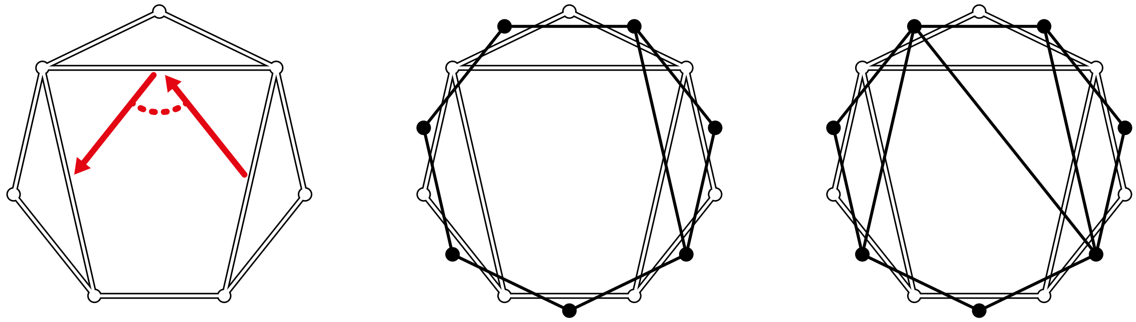

FIGURE 1. A dissection $\mathrm{D}_\circ$ with its quiver $\mathrm{Q}(\mathrm{D}_\circ)$ (left), a $\mathrm{D}_\circ$-accordion diagonal (middle) and a $\mathrm{D}_\circ$-accordion dissection (right).

1. Motivating example: accordion complexes of dissections

Let P be a convex polygon. We call *diagonals* of P the segments connecting two non-consecutive vertices of P. A *dissection* of P is a set D of non-crossing diagonals. It dissects the polygon into *cells*. We denote by $\mathrm{Q}(\mathrm{D})$ the quiver with relations whose vertices are the diagonals of D, whose arrows connect any two counterclockwise consecutive edges of a cell of D, and whose relations are given by triples of counterclockwise consecutive edges of a cell of D. See Figure 1 for an example.

We now consider $2m$ points on the unit circle alternately colored black and white, and let $\mathrm{P}_\circ$ (resp. $\mathrm{P}_\bullet$) denote the convex hull of the white (resp. black) points. We fix an arbitrary reference dissection $\mathrm{D}_\circ$ of $\mathrm{P}_\circ$. A diagonal $\delta_\bullet$ of $\mathrm{P}_\bullet$ is a *$\mathrm{D}_\circ$-accordion diagonal* if it crosses either none or two consecutive edges of any cell of $\mathrm{D}_\circ$. In other words the diagonals of $\mathrm{D}_\circ$ crossed by $\delta_\bullet$ together with the two boundary edges of $\mathrm{P}_\circ$ crossed by $\delta_\bullet$ form an accordion. A *$\mathrm{D}_\circ$-accordion dissection* is a set of non-crossing $\mathrm{D}_\circ$-accordion diagonals. See Figure 1 for an example. We call *$\mathrm{D}_\circ$-accordion complex* the simplicial complex $\mathcal{AC}(\mathrm{D}_\circ)$ of $\mathrm{D}_\circ$-accordion dissections. This complex appeared in recent works of F. Chapoton [**?**], A. Garver and T. McConville [**?**], and T. Manneville and V. Pilaud [**?**].

For a diagonal $\delta_\circ$ of $\mathrm{D}_\circ$ and a $\mathrm{D}_\circ$-accordion diagonal $\delta_\bullet$ intersecting $\delta_\circ$, we consider the three edges (including $\delta_\circ$) crossed by $\delta_\bullet$ in the two cells of $\mathrm{D}_\circ$ containing $\delta_\circ$. We define $\varepsilon\big(\delta_\circ \in \mathrm{D}_\circ \mid \delta_\bullet\big)$ to be 1, -1, or 0 depending on whether these three edges form a Z, a Σ, or a V. The *$\mathbf{g}$-vector* of $\delta_\bullet$ with respect to $\mathrm{D}_\circ$ is the vector $\mathbf{g}(\mathrm{D}_\circ \,|\, \delta_\bullet) \in \mathbb{R}^{\mathrm{D}_\circ}$ whose $\delta_\circ$-coordinate is $\varepsilon\big(\delta_\circ \in \mathrm{D}_\circ \mid \delta_\bullet\big)$.

