Introduction

BOND VALUATION AND ANALYSIS IN R

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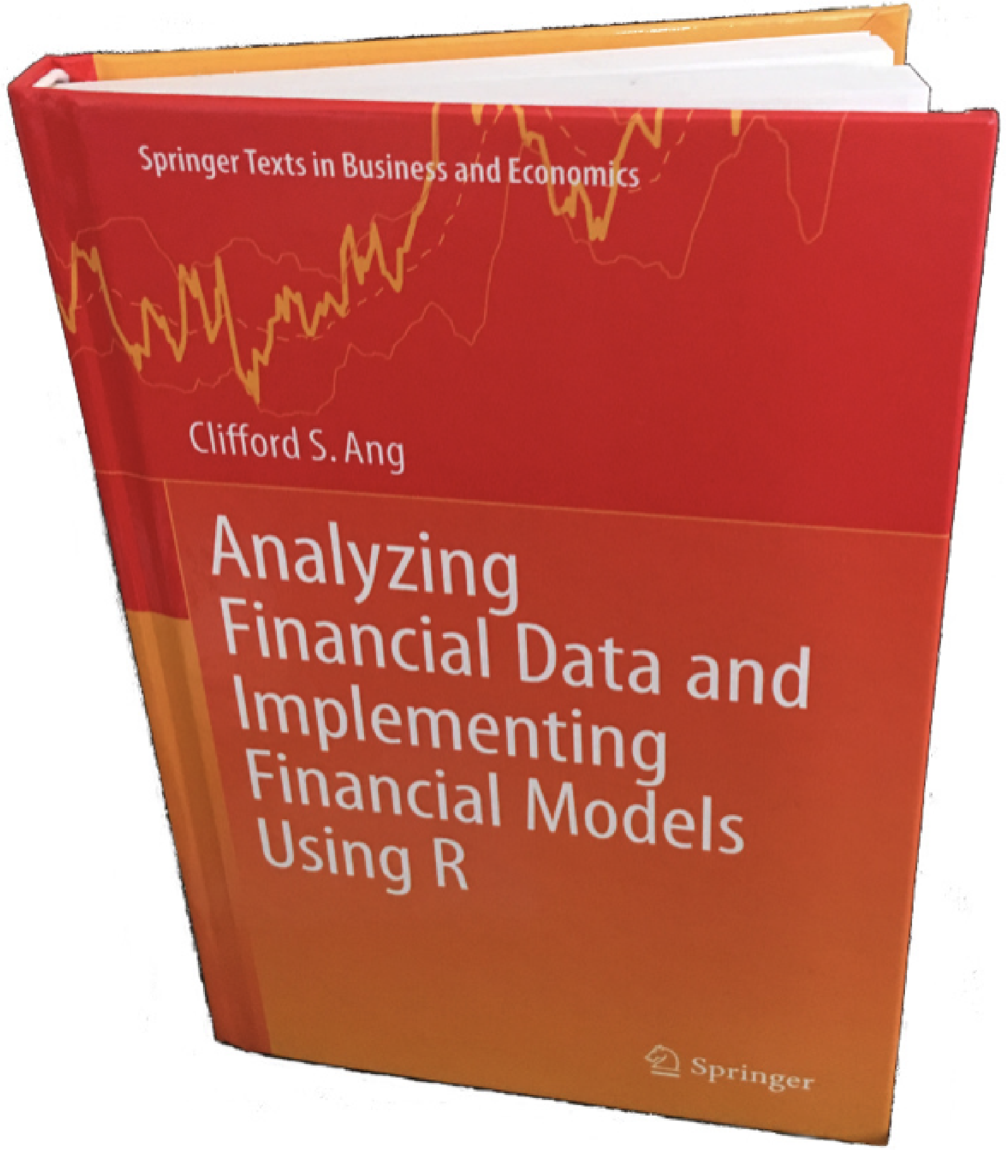
About me

- Advise clients on valuation and other financial issues primarily related to litigation
- Author of Analyzing Financial Data and Implementing Financial Models Using R



Bonds

- Debt instrument
- Repay borrowed amount + interest
- Focus on fundamental concepts of bond valuation

