Value-at-risk and expected shortfall

QUANTITATIVE RISK MANAGEMENT IN R

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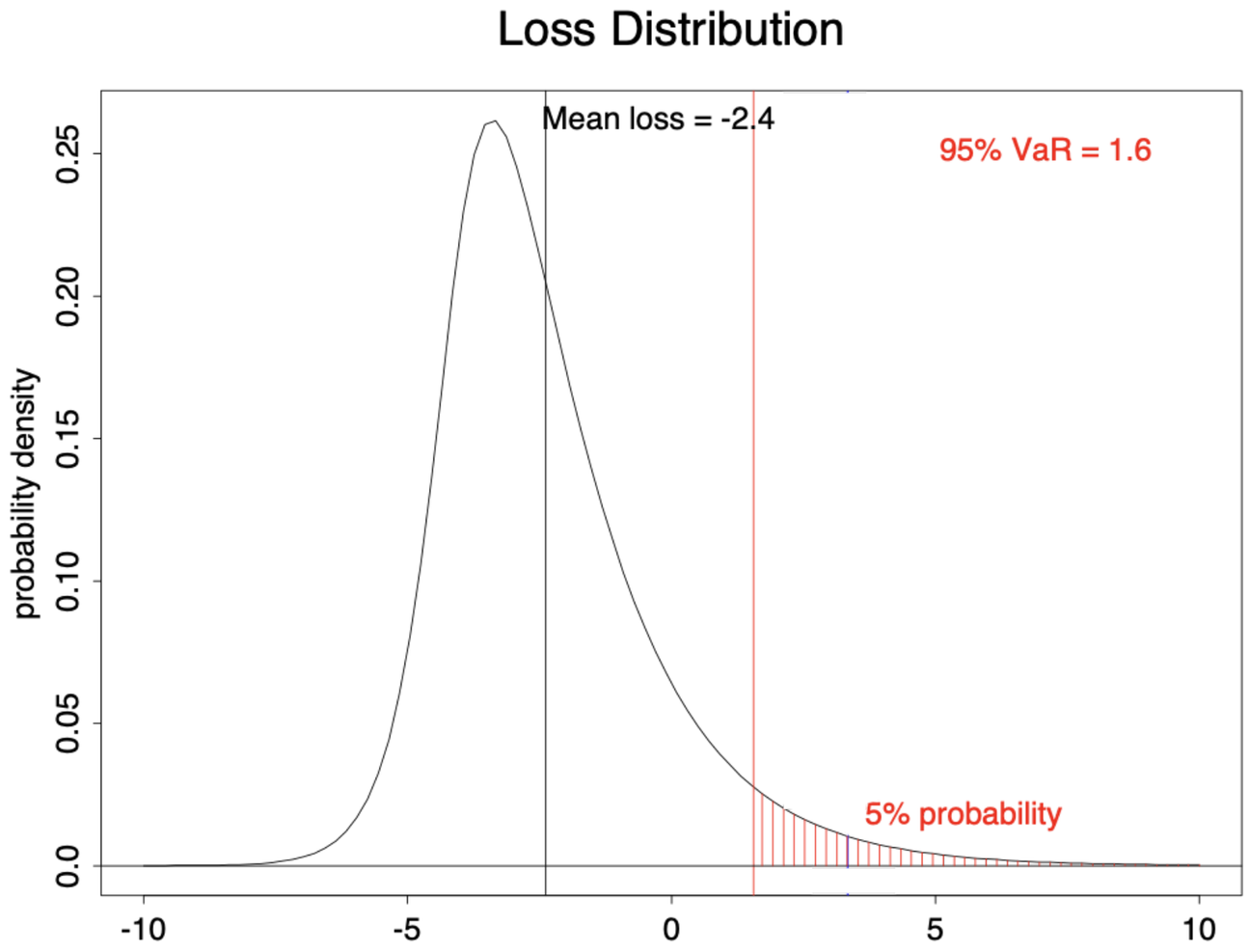
Value-at-risk (VaR)

- Consider the distribution of losses over a fixed time period (day, week, etc.)
- α-VaR is the α-**quantile** of the loss distribution
- α known as **confidence level** (e.g. 95%, 99%)
- Should lose no more than α-VaR with probability α