Example 1. When the reference dissection $\mathrm{D}_\circ$ is a triangulation of $\mathrm{P}_\circ$, any diagonal of $\mathrm{P}_\bullet$ is a $\mathrm{D}_\circ$-accordion diagonal. The $\mathrm{D}_\circ$-accordion complex is thus an n-dimensional associahedron (of type A), where $n = m - 3$. In this case, it is known [**?**] that the $\mathrm{D}_\circ$-accordion complex is isomorphic to the 2-term silting complex of the quiver $\mathrm{Q}(\mathrm{D}_\circ)$ of the triangulation $\mathrm{D}_\circ$ (see Section **??** for definitions). The isomorphism sends a diagonal of $\mathrm{P}_\bullet$ to the 2-term projective complex with the same $\mathbf{g}$-vector.

The initial motivation of this paper was to prove the following extension of Example 1.

Theorem 2. *For any reference dissection $\mathrm{D}_\circ$, the $\mathrm{D}_\circ$-accordion complex is isomorphic to the 2-term silting complex of the quiver $\mathrm{Q}(\mathrm{D}_\circ)$.*

One possible approach to Theorem 2 would be to provide an explicit bijective map between $\mathrm{D}_\circ$-accordion diagonals and 2-term projective complexes for $\mathrm{Q}(\mathrm{D}_\circ)$. Such a map is easy to guess using $\mathbf{g}$-vectors, but the proof that it is actually a bijection and that it preserves compatibility is intricated. This approach was developed in the more general context of non-kissing complexes of gentle quivers in [**?**]. In this paper, we use an alternative simpler strategy to obtain Theorem 2, understanding accordion complexes as certain subcomplexes of the associahedron.

For that, consider two nested dissections $\mathrm{D}_\circ \subset \mathrm{D}'_\circ$. Observe that any $\mathrm{D}_\circ$-accordion diagonal is a $\mathrm{D}'_\circ$-accordion diagonal. Conversely a $\mathrm{D}'_\circ$-accordion diagonal $\delta_\bullet$ is a $\mathrm{D}_\circ$-accordion diagonal if and only if it does not cross any diagonal $\delta'_\circ$ of $\mathrm{D}'_\circ \smallsetminus \mathrm{D}_\circ$ as a Z or a Σ, that is if and only if the $\delta'_\circ$-coordinate of its $\mathbf{g}$-vector $\mathbf{g}(\mathrm{D}'_\circ \,|\, \delta_\bullet)$ vanishes for any $\delta'_\circ \in \mathrm{D}'_\circ \smallsetminus \mathrm{D}_\circ$. This observation shows the following statement.

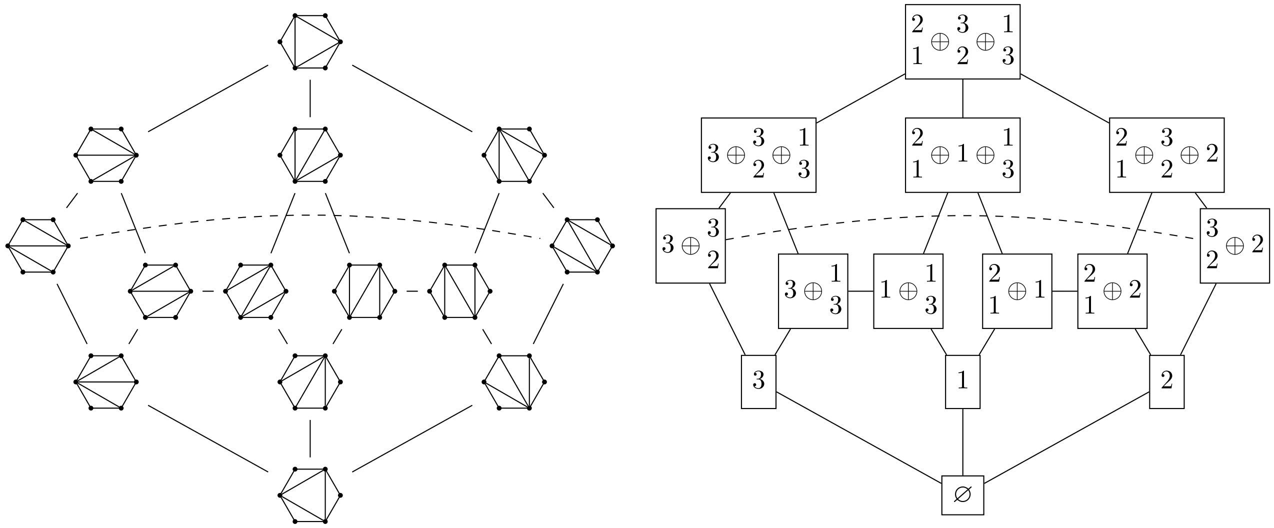

FIGURE 2. The associahedron (left) and the 2-term silting complex of an oriented triangle (right).

