uniformizable space

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Entry type	Definition
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Defines	uniformizable
Defines	completely uniformizable

Let X be a topological space with $\mathcal{T}$ the topology defined on it. X is said to be *uniformizable*

1. there is a uniformity $\mathcal{U}$ defined on X, and
2. $\mathcal{T} = T_{\mathcal{U}}$, the uniform topology induced by $\mathcal{U}$.

It can be shown that a topological space is uniformizable iff it is completely regular.

Clearly, every pseudometric space is uniformizable. The converse is true if the space has a countable basis. Pushing this idea further, one can show that a uniformizable space is metrizable iff it is separating (or Hausdorff) and has a countable basis.

Let X, $\mathcal{T}$, and $\mathcal{U}$ be defined as above. Then X is said to be *completely uniformizable* if $\mathcal{U}$ is a complete uniformity.

Every paracompact space is completely uniformizable. Every completely uniformizable space is completely regular, and hence uniformizable.