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every second countable space is separable

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Theorem 1. *[?] Every second countable space is separable.*

Proof. Let X be a second countable space and let \mathcal{B} be a countable base. For every non-empty set B in \mathcal{B} , choose a point $x_B \in B$. The set A of all such points x_B is clearly countable and it's also dense since any open set intersects it and thus the whole space is the closure of A . That is, A is a countably dense subset of X . Therefore, X is separable. \square

References

- [1] J.L. Kelley, *General Topology*, D. van Nostrand Company, Inc., 1955.