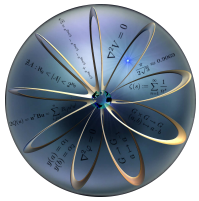

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closed complex plane

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The complex plane $\mathbb{C}$, i.e. the set of the complex numbers z satisfying

$$|z| < \infty,$$

is open but not closed, since it doesn't contain the accumulation points of all sets of complex numbers, for example of the set $\{1, 2, 3, \ldots\}$. One can $\mathbb{C}$ to the *closed complex plane* $\mathbb{C} \cup \{\infty\}$ by adding to $\mathbb{C}$ the infinite point ∞ which the lacking accumulation points. One settles that $|\infty| = \infty$, where the latter ∞ means the real infinity.

The resulting space is the one-point compactification of $\mathbb{C}$. The open sets are the open sets in $\mathbb{C}$ together with sets containing ∞ whose complement is compact in $\mathbb{C}$. Conceptually, one thinks of the additional open sets as those open sets "around ∞".

The one-point compactification of $\mathbb{C}$ is also the complex projective line $\mathbb{CP}^1$, as well as the Riemann sphere.