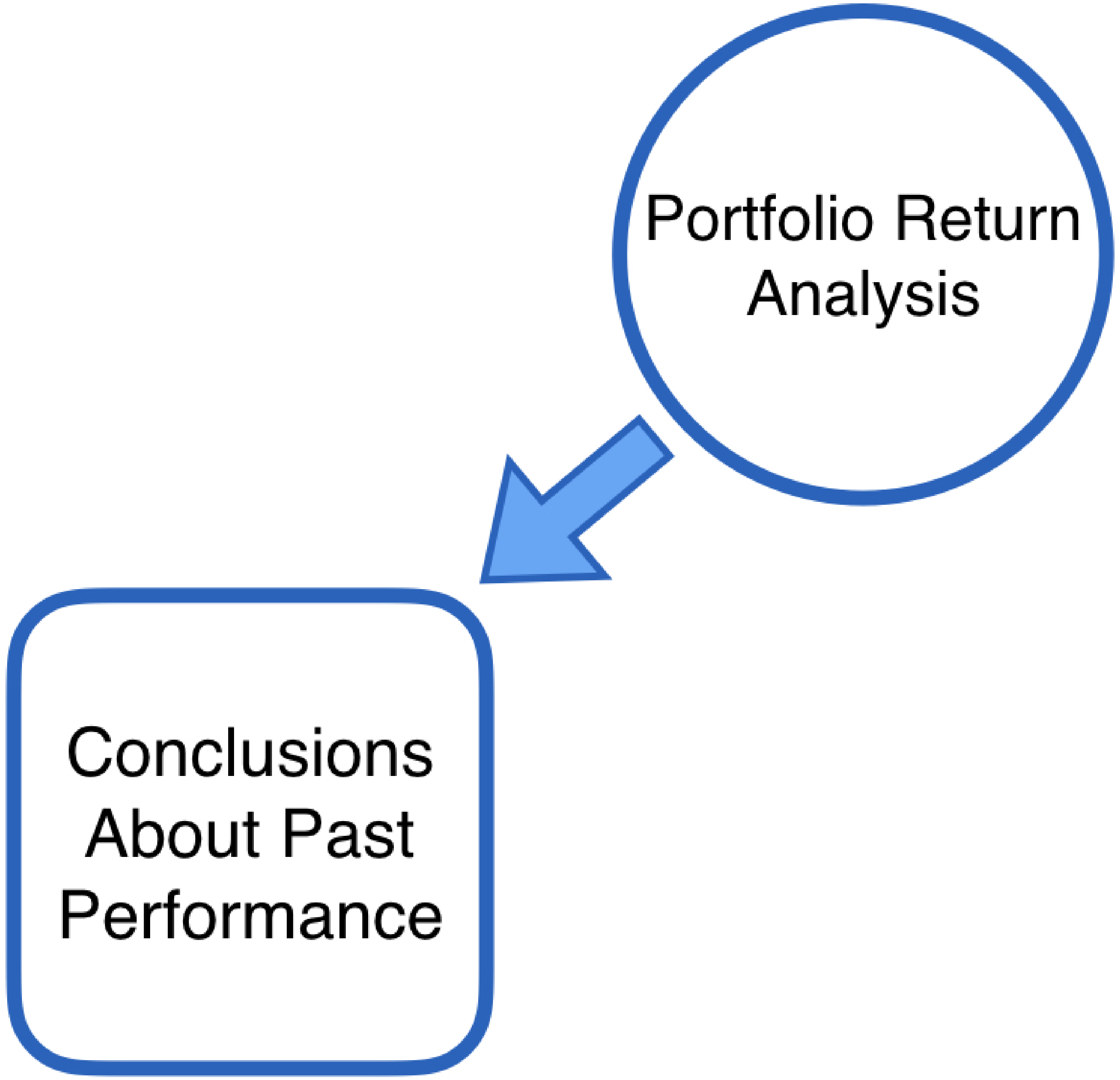
Dimensions of portfolio performance

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

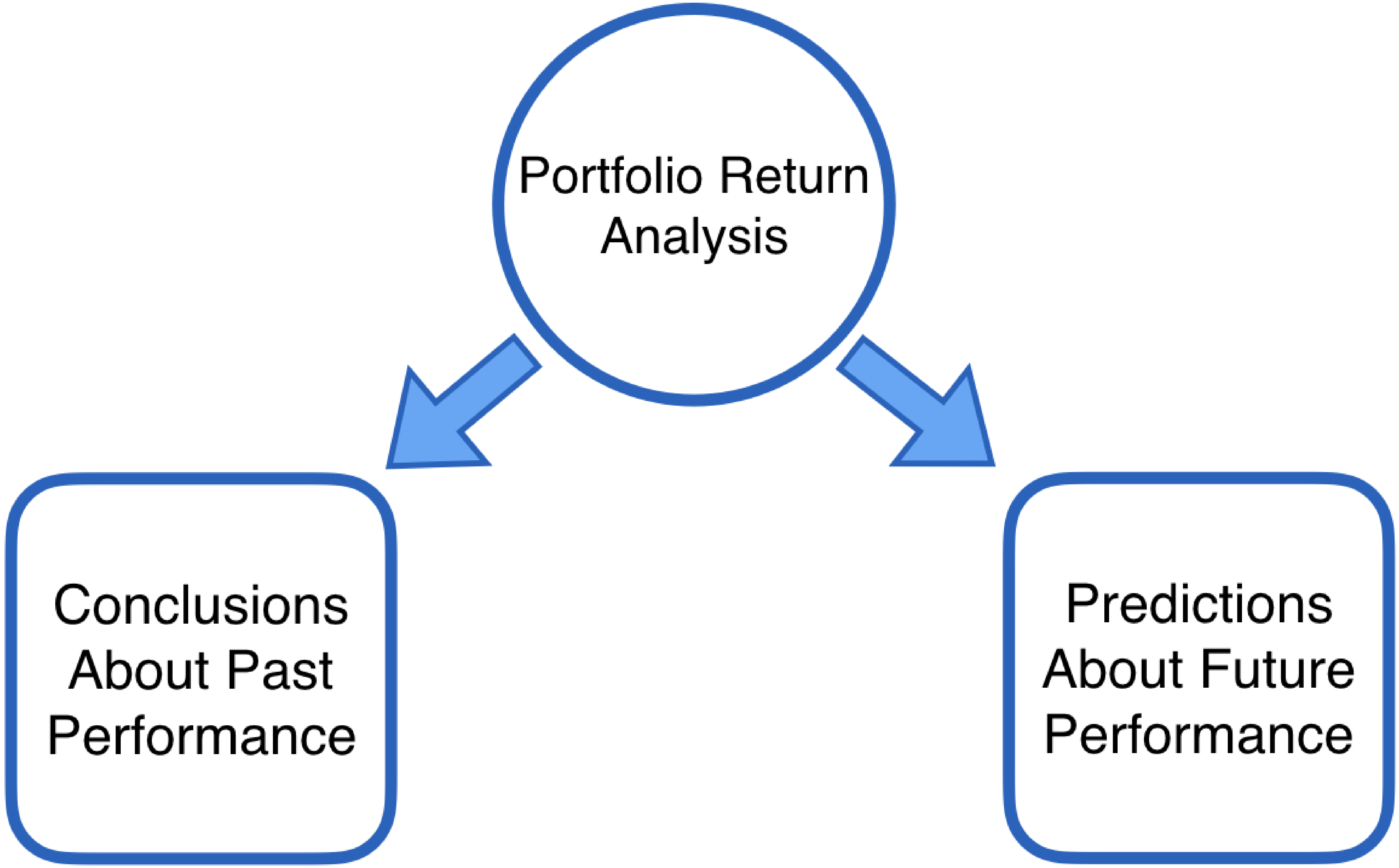
Kris Boudt
Professor, Free University Brussels & Amsterdam

Interpretation of portfolio returns

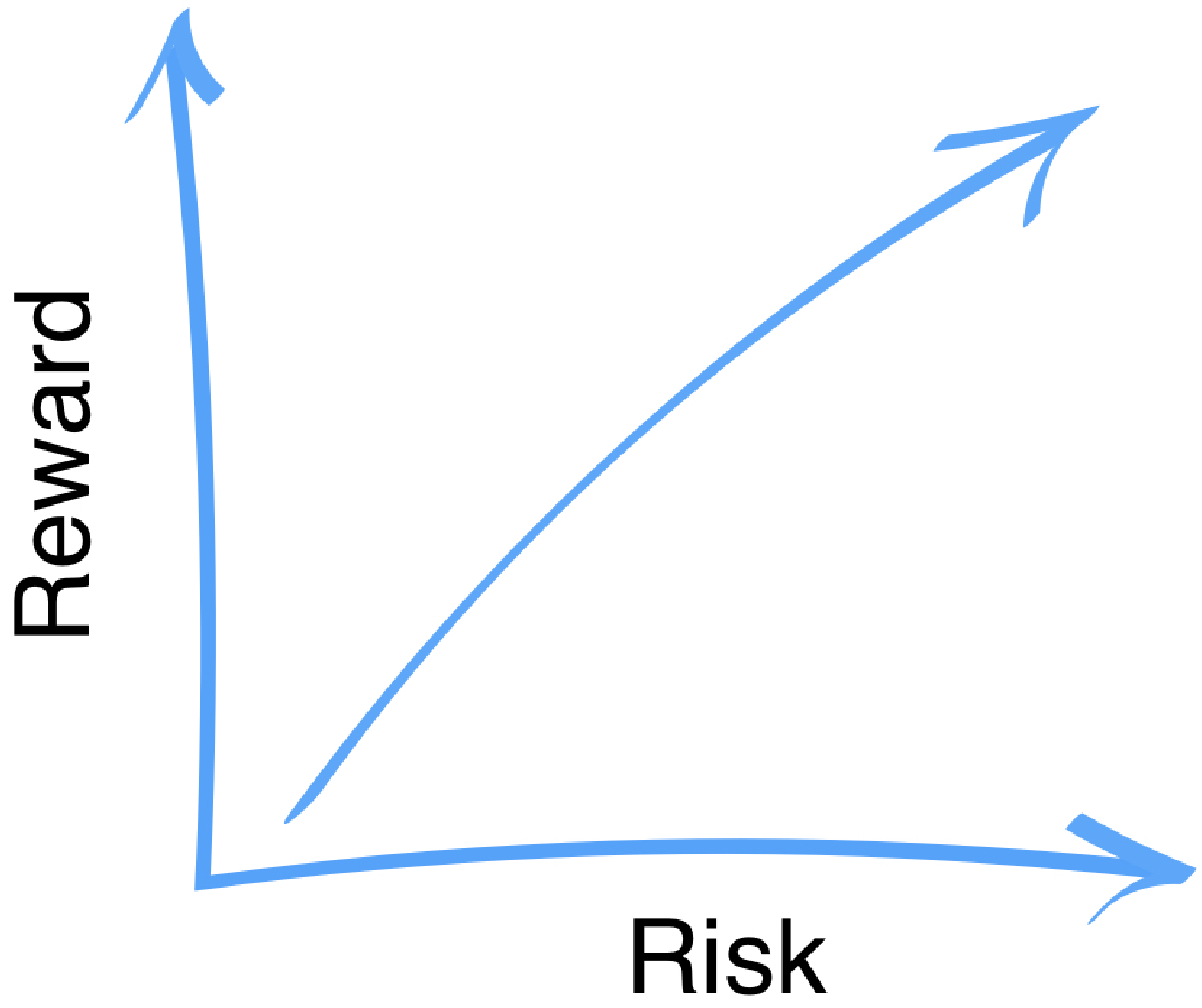
Interpretation of portfolio returns



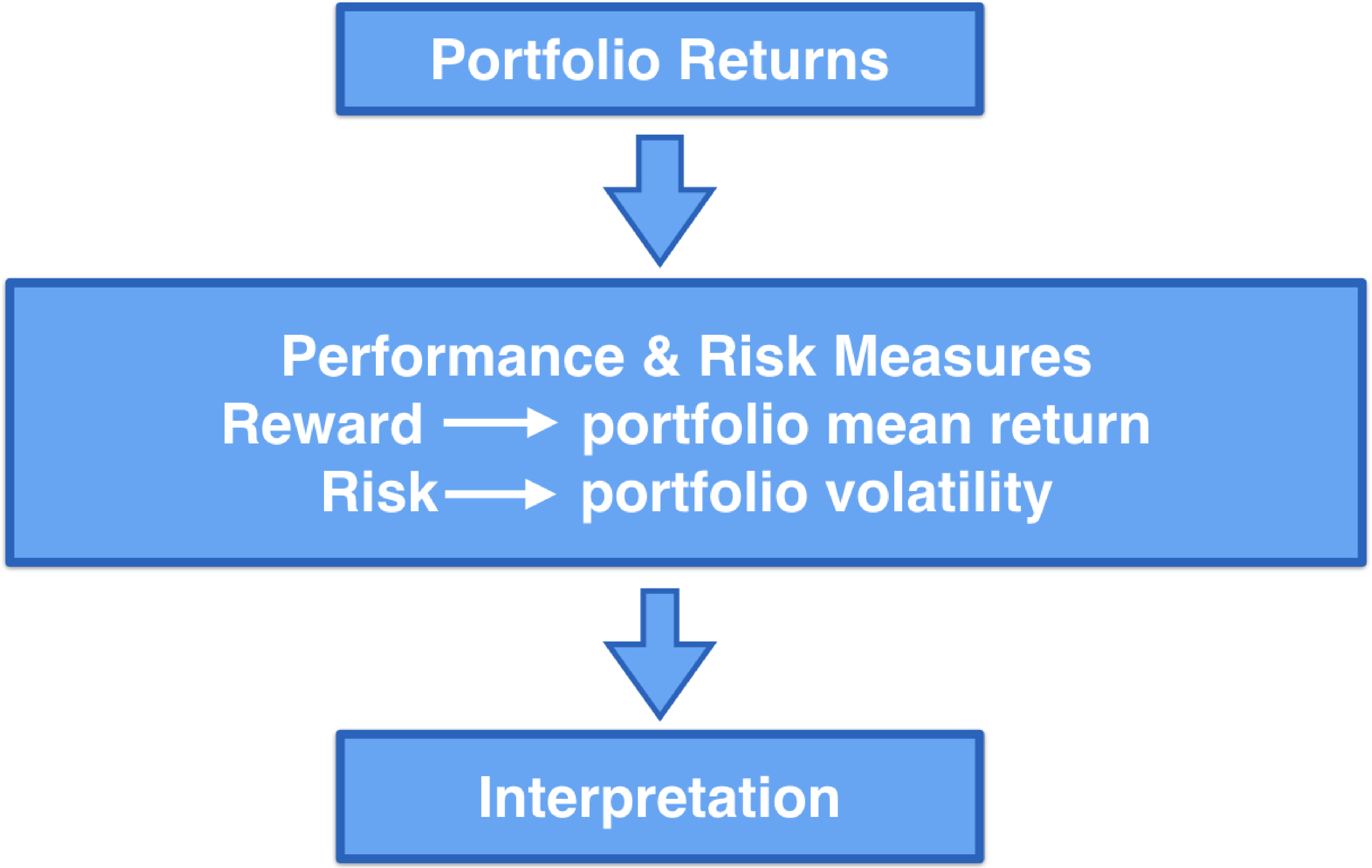
Interpretation of portfolio returns



Risk vs. reward



Need for performance measure



Arithmetic mean return

- Assume a sample of T portfolio return observations:
  - $R_1, R_2, ..., R_T$
- Reward measurement: Arithmetic mean return is given:
  - $\hat{\mu} = \dfrac{R_1, R_2, ..., R_T}{T}$
- It shows how large the portfolio return is on average

Risk: portfolio volatility

- De-meaned return
  - $R_i - \hat{\mu}$
- Variance of the portfolio
  - $\hat{\sigma}^2 = \dfrac{(R_1 - \hat{\mu})^2 + (R_2 - \hat{\mu})^2 + ... + (R_T - \hat{\mu})^2}{T_1}$
- Portfolio volatility:
  - $\hat{\sigma} = \sqrt{\hat{\sigma}^2}$

No linear compensation in return

- Mismatch between average return and effective return

$$\text{final value}=$$
$$\text{initial value} * (1+0.5)*(1-0.5)= 0.75 * \text{initial value}$$

No linear compensation in return

- Mismatch between average return and effective return

final value=
initial value * (1 +0.5)*(1-0.5)= 0.75 * initial value

No linear compensation in return

- Mismatch between average return and effective return

final value=
initial value * (1 +0.5)*(1-0.5)= 0.75 * initial value

Average Return = (0.5 - 0.5) / 2 = 0

Geometric mean return

- Formula for Geometric Mean for a sample of T portfolio return observations $R_1, R_2, ..., R_T$:

Geometric mean = $[(1 + R_1) \cdot (1 + R_2) \cdot ...(1 + R_T)]^{1/T} - 1$

- Example: +50% & -50% return
  - Geometric mean = $[(1 + 0.50) \cdot (1 - 0.50)]^{1/2} - 1$
  - = $0.75^{1/2} - 1$
  - = -13.4%


Google
amazon.com
ebay
priceline.com
E*TRADE
NETFLIX
ALEXION
EXPRESS SCRIPTS
Ford
LEGG MASON
RALPH LAUREN
WHOLE FOODS
comcast
NVIDIA
CITRIX
Ameriprise Financial
JANUS CAPITAL Group
FIFTH THIRD BANK
SUNTRUST
McAfee
Cognizant
IPG
TEXTRON
LEXMARK
TERADYNE
TSYS
QUALCOMM
HALLIBURTON
WYNDHAM WORLDWIDE
POLO RALPH LAUREN
Expedia
CBRE
metroPCS
EA
xerox
Lincoln Financial Group
Morgan Stanley
C.H. ROBINSON
ALLERGAN
AutoNation
EATON
STATE STREET
EXPRESS SCRIPTS
NOVELLUS
Robert Half
Cigna
PROLOGIS
Pfizer
DaVita
KeyCorp
ups
vf
COVENTRY
EXXON Mobil
Microsoft
Walmart
BERKSHIRE HATHAWAY
Johnson & Johnson
GE
WELLS FARGO
Chevron
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P&G
IBM
verizon
Bank of America
ORACLE
f
at&t
Coca-Cola
MERCK
citi
Walt Disney
VISA
Sara Lee
The Washington Post
CME Group
Lorillard
BLACKROCK
Prudential
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LOCKHEED MARTIN
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SIMON
Entergy
PHILIP MORRIS
COACH
HEALTHCARE REIT
Kimberly-Clark
MetLife
CenturyLink
3M
Adobe
AA
BOEING
Capital One
CISCO
COLGATE PALMOLIVE
COSTCO WHOLESALE
DELTA
FOSSIL
GM
THE HOME DEPOT
hp
Kraft
M
MICHAEL KORS
NIKE
ROSS
SEAGATE
SOUTHWEST AIRLINES
Symantec
Time Warner Cable
AMERICAN EXPRESS
CAT
Baxter
BB&T
CSX

