Logistic regression: introduction

CREDIT RISK MODELING IN R

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Final data structure

```
str(training_set)
```

```
'data.frame':\t19394 obs. of  8 variables:
 $ loan_status   : Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 1 1 1 ...
 $ loan_amnt     : int  25000 16000 8500 9800 3600 6600 3000 7500 6000 22750 ...
 $ grade         : Factor w/ 7 levels "A","B","C","D",..: 2 4 1 2 1 1 1 2 1 1 ...
 $ home_ownership: Factor w/ 4 levels "MORTGAGE","OTHER",..: 4 4 1 1 1 3 4 3 4 1 ...
 $ annual_inc    : num  91000 45000 110000 102000 40000 ...
 $ age           : int  34 25 29 24 59 35 24 24 26 25 ...
 $ emp_cat       : Factor w/ 5 levels "0-15","15-30",..: 1 1 1 1 1 2 1 1 1 1 ...
 $ ir_cat        : Factor w/ 5 levels "0-8","11-13.5",..: 2 3 1 4 1 1 1 4 1 1 ...
```

What is logistic regression?

- A regression model with output between 0 and 1

$$P(\text{loan status} = 1 | x_1, ..., x_m) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + ... + \beta_m x_m)}}$$

- $x_1, ..., x_m$:

```
loan_amnt   grade   age   annual_inc   home_ownership   emp_cat   ir_cat
```

- $\beta_0, ...\beta_m$: Parameters to be estimated
- $\beta_0 + \beta_1 x_1 + ... + \beta_m x_m$: Linear predictor

Fitting a logistic model in R

```r
log_model <- glm(loan_status ~ age ,
                 family= "binomial", data = training_set)
log_model
```

```
Call:  glm(formula = loan_status ~ age,
           family = "binomial", data = training_set)
Coefficients:
(Intercept)          age
  -1.793566    -0.009726
Degrees of Freedom: 19393 Total (i.e. Null);  19392 Residual
Null Deviance:\t    13680
Residual Deviance: 13670 \tAIC: 13670
```

$$P(\text{loan status} = 1|\text{age}) = \frac{1}{1 + e^{-(\hat{\beta}_0 + \hat{\beta}_1 \text{age})}}$$

Probabilities of default

$$P(\text{loan status} = 1|x_1, ..., x_m) = \frac{1}{1 + e^{-(\beta_0+\beta_1 x_1+...+\beta_m x_m)}} = \frac{e^{\beta_0+\beta_1 x_1+...+\beta_m x_m}}{1 + e^{\beta_0+\beta_1 x_1+...+\beta_m x_m}}$$

$$P(\text{loan status} = 0|x_1, ..., x_m) = 1 - \frac{e^{\beta_0+\beta_1 x_1+...+\beta_m x_m}}{1 + e^{\beta_0+\beta_1 x_1+...+\beta_m x_m}} = \frac{1}{1 + e^{\beta_0+\beta_1 x_1+...+\beta_m x_m}}$$

$$\frac{P(\text{loan status} = 1|x_1, ..., x_m)}{P(loan\ status = 0|x_1, ..., x_m)} = e^{\beta_0+\beta_1 x_1+...+\beta_m x_m}$$

- Odds in favor of `loan_status = 1`

Interpretation of coefficient

- If variable x_j goes up by 1
  - The odds are multiplied by $e^{\beta j}$
- $\beta_j < 0$
  - $e^{\beta j} < 1$
  - The odds decrease as x_j increases
- $\beta_j > 0$
  - $e^{\beta j} > 1$
  - The odds increase as x_j increases

Applied to our model:

- If variable `age` goes up by 1
  - The odds are multiplied by $e^{-0.009726}$
  - The odds are multiplied by 0.991

Let's practice!

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Logistic regression: predicting the probability of default

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An example with "age" and "home ownership"

```r
log_model_small <- glm(loan_status ~ age + home_ownership,  family = "binomial", data = training_set)
log_model_small
```

```
Call:  glm(formula = loan_status ~ age + home_ownership,
           family = "binomial", data = training_set)
Coefficients:
 (Intercept)             age         home_ownershipOTHER    home_ownershipOWN   home_ownershipRENT
 -1.886396            -0.009308            0.129776             -0.019384            0.158581
Degrees of Freedom: 19393 Total (i.e. Null);  19389 Residual
Null Deviance:    13680
Residual Deviance: 13660 AIC: 13670
```

$$P(\text{loan status} = 1|\text{age}, \text{home ownership}) = \frac{1}{1 + e^{-(\hat{\beta}_0 + \hat{\beta}_1 \text{age} + \hat{\beta}_2 \text{OTHER} + \hat{\beta}_3 \text{OWN} + \hat{\beta}_4 \text{RENT})}}$$

Test set example

$$P(\text{loan status} = 1|\text{age} = 33, \text{home ownership} = \text{RENT})$$

$$= \frac{1}{1 + e^{-(\hat{\beta}_0 + \hat{\beta}_1 33 + \hat{\beta}_2 0 + \hat{\beta}_3 0 + \hat{\beta}_4 1)}}$$

$$= \frac{1}{1 + e^{(-(1.886396 + (-0.009308) \times 33 + (0.158581) \times 1))}}$$

$$= 0.115579$$

```r
test_case <- as.data.frame(test_set[1,])
test_case
```

```
  loan_status loan_amnt  grade home_ownership annual_inc  age  emp_cat   ir_cat
1      0         5000      B          RENT         24000    33   0-15      8-11
```

```r
predict(log_model_small, newdata = test_case)
```

```
       1
-2.03499
```

$$-\hat{\beta}_0 + \hat{\beta}_1 age + \hat{\beta}_2 \text{OTHER} + \hat{\beta}_3 \text{OWN} + \hat{\beta}_4 \text{RENT}$$

```r
predict(log_model_small, newdata = test_case, type = "response")
```

```
        1
0.1155779
```

Let's practice!