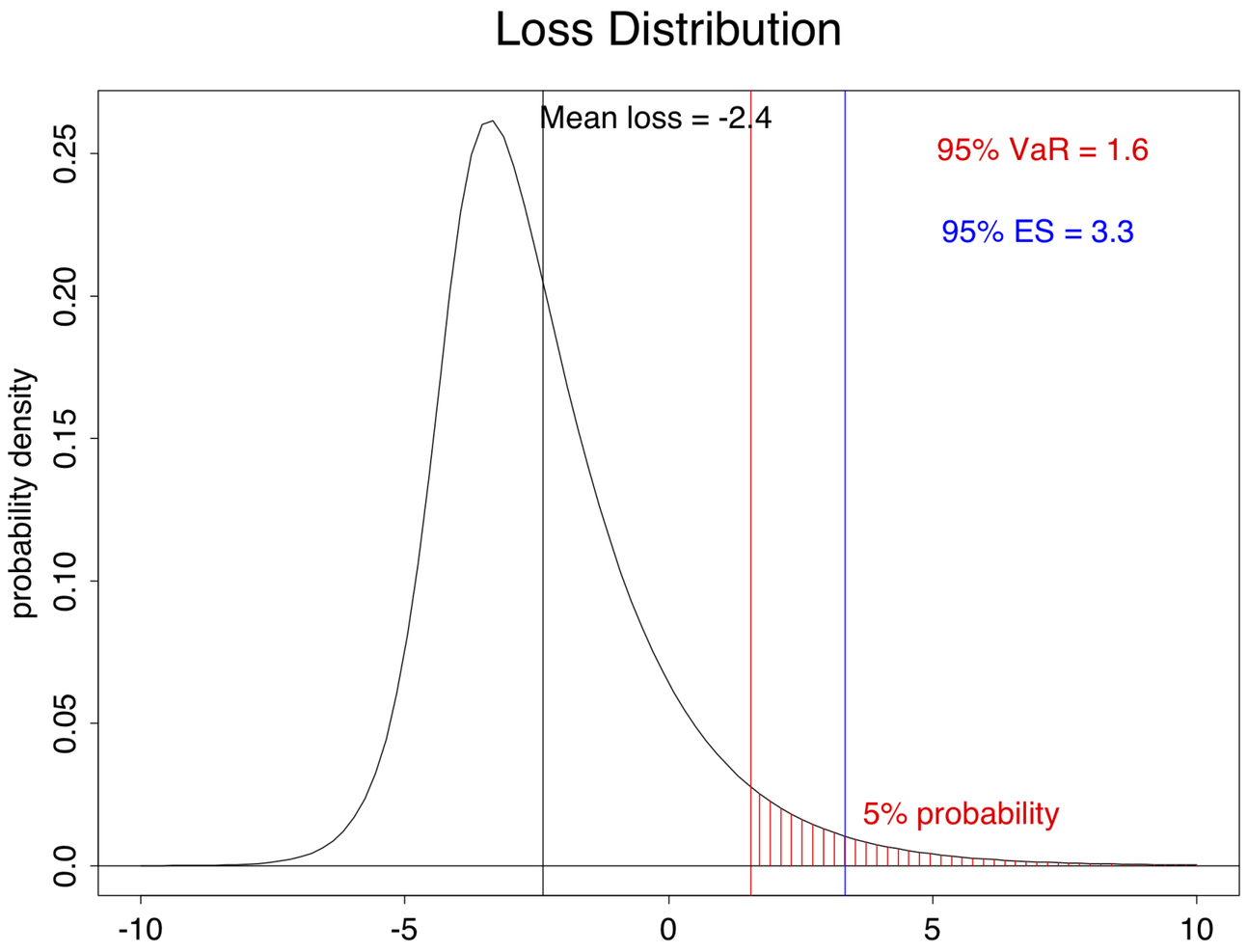
95% VaR illustrated



Expected shortfall (ES)

- Increasingly important in banking regulation
- Tail VaR (TVaR), conditional VaR (CVaR) or **expected shortfall** (ES)
- α-ES is expected loss given that loss exceeds α-VaR
- Expectation of tail of distribution

95% of ES illustrated



Let's practice!