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

The (annualized) Sharpe ratio

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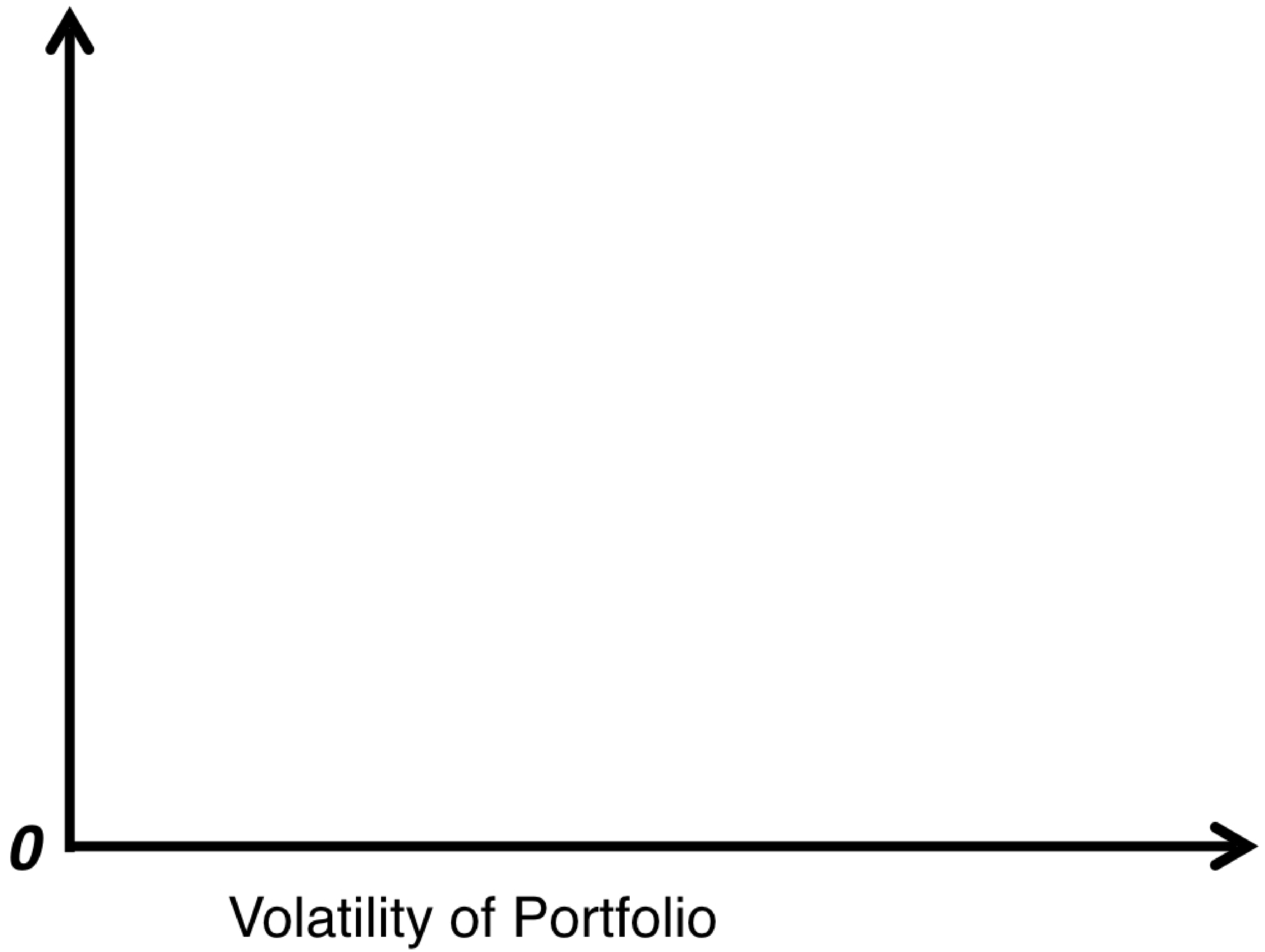
Benchmarking performance

Risky Portfolio E.g: portfolio invested in stocks, bonds, real estate, and commodities
Reward: measured by mean portfolio return
Risk: measured by volatility of the portfolio returns

Benchmarking performance

Risky Portfolio E.g: portfolio invested in stocks, bonds, real estate, and commodities	Risk Free Asset E.g: US Treasury Bill
Reward: measured by mean portfolio return	Reward: measured by risk free rate
Risk: measured by volatility of the portfolio returns	Risk: No risk, volatility = 0 return = risk free rate

Risk-return trade-off



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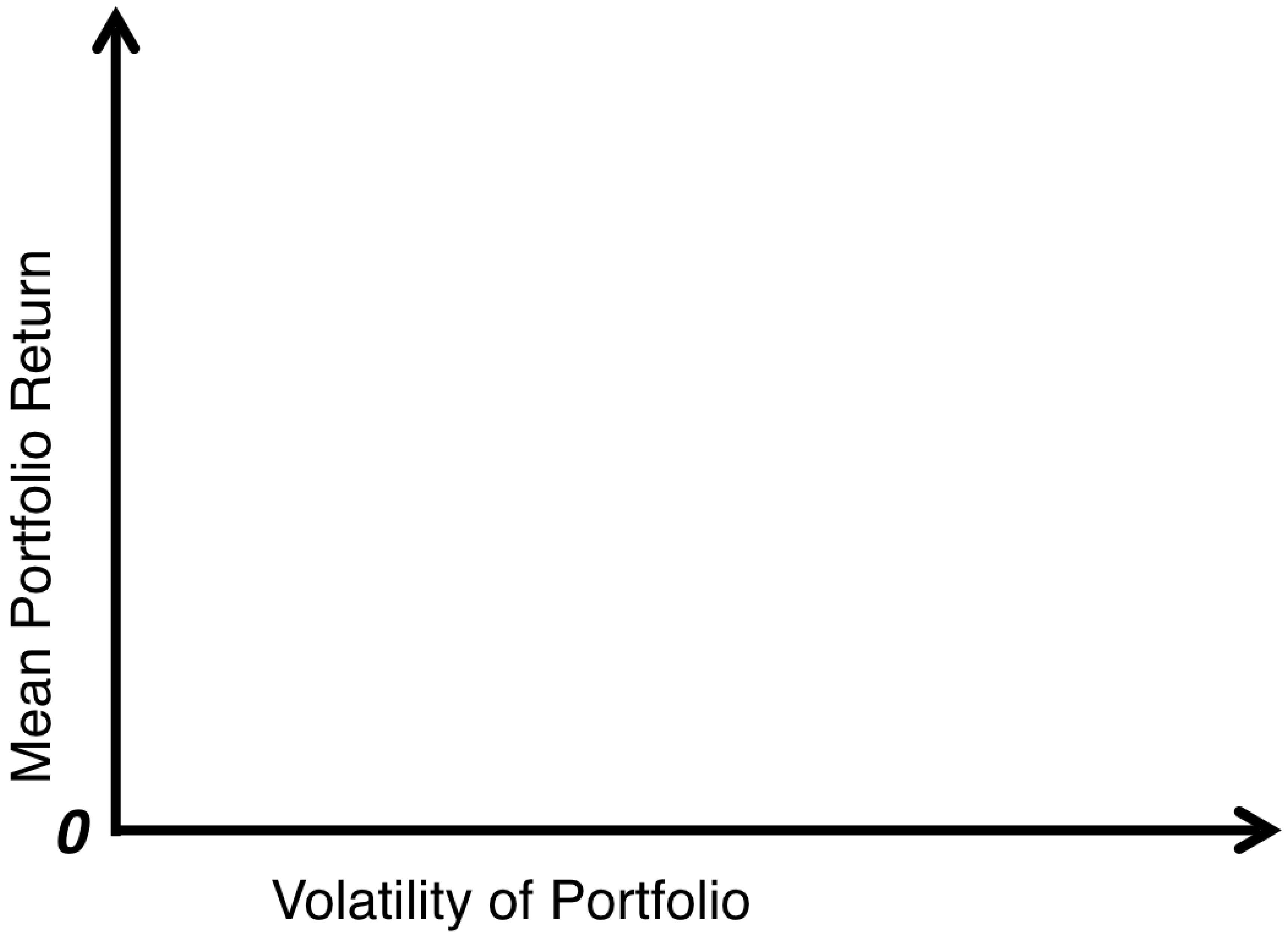

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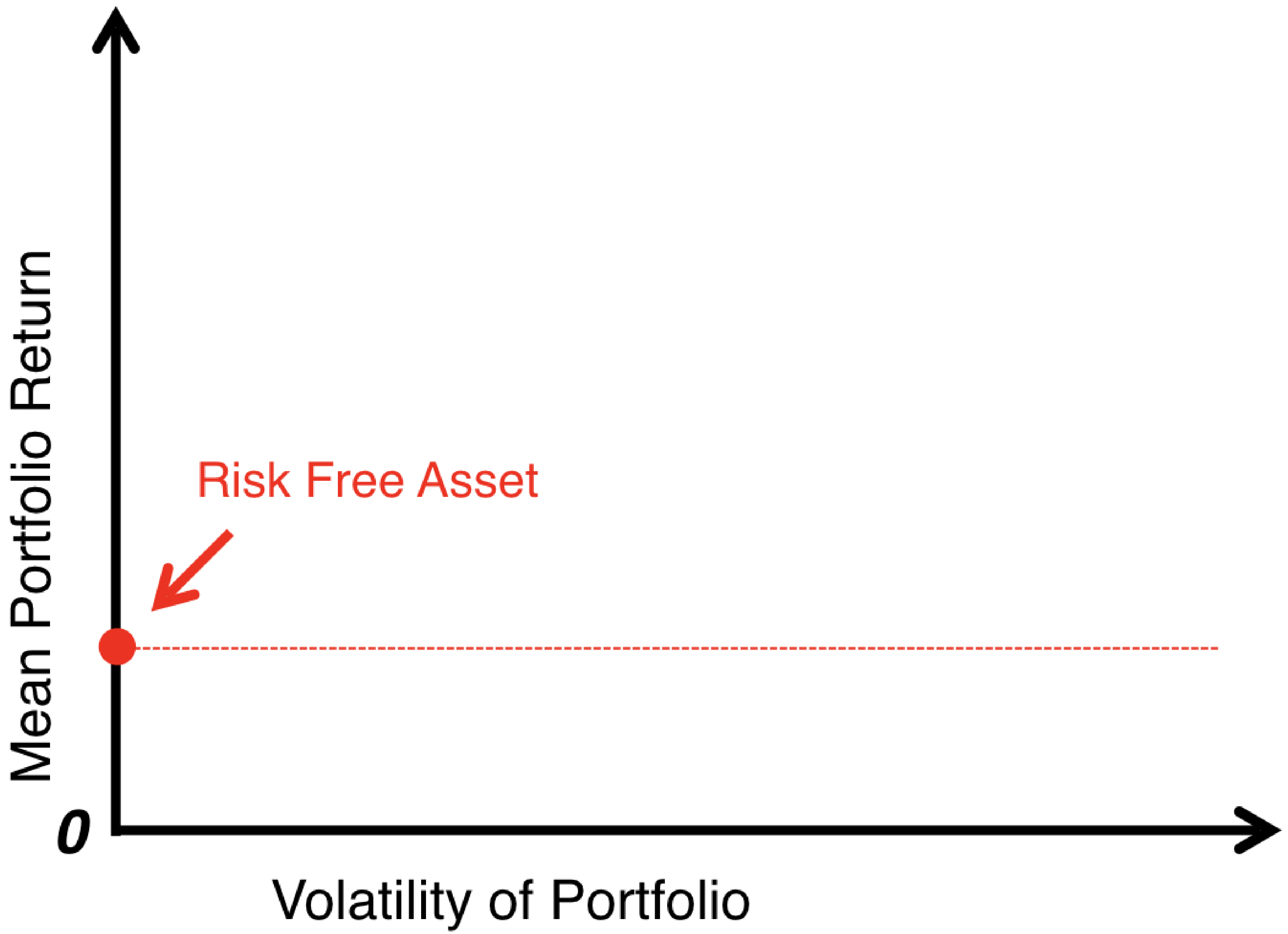

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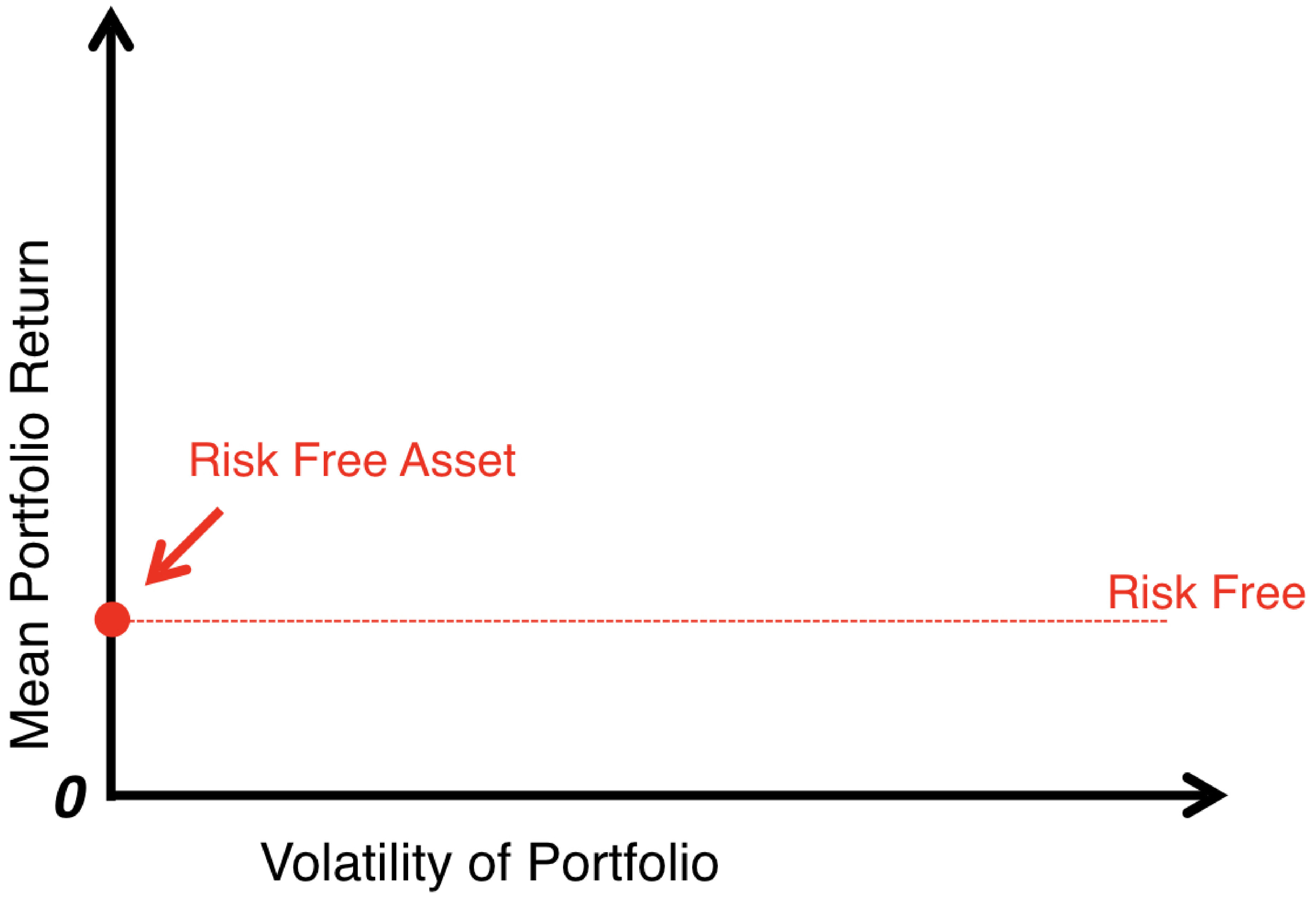

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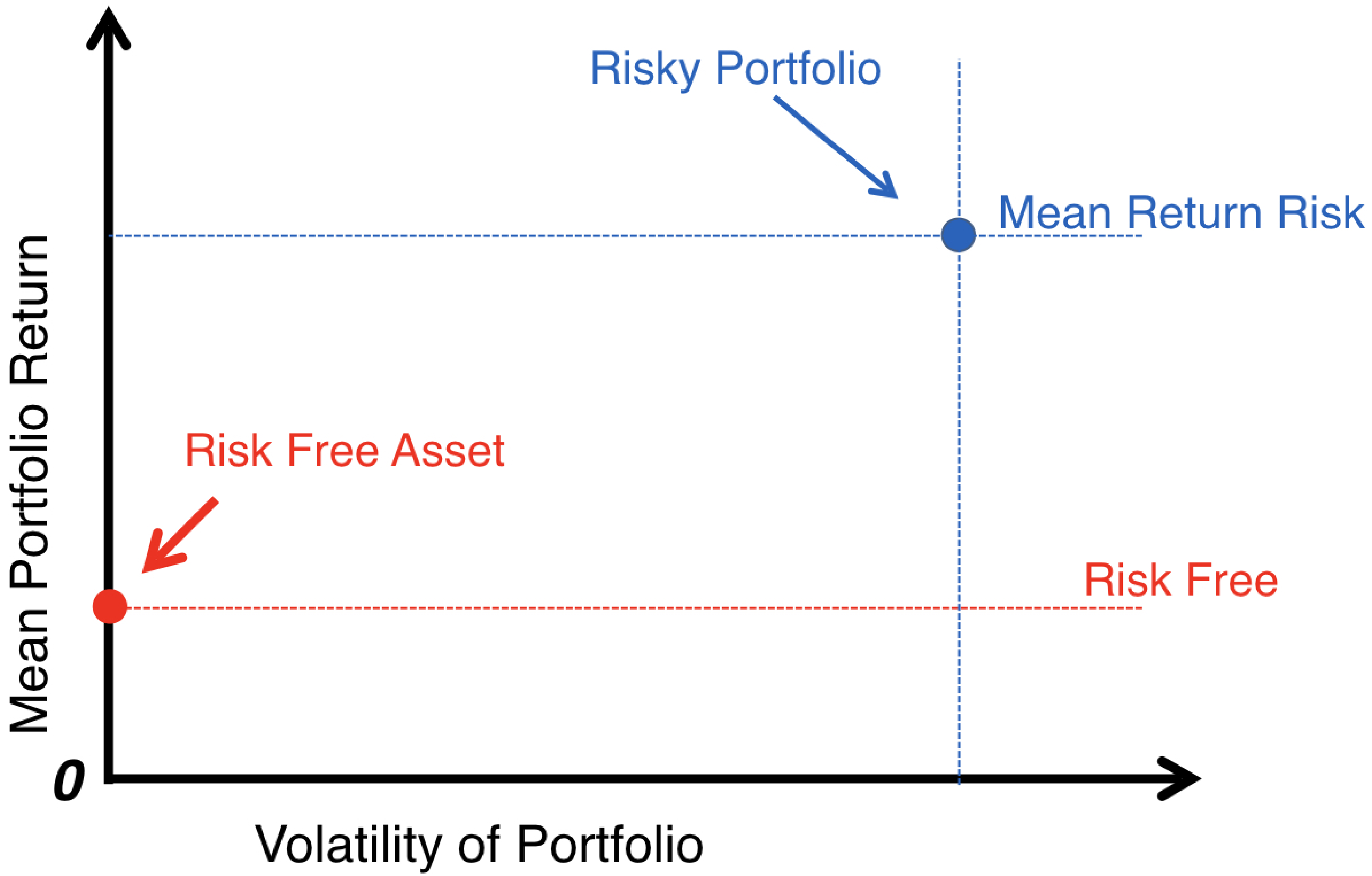