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Evaluating the logistic regression model result

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Recap: model evaluation

```
test_set$loan_status    model_prediction
                 ...                ...
[8066,]            1                  1
[8067,]            0                  0
[8068,]            0                  0
[8069,]            0                  0
[8070,]            0                  0
[8071,]            0                  1
[8072,]            1                  0
[8073,]            1                  1
[8074,]            0                  0
[8075,]            0                  0
[8076,]            0                  0
[8077,]            1                  1
[8078,]            0                  0
[8079,]            0                  1
        ...                         ...
```

Actual loan status v. Model prediction

	No default (0)	Default (1)
No default (0)	8	2
Default (1)	1	3

In reality...

```
test_set$loan_status    model_prediction
                ...                 ....
[8066,]           1           0.09881492
[8067,]           0           0.09497852
[8068,]           0           0.21071984
[8069,]           0           0.04252119
[8070,]           0           0.21110838
[8071,]           0           0.08668856
[8072,]           1           0.11319341
[8073,]           1           0.16662207
[8074,]           0           0.15299176
[8075,]           0           0.08558058
[8076,]           0           0.08280463
[8077,]           1           0.11271048
[8078,]           0           0.08987446
[8079,]           0           0.08561631
          ....                     ....
```

Actual loan status v. Model prediction

	No default (0)	Default (1)
No default (0)	?	?
Default (1)	?	?

In reality...

```
test_set$loan_status    model_prediction
         ....                     ....
[8066,]            1          0.09881492
[8067,]            0          0.09497852
[8068,]            0          0.21071984
[8069,]            0          0.04252119
[8070,]            0          0.21110838
[8071,]            0          0.08668856
[8072,]            1          0.11319341
[8073,]            1          0.16662207
[8074,]            0          0.15299176
[8075,]            0          0.08558058
[8076,]            0          0.08280463
[8077,]            1          0.11271048
[8078,]            0          0.08987446
[8079,]            0          0.08561631
         ....                     ....
```

Cutoff or threshold value

- Between 0 and 1

Cutoff = 0.5

```
test_set$loan_status    model_prediction
         ...                        ...
  [8066,]          1                   0
  [8067,]          0                   0
  [8068,]          0                   0
  [8069,]          0                   0
  [8070,]          0                   0
  [8071,]          0                   0
  [8072,]          1                   0
  [8073,]          1                   0
  [8074,]          0                   0
  [8075,]          0                   0
  [8076,]          0                   0
  [8077,]          1                   0
  [8078,]          0                   0
  [8079,]          0                   0
        ...                         ...
```

Cutoff = 0.5

```
test_set$loan_status    model_prediction
       ...                      ...
  [8066,]          1                  0
  [8067,]          0                  0
  [8068,]          0                  0
  [8069,]          0                  0
  [8070,]          0                  0
  [8071,]          0                  0
  [8072,]          1                  0
  [8073,]          1                  0
  [8074,]          0                  0
  [8075,]          0                  0
  [8076,]          0                  0
  [8077,]          1                  0
  [8078,]          0                  0
  [8079,]          0                  0
      ...                       ...
```

Actual loan status v. Model prediction

	No default (0)	Default (1)
No default (0)	10	0
Default (1)	4	0

$$\text{Sensitivity} = 0/(4+0) = 0\%$$

$$\text{Accuracy} = 10/(10+4+0+0) = 71.4\%$$

Cutoff = 0.1

```
test_set$loan_status    model_prediction
        ...                    ...
  [8066,]          1                  0
  [8067,]          0                  0
  [8068,]          0                  0
  [8069,]          0                  0
  [8070,]          0                  0
  [8071,]          0                  0
  [8072,]          1                  0
  [8073,]          1                  0
  [8074,]          0                  0
  [8075,]          0                  0
  [8076,]          0                  0
  [8077,]          1                  0
  [8078,]          0                  0
  [8079,]          0                  0
       ...                     ...
```

Actual loan status v. Model prediction

	No default (0)	Default (1)
No default (0)	7	3
Default (1)	1	3

$$\text{Sensitivity} = 3/(3+1) = 75\%$$

$$\text{Accuracy} = 10/(10+4+0+0) = 71.4\%$$

Let's practice!

