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connected space

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Related topic	SemilocallySimplyConnected
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Related topic	ExampleOfAConnectedSpaceWhichIsNotPathConnected
Related topic	LocallyConnected
Related topic	ProofOfGeneralizedIntermediateValueTheorem
Related topic	AConnectedNormalSpaceWithMoreThanOnePointIsUncountable2
Related topic	AConnectedNormalSpaceWithMoreThanOn
Defines	connected
Defines	connected components
Defines	disconnected
Defines	connectedness

A topological space X is said to be *connected* if there is no pair of nonempty subsets U, V such that both U and V are open in X, $U \cap V = \emptyset$ and $U \cup V = X$. If X is not connected, i.e. if there are sets U and V with the above properties, then we say that X is *disconnected*.

Every topological space X can be viewed as a collection of subspaces each of which are connected. These subspaces are called the *connected components* of X. Slightly more rigorously, we define an equivalence relation $\sim$ on points in X by declaring that $x \sim y$ if there is a connected subset Y of X such that x and y both lie in Y. Then a connected component of X is defined to be an equivalence class under this relation.