Characteristics of a bond

- **Issuer**: the entity that borrows the money
  - Corporations
  - Governments
  - Municipalities
- **Principal**: the amount borrowed
  - Also called *par value* or *face value*
- **Coupon rate:** the amount of interest the issuer agrees to pay
  - Annually, semi-annually, or quarterly
  - Fixed or floating rate

Characteristics of a bond

- **Maturity date:** data when principal amount is returned to investor
  - Some bonds do not mature
- **Embedded options**
  - Could affect bond's cash flow profile, i.e., can change amount and timing of cash flow
    - Callable bond: issuer can buyback bond earlier than maturity at a pre-agreed price
  - More complex analysis is required

Bonds in this course

- Annual coupons
- Fixed rate
- Fixed maturity
- No embedded options

Price vs. value

- We will use "price" and "value" interchangeably, but there are distinctions:
  - **Price**: amount paid to acquire asset
  - **Value**: how much the asset is worth
- For actively traded assets, price *may* be considered the best estimate of value

Let's practice!

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Time value of money

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Time value of money (TVM)

- $\$1$ today is worth more than $\$1$ tomorrow
- Suppose you won $10,000 in a game, what would you choose?
  - Receive the $10,000 today?
  - Receive the $10,000 one year from now?

Future value

- The future value is the value of $1 at some point in the future
- Prefer $1 today, so would have to be compensated to agree to receive the cash flow in the future
- Future value (`fv`) one and two years from now can be calculated as:

```
fv1 <- pv * (1 + r)
fv2 <- pv * (1 + r) * (1 + r)
```

`r` - interest rate

`pv` - present value

Present value

- Reverse logic of future values
- The value of $1 in the future is worth less today
- So you will be willing to take less than $1 today instead of waiting to receive $1 one or two years from now
- This can be calculated as follows:

```
pv <- fv1 / (1 + r)
pv <- fv2 / ((1 + r) * (1 + r))
```