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Wrap-up and remarks

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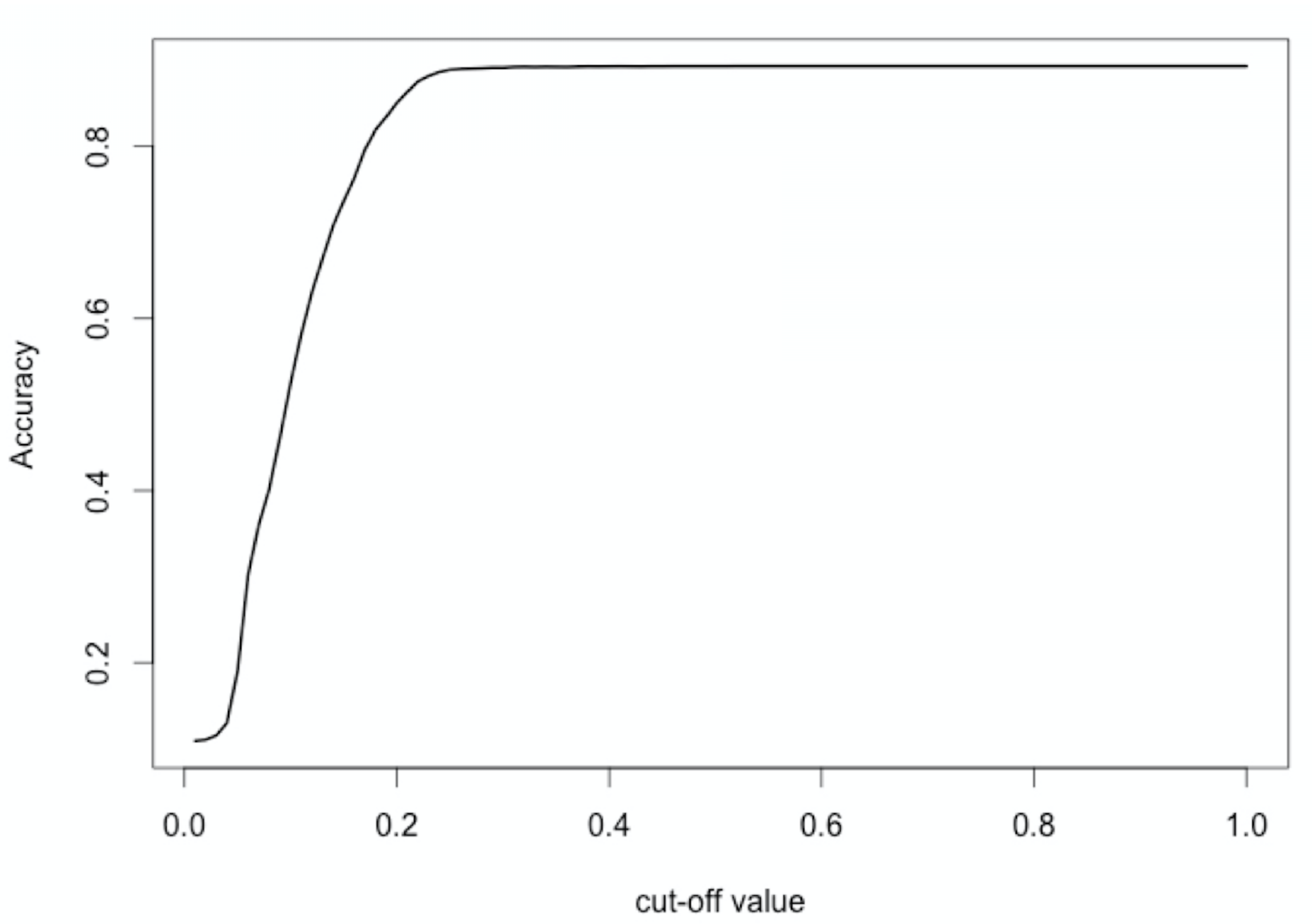
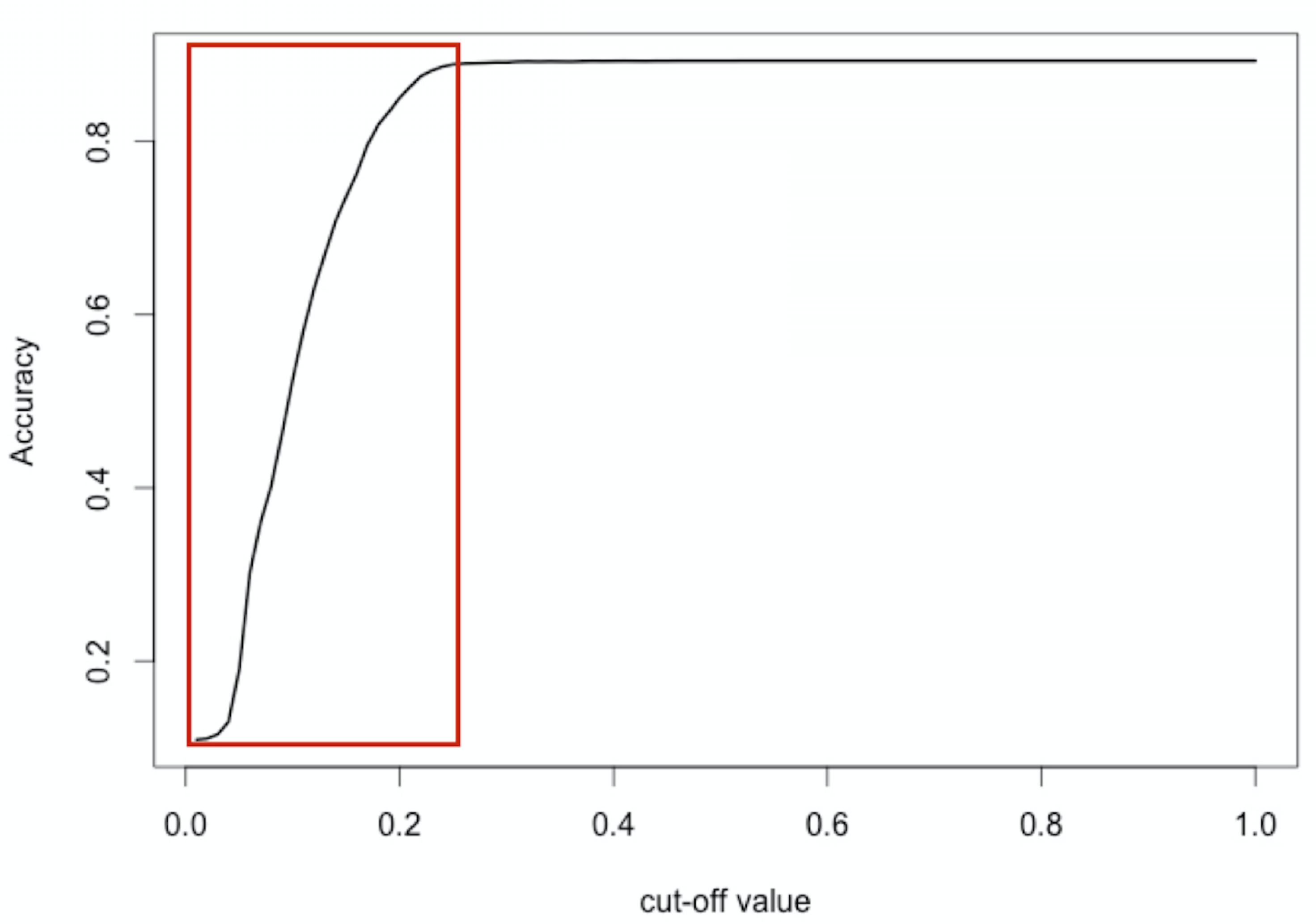
Best cut-off for accuracy?