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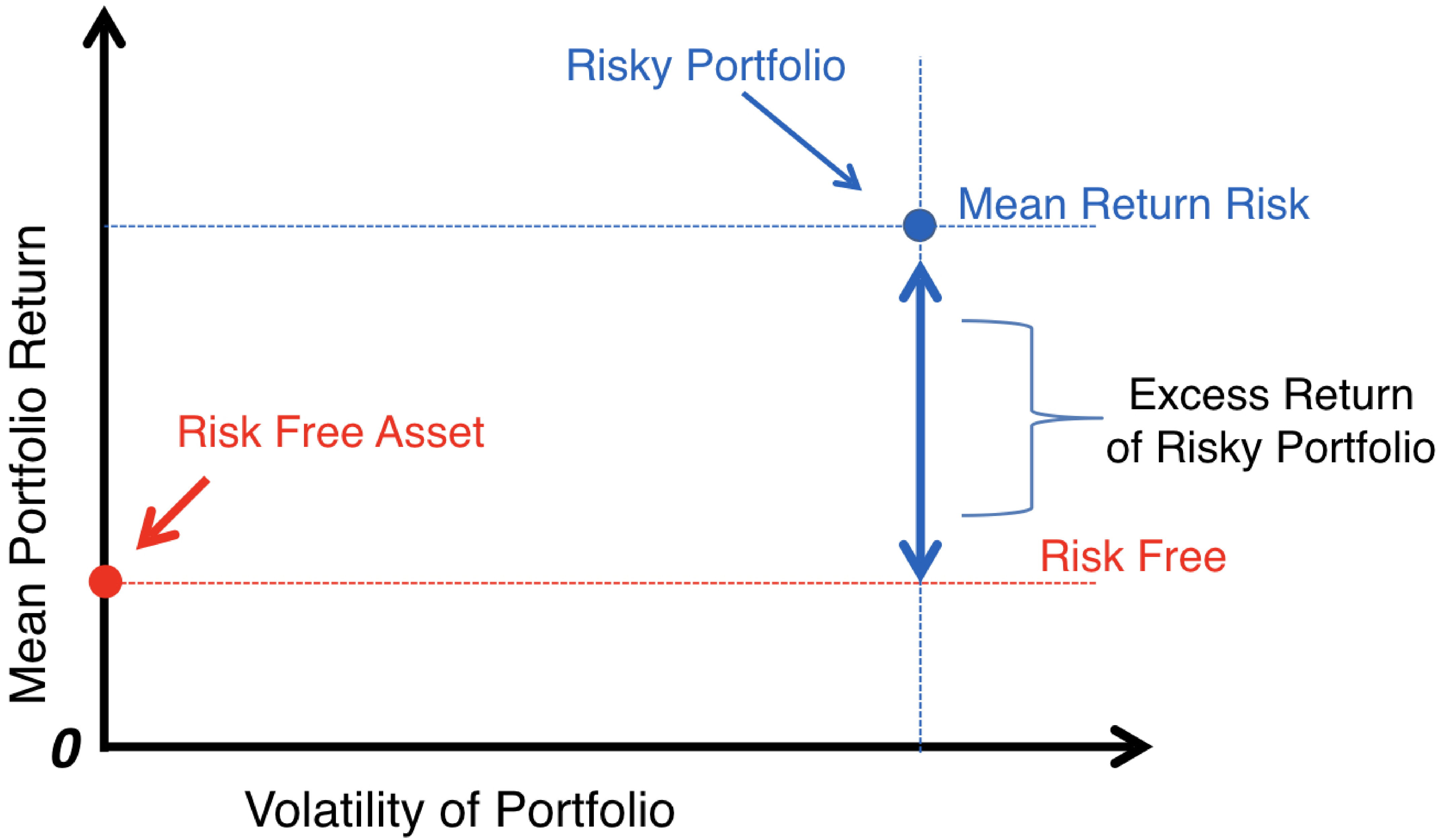
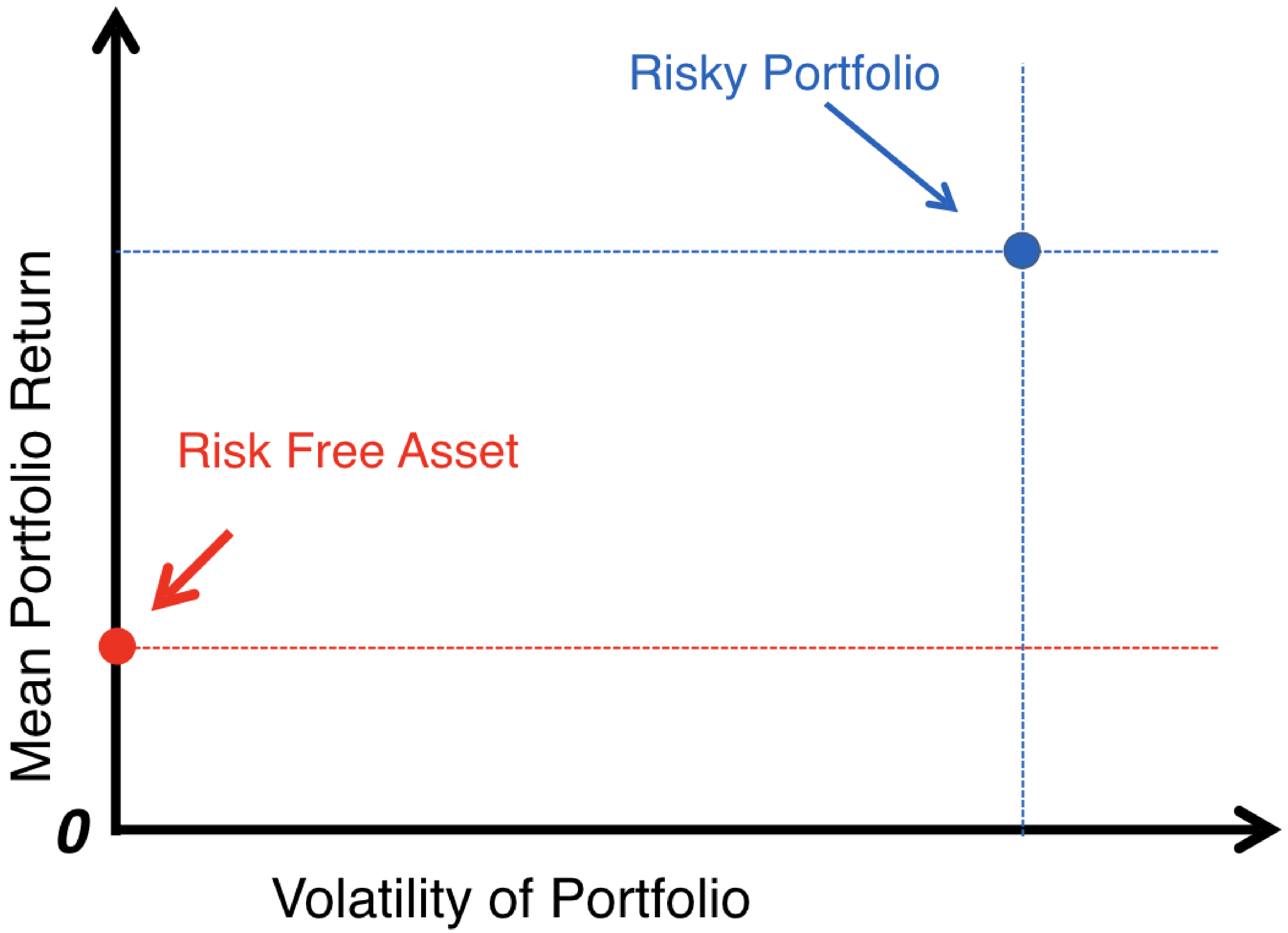
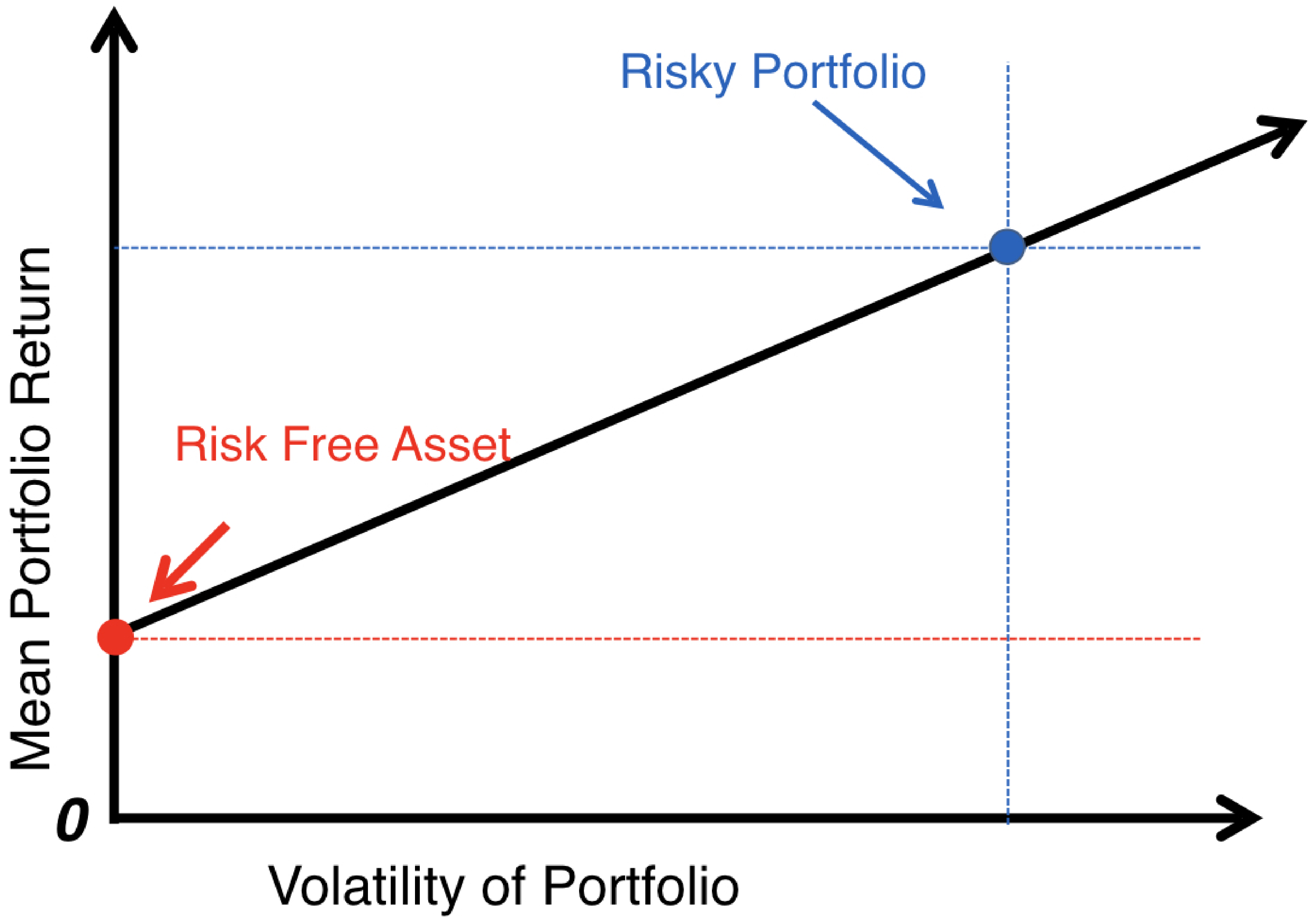


