a connected and locally path connected space is path connected

Canonical name	AConnectedAndLocallyPathConnectedSpaceIsPathConnected
Date of creation	2013-03-22 16:50:43
Last modified on	2013-03-22 16:50:43
Owner	Mathprof (13753)
Last modified by	Mathprof (13753)
Numerical id	6
Author	Mathprof (13753)
Entry type	Theorem
Classification	msc 54D05

Theorem. A connected, locally path connected topological space is path connected.

Proof. Let X be the space and fix $p \in X$. Let C be the set of all points in X that can be joined to p by a path. C is nonempty so it is enough to show that C is both closed and open.

To show first that C is open: Let c be in C and choose an open path connected neighborhood U of c. If $u \in U$ we can find a path joining u to c and then join that path to a path from p to c. Hence u is in C.

To show that C is closed: Let c be in $\overline{C}$ and choose an open path connected neighborhood U of c. Then $C \cap U \neq \varnothing$. Choose $q \in C \cap U$. Then c can be joined to q by a path and q can be joined to p by a path, so by addition of paths, p can be joined to c by a path, that is, $c \in C$.