complete uniform space

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Let X be a uniform space with uniformity $\mathcal{U}$. A filter $\mathcal{F}$ on X is said to be a *Cauchy filter* if for each entourage V in $\mathcal{U}$, there is an $F \in \mathcal{F}$ such that $F \times F \subseteq V$.

We say that X is *complete* if every Cauchy filter is a convergent filter in the topology $T_{\mathcal{U}}$ `http://planetmath.org/TopologyInducedByUniformStructure`induced by $\mathcal{U}$. $\mathcal{U}$ in this case is called a *complete uniformity*.

A *Cauchy sequence* $\{x_i\}$ in a uniform space X is a sequence in X whose section filter is a Cauchy filter. A Cauchy sequence is said to be convergent if its section filter is convergent. X is said to be *sequentially complete* if every Cauchy sequence converges (every section filter of it converges).

Remark. This is a generalization of the concept of completeness in a metric space, as a metric space is a uniform space. As we see above, in the course of this generalization, two notions of completeness emerge: that of completeness and sequentially completeness. Clearly, completeness always imply sequentially completeness. In the context of a metric space, or a metrizable uniform space, the two notions are indistinguishable: sequentially completeness also implies completeness.