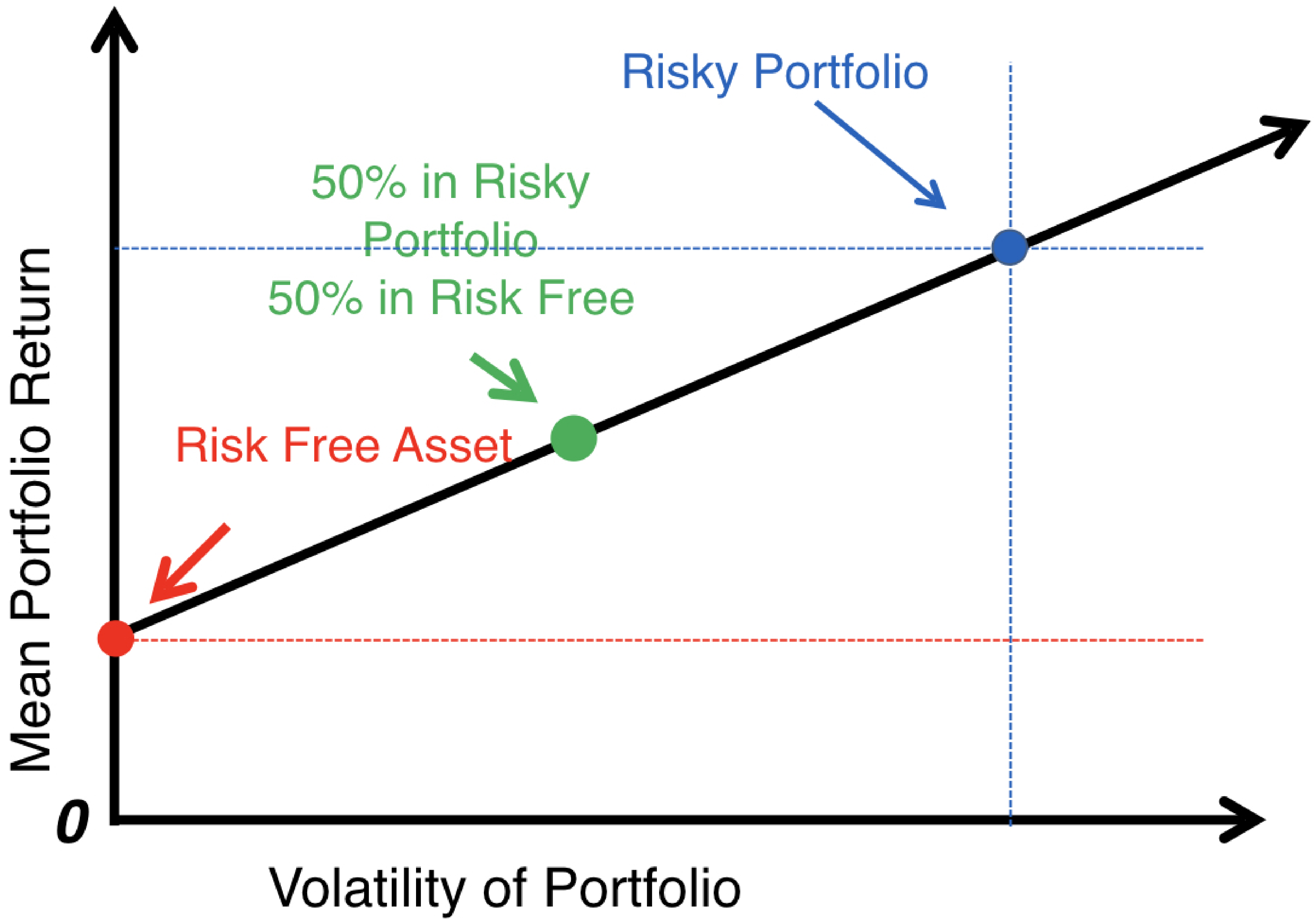
Capital allocation line



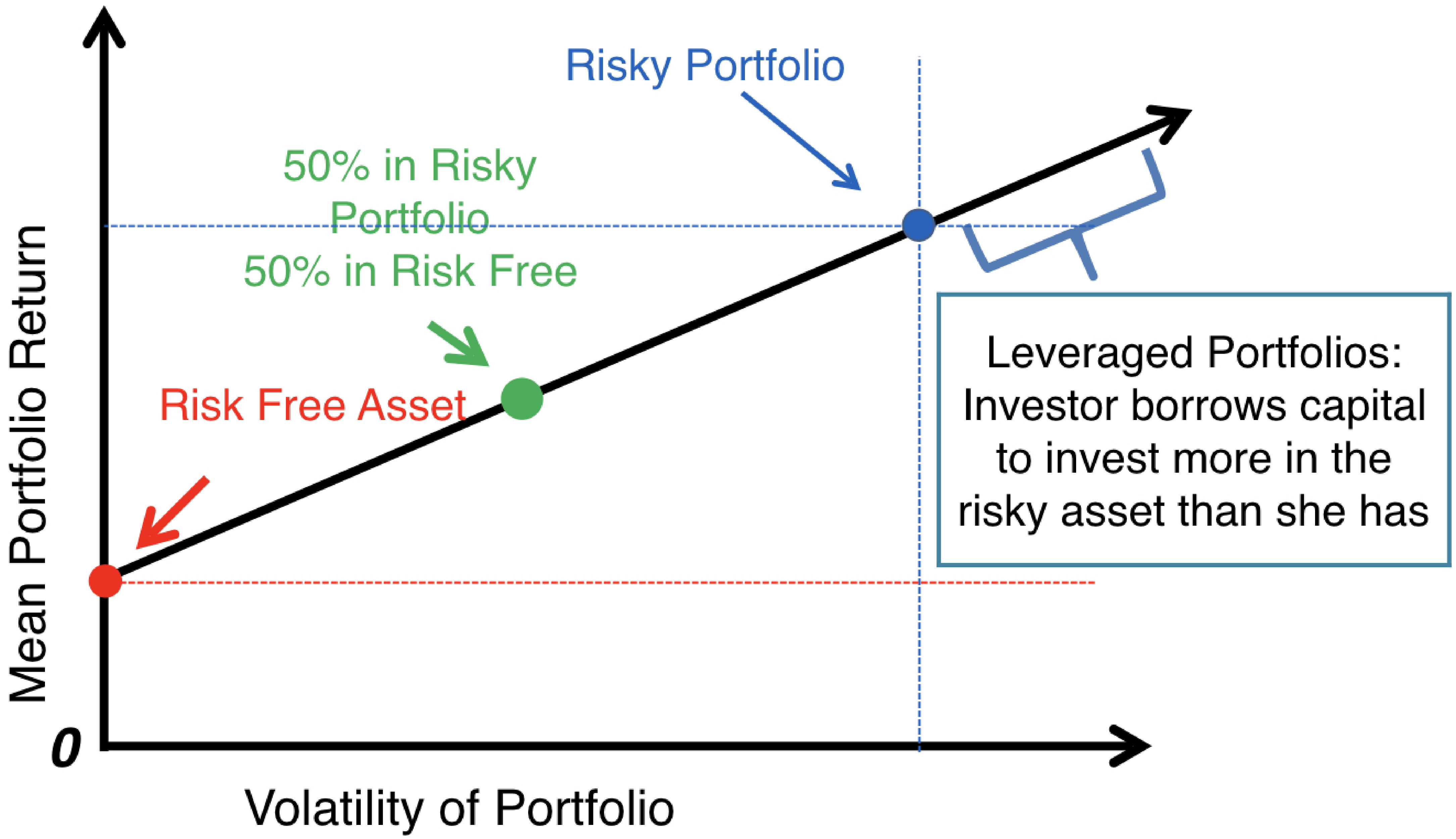
Capital allocation line



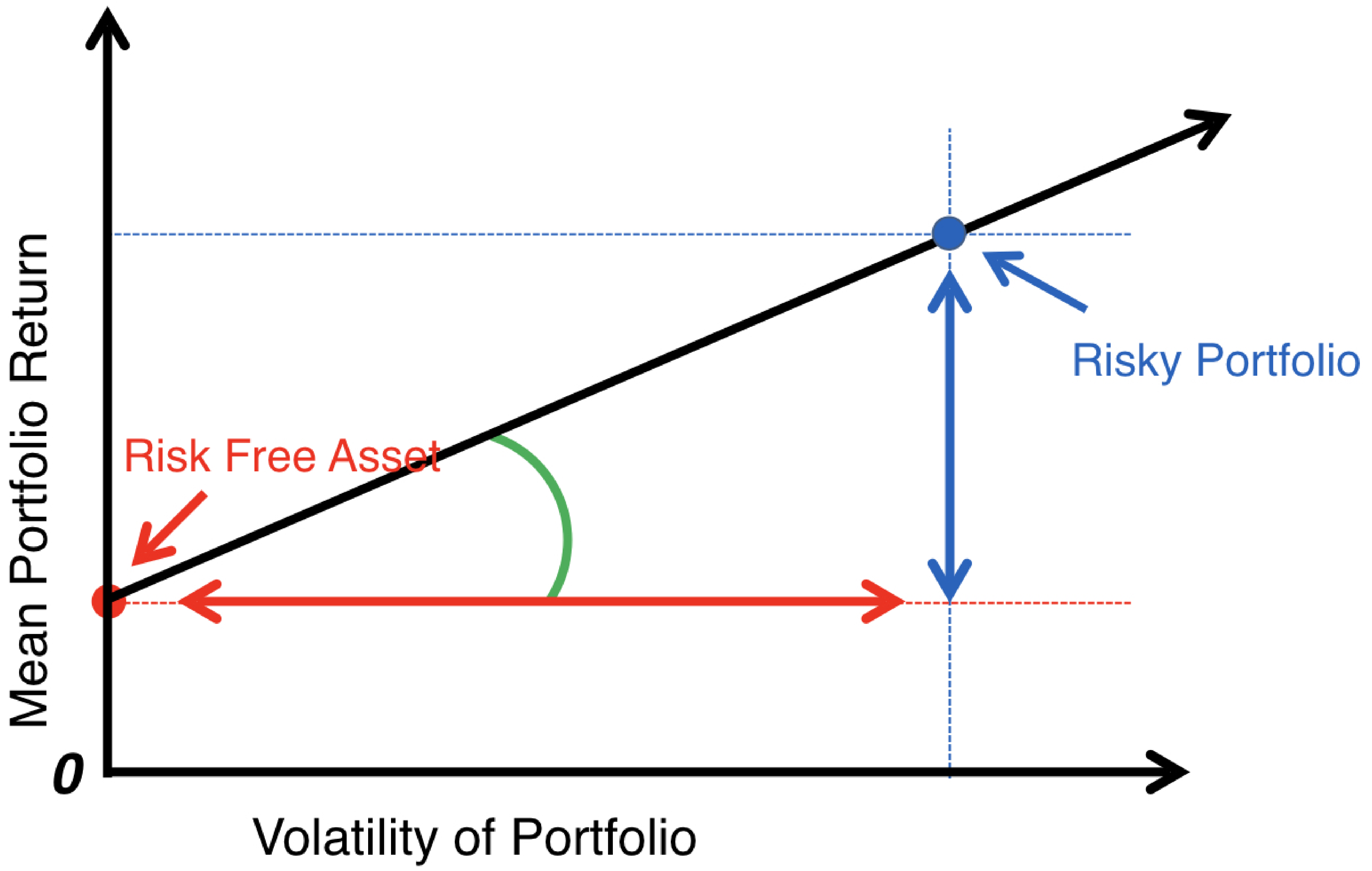
Capital allocation line



Capital allocation line



The Sharpe ratio



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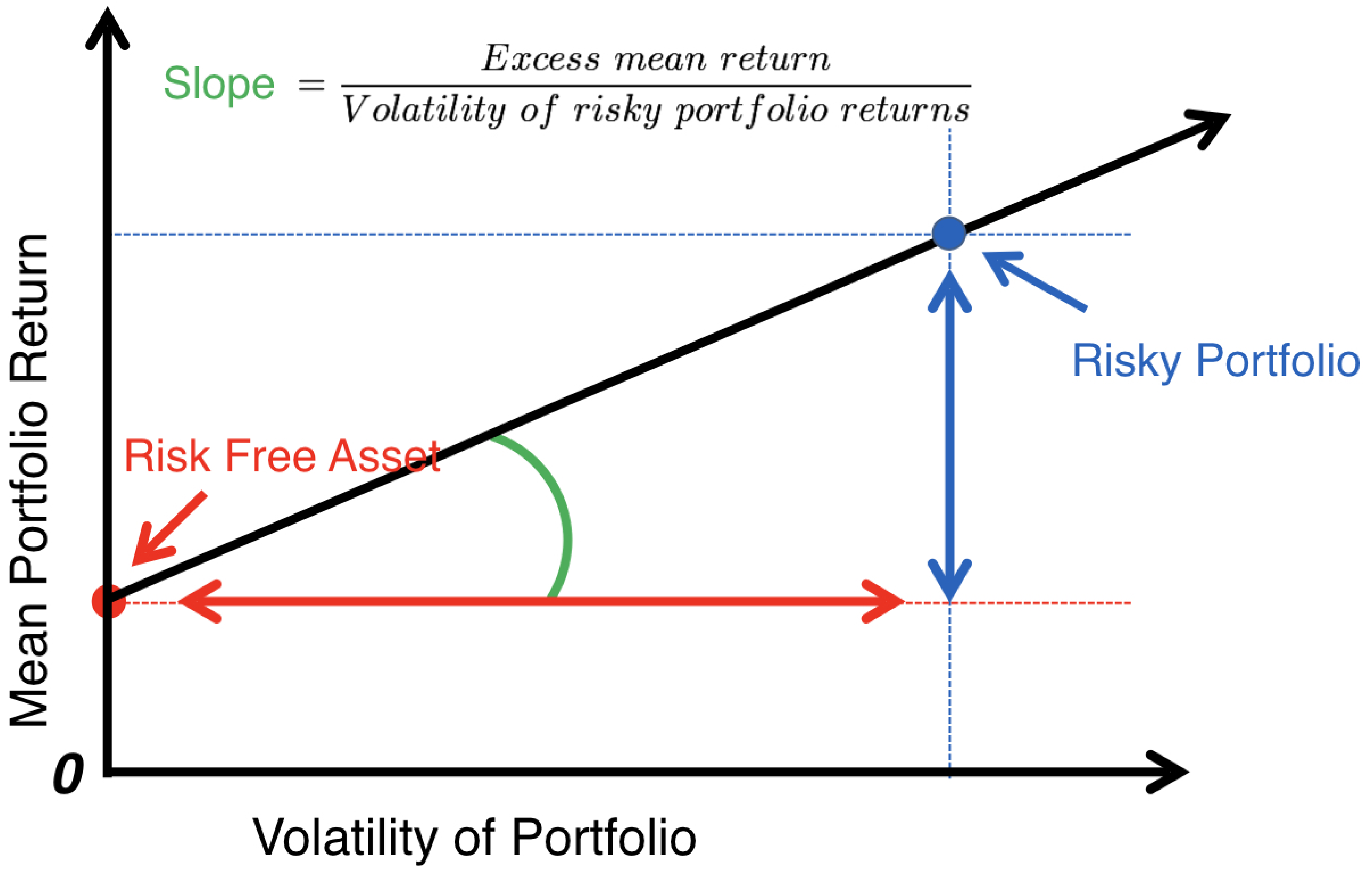

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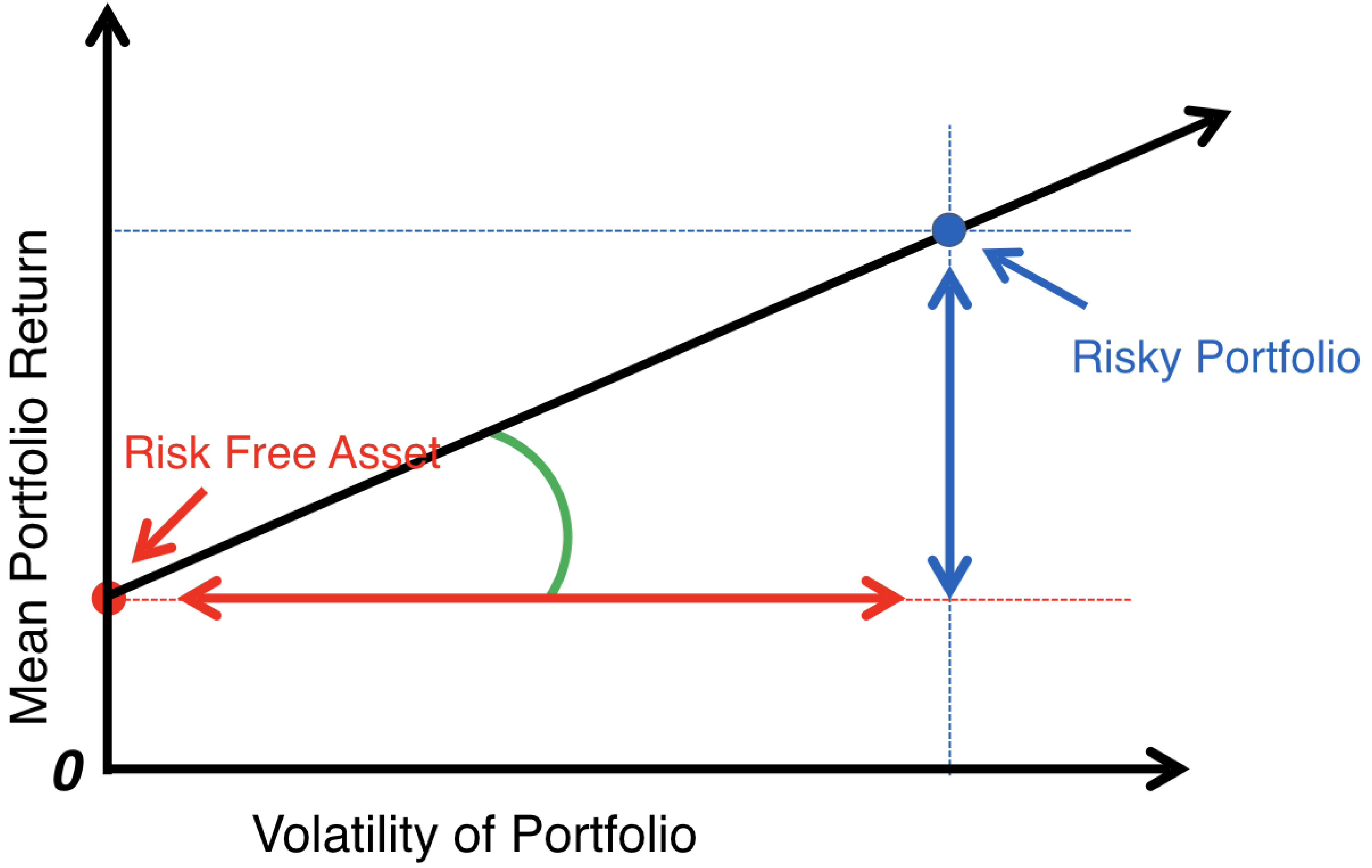
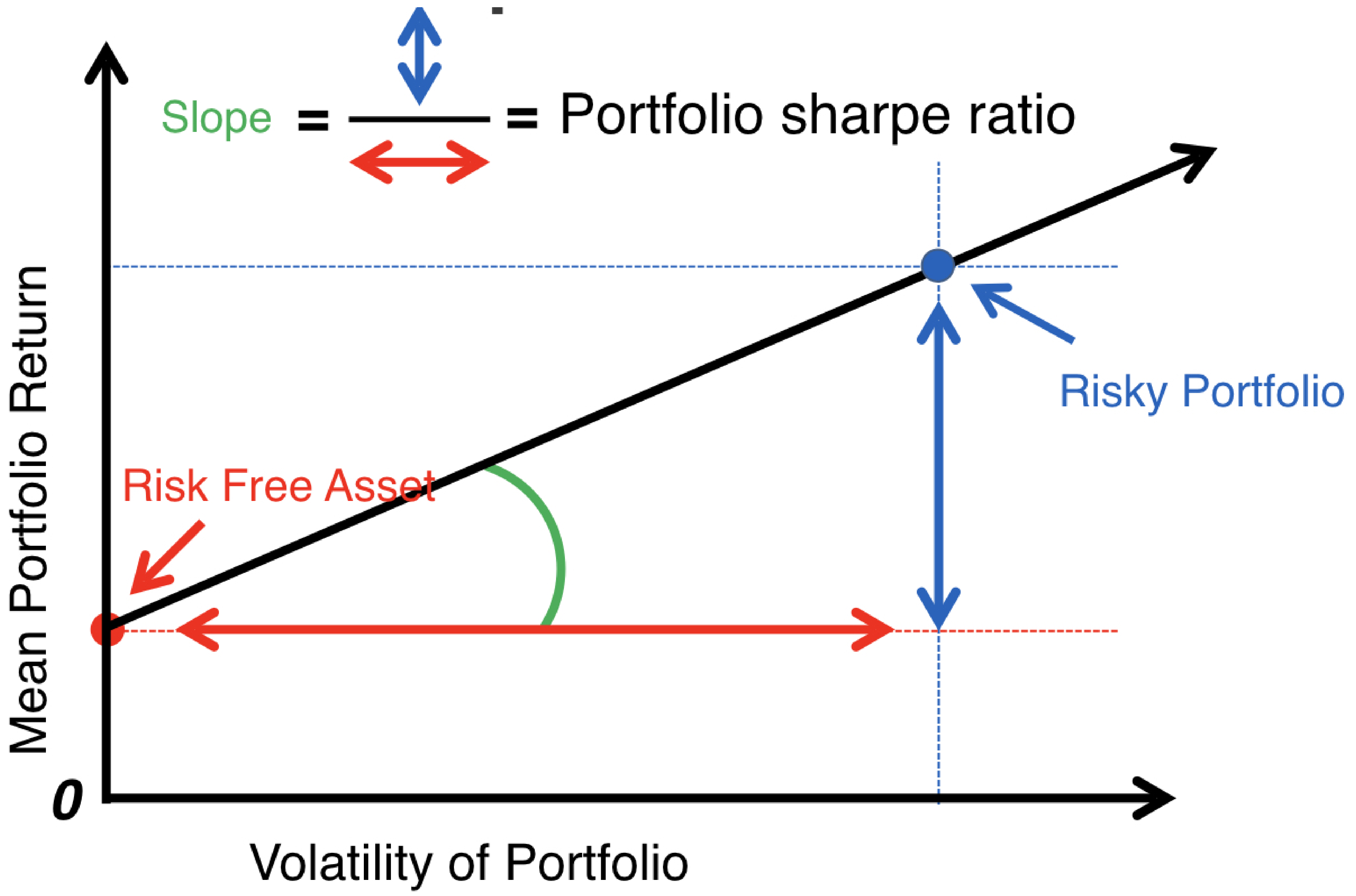


The Sharpe ratio



Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c(-0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
```

returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	
geometric mean	
volatility	
sharpe ratio	

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c(-0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
mean(sample_returns)
```

returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	0.015
geometric mean	
volatility	
sharpe ratio	

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c(-0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
mean.geometric(sample_returns)
```

returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	0.015
geometric mean	0.01468148
volatility	
sharpe ratio	

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c(-0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
StdDev(sample_returns)
```

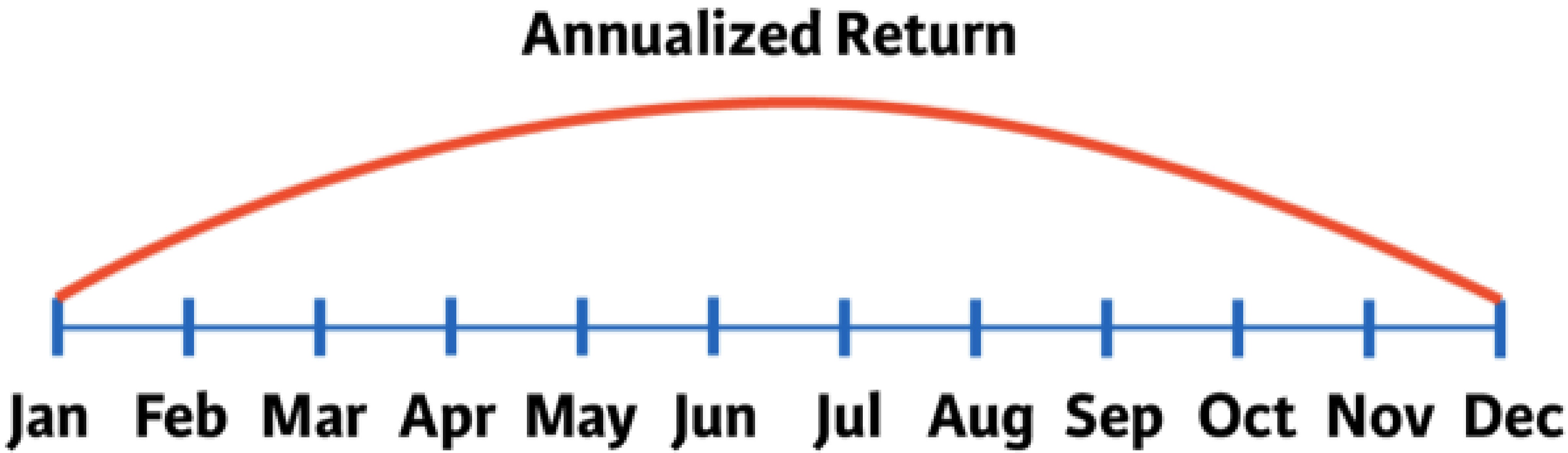
returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	0.015
geometric mean	0.01468148
volatility	0.02725541
sharpe ratio	

