# contractive maps are uniformly continuous



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**Theorem** A contraction mapping is uniformly continuous.

**Proof** Let $T : X \to X$ be a contraction mapping in a metric space $X$ with metric $d$. Thus, for some $q \in [0, 1)$, we have for all $x, y \in X$,

$$d(Tx, Ty) \leq qd(x, y).$$

To prove that $T$ is uniformly continuous, let $\varepsilon > 0$ be given. There are two cases. If $q = 0$, our claim is trivial, since then for all $x, y \in X$,

$$d(Tx, Ty) = 0 < \varepsilon.$$

On the other hand, suppose $q \in (0, 1)$. Then for all $x, y \in X$ with $d(x, y) < \varepsilon/q$, we have

$$d(Tx, Ty) \leq qd(x, y) < \varepsilon.$$

In conclusion, $T$ is uniformly continuous. □

The result is stated without proof in [**?**], pp. 221.

# References


[1] W. Rudin, *Principles of Mathematical Analysis*, McGraw-Hill Inc., 1976.