locally homeomorphic

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Let X and Y be topological spaces. Then X is *locally homeomorphic* to Y, if for every $x \in X$ there is a neighbourhood $U \subseteq X$ of x and an http://planetmath.org/node/380open set $V \subseteq Y$, such that U and V with their respective subspace topology are homeomorphic.

Examples

- Let $X = \{1\}$ and $Y = \{2, 3\}$ be discrete spaces with one resp. two elements. Since X and Y have different cardinalities, they cannot be homeomorphic. They are, however, locally homeomorphic to each other.

- Again, let $X = \{1\}$ be a discrete space with one element, but now let $Y = \{2, 3\}$ the space with topology $\{\emptyset, \{2\}, Y\}$. Then X is still locally homeomorphic to Y, but Y is not locally homeomorphic to X, since the smallest neighbourhood of 3 already has more elements than X.

- Now, let X be as in the previous examples, and $Y = \{2, 3\}$ be http://planetmath.org/node/
Then neither X is locally homeomorphic to Y nor the other way round.

- Non-trivial examples arise with locally Euclidean spaces, especially manifolds.