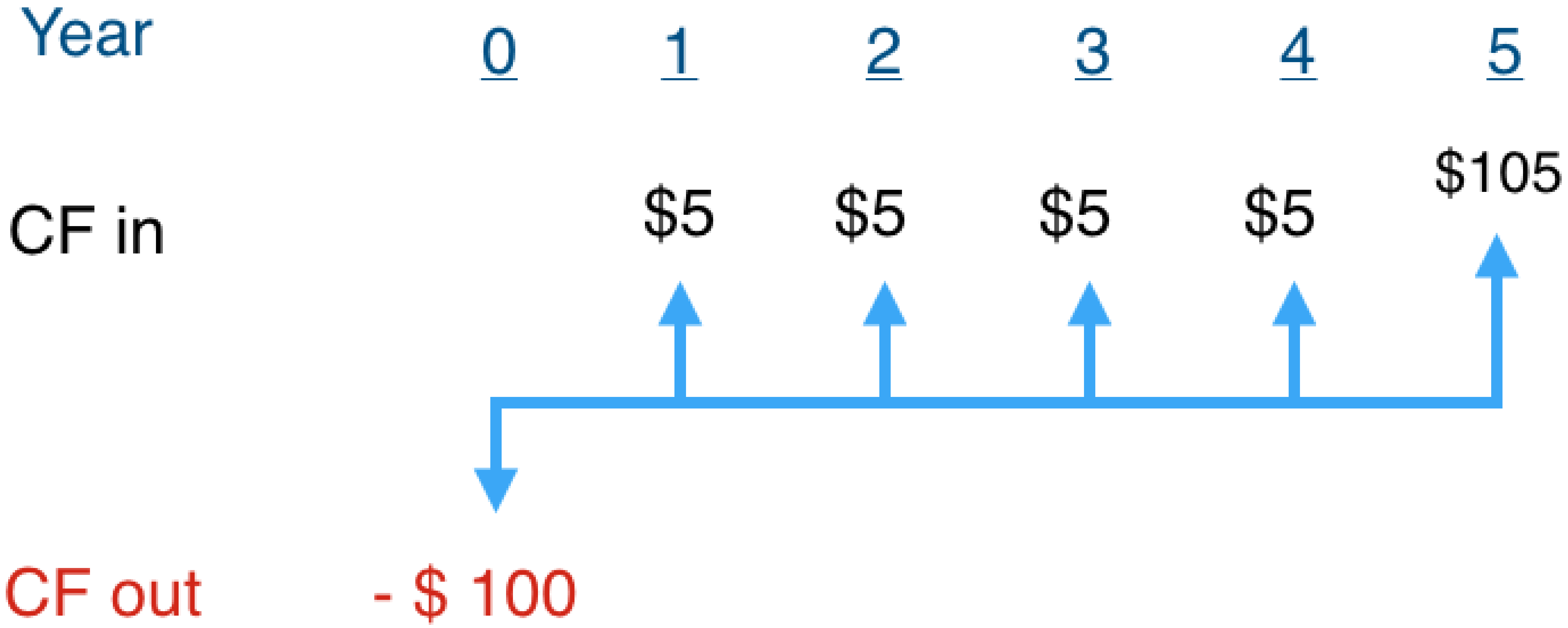
`r` - interest rate

`fv1` - future value 1 year from now, `fv2` - future value 2 years from now

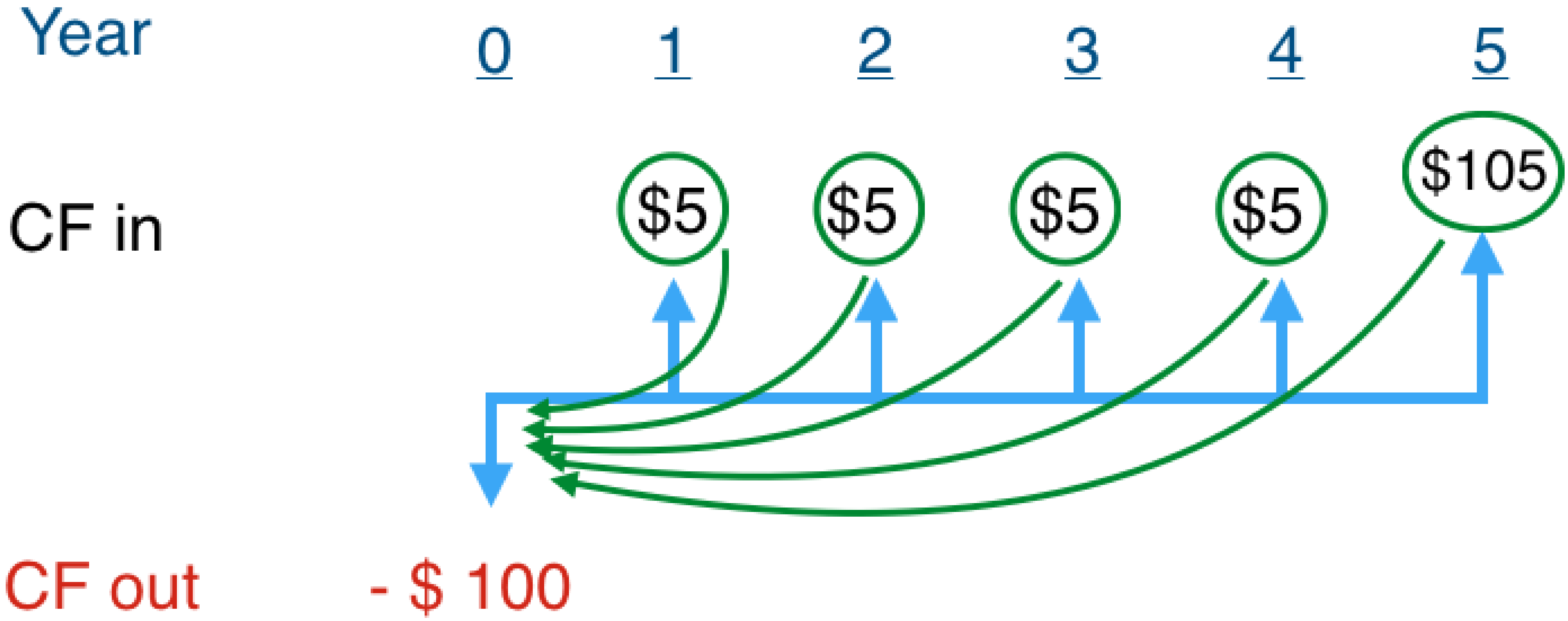
TVM applied to bonds

- We can apply this time value of money concept to bonds
- Example:
  - $100 par value, 5% coupon rate (= $5), 5 years to maturity
  - Price = $100 today

