isometry

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Related topic	IsometricIsomorphism
Defines	isometric
Defines	isometric mapping
Defines	isometric embedding
Defines	isometry group
Defines	group of isometries

Let (X_1, d_1) and (X_2, d_2) be metric spaces. A function $f\colon X_1 \to X_2$ is said to be an *isometric mapping* (or *isometric embedding*) if

$$d_1(x, y) = d_2(f(x), f(y))$$

for all $x, y \in X_1$.

Every isometric mapping is injective, for if $x, y \in X_1$ with $x \neq y$ then $d_1(x, y) > 0$, and so $d_2(f(x), f(y)) > 0$, and then $f(x) \neq f(y)$. One can also easily show that every isometric mapping is continuous.

An isometric mapping that is surjective (and therefore bijective) is called an *isometry*. (Readers are warned, however, that some authors do not require isometries to be surjective; that is, they use the term *isometry* for what we have called an isometric mapping.) Every isometry is a homeomorphism.

If there is an isometry between the metric spaces (X_1, d_1) and (X_2, d_2), then they are said to be *isometric*. Isometric spaces are essentially identical as metric spaces, and in particular they are homeomorphic.

Given any metric space (X, d), the set of all isometries $X \to X$ forms a group under composition. This group is called the *isometry group* (or *group of isometries*) of X, and may be denoted by $\mathrm{Iso}(X)$ or $\mathrm{Isom}(X)$. In general, *an* (as opposed to *the*) isometry group (or group of isometries) of X is any subgroup of $\mathrm{Iso}(X)$.