$$\mu_k(B) = E^B_{\varepsilon,k}\, B\, F^B_{\varepsilon,k} = \left(J_k + [\varepsilon B]^{\bullet k}_+\right) B \left(J_k + [-\varepsilon B]^{k\bullet}_+\right) \quad \text{either } \varepsilon$$

Ex: $\begin{bmatrix} 0 & -2 & -1 \\ 3 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ $\quad k=2$, $\varepsilon = +$

$$\begin{bmatrix} 1 & & \\ & -1 & \\ & & 1 \end{bmatrix} B \begin{bmatrix} 1 & & \\ & -1 & \\ & & 1 \end{bmatrix} = \begin{bmatrix} 1 & & \\ & -1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 0 & +2 & -1 \\ 3 & 0 & 1 \\ 1 & +1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & -1 \\ -3 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$\varepsilon = -$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} B \begin{bmatrix} 1 & 0 & 0 \\ 3 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -6 & 2 & -3 \\ 3 & 0 & 1 \\ -2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & -1 \\ -3 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\left(G_t^{B_0,t_0}\right)^{-1} B_0\, C_t^{B_0,t_0} = B_t \quad \xrightarrow{\text{So}} \quad G_t^{B_0,t_0} B_t = B_0\, C_t^{B_0,t_0}$$

Choose λ_t in positive chamber and mutate $k_1 \cdots k_\ell$ to get the initial λ.
Signs ε_i come from λ and $\eta(\lambda)\ldots$

$$t_1 \xrightarrow{k_1} t_2 \xrightarrow{k_2} \cdots \xrightarrow{k_{\ell-1}} t_\ell \xrightarrow{k_\ell} t \qquad\qquad E^i_{\varepsilon_i} = E^{B_{t_i}}_{k_i,\varepsilon_i}$$

$F^i_{\varepsilon_i}$ = same

$$A = E^{\ell-1}_{\varepsilon_{\ell-1}} E^{\ell-2}_{\varepsilon_{\ell-2}} \cdots E^1_{\varepsilon_1} B_1 = B_\ell\, F^{\ell-1}_{\varepsilon_{\ell-1}} \cdots F^1_{\varepsilon_1}$$

Want: posspan $\left(\text{columns of } E^\ell_{-\varepsilon_\ell} A : \text{sign of their } k_i\text{-entry is } -\varepsilon_\ell\right)$

$$\subseteq \text{pos span}\left(\text{columns of } E^\ell_{\varepsilon_\ell} A\right)$$

Know: $\operatorname{posspan}\left(\text{columns of } E^{\ell}_{-\varepsilon_\ell} B_\ell : \text{sign of their } k_\ell\text{-entry is } -\varepsilon_\ell\right)$

$\subseteq \operatorname{posspan}\left(\text{columns of } E^{\ell}_{\varepsilon_\ell} B_\ell\right)$

Could this possibly hold up under applying $F^{\ell-1}_{\varepsilon_{\ell-1}} \cdots F^{1}_{\varepsilon_1}$ on the right?

<u>Ex:</u>

$$\underset{t_1}{\overset{B_1}{\begin{bmatrix} 0 & 2 & -1 \\ -1 & 0 & 1 \\ 1 & -2 & 0 \end{bmatrix}}} \xrightarrow{2} \underset{t_2}{\overset{B_2}{\begin{bmatrix} 0 & -2 & 1 \\ 1 & 0 & -1 \\ -1 & 2 & 0 \end{bmatrix}}} \xrightarrow{1} \underset{t_3}{\overset{B_3}{\begin{bmatrix} 0 & 2 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}}}$$

$k_1 = 2 \quad k_2 = 1 \quad k_3 = 2$

$\varepsilon_1 = + \quad \varepsilon_2 = - \quad \varepsilon_3 = -$

$$E^1_{2,+} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E^2_{1-} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad E^3_{2-} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E^3_{2+} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F^1_{2,+} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad F^2_{1-} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = E^2_{1-} E^1_{2+} B_1 = E^2_{1-} \begin{bmatrix} -2 & 2 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -1 \\ 1 & 0 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$E^3_{2-} A = \begin{bmatrix} 2 & -2 & -1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{columns with } 2^{\text{nd}} \text{ entry} \leq 0 \quad \begin{bmatrix} 2 & -2 \\ -1 & 0 \\ -1 & 0 \end{bmatrix}$$

$$E^3_{2+} A = \begin{bmatrix} 4 & -2 & -3 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{columns with } 2^{\text{nd}} \text{ entry} \geq 0 \quad \begin{bmatrix} -2 & -3 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$

$$E^3_{2-} A F = \begin{bmatrix} 2-a & -2-b & 1 \\ -1+a & b & -1 \\ -1+a & b & -1 \end{bmatrix} \quad \begin{array}{l} a \geq 1 : 3^{\text{rd}} \text{ col} \\ a \leq 1 \quad 1^{\text{st}}, 3^{\text{rd}} \text{ col} \end{array}$$

$$E^3_{2+} A F = \begin{bmatrix} 4-3a & -2-3b & 3 \\ -1+a & b & -1 \\ -1+a & b & -1 \end{bmatrix} \quad \begin{array}{l} a \geq 1 : 1^{\text{st}}, 2^{\text{nd}} \\ a \leq 1 : 2^{\text{nd}} \end{array}$$

$$\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \quad \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -2b \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -2a \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -a \\ a \\ a \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ -1 \\ -4 \end{bmatrix}$$

Can't get 1st column!