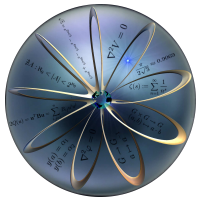

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precompact set

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Synonym	precompact
Synonym	relatively compact

Definition 1. *A subset in a topological space is* precompact *if its closure is compact [?].*

For metric spaces, we have the following theorem due to Hausdorff [?].

Theorem Suppose K is a set in a complete metric space X. Then K relatively compact if and only if for any $\varepsilon > 0$ there is a finite `http://planetmath.org/VarepsilonNet` net for K.

Examples

1. In $\mathbb{R}^n$ every point has a precompact neighborhood.

2. On a manifold, every point has a precompact neighborhood. This follows from the previous example, since a homeomorphism commutes with the closure operator, and since the continuous image of a compact set is compact.

Notes

A synonym is *relatively compact* [?, ?].

Some authors (notably Bourbaki see [?]) use precompact differently - as a synonym for `http://planetmath.org/TotallyBounded`totally bounded (in the generality of topological groups). "Relatively compact" is then used to mean "precompact "as it is defined here

References

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[3] E. Kreyszig, *Introductory Functional Analysis With Applications*, John Wiley & Sons, 1978.

[4] N. Bourbaki, *Topological Vector Spaces* Springer-Verlag, 1981