topology of the complex plane

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Defines	accumulation point
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Defines	open
Defines	closed
Defines	bounded
Defines	compact

The usual topology for the complex plane $\mathbb{C}$ is the topology induced by the metric

$$d(x,\,y) := |x-y|$$

for $x,\,y \in \mathbb{C}$. Here, $|\cdot|$ is the http://planetmath.org/ModulusOfComplexNumbercomplex modulus.

If we identify $\mathbb{R}^2$ and $\mathbb{C}$, it is clear that the above topology coincides with topology induced by the Euclidean metric on $\mathbb{R}^2$.

Some basic topological concepts for $\mathbb{C}$:

1. The open balls

   $$B_r(\zeta) \;=\; \{z \in \mathbb{C} \colon |z-\zeta| < r\}$$

   are often called *open disks*.

2. A point ζ is an *accumulation point* of a subset A of $\mathbb{C}$, if any open disk $B_r(\zeta)$ contains at least one point of A distinct from ζ.

3. A point ζ is an *interior point* of the set A, if there exists an open disk $B_r(\zeta)$ which is contained in A.

4. A set A is *open*, if each of its points is an interior point of A.

5. A set A is *closed*, if all its accumulation points belong to A.

6. A set A is *bounded*, if there is an open disk $B_r(\zeta)$ containing A.

7. A set A is *compact*, if it is closed and bounded.