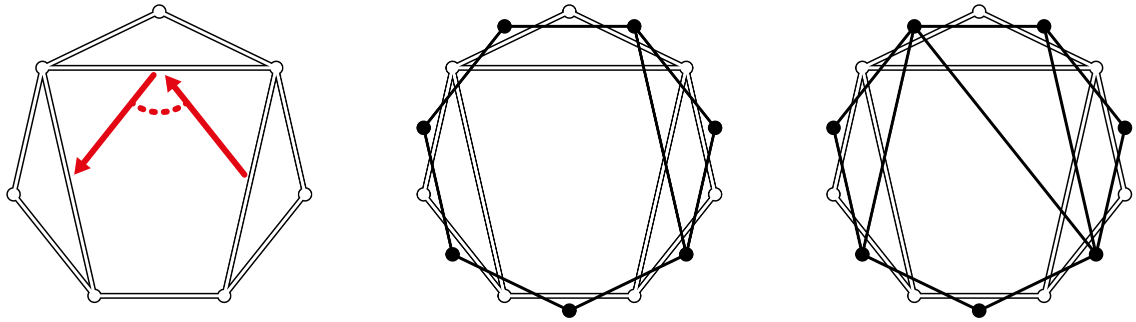

FIGURE 1. A dissection $\mathrm{D}_\circ$ with its quiver $\mathrm{Q}(\mathrm{D}_\circ)$ (left), a $\mathrm{D}_\circ$-accordion diagonal (middle) and a $\mathrm{D}_\circ$-accordion dissection (right).

## 1. Motivating example: accordion complexes of dissections

Let P be a convex polygon. We call *diagonals* of P the segments connecting two non-consecutive vertices of P. A *dissection* of P is a set D of non-crossing diagonals. It dissects the polygon into *cells*. We denote by Q(D) the quiver with relations whose vertices are the diagonals of D, whose arrows connect any two counterclockwise consecutive edges of a cell of D, and whose relations are given by triples of counterclockwise consecutive edges of a cell of D. See Figure 1 for an example.

We now consider $2m$ points on the unit circle alternately colored black and white, and let $\mathrm{P}_\circ$ (resp. $\mathrm{P}_\bullet$) denote the convex hull of the white (resp. black) points. We fix an arbitrary reference dissection $\mathrm{D}_\circ$ of $\mathrm{P}_\circ$. A diagonal $\delta_\bullet$ of $\mathrm{P}_\bullet$ is a *$\mathrm{D}_\circ$-accordion diagonal* if it crosses either none or two consecutive edges of any cell of $\mathrm{D}_\circ$. In other words the diagonals of $\mathrm{D}_\circ$ crossed by $\delta_\bullet$ together with the two boundary edges of $\mathrm{P}_\circ$ crossed by $\delta_\bullet$ form an accordion. A *$\mathrm{D}_\circ$-accordion dissection* is a set of non-crossing $\mathrm{D}_\circ$-accordion diagonals. See Figure 1 for an example. We call *$\mathrm{D}_\circ$-accordion complex* the simplicial complex $\mathcal{AC}(\mathrm{D}_\circ)$ of $\mathrm{D}_\circ$-accordion dissections. This complex appeared in recent works of F. Chapoton [Cha00], A. Garver and T. McConville [GM16], and T. Manneville and V. Pilaud [MP17].

For a diagonal $\delta_\circ$ of $\mathrm{D}_\circ$ and a $\mathrm{D}_\circ$-accordion diagonal $\delta_\bullet$ intersecting $\delta_\circ$, we consider the three edges (including $\delta_\circ$) crossed by $\delta_\bullet$ in the two cells of $\mathrm{D}_\circ$ containing $\delta_\circ$. We define $\varepsilon\big(\delta_\circ \in \mathrm{D}_\circ \mid \delta_\bullet\big)$ to be $1$, $-1$, or $0$ depending on whether these three edges form a Z, a S, or a V. The *$\mathbf{g}$-vector* of $\delta_\bullet$ with respect to $\mathrm{D}_\circ$ is the vector $\mathbf{g}(\mathrm{D}_\circ \,|\, \delta_\bullet) \in \mathbb{R}^{\mathrm{D}_\circ}$ whose $\delta_\circ$-coordinate is $\varepsilon\big(\delta_\circ \in \mathrm{D}_\circ \mid \delta_\bullet\big)$.