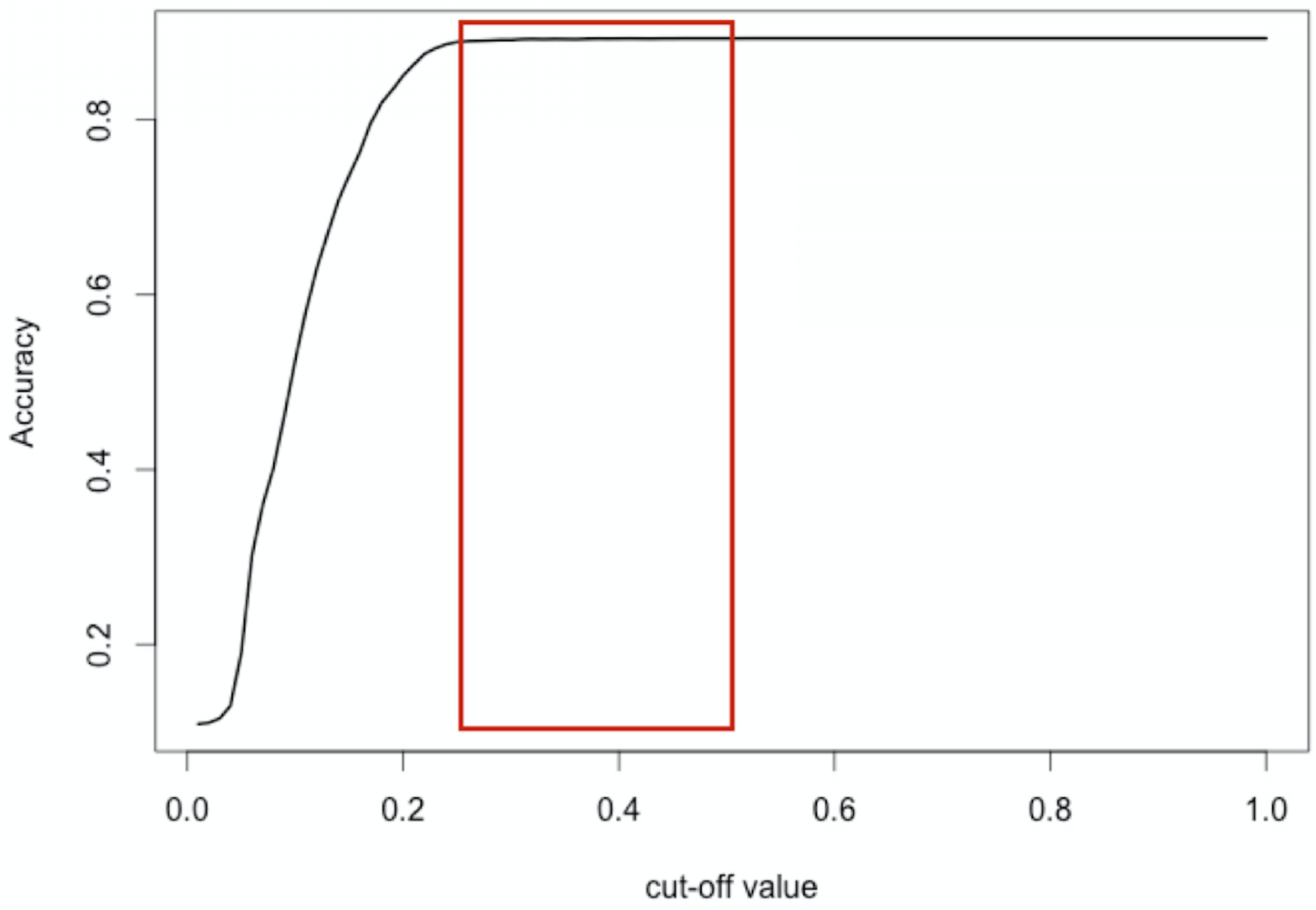
$$\text{Accuracy} = \frac{TP+TN}{TP+FP+TN+FN}$$

Best cut-off for accuracy?



