Characteristics of volatile return series

QUANTITATIVE RISK MANAGEMENT IN R

Alexander McNeil
Professor, University of York

Log-returns compared with iid data

- Can financial returns be modeled as **independent and identically distributed** (iid)?
- Random walk model for log asset prices
- Implies that future price behavior cannot be predicted
- Instructive to compare real returns with iid data
- Real returns often show **volatility clustering**

Let's practice!

QUANTITATIVE RISK MANAGEMENT IN R

Estimating serial correlations

QUANTITATIVE RISK MANAGEMENT IN R

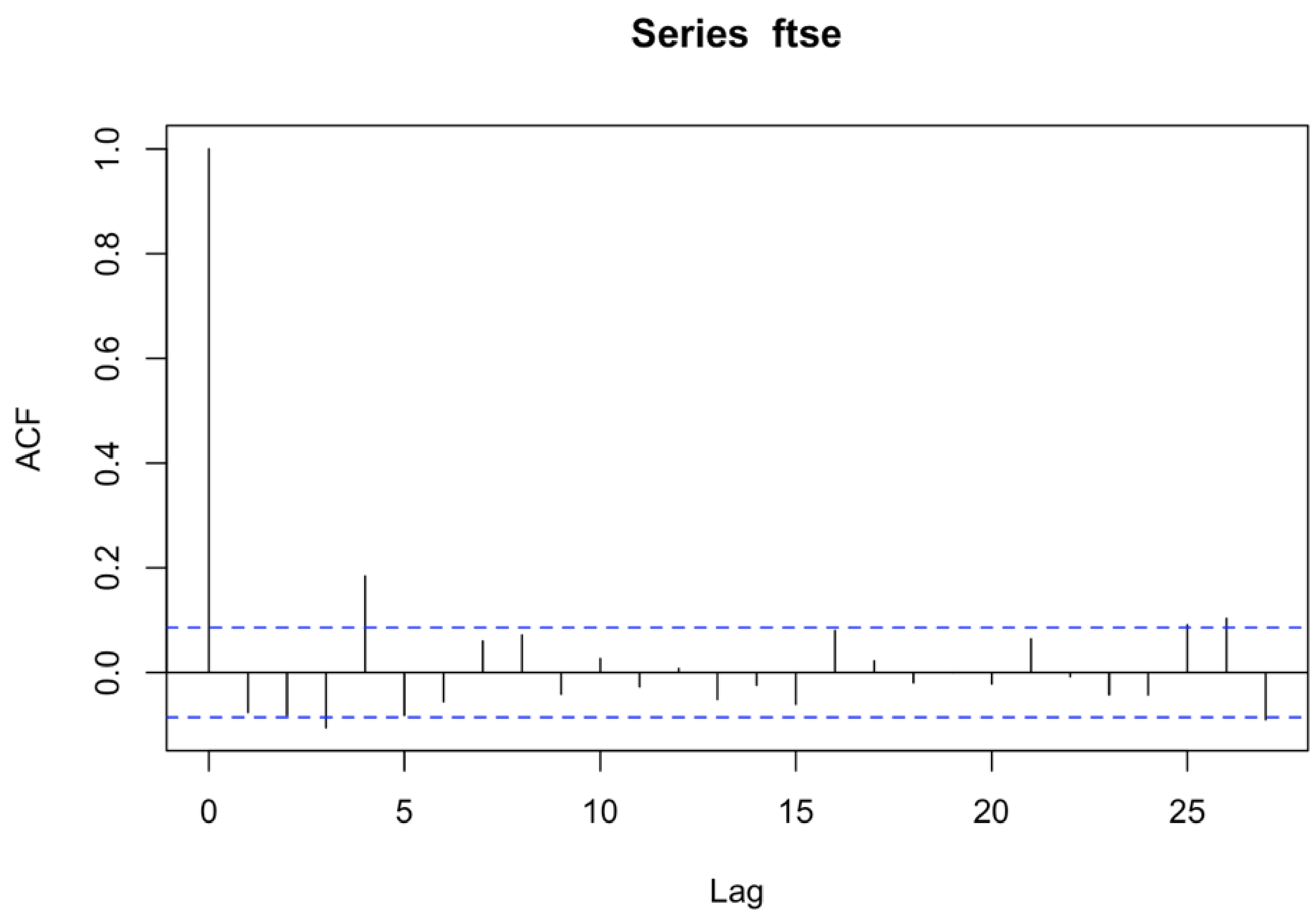
Alexander McNeil
Professor, University of York

