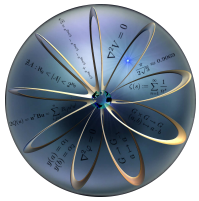

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finite and countable discrete spaces

Canonical name	FiniteAndCountableDiscreteSpaces
Date of creation	2013-03-22 15:17:11
Last modified on	2013-03-22 15:17:11
Owner	matte (1858)
Last modified by	matte (1858)
Numerical id	9
Author	matte (1858)
Entry type	Theorem
Classification	msc 54-00

Theorem 1. *Suppose $X \neq \emptyset$ is equipped with the discrete topology.*

1. *If X if finite, then X is homeomorphic to $\{1, \ldots, n\}$ for some $n \geq 1$.*
2. *If X if countable, then X is homeomorphic to $\mathbb{Z}$.*

Here, $\{1, \ldots, n\}$ and $\mathbb{Z}$ are endowed with the discrete topology (or, equivalently, the subspace topology from $\mathbb{R}$).

Proof. The first claim will be proven. If

$$X = \{a_1, \ldots, a_n\}$$

let $\Phi \colon \{1, \ldots, n\} \to X$ be

$$\Phi(i) = a_i, \quad i = 1, \ldots, n.$$

Since Φ is a bijection, it is a homeomorphism.

The proof of the second claim is to that of the first. □