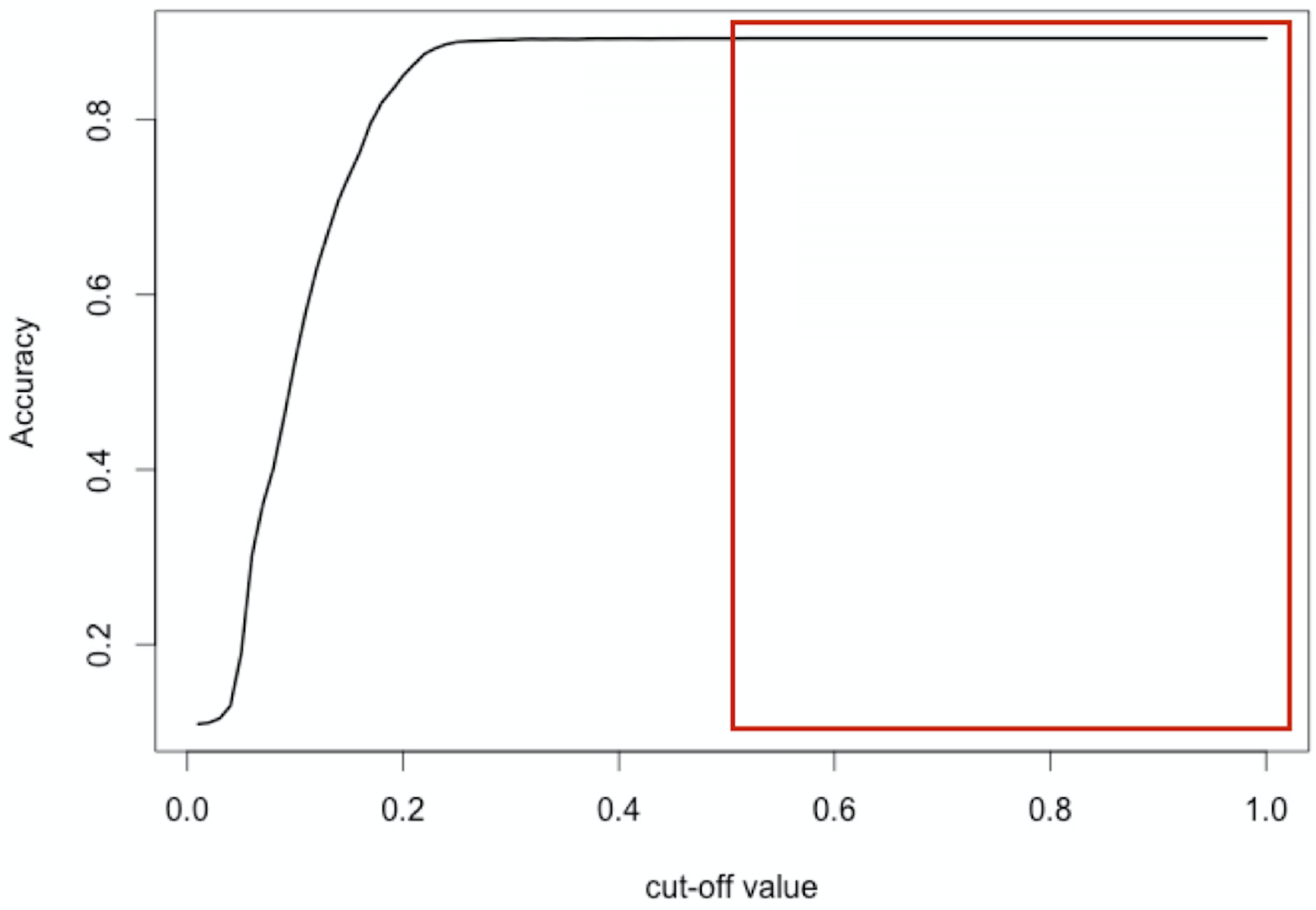
$$\text{Accuracy} = \frac{TP+TN}{TP+FP+TN+FN}$$

Best cut-off for accuracy?