**Example 1.** When the reference dissection $\mathrm{D}_\circ$ is a triangulation of $\mathrm{P}_\circ$, any diagonal of $\mathrm{P}_\bullet$ is a $\mathrm{D}_\circ$-accordion diagonal. The $\mathrm{D}_\circ$-accordion complex is thus an $n$-dimensional associahedron (of type $A$), where $n = m - 3$. In this case, it is known [**?**] that the $\mathrm{D}_\circ$-accordion complex is isomorphic to the 2-term silting complex of the quiver $\mathrm{Q}(\mathrm{D}_\circ)$ of the triangulation $\mathrm{D}_\circ$ (see Section **??** for definitions). The isomorphism sends a diagonal of $\mathrm{P}_\bullet$ to the 2-term projective complex with the same $\mathbf{g}$-vector. See Figure 2 for an illustration.

The initial motivation of this paper was to prove the following extension of Example 1, suggested by the similarity of the two pictures of Figure 3 pointed out to us by F. Chapoton.

**Theorem 2.** *For any reference dissection* $\mathrm{D}_\circ$*, the* $\mathrm{D}_\circ$*-accordion complex is isomorphic to the 2-term silting complex of the quiver* $\mathrm{Q}(\mathrm{D}_\circ)$*.*

One possible approach to Theorem 2 would be to provide an explicit bijective map between $\mathrm{D}_\circ$-accordion diagonals and 2-term projective complexes for $\mathrm{Q}(\mathrm{D}_\circ)$. Such a map is easy to guess using $\mathbf{g}$-vectors, but the proof that it is actually a bijection and that it preserves compatibility is intricated. This approach was developed in the more general context of non-kissing complexes of gentle quivers in [YP17]. In this paper, we use an alternative simpler strategy to obtain Theorem 2, understanding accordion complexes as certain subcomplexes of the associahedron.

For that, consider two nested dissections $\mathrm{D}_\circ \subset \mathrm{D}'_\circ$. Observe that any $\mathrm{D}_\circ$-accordion diagonal is a $\mathrm{D}'_\circ$-accordion diagonal. Conversely a $\mathrm{D}'_\circ$-accordion diagonal $\delta_\bullet$ is a $\mathrm{D}_\circ$-accordion diagonal if

