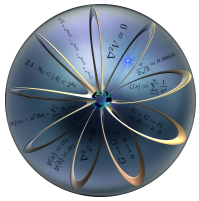

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a connected normal space with more than one point is uncountable

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The proof of the following result is an application of the generalized intermediate value theorem (along with Urysohn's lemma):

Proposition. *A connected normal space with more than one point is uncountable.*

Proof. Let X be a http://planetmath.org/node/941connected http://planetmath.org/node/1 space with at least two distinct points x_1 and x_2. As the sets $\{x_1\}$ and $\{x_2\}$ are http://planetmath.org/node/2739closed and disjoint, Urysohn's lemma furnishes a continuous function $f : X \to [0, 1]$ such that $f(x_1) = 0$ and $f(x_2) = 1$. Because X is connected, the generalized intermediate value theorem implies that f is surjective. Thus f may be suitably to give a bijection between a subset of X and the uncountable set $[0, 1]$, from which it follows that X is uncountable. □