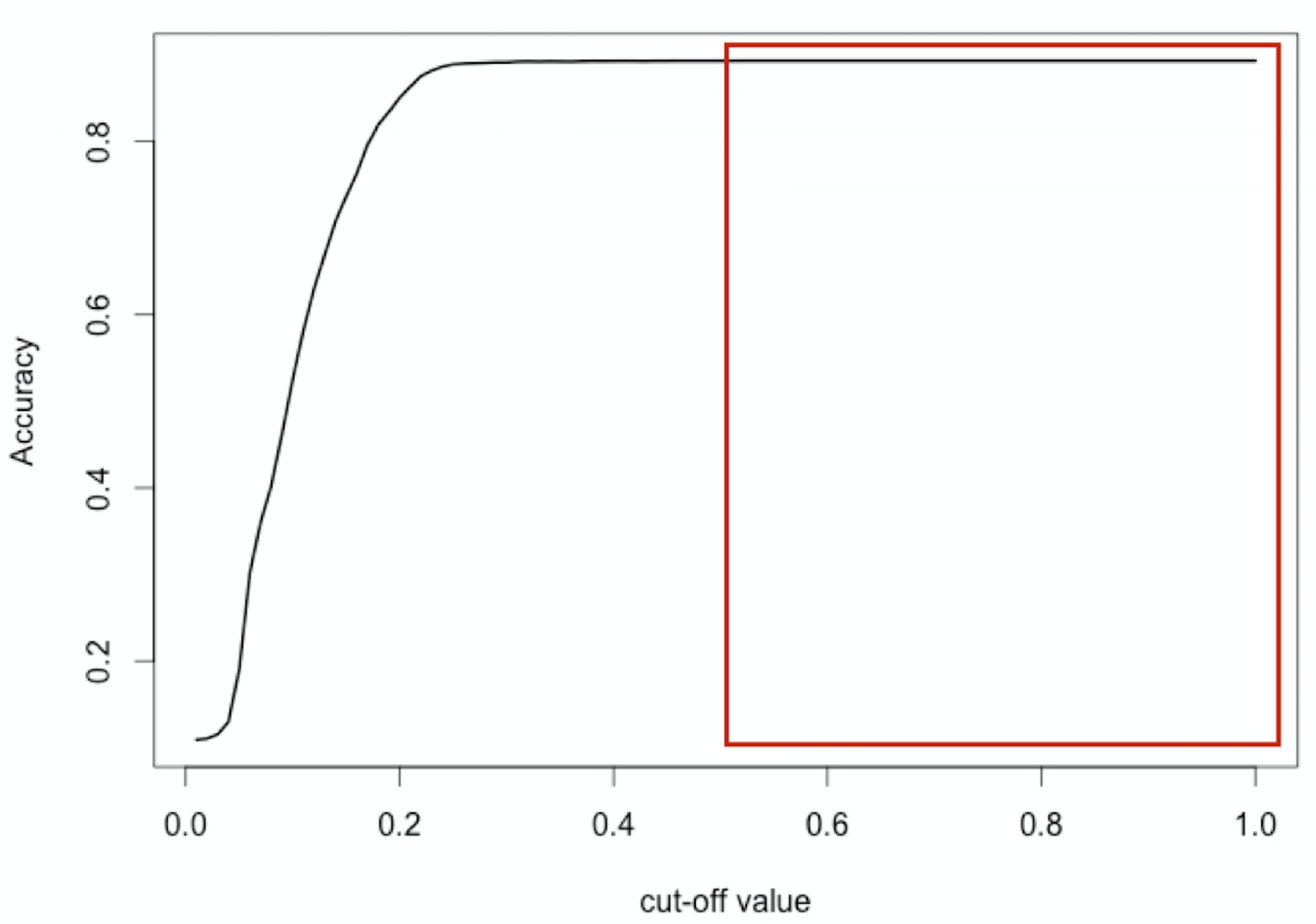
$$\text{Accuracy} = \frac{TP+TN}{TP+FP+TN+FN}$$

Best cut-off for accuracy?



$$\text{Accuracy} = \frac{TP+TN}{TP+FP+TN+FN}$$

Best cut-off for accuracy?