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c(-0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
(mean(sample_returns)-0.004)/StdDev(sample_returns)
```

returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	0.015
geometric mean	0.01468148
volatility	0.02725541
sharpe ratio	0.4035897

Annualize monthly performance



- Arithmetic mean: monthly mean * 12
- Geometric mean, when R_i are monthly returns:
  - $[(1+R_1)\cdot(1+R_2)\cdot\ldots\cdot(1+R_T)]^{12/T}-1$
- Volatility: monthly volatility * sqrt(12)

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015		
geometric mean	0.01468148		
volatility	0.02725541		
sharpe ratio	0.4035897		

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
Return.annualized(sample_returns, scale = 12, geometric = FALSE)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015	12	0.18
geometric mean	0.01468148		
volatility	0.02725541		
sharpe ratio	0.4035897		

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
Return.annualized(sample_returns, scale = 12, geometric = TRUE)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015	12	0.18
geometric mean	0.01468148	$[0.98 * ... * 1.04]^{\frac{12}{8}} - 1$	0.1911235
volatility	0.02725541		
sharpe ratio	0.4035897		

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
Std.Dev.annualized(sample_returns, scale = 12)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015	12	0.18
geometric mean	0.01468148	$[0.98 * ... * 1.04]^{\frac{12}{8}} - 1$	0.1911235
volatility	0.02725541	sqrt(12)	0.0944155
