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dense set

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A subset D of a topological space X is said to be *dense* (or *everywhere dense*) in X if the closure of D is equal to X. Equivalently, D is dense if and only if D intersects every nonempty open set.

In the special case that X is a metric space with metric d, then this can be rephrased as: for all $\varepsilon > 0$ and all $x \in X$ there is $y \in D$ such that $d(x, y) < \varepsilon$.

For example, both the rationals $\mathbb{Q}$ and the irrationals $\mathbb{R} \setminus \mathbb{Q}$ are dense in the reals $\mathbb{R}$.

The least cardinality of a dense set of a topological space is called the *density* of the space. It is conventional to take the density to be $\aleph_0$ if it would otherwise be finite; with this convention, the spaces of density $\aleph_0$ are precisely the separable spaces. The density of a topological space X is denoted $d(X)$. If X is a Hausdorff space, it can be shown that $|X| \leq 2^{2^{d(X)}}$.