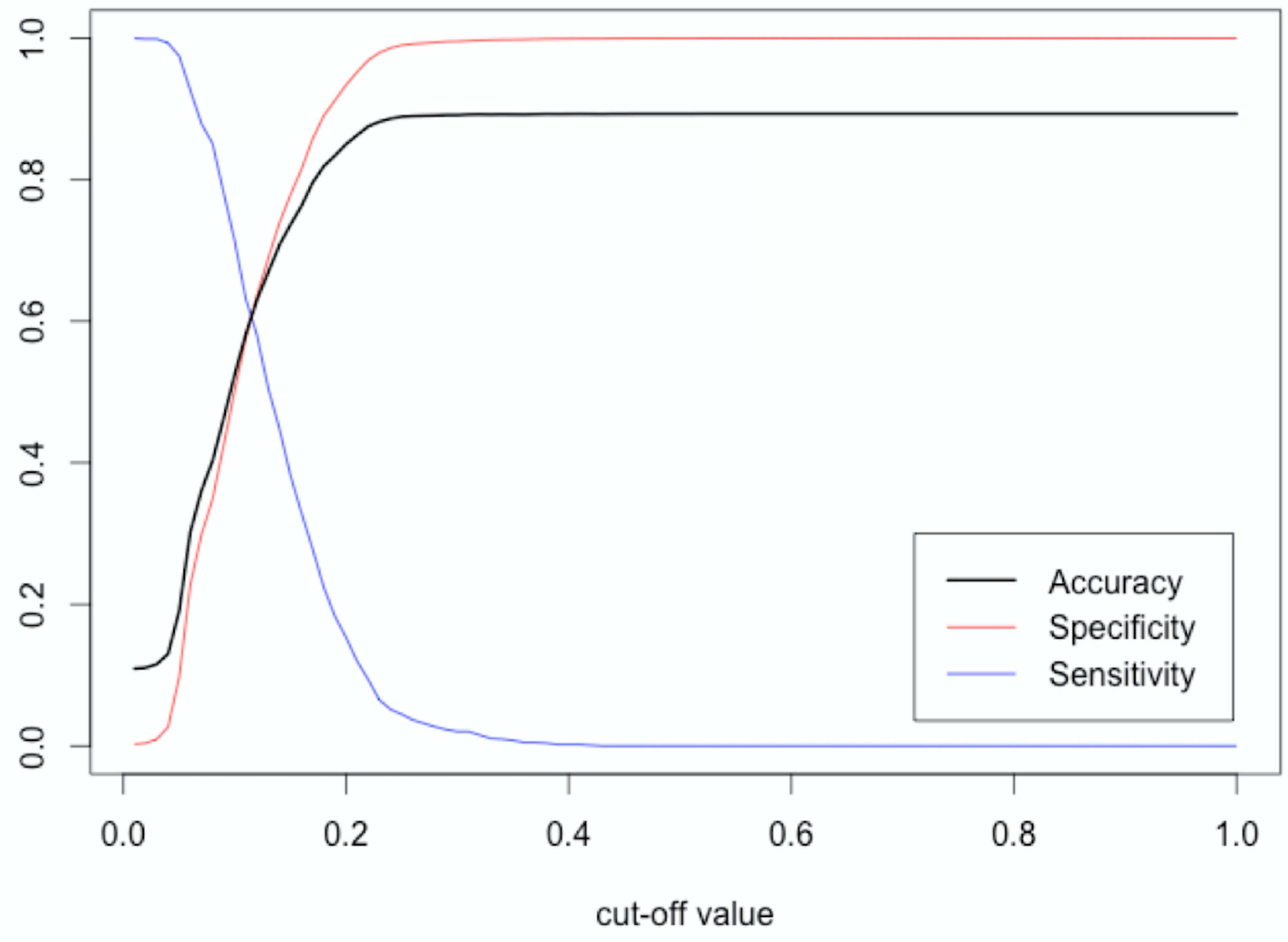


$$\text{Accuracy} = 89.31\%$$

$$\text{Actual defaults in test set} = 10.69\%$$

$$= (100 - 89.31)\%$$

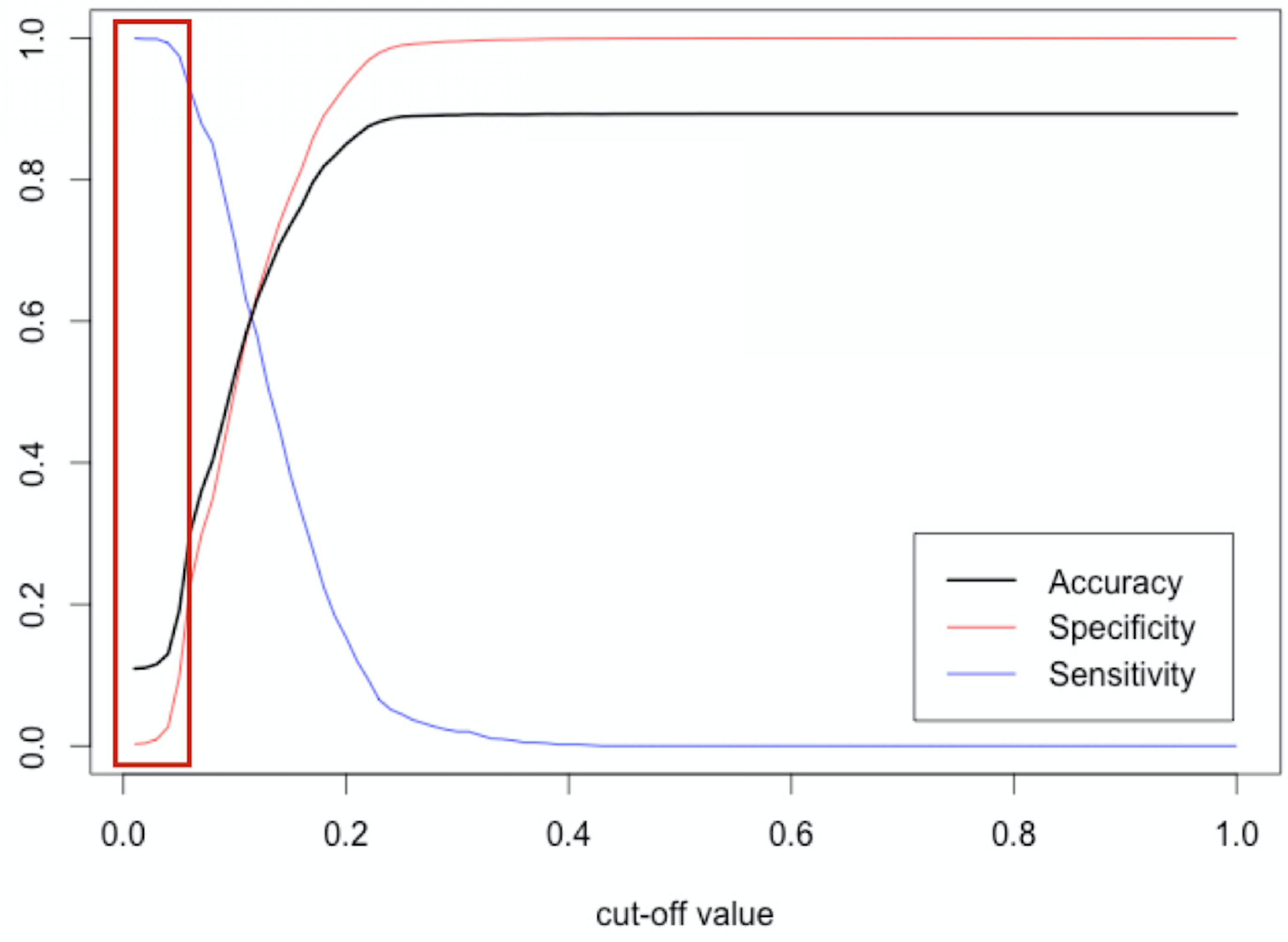
What about sensitivity or specificity?



