



Math for the people, by the people.

when are balls separated

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Let  $(X, d)$  be a metric space, and let  $B_r(x)$  be the  $x$ -centered open ball of radius  $r$ . If  $d(x, y) \geq r + s$ , then the balls  $B_r(x)$  and  $B_s(y)$  are separated.

To prove this, suppose that  $B_r(x)$  and  $B_s(y)$  are not separated. Then there exists a  $z \in X$  such that either

$$d(x, z) < r, \quad d(y, z) \leq s,$$

or

$$d(x, z) \leq r, \quad d(y, z) < s.$$

In either case,

$$d(x, y) \leq d(x, z) + d(z, y) < r + s.$$