Sample autocorrelations

- **Sample autocorrelation function** (acf) measures correlation between variables separated by **lag** (k)
- **Stationarity** is implicitly assumed:
  - Expected return constant over time
  - Variance of return distribution always the same
  - Correlation between returns k apart always the same
- Notation for sample autocorrelation: $\hat{\rho}(k)$

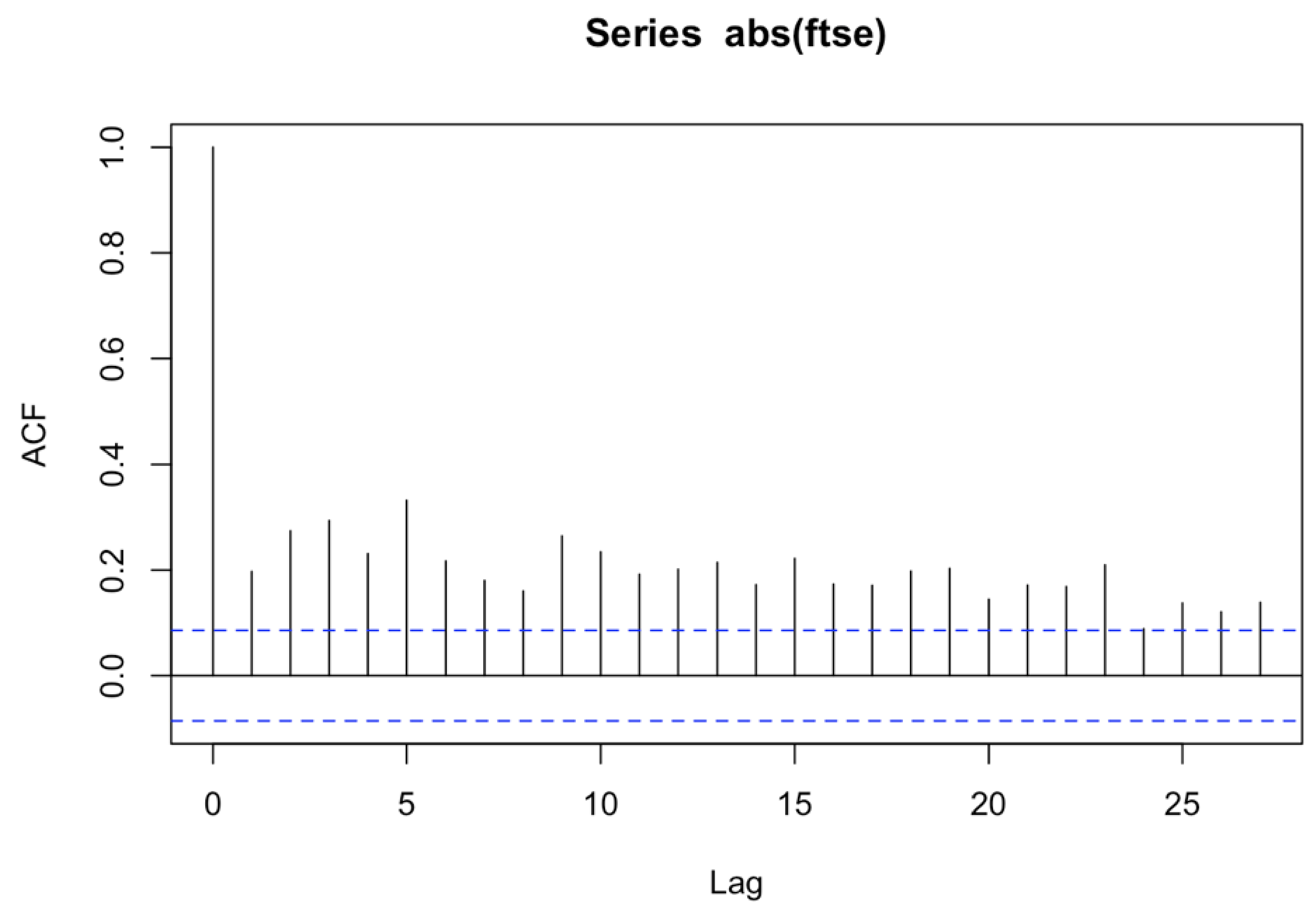
The sample acf plot or correlogram

```
acf(ftse)
```



