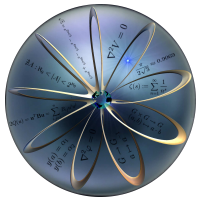

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topological transformation group

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Defines	effective topological transformation group

Let G be a topological group and X any topological space. We say that G is a *topological transformation group* of X if G *acts* on X continuously, in the following sense:

1. there is a continuous function $\alpha : G \times X \to X$, where $G \times X$ is given the product topology
2. $\alpha(1, x) = x$, and
3. $\alpha(g_1 g_2, x) = \alpha(g_1, \alpha(g_2, x))$.

The function α is called the *(left) action* of G on X. When there is no confusion, $\alpha(g, x)$ is simply written gx, so that the two conditions above read $1x = x$ and $(g_1 g_2)x = g_1(g_2 x)$.

If a topological transformation group G on X is *effective*, then G can be viewed as a group of homeomorphisms on X: simply define $h_g : X \to X$ by $h_g(x) = gx$ for each $g \in G$ so that h_g is the identity function precisely when $g = 1$.

Some Examples.

1. Let $X = \mathbb{R}^n$, and G be the group of $n \times n$ matrices over $\mathbb{R}$. Clearly X and G are both topological spaces with the usual topology. Furthermore, G is a topological group. G acts on X continuous if we view elements of X as column vectors and take the action to be the matrix multiplication on the left.

2. If G is a topological group, G can be considered a topological transformation group on itself. There are many continuous actions that can be defined on G. For example, $\alpha : G \times G \to G$ given by $\alpha(g, x) = gx$ is one such action. It is continuous, and satisfies the two action axioms. G is also effective with respect to α.