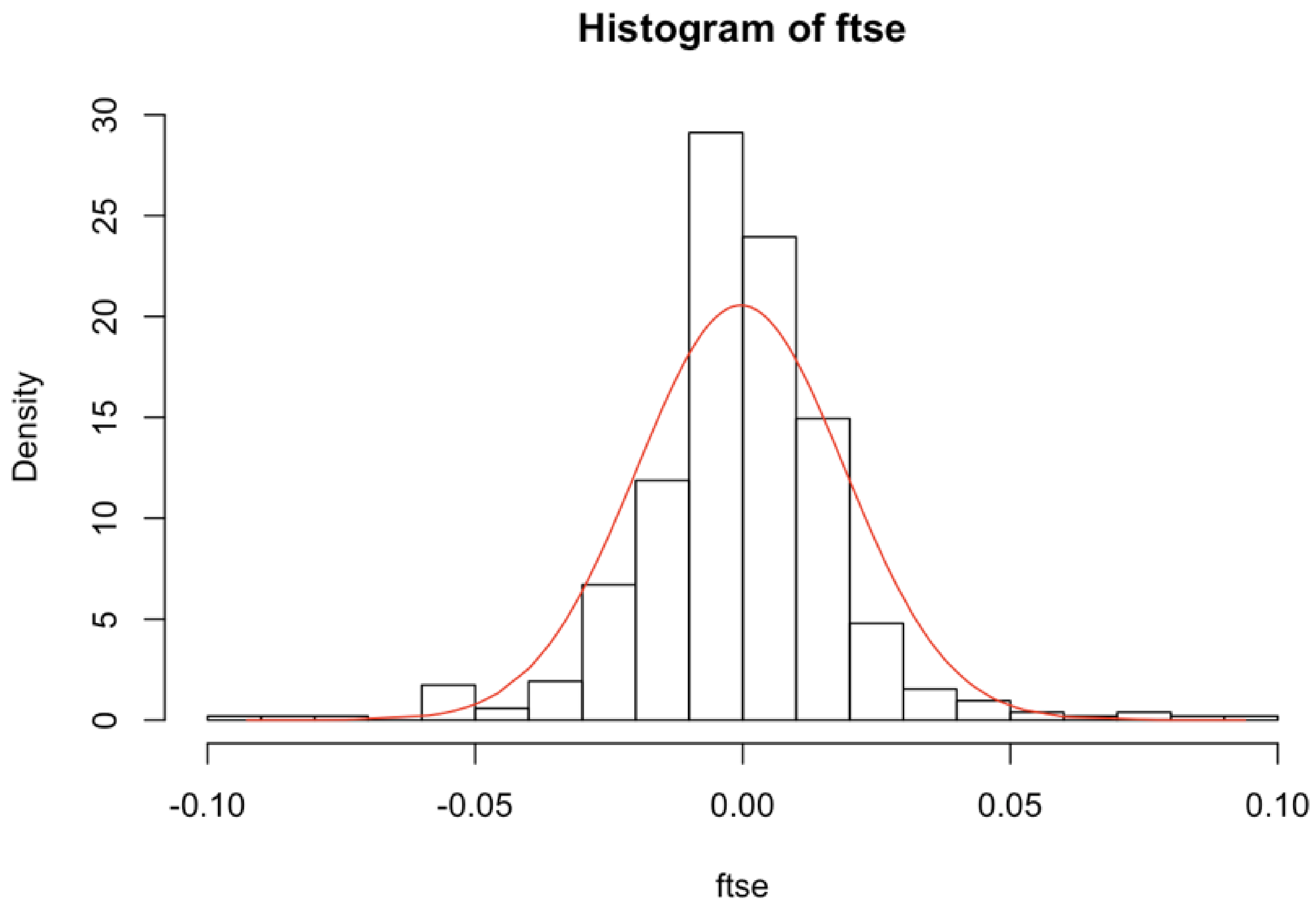
```
library(moments)

skewness(ftse)
```

```
-0.01187921
```

```
kurtosis(ftse)
```

```
7.437121
```



The Jarque-Bera test

- Compares skewness and kurtosis of data with theoretical normal values (0 and 3)
- Detects skewness, heavy tails, or both

$$T = \frac{1}{6}n\left(b^2 + \frac{1}{4}(k-3)^2\right)$$

```
jarque.test(ftse)
```

```
    Jarque-Bera Normality Test
data: ftse
JB = 428.23, p-value < 2.2e-16
alternative hypothesis: greater
```

Longer-interval and overlapping returns

- Daily returns are usually very non-normal
- What about longer-intervals returns?
- Weekly, monthly, quarterly returns obtained by summation
- Recall CLT - suggests they may be more normal
- Reduce quantity of data so tests are weaker
- Can also analyze **overlapping** or **moving sums** of returns

Let's practice!