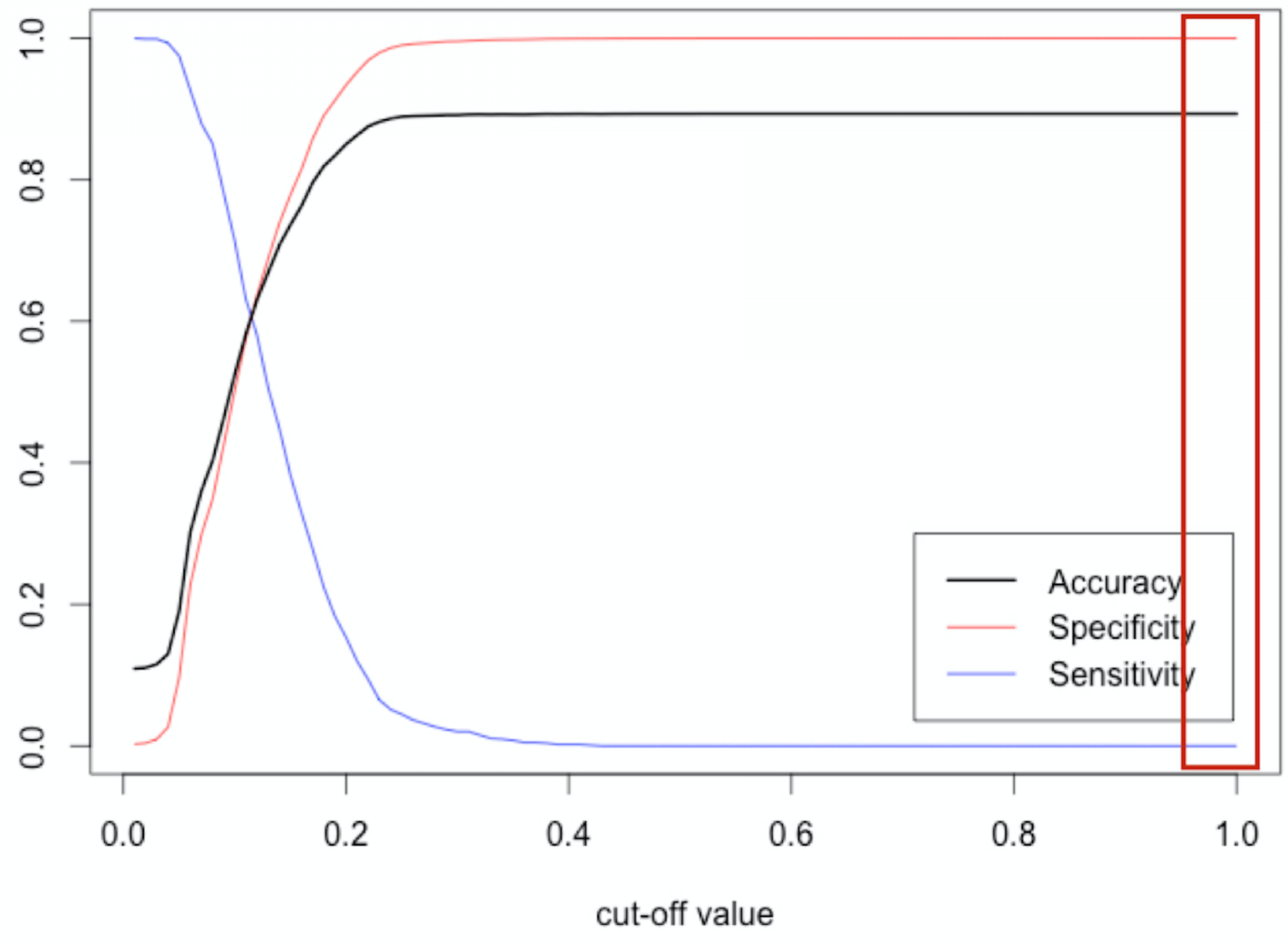
$$\text{Sensitivity} = 1037/(1037 + 0) = 100\%$$

$$\text{Specificity} = 0/(0 + 864) = 0\%$$

What about sensitivity or specificity?



What about sensitivity or specificity?



$$\text{Sensitivity} = 0/(0 + 1037) = 0\%$$

$$\text{Specificity} = 8640/(8640 + 0) = 100\%$$

About logistic regression...

```
log_model_full <- glm(loan_status ~ ., family = "binomial", data = training_set)
```

Is the same as:

```
log_model_full <- glm(loan_status ~ ., family = binomial(link = logit), data = training_set)
```

Recall:

$$P(\text{loan status} = 1 | x_1, ..., x_m) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + ... + \beta_m x_m)}}$$

```
log_model_full <- glm(loan_status ~ .,
                      family = binomial(link = probit),
                      data = training_set)

log_model_full <- glm(loan_status ~ .,
                      family = binomial(link = cloglog),
                      data = training_set)
```

- $\beta_j < 0$
  - The probability of default decreases as x_j increases
- $\beta_j > 0$
  - The probability of default increases as x_j increases

$$P(\text{loan status} = 1 | x_1, ..., x_m) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + ... + \beta_m x_m)}}$$

Let's practice!

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