The sample acf plot or correlogram

```r
acf(abs(ftse))
```



Let's practice!

QUANTITATIVE RISK MANAGEMENT IN R

The Ljung-Box test

QUANTITATIVE RISK MANAGEMENT IN R

Alexander McNeil
Professor, University of York

Testing the iid hypothesis with the Ljung-Box test

- Numerical test calculated from squared sample autocorrelations up to certain lag
- Compared with chi-squared distribution with **degrees of freedom** (df)
- Should also be carried out on absolute terms

$$X^2 = n(n+2)\sum_{j=1}^{k}\frac{\hat{\rho}(j)^2}{n-j}$$

Example of Ljung-Box test

```
Box.test(ftse, lag = 10, type = "Ljung")
```

```
	Box-Ljung test


data: ftse
X-squared = 41.602, df = 10, p-value = 8.827e-06
```

```
Box.test(abs(ftse), lag = 10, type = "Ljung")
```

```
	Box-Ljung test


data: abs(ftse)
X-squared = 314.62, df = 10, p-value < 2.2e-16
```

Applying Ljung-Box to longer-interval returns

```
ftse_w <- apply.weekly(ftse, FUN = sum)
head(ftse_w, n = 3)
```

```
                  ^FTSE
2008-01-04 -0.01693075
2008-01-11 -0.02334674
2008-01-18 -0.04963134
```

```
Box.test(ftse_w, lag = 10, type = "Ljung")
```

```
    Box-Ljung test

data: ftse_w
X-squared = 18.11, df = 10, p-value = 0.05314
```

```
Box.test(abs(ftse_w), lag = 10, type = "Ljung")
```

```
    Box-Ljung test

data: abs(ftse_w)
X-squared = 34.307, df = 10, p-value = 0.0001638
```

Let's practice!

QUANTITATIVE RISK MANAGEMENT IN R

Looking at the extremes in volatile return series

QUANTITATIVE RISK MANAGEMENT IN R

Alexander McNeil
Professor, University of York

Extracting the extreme of return series

- Extract the most extreme negative log-returns exceeding 0.025

```
ftse <- diff(log(FTSE))["1991-01-02/2010-12-31"]
ftse_losses <- -ftse
ftse_extremes <- ftse_losses[ftse_losses > 0.025]

head(ftse_extremes)
```