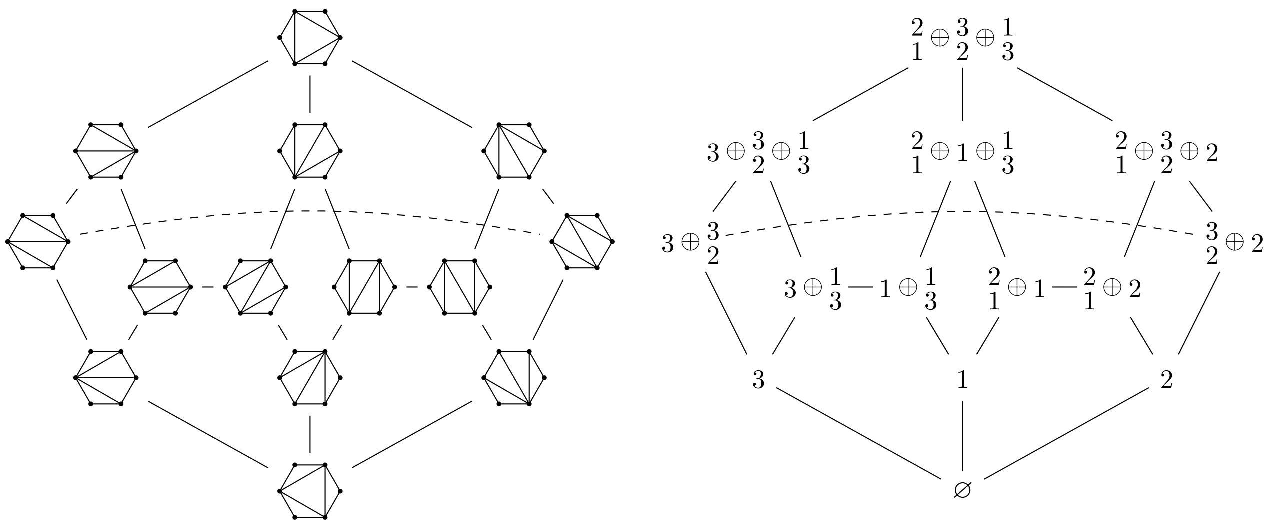


FIGURE 2. The associahedron (left) and the 2-term silting complex of an oriented triangle (right).

and only if it does not cross any diagonal $\delta'_\circ$ of $\mathrm{D}'_\circ \smallsetminus \mathrm{D}_\circ$ as a $\mathsf{Z}$ or a $\Sigma$, that is if and only if the $\delta'_\circ$-coordinate of its $\mathbf{g}$-vector $\mathbf{g}(\mathrm{D}'_\circ \,|\, \delta_\bullet)$ vanishes for any $\delta'_\circ \in \mathrm{D}'_\circ \smallsetminus \mathrm{D}_\circ$. This observation shows the following statement.

**Theorem 3** ([MP17]). *For any two nested dissections $\mathrm{D}_\circ \subset \mathrm{D}'_\circ$, the accordion complex $\mathcal{AC}(\mathrm{D}_\circ)$ is isomorphic to the subcomplex of $\mathcal{AC}(\mathrm{D}'_\circ)$ induced by $\mathrm{D}'_\circ$-accordion diagonals $\delta_\bullet$ whose $\mathbf{g}$-vector $\mathbf{g}(\mathrm{D}'_\circ \,|\, \delta_\bullet)$ lie in the coordinate subspace spanned by elements in $\mathrm{D}_\circ$.*

Consider now any quiver with relations Q and any subset $J$ of vertices of Q. We call *shortcut quiver* the quiver with relations $\mathrm{Q}/J$ whose vertices are the vertices of Q not in $J$, whose arrows are the paths in Q with internal vertices in $J$, and whose relations are inherited from those of Q. For example, quivers of subdissections are shortcut quivers: if $\mathrm{D}_\circ \subset \mathrm{D}'_\circ$, then $\mathrm{Q}(\mathrm{D}_\circ) = \mathrm{Q}(\mathrm{D}'_\circ)/(\mathrm{D}'_\circ \smallsetminus \mathrm{D}_\circ)$. The main result of this paper is the following statement.

**Theorem 4.** *For any quiver with relations Q and any subset $J$ of vertices of Q, the 2-term silting complex $\mathcal{SC}(\mathrm{Q}/J)$ for the shortcut quiver $\mathrm{Q}/J$ is isomorphic to the subcomplex of the 2-term silting complex $\mathcal{SC}(\mathrm{Q})$ induced by 2-term projective complexes whose $\mathbf{g}$-vector lie in the coordinate subspace spanned by vertices not in $J$.*

Combining Theorems 3 and 4 together with Example 1 proves Theorem 2.

