

planetmath.org

Math for the people, by the people.

proof that a path connected space is connected

 ${\bf Canonical\ name} \quad {\bf ProofThat APath Connected Space Is Connected}$

Date of creation 2013-03-22 12:46:30 Last modified on 2013-03-22 12:46:30

Owner n3o (216) Last modified by n3o (216)

Numerical id 6

Author n3o (216) Entry type Proof Classification msc 54D05 Let X be a path connected topological space. Suppose that $X = A \cup B$, where A and B are non empty, disjoint, open sets. Let $a \in A$, $b \in B$, and let $\gamma: I \to X$ denote a path from a to b.

We have $I = \gamma^{-1}(A) \cup \gamma^{-1}(B)$, where $\gamma^{-1}(A), \gamma^{-1}(B)$ are non empty, open and disjoint. Since I is connected, this is a contradiction, which concludes the proof.