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cover

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Definition ([**?**], pp. 49) Let Y be a subset of a set X. A **cover** for Y is a collection of sets $\mathcal{U} = \{U_i\}_{i \in I}$ such that each U_i is a subset of X, and

$$Y \subset \bigcup_{i \in I} U_i.$$

The collection of sets can be arbitrary, that is, I can be finite, countable, or uncountable. The cover is correspondingly called a **finite cover**, **countable cover**, or **uncountable cover**.

A **subcover** of $\mathcal{U}$ is a subset $\mathcal{U}' \subset \mathcal{U}$ such that $\mathcal{U}'$ is also a cover of X.

A **refinement** $\mathcal{V}$ of $\mathcal{U}$ is a cover of X such that for every $V \in \mathcal{V}$ there is some $U \in \mathcal{U}$ such that $V \subset U$. When $\mathcal{V}$ refines $\mathcal{U}$, it is usually written $\mathcal{V} \preceq \mathcal{U}$. $\preceq$ is a preorder on the set of covers of any topological space X.

If X is a topological space and the members of $\mathcal{U}$ are open sets, then $\mathcal{U}$ is said to be an *open cover*. Open subcovers and open refinements are defined similarly.

Examples

1. If X is a set, then $\{X\}$ is a cover of X.

2. The power set of a set X is a cover of X.

3. A topology for a set is a cover of that set.

References

[1] J.L. Kelley, *General Topology*, D. van Nostrand Company, Inc., 1955.