QUANTITATIVE RISK MANAGEMENT IN R

The Student t distribution

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The Student t distribution

$$f_X(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\sigma\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})}\left(1 + \frac{(x-\mu)^2}{\nu\sigma^2}\right)^{-\frac{\nu+1}{2}}$$

- This distribution has three parameters: μ, σ, ν
- Small values of ν give heavier tails
- As ν gets larger the distribution tends to normal

Fitting the Student t distribution

- Method of **maximum likelihood** (ML)
- `fit.st()` in QRM package
- Small ν value (2.95) for FTSE log-returns from 2008-09

```
library(QRM)

tfit <- fit.st(ftse)
tpars <- tfit$par.ests
tpars
```

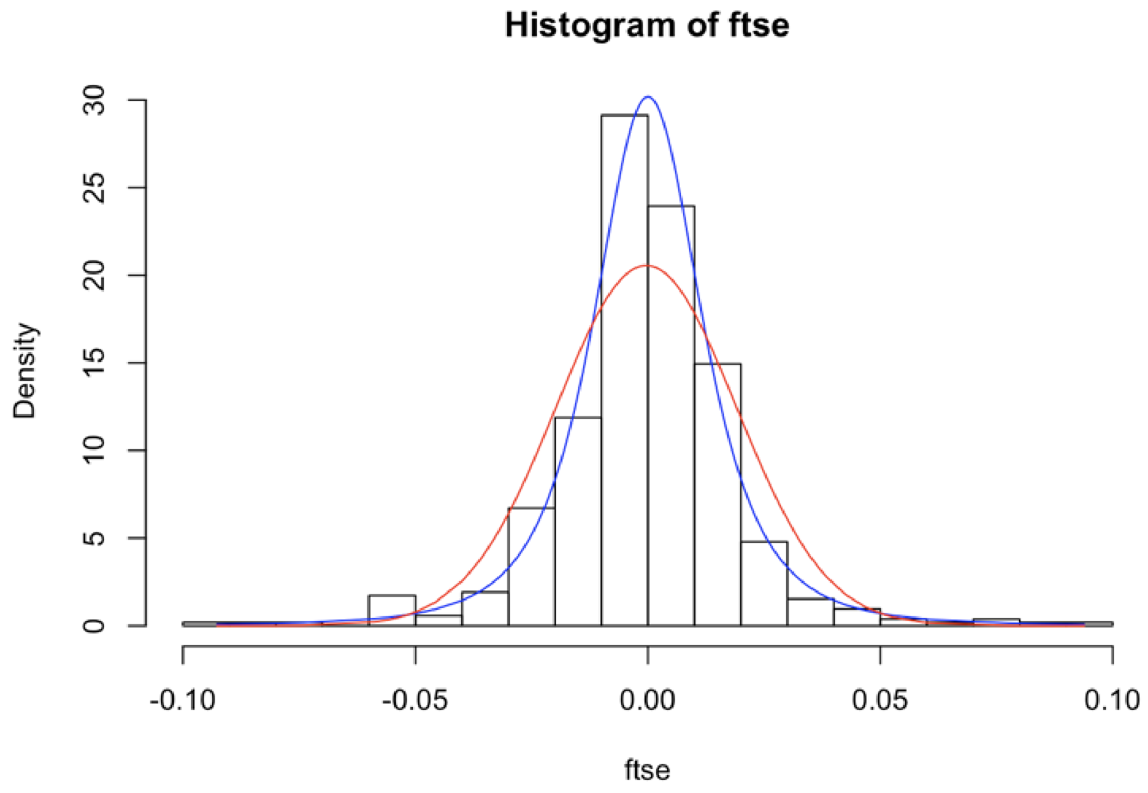
```
          nu            mu         sigma
2.949514e+00  4.429863e-05  1.216422e-02
```

```
nu <- tpars[1]
mu <- tpars[2]
sigma <- tpars[3]
```

Displaying the fitted Student t distribution

```
hist(ftse, nclass = 20, probability = TRUE)
lines(ftse, dnrom(ftse, mean = mean(ftse), sd = sd(ftse)), col = "red")

yvals <- dt((ftse - mu)/sigma, df = nu)/sigma
lines(ftse, yvals, col = "blue")
```



Let's practice!

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