partition of unity

Canonical name	PartitionOfUnity
Date of creation	2013-03-22 13:29:23
Last modified on	2013-03-22 13:29:23
Owner	mhale (572)
Last modified by	mhale (572)
Numerical id	10
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Entry type	Definition
Classification	msc 54D20
Classification	msc 58A05
Defines	locally finite partition of unity
Defines	subordinate to an open cover

Let X be a topological space. A **partition of unity** is a collection of continuous functions $\{\varepsilon_i \colon X \to [0,1]\}$ such that

$$\sum_i \varepsilon_i(x) = 1 \quad \text{for all } x \in X. \tag{1}$$

A partition of unity is **locally finite** if each x in X is contained in an open set on which only a finite number of ε_i are non-zero. That is, if the cover $\{\varepsilon_i^{-1}((0,1])\}$ is locally finite.

A partition of unity is **subordinate to an open cover** $\{U_i\}$ of X if each ε_i is zero on the complement of U_i.

Example 1 (Circle)
A partition of unity for $\mathbb{S}^1$ *is given by* $\{\sin^2(\theta/2), \cos^2(\theta/2)\}$ *subordinate to the covering* $\{(0, 2\pi), (-\pi, \pi)\}$.

Application to integration

Let M be an orientable manifold with volume form ω and a partition of unity $\{\varepsilon_i(x)\}$. Then, the integral of a function $f(x)$ over M is given by

$$\int_M f(x)\omega = \sum_i \int_{U_i} \varepsilon_i(x) f(x)\omega.$$

It is of the choice of partition of unity.