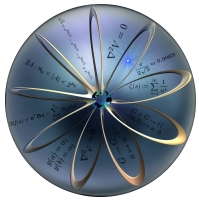

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boundary / frontier

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Definition. Let X be a topological space and let A be a subset of X. The *boundary* (or *frontier*) of A is the set $\partial A = \overline{A} \cap \overline{X \backslash A}$, where the overline denotes the closure of a set. Instead of ∂A, many authors use some other notation such as $\mathrm{bd}(A)$, $\mathrm{fr}(A)$, A^b or $\beta(A)$. Note that the ∂ symbol is also used for other meanings of 'boundary'.

From the definition, it follows that the boundary of any set is a closed set. It also follows that $\partial A = \partial(X \backslash A)$, and $\partial X = \varnothing = \partial \varnothing$.

The term 'boundary' (but not 'frontier') is used in a different sense for topological manifolds: the boundary ∂M of a topological n-manifold M is the set of points in M that do not have a neighbourhood homeomorphic to $\mathbb{R}^n$. (Some authors define topological manifolds in such a way that they necessarily have empty boundary.) For example, the boundary of the topological 1-manifold $[0, 1]$ is $\{0, 1\}$.