

In a metric space X , let $\overline{B}_r(x)$ be the closed ball centered at $x \in X$ with radius $r > 0$.

Theorem 1 (Nested sphere theorem [?]). *A metric space X is complete if and only if every sequence $\{\overline{B}_{r_n}(x_n)\}_n$ such that $\overline{B}_{r_{i+1}}(x_{i+1}) \subseteq \overline{B}_{r_i}(x_i)$ and $r_n \rightarrow 0$ when $n \rightarrow \infty$ has a nonempty intersection (i.e. $\bigcap_{n=1}^{\infty} \overline{B}_{r_n}(x_n) \neq \emptyset$).*

References

- [KF] Kolmogorov, A.N. & Fomin, S.V.: Introductory Real Analysis, Translated & Edited by Richard A. Silverman. Dover Publications, Inc. New York, 1970.