sharpe ratio	0.4035897		

Performance statistics in action

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library(PerformanceAnalytics)
sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
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    Std.Dev.annualized(sample_returns, scale = 12)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015	12	0.18
geometric mean	0.01468148	$[0.98 * ... * 1.04]^{\frac{12}{8}} - 1$	0.1911235
volatility	0.02725541	sqrt(12)	0.0944155
sharpe ratio	0.4035897	sqrt(12)	1.398076

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Time-variation in portfolio performance

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

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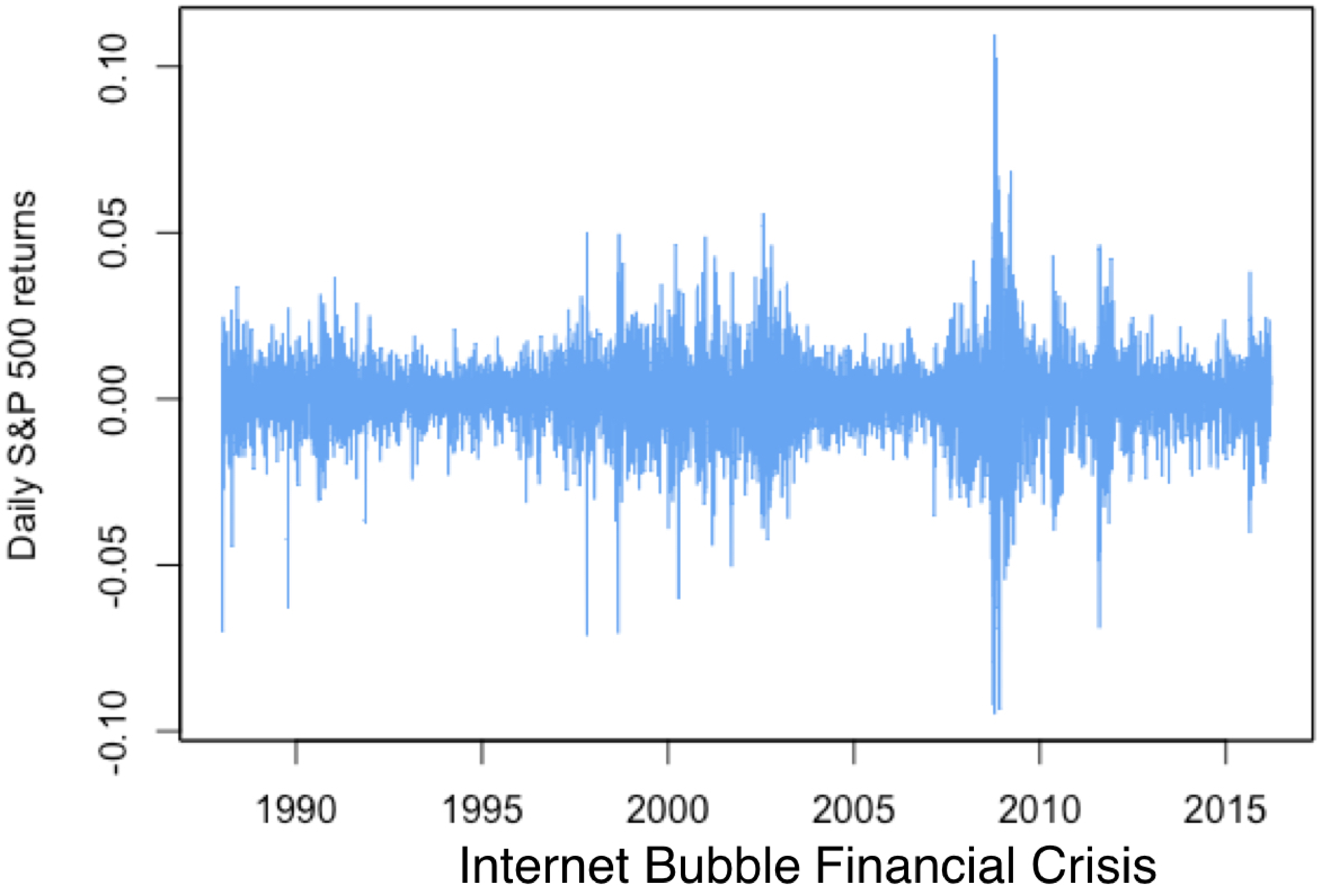
Bulls & bears

- Business cycle, news, and swings in the market psychology affect the market

Clusters of high & low volatility



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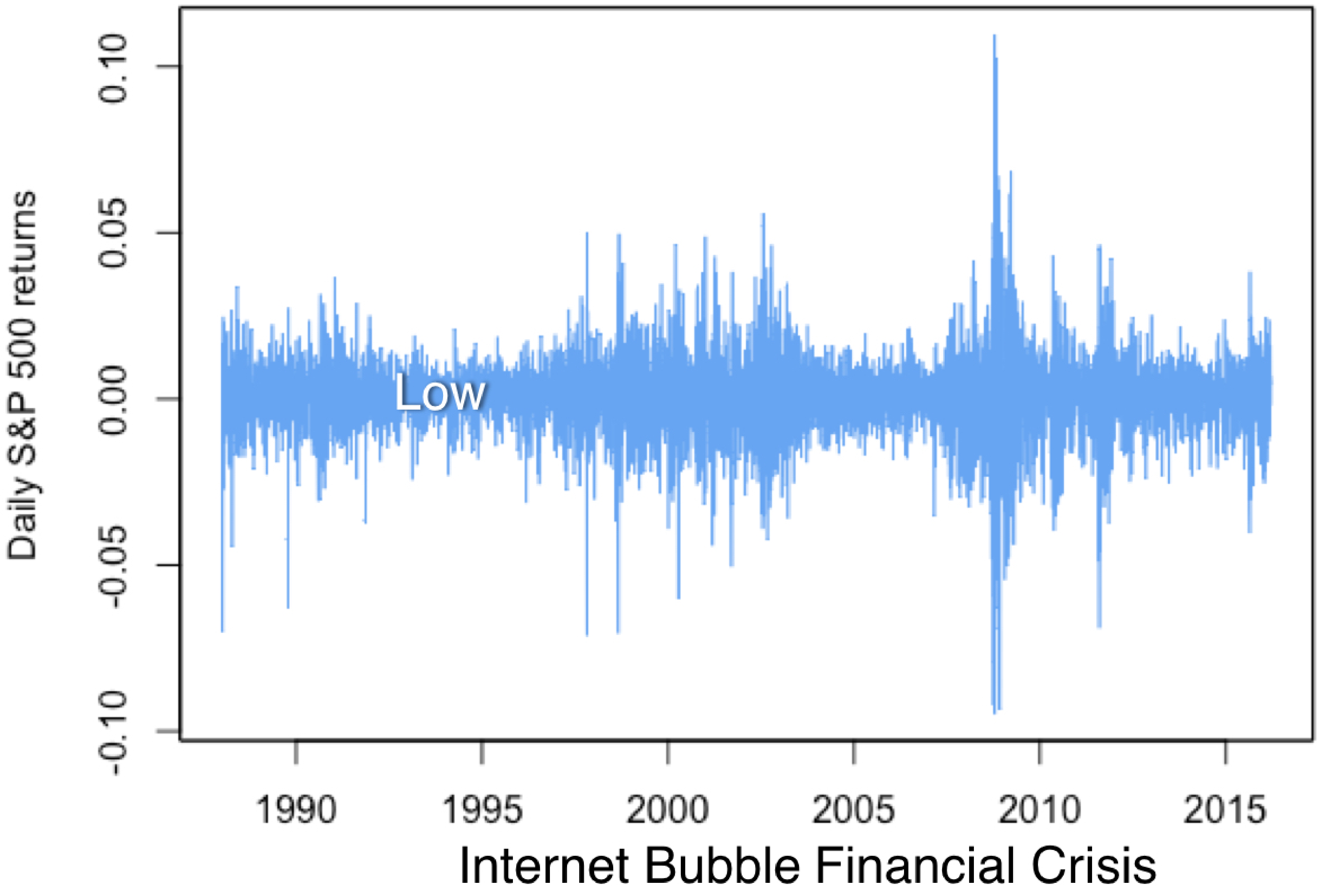

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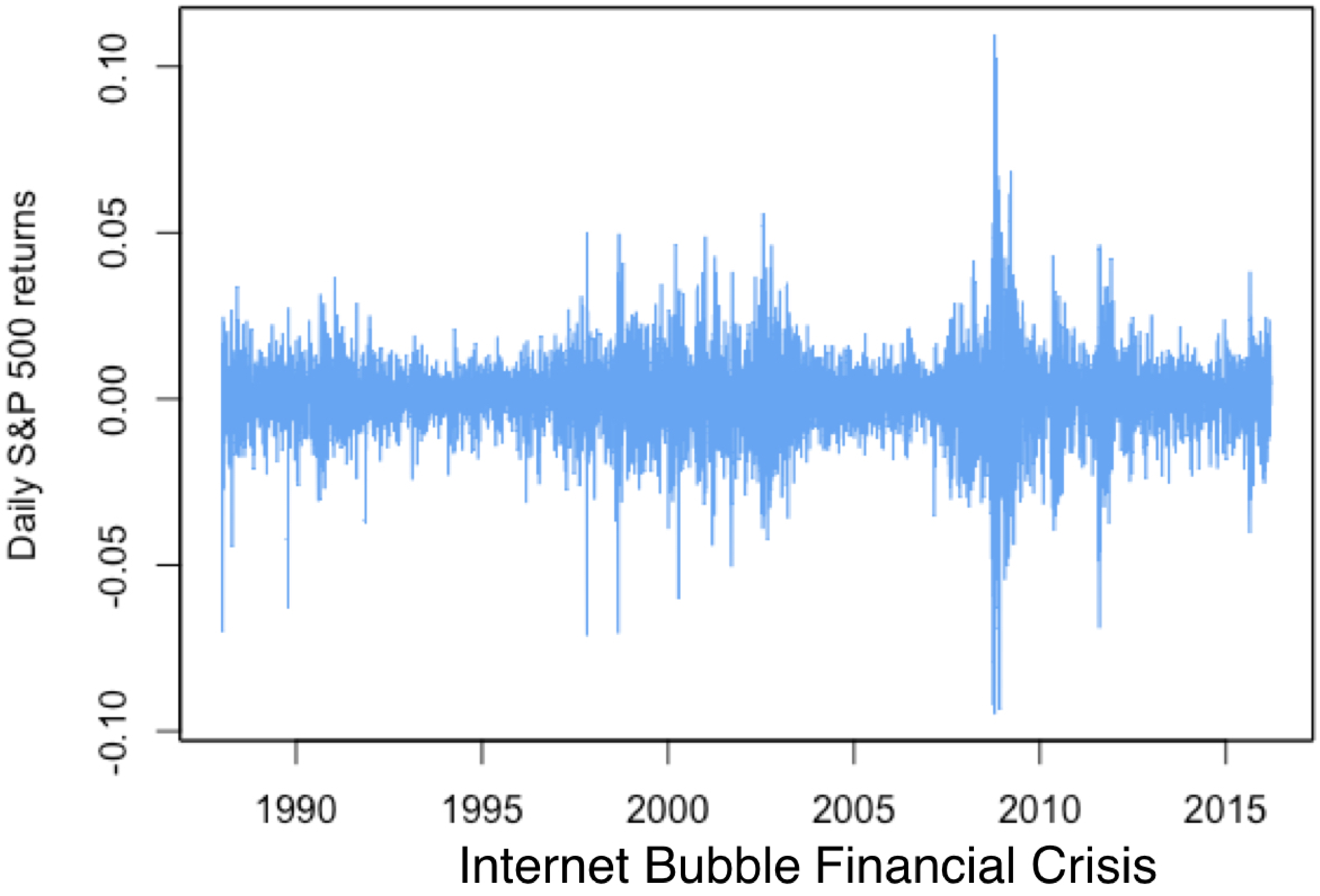

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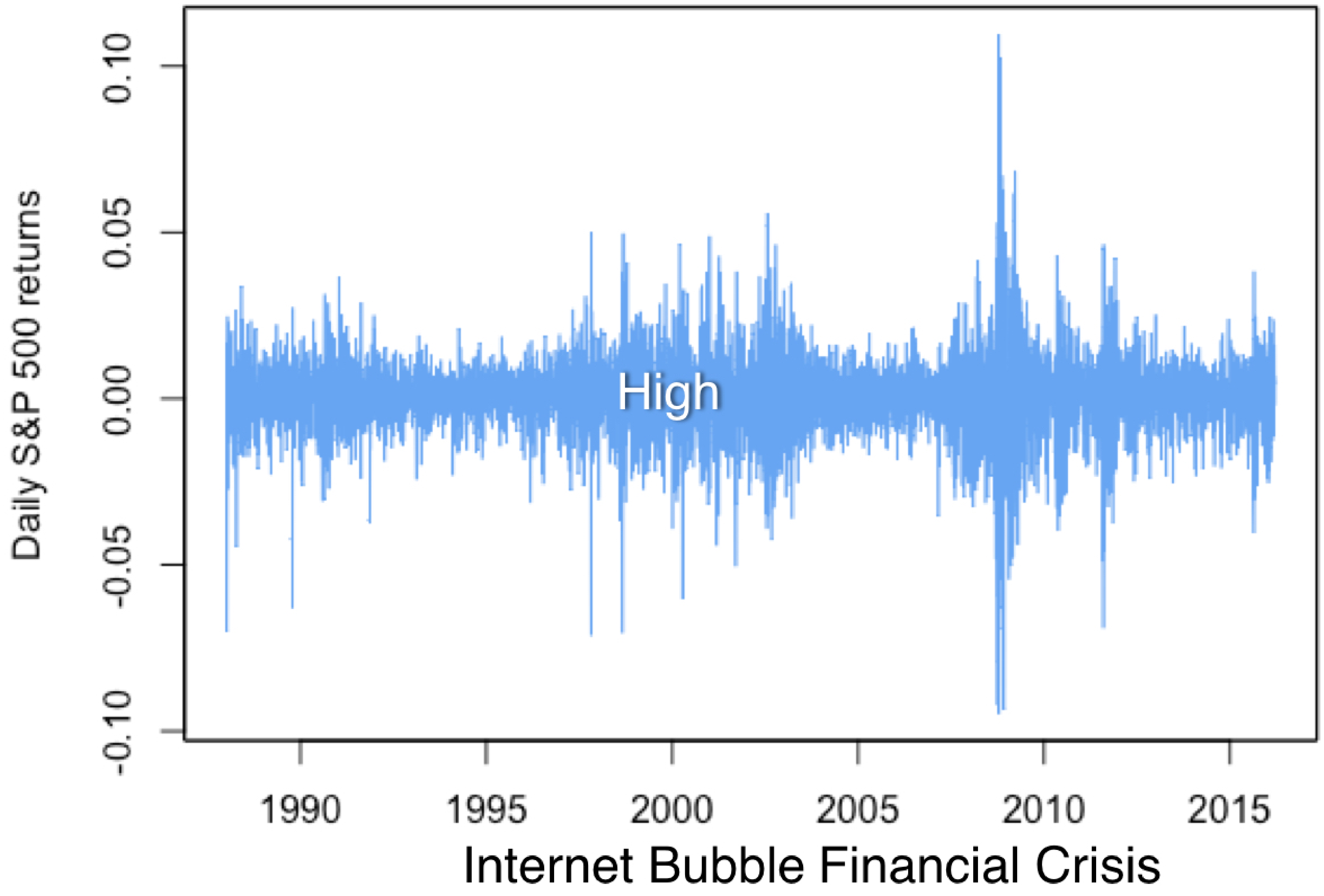

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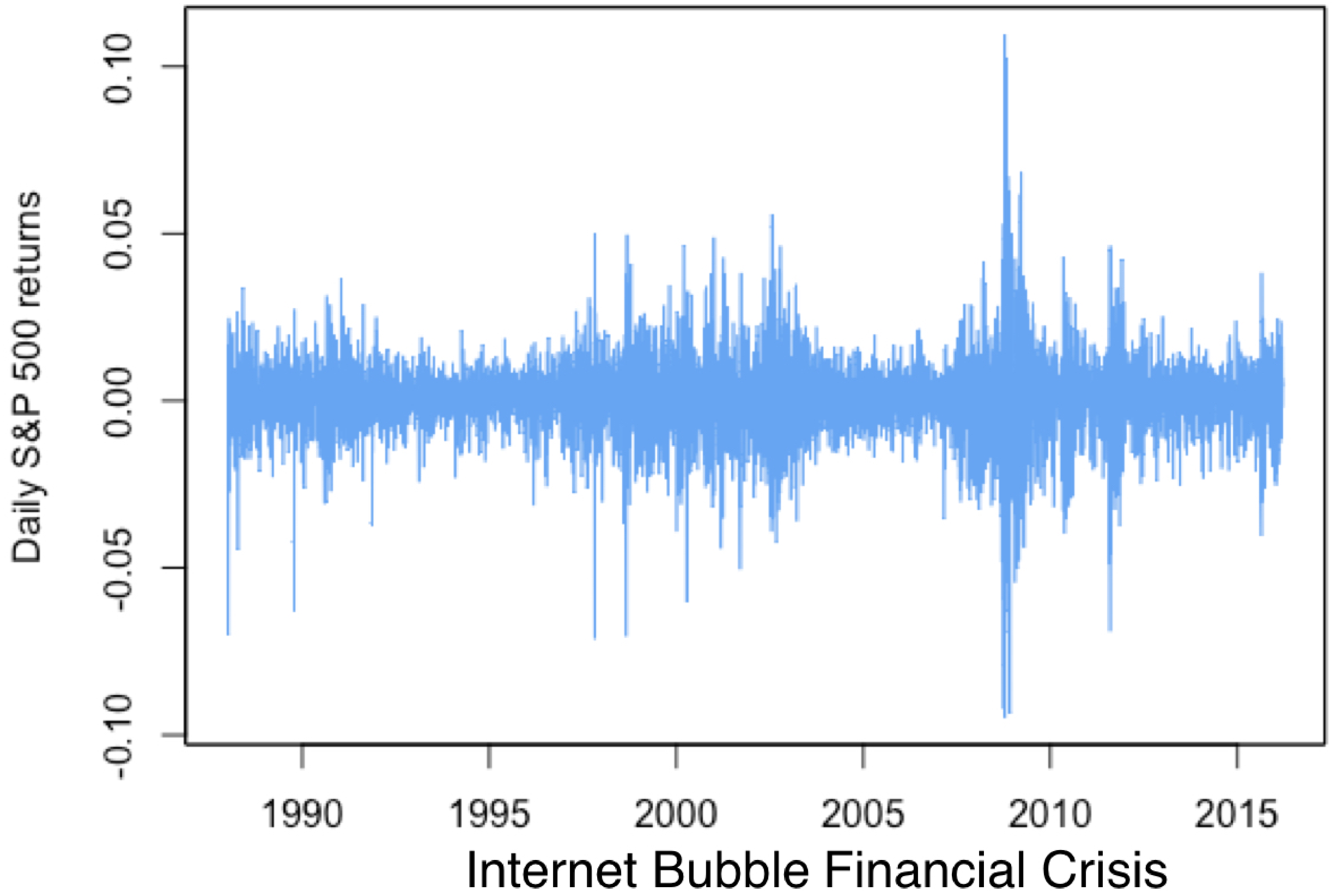

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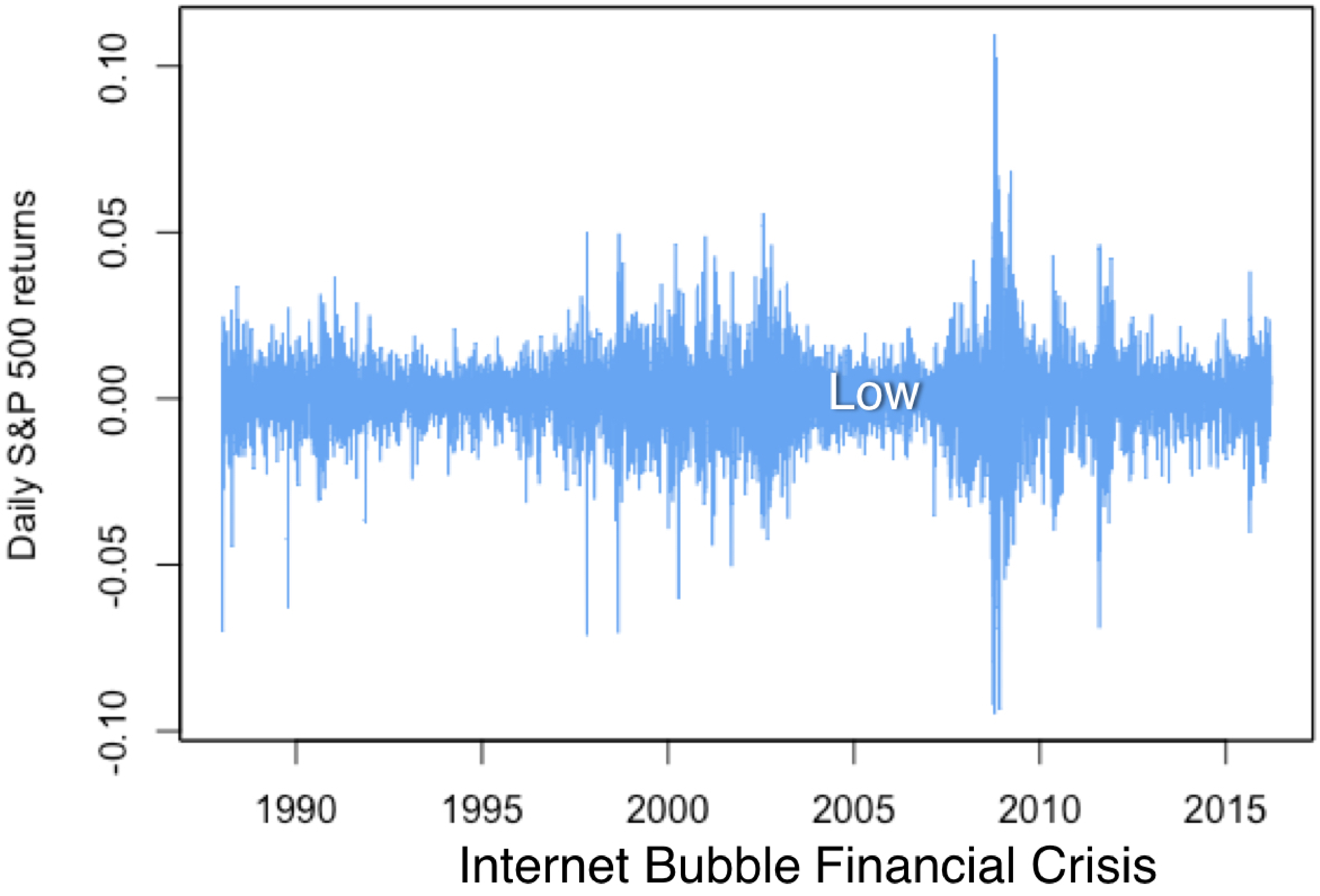
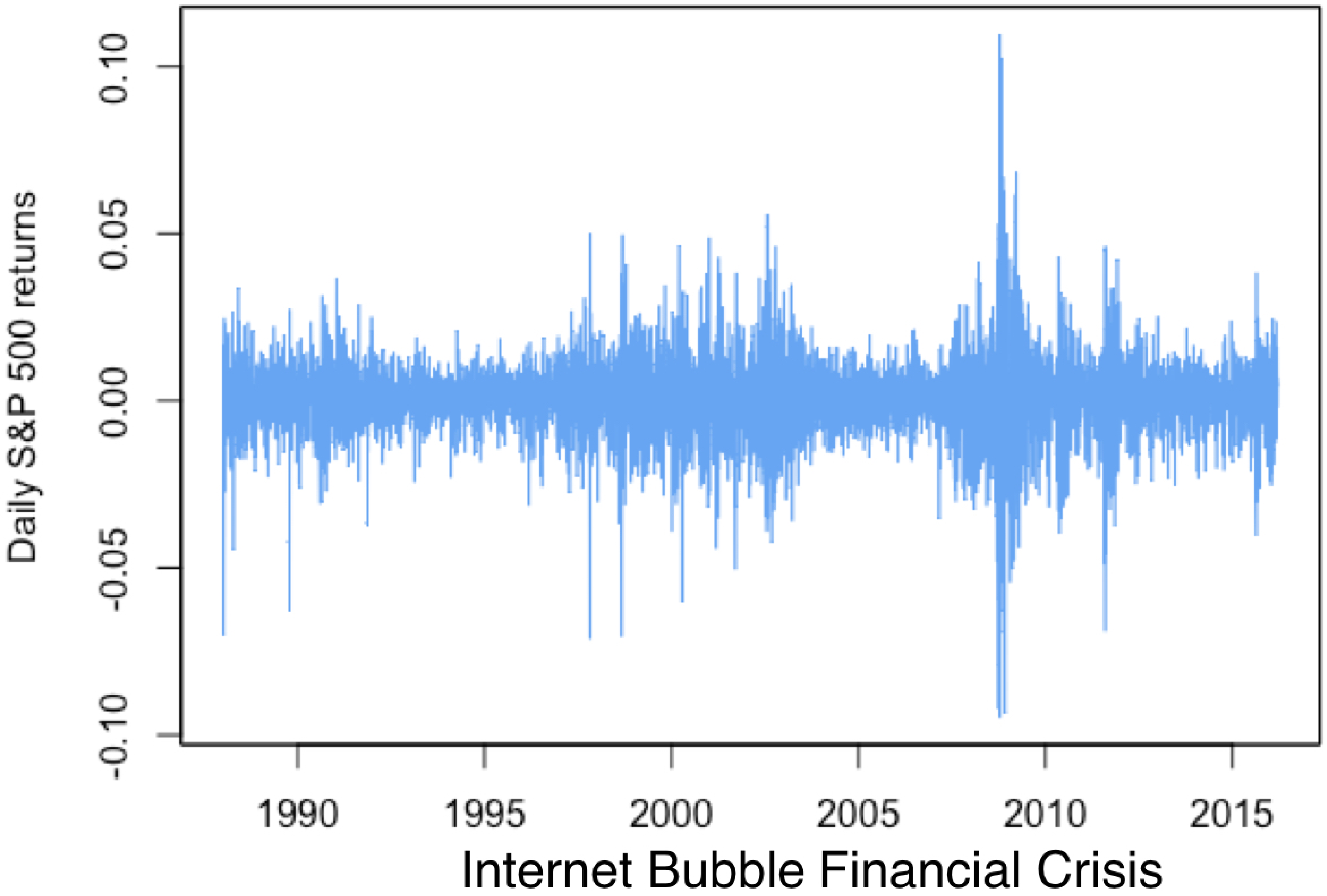
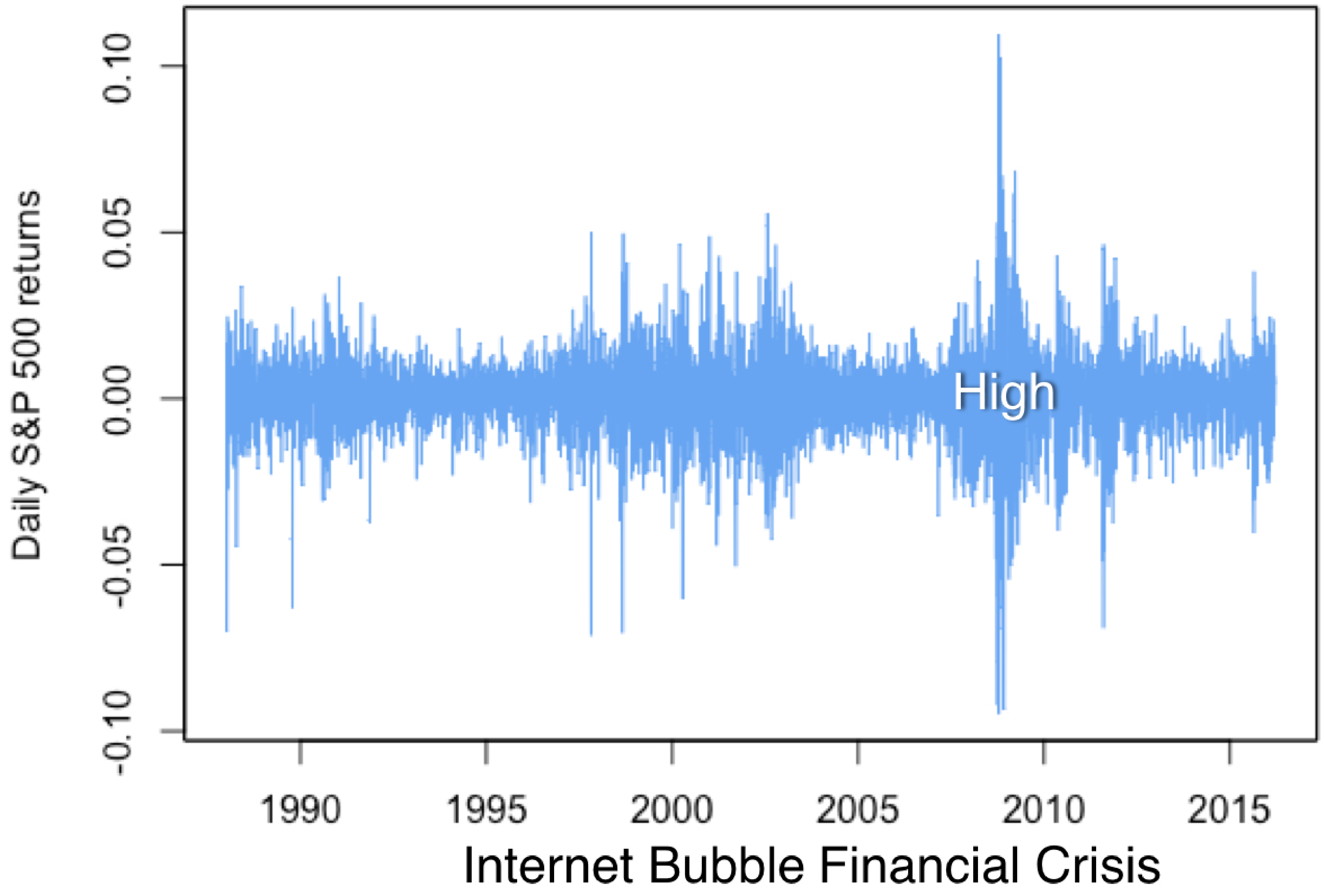


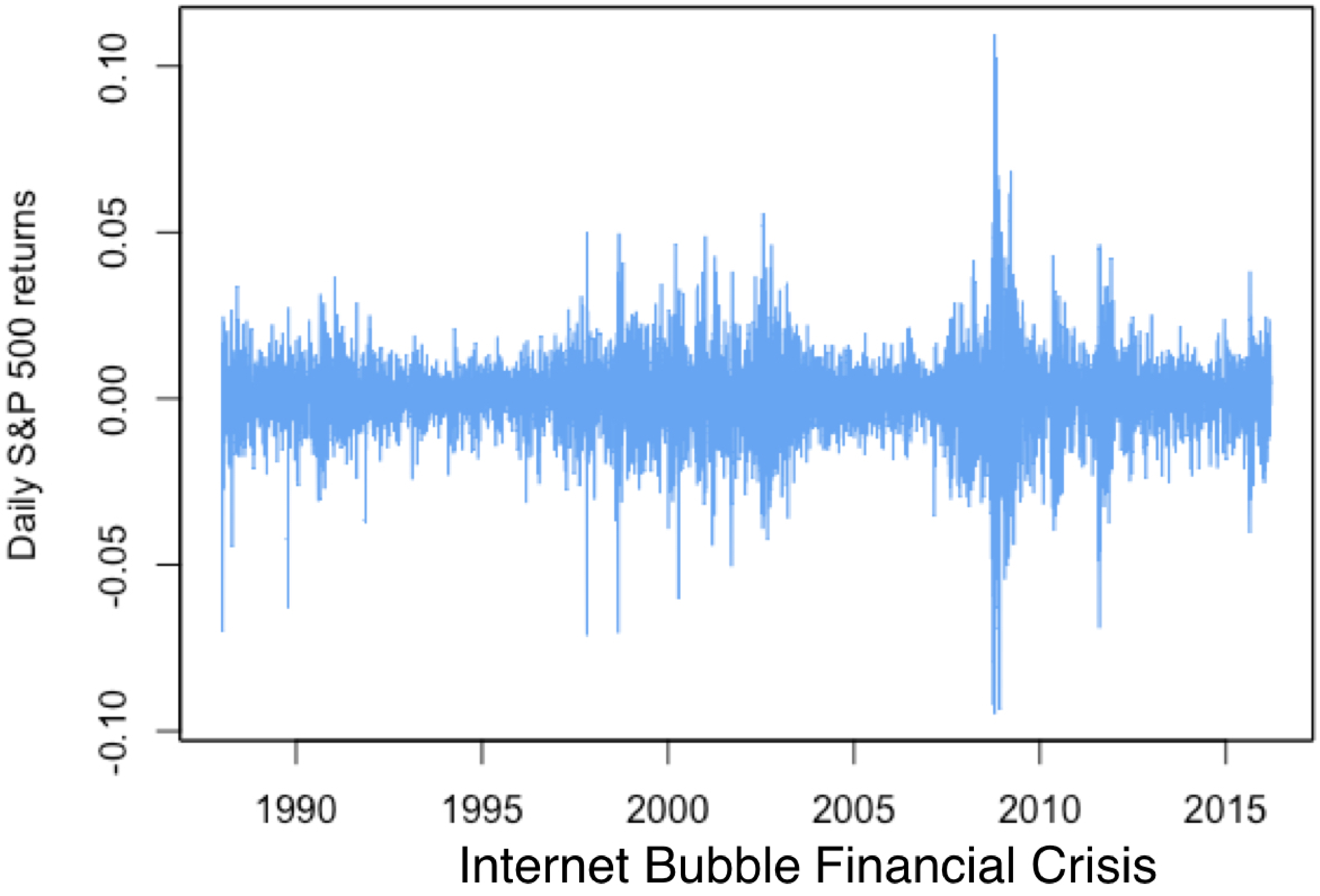