Theorem 3 ([?])**.** *For any two nested dissections* $\mathrm{D}_\circ \subset \mathrm{D}'_\circ$*, the accordion complex* $\mathcal{AC}(\mathrm{D}_\circ)$ *is isomorphic to the subcomplex of* $\mathcal{AC}(\mathrm{D}'_\circ)$ *induced by* $\mathrm{D}'_\circ$*-accordion diagonals* $\delta_\bullet$ *whose* $\mathbf{g}$*-vector* $\mathbf{g}(\mathrm{D}'_\circ \mid \delta_\bullet)$ *lie in the coordinate subspace spanned by elements in* $\mathrm{D}_\circ$*.*

Consider now any quiver with relations Q and any subset J of vertices of Q. We call *shortcut quiver* the quiver with relations Q/J whose vertices are the vertices of Q not in J, whose arrows are the paths in Q with internal vertices in J, and whose relations are inherited from those of Q. For example, quivers of subdissections are shortcut quivers: if $\mathrm{D}_\circ \subset \mathrm{D}'_\circ$, then $\mathrm{Q}(\mathrm{D}_\circ) = \mathrm{Q}(\mathrm{D}'_\circ)/(\mathrm{D}'_\circ \smallsetminus \mathrm{D}_\circ)$. The main result of this paper is the following statement.

Theorem 4. *For any quiver with relations* Q *and any subset* J *of vertices of* Q*, the* 2*-term silting complex* $\mathcal{SC}(\mathrm{Q}/J)$ *for the shortcut quiver* Q/J *is isomorphic to the subcomplex of the* 2*-term silting complex* $\mathcal{SC}(\mathrm{Q})$ *induced by* 2*-term projective complexes whose* $\mathbf{g}$*-vector lie in the coordinate subspace spanned by vertices not in* J*.*

Combining Theorems 3 and 4 together with Example 1 proves Theorem 2.

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Acknoledgements

We are grateful to F. Chapoton for various conversations on quadrangulations and Stokes posets.

References

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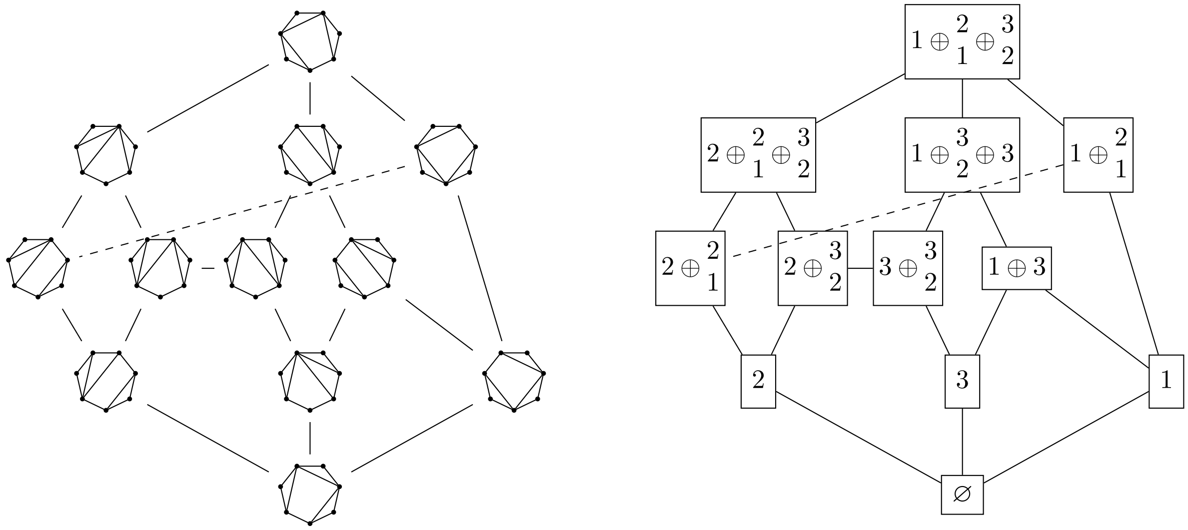

FIGURE 3. The $D_\circ$-accordion complex of the dissection $D_\circ$ of Figure 1 (left) and the 2-term silting complex of the quiver $Q(D_\circ)$ (right).