quotient space

Canonical name	QuotientSpace
Date of creation	2013-03-22 12:39:40
Last modified on	2013-03-22 12:39:40
Owner	djao (24)
Last modified by	djao (24)
Numerical id	5
Author	djao (24)
Entry type	Definition
Classification	msc 54B15
Related topic	AdjunctionSpace
Defines	quotient topology
Defines	quotient map

Let X be a topological space, and let $\sim$ be an equivalence relation on X. Write X^* for the set of equivalence classes of X under $\sim$. The *quotient topology* on X^* is the topology whose open sets are the subsets $U \subset X^*$ such that

$$\bigcup U \subset X$$

is an open subset of X. The space X^* is called the *quotient space* of the space X with respect to $\sim$. It is often written $X/\sim$.

The projection map $\pi : X \longrightarrow X^*$ which sends each element of X to its equivalence class is always a continuous map. In fact, the map π satisfies the stronger property that a subset U of X^* is open if and only if the subset $\pi^{-1}(U)$ of X is open. In general, any surjective map $p : X \longrightarrow Y$ that satisfies this stronger property is called a *quotient map*, and given such a quotient map, the space Y is always homeomorphic to the quotient space of X under the equivalence relation

$$x \sim x' \iff p(x) = p(x').$$

As a set, the construction of a quotient space collapses each of the equivalence classes of $\sim$ to a single point. The topology on the quotient space is then chosen to be the strongest topology such that the projection map π is continuous.

For $A \subset X$, one often writes X/A for the quotient space obtained by identifying all the points of A with each other.