Performance statistics in action



Performance statistics in action



Performance statistics in action



Rolling estimation samples

- Rolling samples of K observations:
  - Discard the most distant and include the most recent

R_{t-k+1}	R_{t-k+2}	R_{t-k+3}	...	R_t	R_{t+1}	R_{t+2}	R_{t+3}

Rolling estimation samples

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Rolling estimation samples

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Rolling estimation samples

- Rolling samples of K observations:
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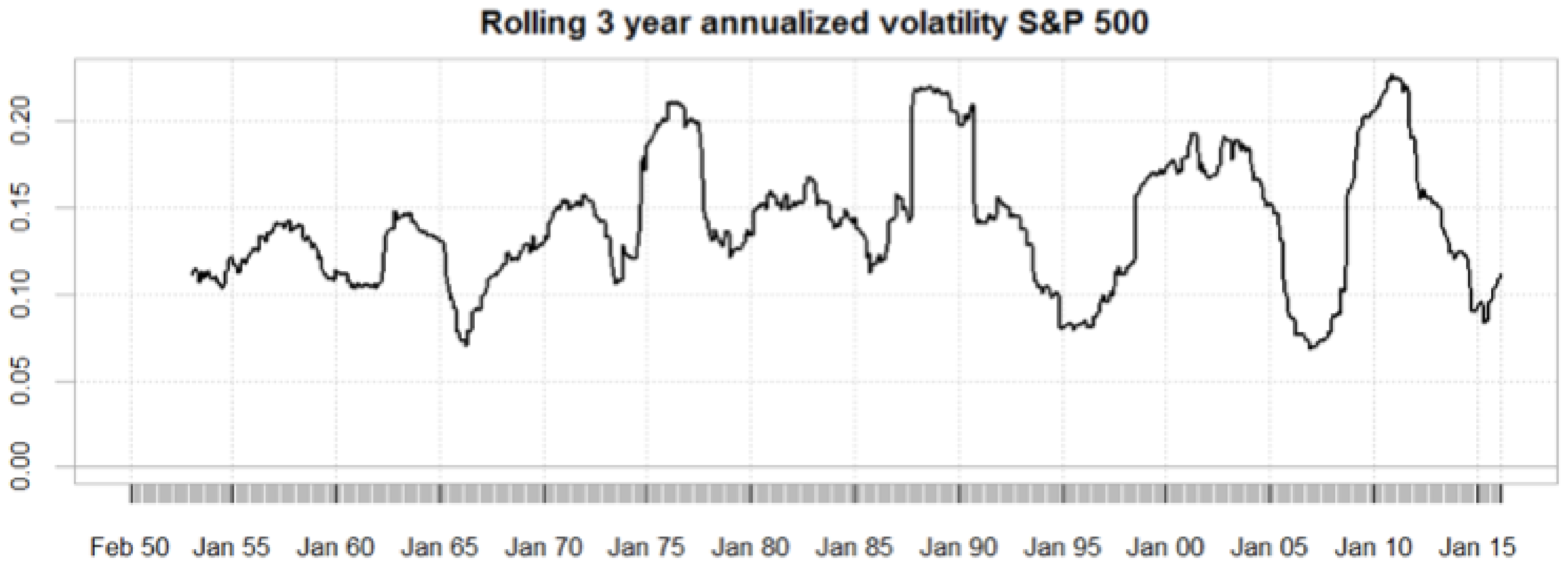
R_{t-k+1}	R_{t-k+2}	R_{t-k+3}	...	R_t	R_{t+1}	R_{t+2}	R_{t+3}

Rolling performance calculation



Choosing window length

- Need to balance noise (long samples) with recency (shorter samples)
- Longer sub-periods smooth highs and lows
- Shorter sub-periods provide more information on recent observations

Let's practice!

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Non-normality of the return distribution

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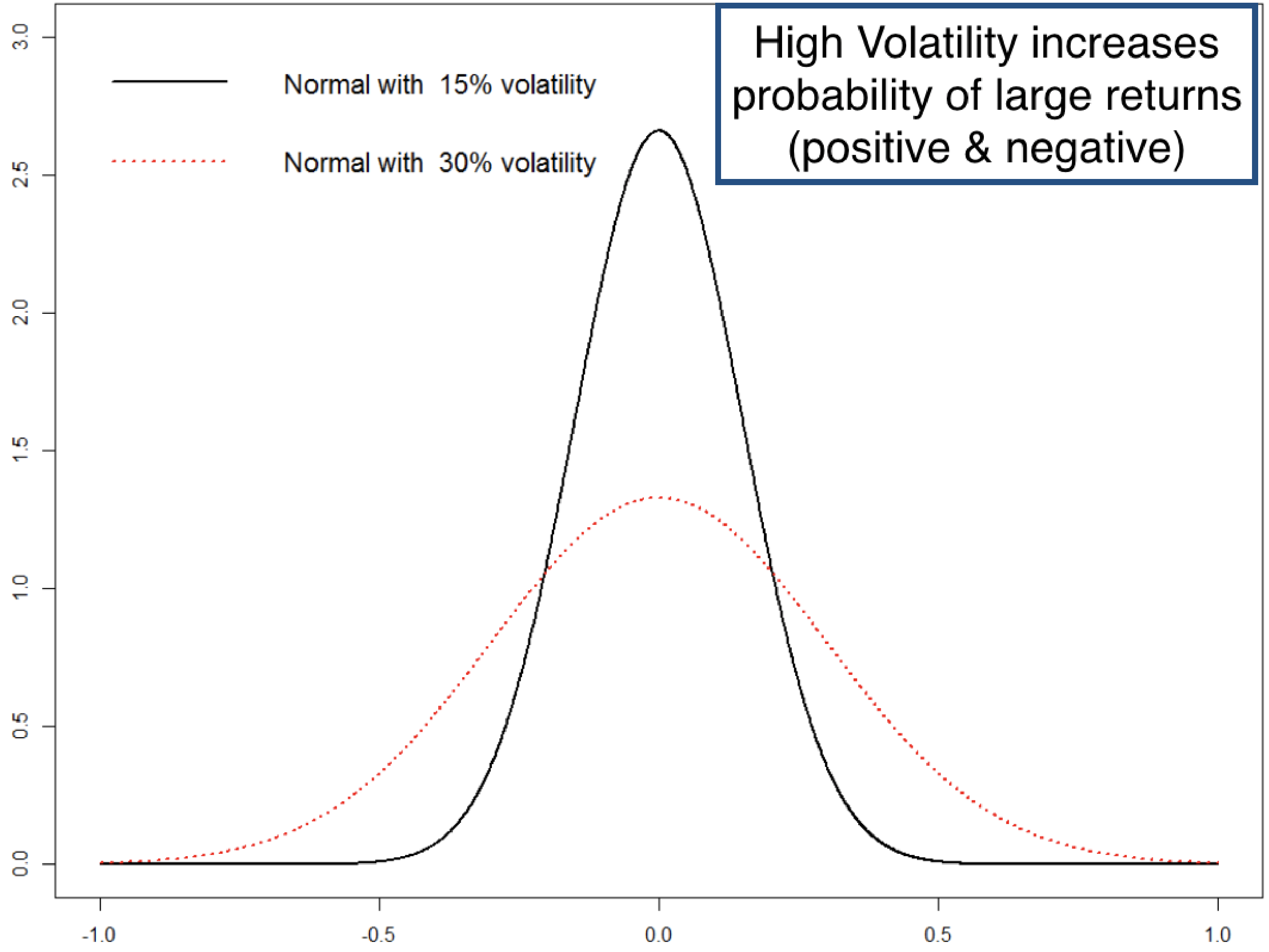
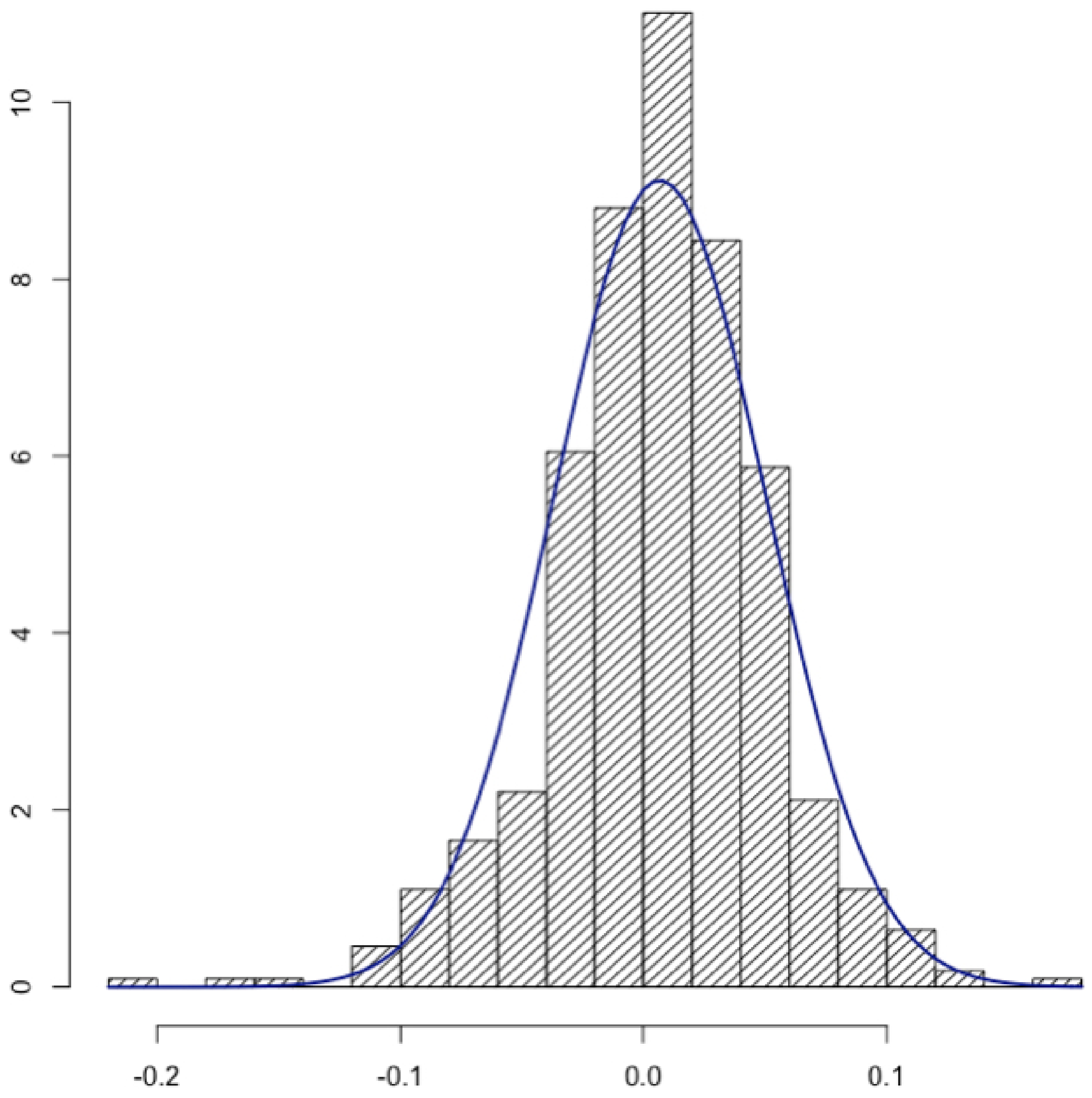
Volatility describes "normal" risk



Non-normality of return



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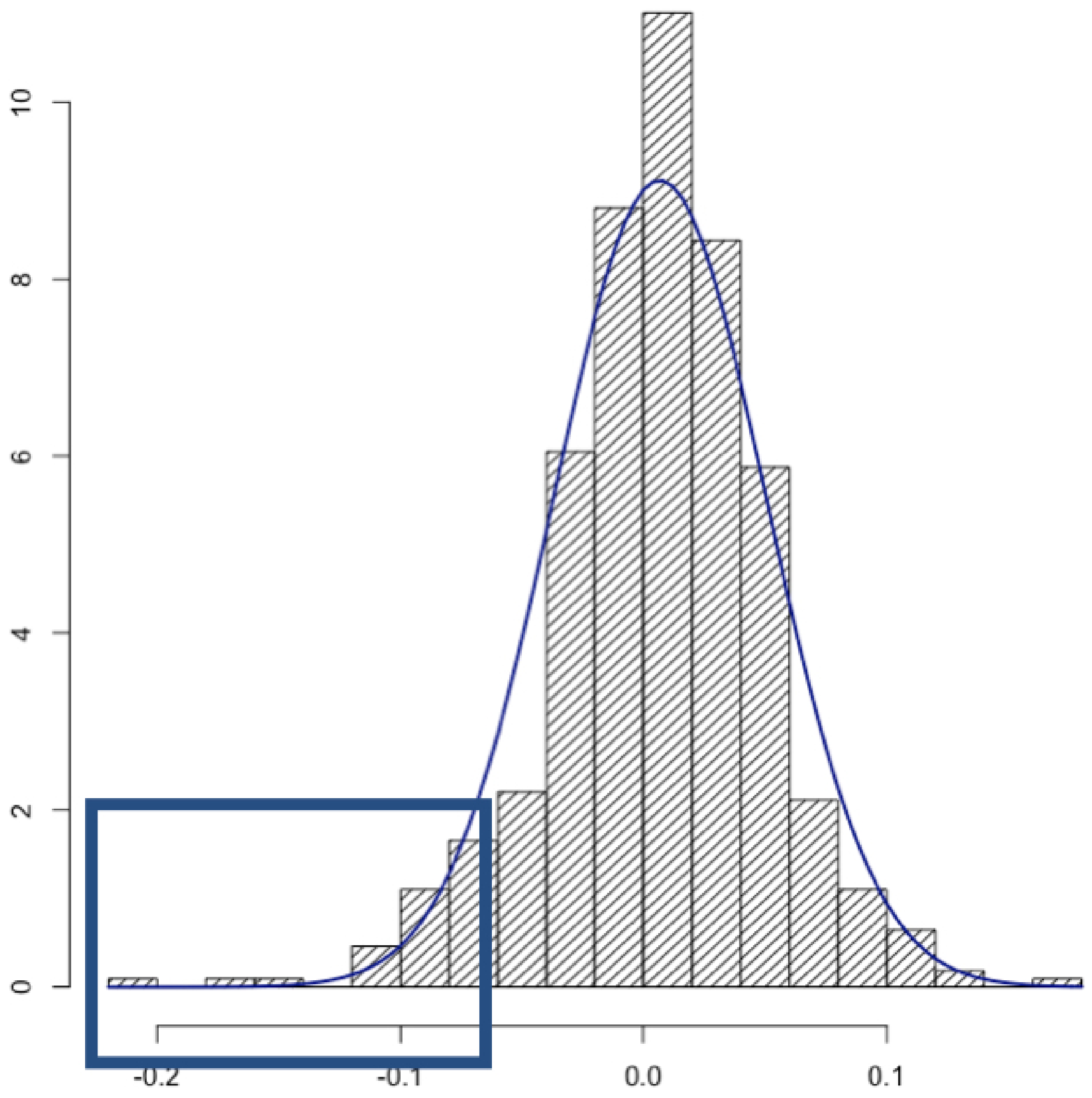

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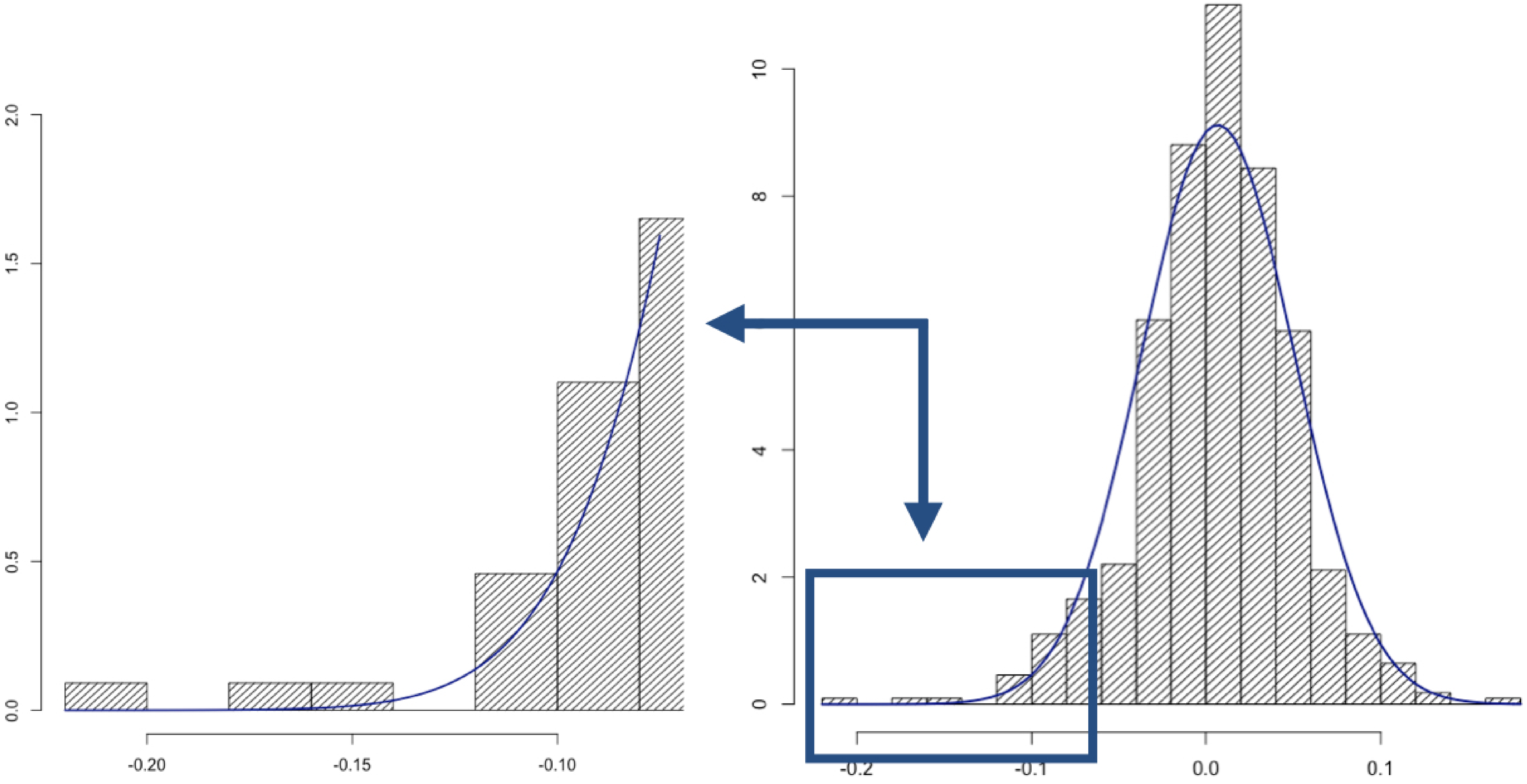


Portfolio return semi-deviation

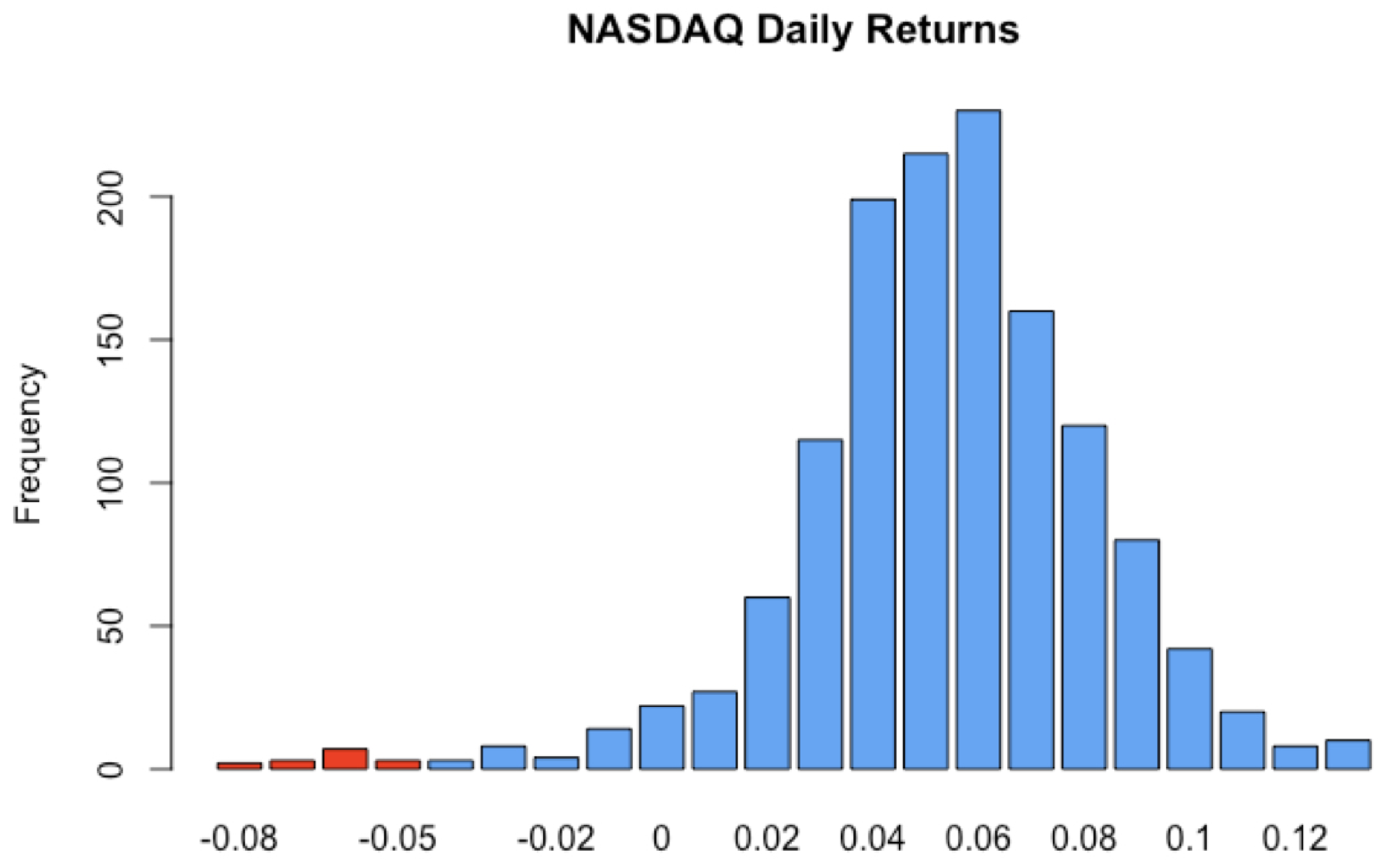
- Standard Deviation of portfolio returns:
  - Take the full sample of returns

$$SD = \sqrt{\frac{(R_1 - \mu)^2 + (R_2 - \mu)^2 + ... + (R_T - \mu)^2}{T - 1}}$$

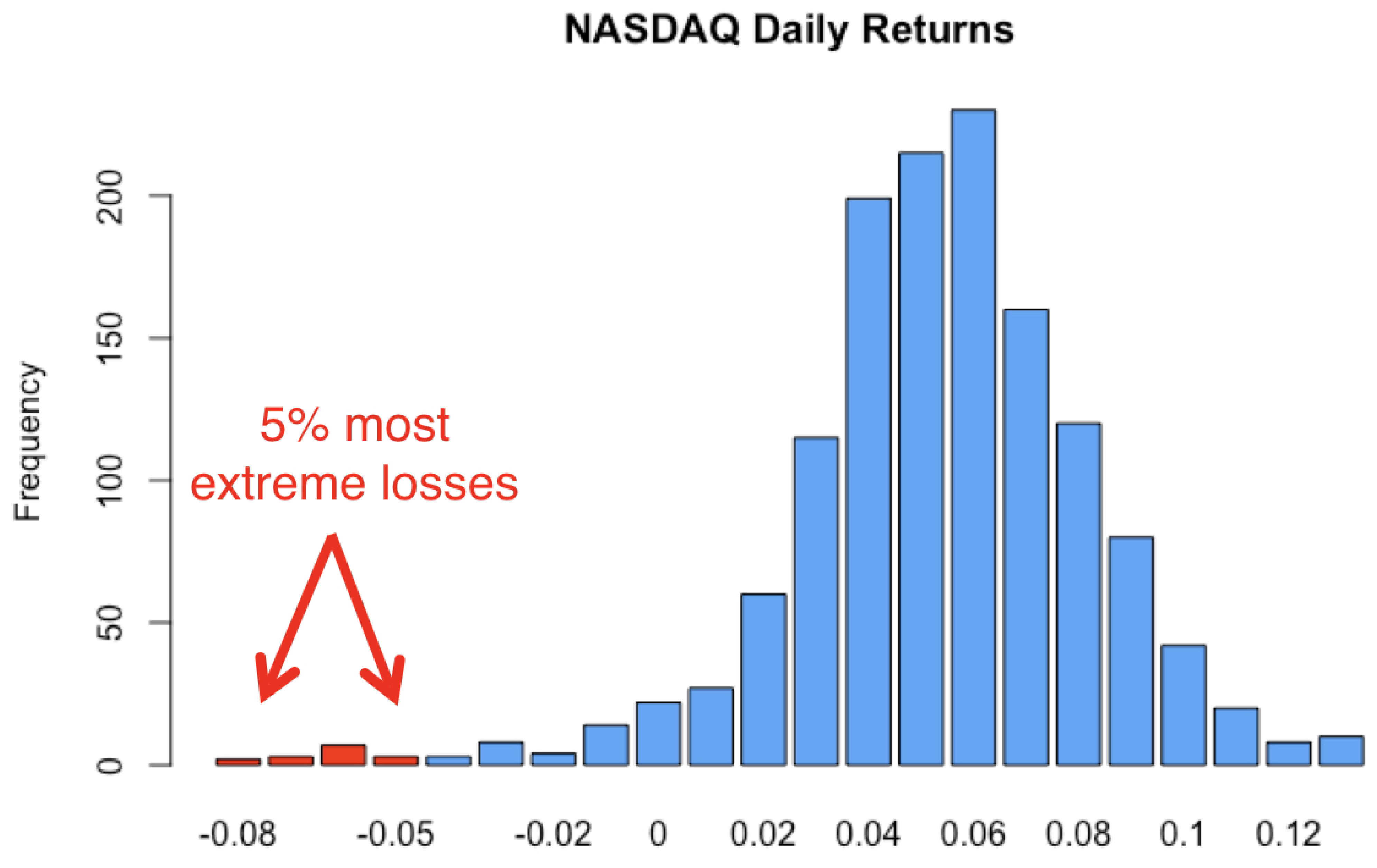
- Semi-Deviation of portfolio returns:
  - Take the subset of returns **below the mean**

$$SemiDev = \sqrt{\frac{(Z_1 - \mu)^2 + (Z_2 - \mu)^2 + ... + (Z_n - \mu)^2}{n}}$$

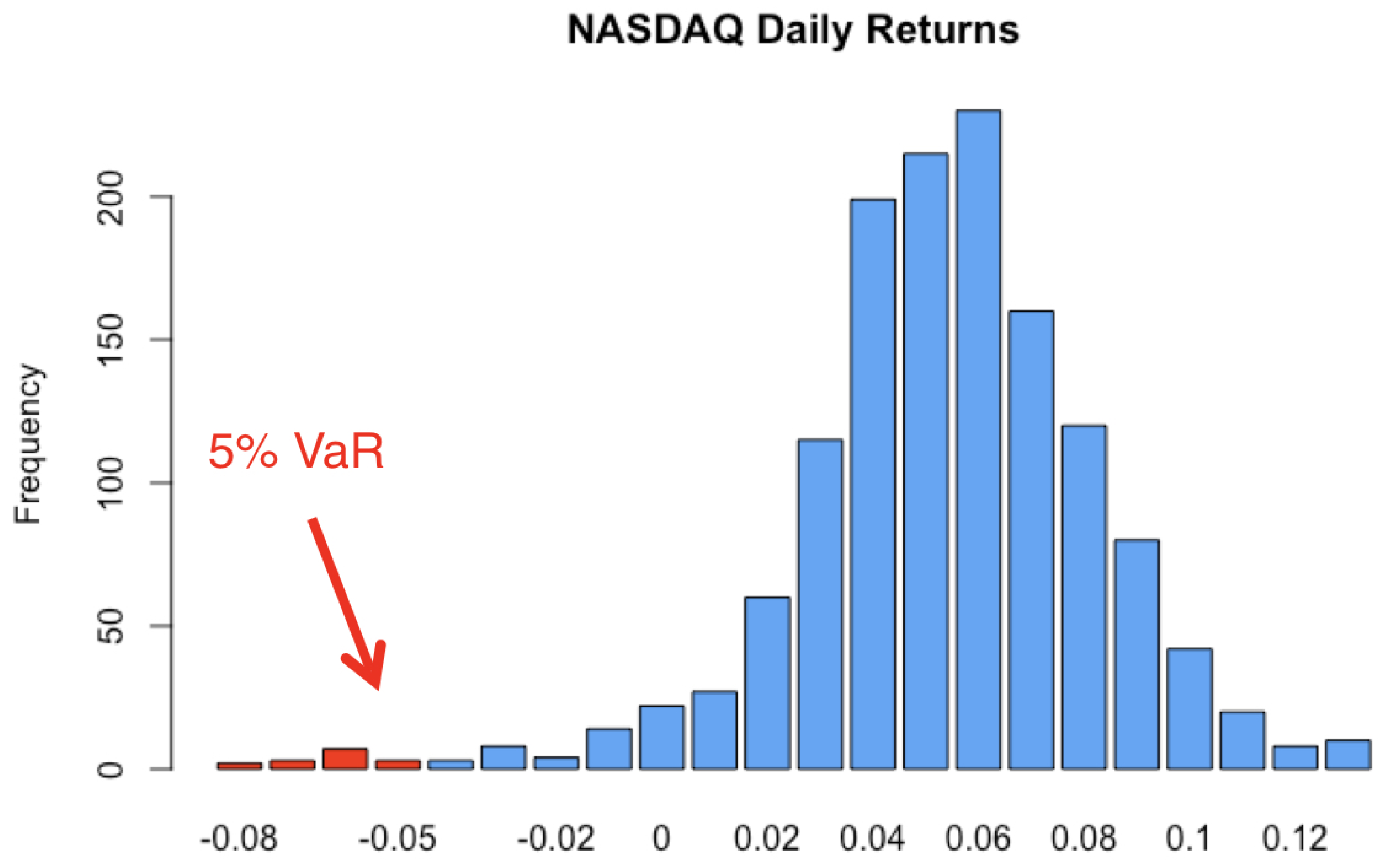
Value-at-risk & expected shortfall



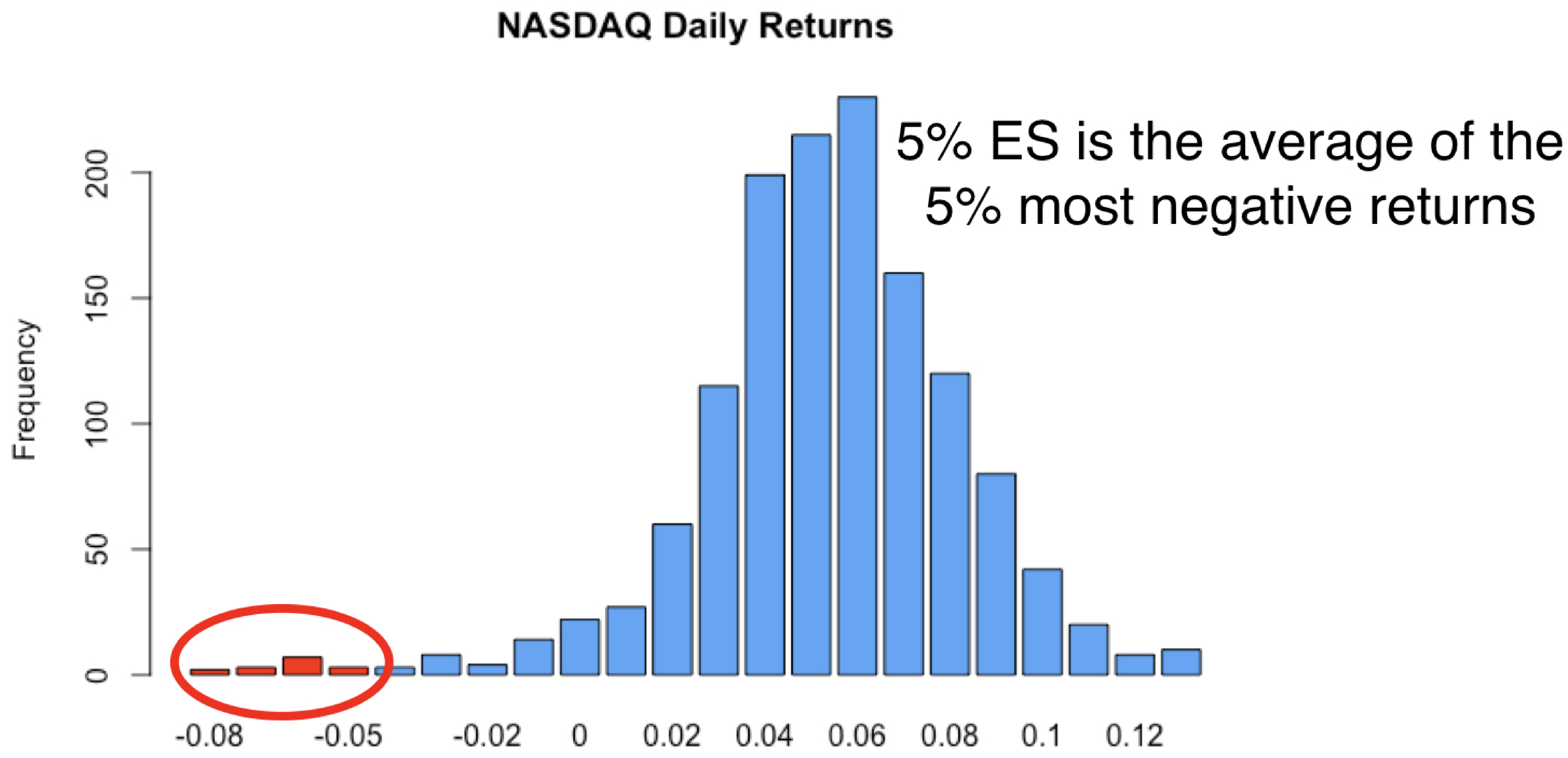
Value-at-risk & expected shortfall



Value-at-risk & expected shortfall



Value-at-risk & expected shortfall

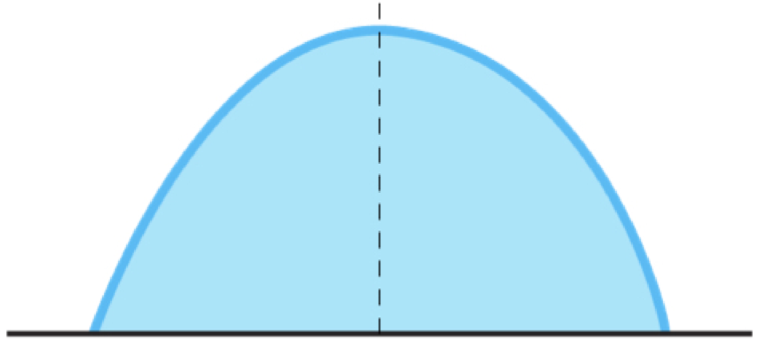


Shape of the distribution

- Is it symmetric?
  - Check the skewness
- Are the tails fatter than those of the normal distribution?
  - Check the excess kurtosis

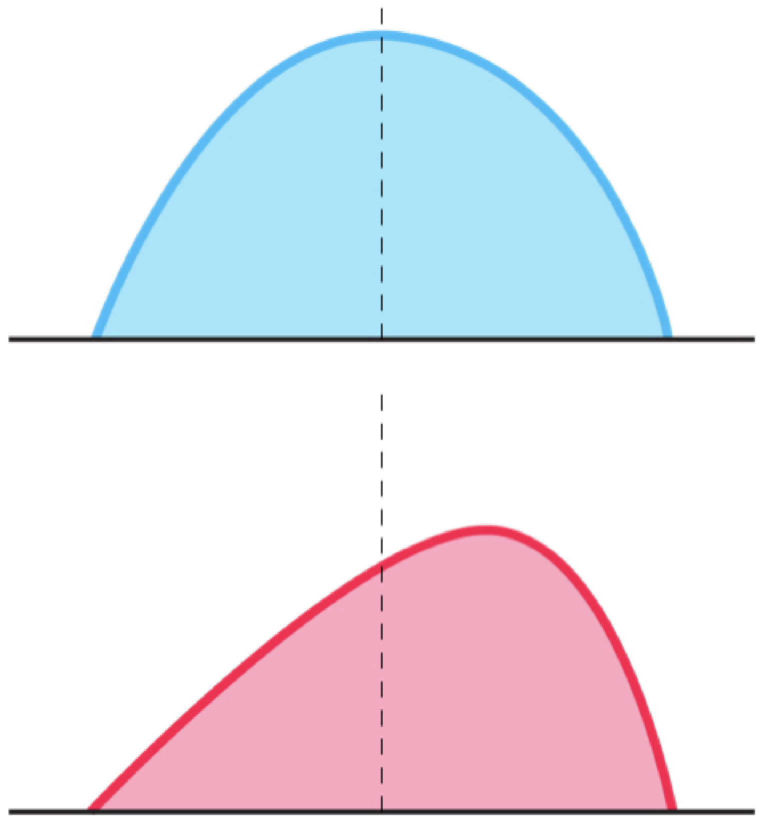
Skewness

- **Zero Skewness**
  - Distribution is symmetric



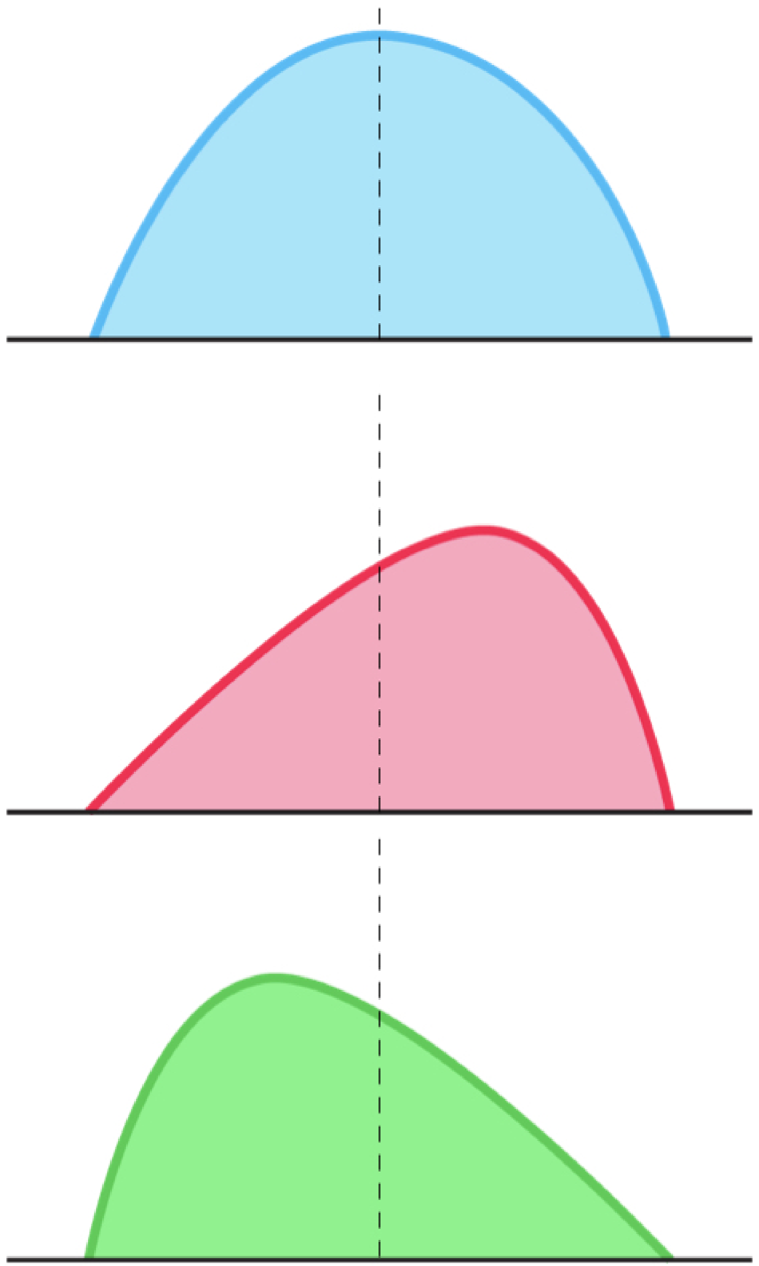
Skewness

- **Zero Skewness**
  - Distribution is symmetric
- **Negative Skewness**
  - Large negative returns occur more often than large positive returns



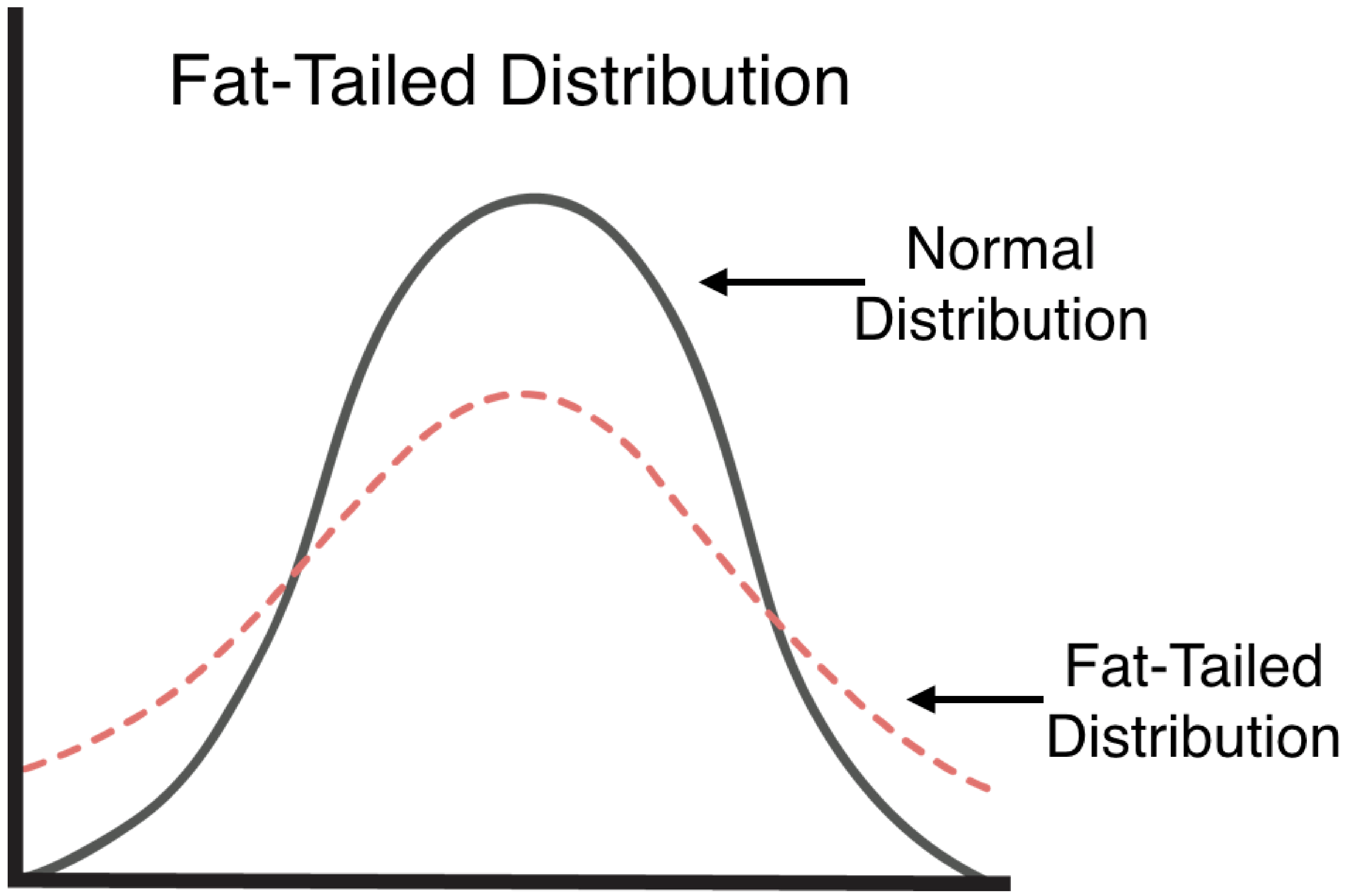
Skewness

- **Zero Skewness**
  - Distribution is symmetric
- **Negative Skewness**
  - Large negative returns occur more often than large positive returns
- **Positive Skewness**
  - Large positive returns occur more often than large negative returns



Kurtosis

- The distribution is fat-tailed when the excess kurtosis > 0



Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R