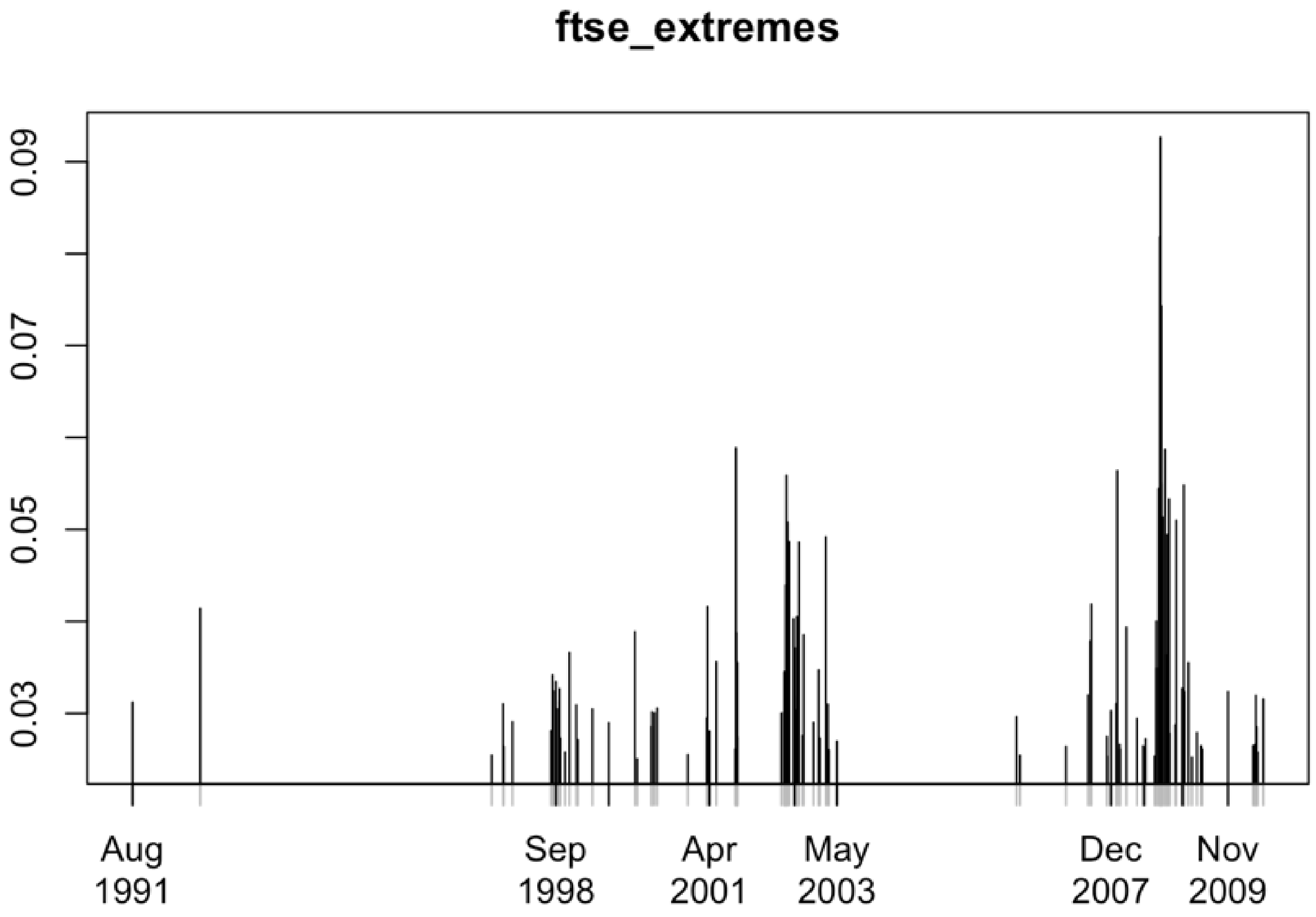
```
                ^FTSE
1991-08-19 0.03119501
1992-10-05 0.04139899
1997-08-15 0.02546526
1997-10-23 0.03102717
```

```
length(ftse_extremes)
```

```
115
```

Plotting the extremes values

```
plot(ftse_extremes, type = "h", auto.grid = FALSE)
```



Let's practice!

QUANTITATIVE RISK MANAGEMENT IN R

The stylized facts of return series

QUANTITATIVE RISK MANAGEMENT IN R

Alexander McNeil
Professor, University of York

The stylized facts

1. Return series are heavier-tailed than normal, or **leptokurtic**
2. The volatility of return series appears to vary over time
3. Return series show relatively little serial correlation
4. Series of absolute returns show profound serial correlation
5. Extreme returns appear in clusters
6. Returns aggregated over longer periods tend to become more normal and less serially dependent

Let's practice!

QUANTITATIVE RISK MANAGEMENT IN R