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International equity portfolio

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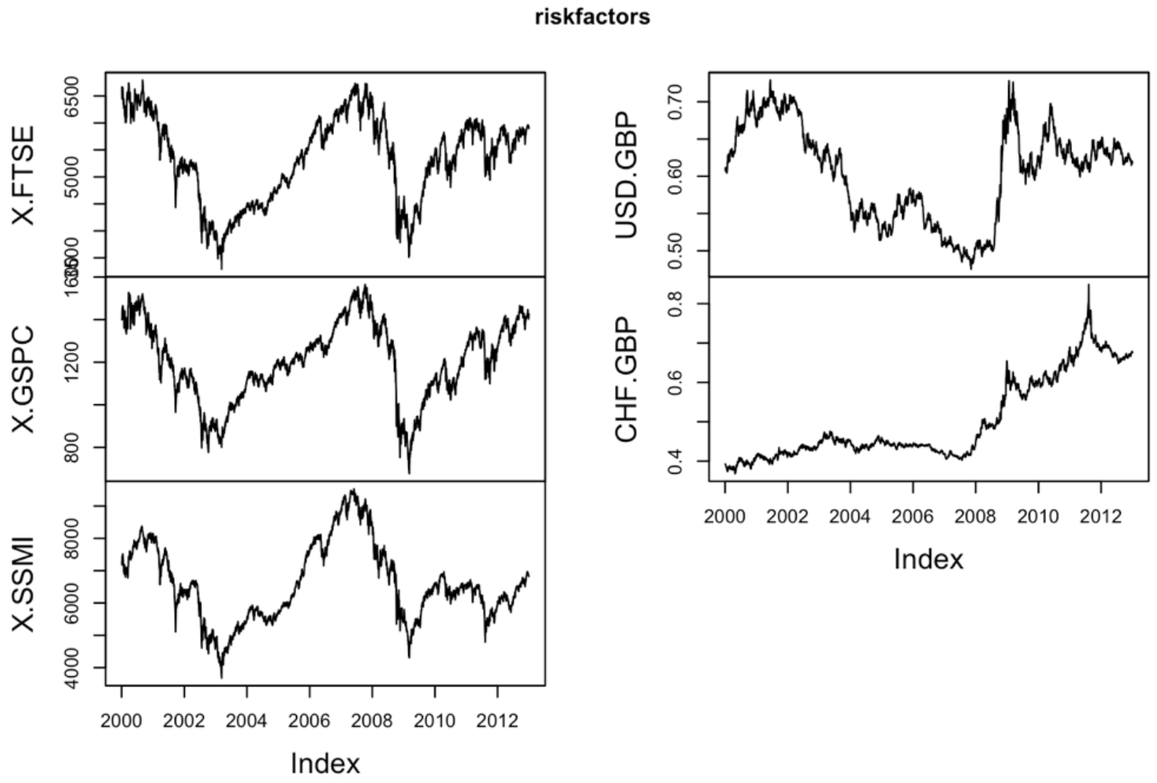
International equity portfolio

- Imagine a UK investor who has invested her wealth:
  - 30% FTSE, 40% S&P 500, 30% SMI
- 5 risk factors: FTSE, S&P 500 and SMI indexes, GBP/USD and GBP/CHF exchange rate

```
riskfactors <- merge(FTSE, SP500, SMI, USD_GBP, CHF_GBP, all = FALSE)["/2012-12-31", ]
```

Displaying the risk factors

```
plot.zoo(riskfactors)
```



Historical simulation

- Simple method that is widely used in financial industry
- Resample historical risk-factor returns and examine their effect on current portfolio
- **Loss operator** shows effect of different risk-factor returns on the portfolio
- Loss operator functions will be provided in the exercises

Empirical estimates of VaR and ES

```
losses <- rnorm(100)
losses_o <- sort(losses, decreasing = TRUE)
head(losses_o, n = 8)
```

```
1.836163 1.775163 1.745427 1.614479 1.602120 1.590034 1.483691 1.408354
```

```
quantile(losses, 0.95)
```

```
     95%
1.590638
```

```
qnorm(0.95)
```

```
1.644854
```

Empirical estimates of VaR and ES

```
mean(losses[losses > quantile(losses, 0.95)])
```

```
1.714671
```

```
ESnorm(0.95)
```

```
2.062713
```

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Option portfolio and Black Scholes

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European options and Black-Scholes

- European **call** option: gives right but not obligation to buy stock for price K at time T
- European **put** option: gives right but not obligation to sell stock for price K at T
- Value at time t < T depends on:
  - Stock price S, time to maturity T-t, interest rate r, annualized volatility σ or sigma
  - Pricing by **Black-Scholes** formula

