1. Motivating example: accordion complexes of dissections

Let P be a convex polygon. We call *diagonals* of P the segments connecting two non-consecutive vertices of P. A *dissection* of P is a set D of non-crossing diagonals. It dissects the polygon into *cells*. We denote by Q(D) the quiver with relations whose vertices are the diagonals of D, whose arrows connect any two counterclockwise consecutive edges of a cell of D, and whose relations are given by triples of counterclockwise consecutive edges of a cell of D. See Figure **??** for an example.

We now consider $2m$ points on the unit circle alternatly colored black and white, and let $\mathrm{P}_\circ$ (resp. $\mathrm{P}_\bullet$) denote the convex hull of the white (resp. black) points. We fix an arbitrary reference dissection $\mathrm{D}_\circ$ of $\mathrm{P}_\circ$. A solid diagonal $\delta_\bullet$ of $\mathrm{P}_\bullet$ is a *$\mathrm{D}_\circ$-accordion diagonal* if it does not enter and exit any cell of $\mathrm{D}_\circ$ crossing two non-consecutive of its edges. In other words the diagonals of $\mathrm{D}_\circ$ crossed by $\delta_\bullet$ form an accordion. A *$\mathrm{D}_\circ$-accordion dissection* is a set of non-crossing $\mathrm{D}_\circ$-accordion diagonals. We call *$\mathrm{D}_\circ$-accordion complex* the simplicial complex $\mathcal{AC}(\mathrm{D}_\circ)$ of $\mathrm{D}_\circ$-accordion dissections.

For a diagonal $\delta_\circ$ of $\mathrm{D}_\circ$ and a $\mathrm{D}_\circ$-accordion diagonal $\delta_\bullet$ intersecting $\delta_\circ$, we consider the three edges (including $\delta_\circ$) crossed by $\delta_\bullet$ in the two cells of $\mathrm{D}_\circ$ containing $\delta_\circ$. We define $\varepsilon\big(\delta_\circ \in \mathrm{D}_\circ \mid \delta_\bullet\big)$ to be 1, -1, or 0 depending on whether these three edges form a Z, a $\mathsf{\Sigma}$, or a V. The *$\mathbf{g}$-vector* of $\delta_\bullet$ with respect to $\mathrm{D}_\circ$ is the vector $\mathbf{g}(\mathrm{D}_\circ \,|\, \delta_\bullet) \in \mathbb{R}^{\mathrm{D}_\circ}$ whose $\delta_\circ$-coordinate is $\varepsilon\big(\delta_\circ \in \mathrm{D}_\circ \mid \delta_\bullet\big)$. For example, ...

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Example 1. When the reference dissection $\mathrm{D}_\circ$ is a triangulation of $\mathrm{P}_\circ$, any diagonal of $\mathrm{P}_\bullet$ is a $\mathrm{D}_\circ$-accordion diagonal. The $\mathrm{D}_\circ$-accordion complex is thus an n-dimensional associahedron (of type A), where $n = m - 3$. In this case, it is known that the $\mathrm{D}_\circ$-accordion complex is isomorphic to the 2-term silting complex of the quiver $\mathrm{Q}(\mathrm{D}_\circ)$ of the triangulation $\mathrm{D}_\circ$ (see Section **??** for definitions).

The initial motivation of this paper was to prove the following extension of Example 1.

Theorem 2. *For any reference dissection $\mathrm{D}_\circ$, the $\mathrm{D}_\circ$-accordion complex is isomorphic to the 2-term silting complex of the quiver $\mathrm{Q}(\mathrm{D}_\circ)$.*

One possible approach to Theorem 2 would be to provide an explicit bijective map between $\mathrm{D}_\circ$-accordion diagonals and 2-term projective complexes for $\mathrm{Q}(\mathrm{D}_\circ)$. Such a map is easy to guess using $\mathbf{g}$-vectors, but the proof that this map is actually a bijection and that it preserves compatibility is intricated. This approach was developed in the more general context of non-kissing complexes of gentle quivers in [**?**]. In this paper, we use an alternative simpler strategy to obtain Theorem 2, understanding accordion complexes as certain subcomplexes of the associahedron.

For that, consider two nested dissections $\mathrm{D}_\circ \subset \mathrm{D}'_\circ$. Observe that any $\mathrm{D}_\circ$-accordion diagonal is a $\mathrm{D}'_\circ$-accordion diagonal. Conversely a $\mathrm{D}'_\circ$-accordion diagonal $\delta_\bullet$ is a $\mathrm{D}_\circ$-accordion diagonal if and only if it does not cross any diagonal of $\mathrm{D}'_\circ \smallsetminus \mathrm{D}_\circ$ as a Z or a $\mathsf{\Sigma}$, that is if and only if its $\mathbf{g}$-vector $\mathbf{g}(\mathrm{D}'_\circ \,|\, \delta_\bullet)$ belongs to the subspace spanned by elements in $\mathrm{D}_\circ$. This observation shows the following statement.

Theorem 3 ([**?**])**.** *For any two nested dissections $\mathrm{D}_\circ \subset \mathrm{D}'_\circ$, the accordion complex $\mathcal{AC}(\mathrm{D}_\circ)$ is isomorphic to the subcomplex of $\mathcal{AC}(\mathrm{D}'_\circ)$ induced by $\mathrm{D}'_\circ$-accordion diagonals $\delta_\bullet$ whose $\mathbf{g}$-vector $\mathbf{g}(\mathrm{D}'_\circ \,|\, \delta_\bullet)$ lie in the coordinate subspace spanned by elements in $\mathrm{D}_\circ$.*

Consider now any quiver with relations Q and any subset J of vertices of Q. We call *shortcut quiver* the quiver with relations Q/J whose vertices are the vertices of Q not in J, whose arrows are the paths in Q with internal vertices in J, and whose relations are inherited from those of Q. For example, quivers of subdissections are shortcut quivers: if $\mathrm{D}_\circ \subset \mathrm{D}'_\circ$, then $\mathrm{Q}(\mathrm{D}_\circ) = \mathrm{Q}(\mathrm{D}'_\circ)/(\mathrm{D}'_\circ \smallsetminus \mathrm{D}_\circ)$. This paper proves the following statement.

Theorem 4. *For any quiver with relations Q and any subset J of vertices of Q, the 2-term silting complex $\mathcal{SC}(\mathrm{Q}/J)$ for the shortcut quiver Q/J is isomorphic to the subcomplex of the 2-term silting complex $\mathcal{SC}(\mathrm{Q})$ induced by 2-term projective complexes whose $\mathbf{g}$-vector lie in the coordinate subspace spanned by vertices not in J.*

Combining Theorems 3 and 4 together with Example 1 proves Theorem 2.

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References

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