We conclude this section with a geometric interpretation of the common phenomenon described in Theorems 3 and 4. For a $\mathrm{D}_\circ$-accordion dissection $\mathrm{D}_\bullet$, denote by $\mathbb{R}_{\ge 0}\,\mathbf{g}(\mathrm{D}_\circ \,|\, \mathrm{D}_\bullet)$ the polyhedral cone generated by the set of $\mathbf{g}$-vectors $\mathbf{g}(\mathrm{D}_\circ \,|\, \mathrm{D}_\bullet) := \{\mathbf{g}(\mathrm{D}_\circ \,|\, \delta_\bullet) \mid \delta_\bullet \in \mathrm{D}_\bullet\}$. The collection $\mathcal{F}^{\mathbf{g}}(\mathrm{D}_\circ)$ of cones $\mathbb{R}_{\ge 0}\mathbf{g}(\mathrm{D}_\circ \,|\, \mathrm{D}_\bullet)$ for all $\mathrm{D}_\circ$-accordion dissections $\mathrm{D}_\bullet$ is a complete simplicial fan called ***g**-vector fan* of $\mathrm{D}_\circ$ [MP17]. The crucial feature of this fan is that no coordinate hyperplane meets the interior of any of its maximal cones. This is often referred to as the *sign-coherence property* of $\mathbf{g}$-vectors. It implies that for any two nested dissections $\mathrm{D}_\circ \subset \mathrm{D}'_\circ$, the section of $\mathcal{F}^{\mathbf{g}}(\mathrm{D}'_\circ)$ with the coordinate subspace $\mathbb{R}^{\mathrm{D}_\circ}$ is a subfan $\mathcal{F}^{\mathbf{g}}(\mathrm{D}'_\circ)$. The content of Theorem 3 is that this subfan is the $\mathbf{g}$-vector fan $\mathcal{F}^{\mathbf{g}}(\mathrm{D}_\circ)$. A similar statement holds for Theorem 4.

## Acknoledgements

We are grateful to F. Chapoton for pointing out to us the similarity between the two pictures of Figure 3 which gave us the motivation for the present paper.

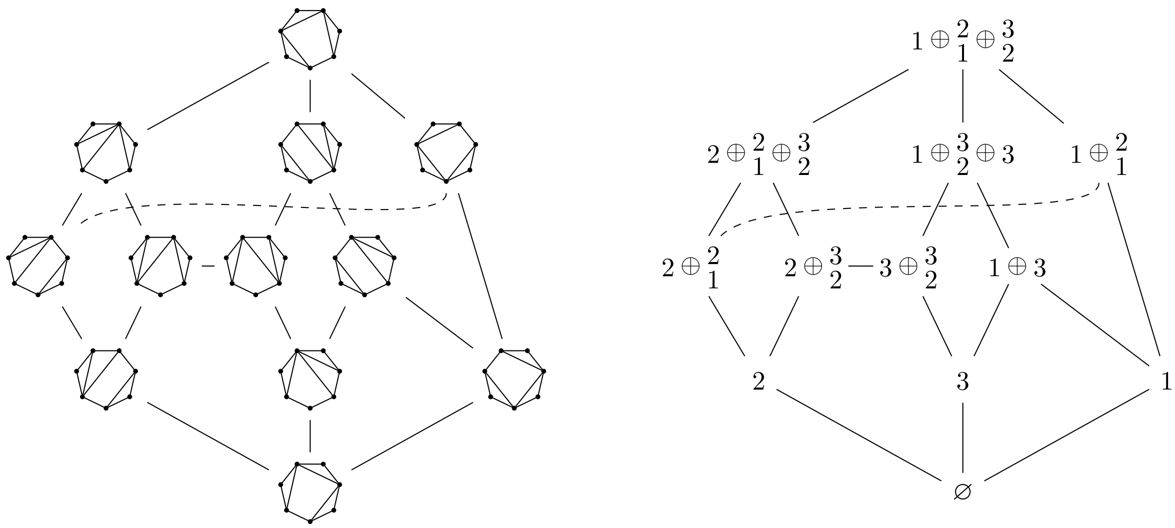


FIGURE 3. The $\mathrm{D}_\circ$-accordion complex of the dissection $\mathrm{D}_\circ$ of Figure 1 (left) and the 2-term silting complex of the quiver $\mathrm{Q}(\mathrm{D}_\circ)$ (right).

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