Pricing a first call option

```
K <- 50
T <- 2
t <- 0
S <- 40
r <- 0.005
sigma <- 0.25
Black_Scholes(t, S, r, sigma, K, T, "call")
```

```
2.619183
```

```
Black_Scholes(t, S, r, sigma*1.2, K, T, "call")
```

```
3.677901
```

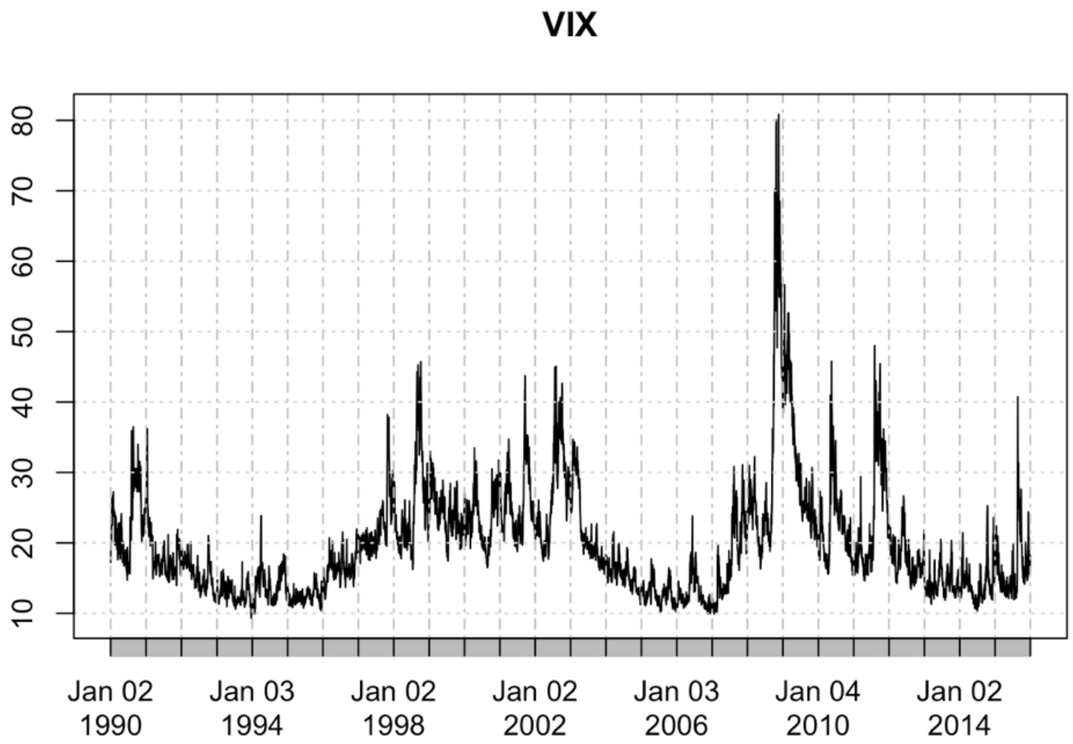
- Price increases with volatility
- Option above is out-of-the-money

Implied volatility X needs change

- Volatility not directly observable
- Market participants use **implied volatility**, the value of volatility implied by quoted option price

The VIX index

```
plot(VIX)
```



Let's practice!

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Historical simulation for the option example

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Historical simulation

- Portfolio: single European call option on equity index
- Consider losses and profits over one day
- Changes to index value S, implied volatility σ and interest rate r affect value of portfolio
- We consider S and σ (and assume r stays constant)
- Create loss operator taking S and σ as input and giving the loss or profit as output

Estimating VaR and ES

- Apply loss operator `lossop()` to historical log-returns of S&P 500 and VIX to get simulated losses
- Estimate VaR by sample quantile as before
- Estimate ES by average of losses exceeding VaR estimate

Let's practice!

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Wrap-up

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Not the end of the story...

Consider two things:

1. Can we improve risk sensitivity of VaR and ES estimates?
   - Filtered historical simulation
   - GARCH models
   - EWMA volatility filters
2. Can we improve simple empirical estimates of VaR and ES?
   - Parametric tail models, heavy-tailed distributions, extreme value theory

Thanks for taking the course!

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