Bond investor's trade-off



Comparing cash flows



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Bond valuation

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Bond valuation

- Fixed annual coupon rate
- Fixed maturity date
- Option-free

Value of an asset

- The value of an asset = present value of expected future cash flows
- *Cash flows:* discounted at the appropriate risk-adjusted discount rate

$$V = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t}$$

- CF: cash flows
- y: discount rate

Laying out a bond's cash flows

- Prior to maturity, the investor receives coupon payments

$$\sum_{t=1}^{T-1} \frac{CF_t}{(1+y)^t}$$

- *NB: sum up to* $T-1$
- At maturity, the investor receives the last coupon payment and the par value

$$V = \sum_{t=1}^{T-1} \frac{CF_t}{(1+y)^t} + \frac{C_T + P}{(1+y)^T}$$

Creating a cash flow vector

$$V = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} + \frac{C_T + P}{(1+y)^T}$$

```
cf <- c(c1, c2, c3, c4, c5, ...)
```

Converting to dataframe

- So we can add additional columns, we need to convert the cash flow vector into a data frame
- Use the `data.frame()` command

```
cf <- data.frame(cf)
```

Creating a time index

- Each cash flow occurs at a certain period of time
  - The unit of the periods will be in years
- We create a variable that creates a time index

```
cf$t <- c(1, 2, 3, 4, 5, ...)
```

Calculating the PV factors

- To discount the cash flows, we need a "discount rate"
  - For bonds, the discount rate is called a "yield"
- We create a present value factor used for discounting

```
cf$pv_factor <- 1 / (1 + y)^cf$t


pv_factor <- 1 / (1 + .10)^2
pv_factor
```

```
0.8264463
```

PV of cash flows

- We calculate each cash flow's present value

```
cf$pv <- cf$cf * cf$pv_factor
```

- The sum of the present values of the bond's cash flow is equal to the bond's value

```
sum(cf$pv)
```

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Convert your code into a function

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Bond valuation function

- We will value many bonds in this course
- Steps described in prior chapter will be repeated
- We will create the `bondprc()` function to simplify calculations

Steps in bond valuation

- Generalize these inputs:
  - `p` for par value
  - `r` for coupon rate
  - `ttm` for time to maturity
  - `y` for yield
- We also make some of the code more generic

Steps in bond valuation

```
cf <- c(rep(p * r, ttm - 1), p * (1 + r))
```

- `rep(x, y)` - repeats y times the value of x
- `x = p * r` = coupon payment
- `y = ttm - 1` = bond's time to maturity minus one year
- `p * (1 + r)` = principal + final coupon payment

Steps in bond valuation

```
cf <- data.frame(cf)
```

- Convert to data frame so we can add variables to the data (same as last section)

```
cf$t <- as.numeric(rownames(cf))
```

- Create time index used for discounting
  - `rownames()` of `cf` vector is equal to 1, 2, 3, 4, until the `ttm` of bond
  - `as.numeric()` needed to ensure values are read as numbers

Steps in bond valuation

```
cf$pv_factor <- 1 / (1 + y)^cf$t
```

- Calculate PV factor

```
cf$pv <- cf$cf * cf$pv_factor
```

- Calculate PV of each cash flow

```
sum(cf$pv)
```

- Sum PV to arrive at bond's value

Wrap the code

- Create the `bondprc()` function
- This will take as inputs `p`, `r`, `ttm`, and `y`

```
bondprc <- function(p, r, ttm, y){
    cf <- c(rep(p * r, ttm - 1), p * (1 + r))
    cf <- data.frame(cf)
    cf$t <- as.numeric(rownames(cf))
    cf$pv_factor <- 1 / (1 + y)^cf$t
    cf$pv <- cf$cf * cf$pv_factor
    sum(cf$pv)
}
```

Let's practice!

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