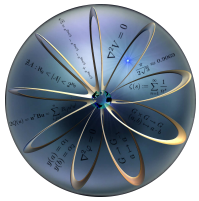

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example of continuous bijections which are not homeomorphisms

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Example 1. Assume that X is a topological space, which neither discrete nor antidiscrete. We will show that there are topological spaces Y and Z such that there are continuous bijections $X \to Y$ and $Z \to X$ which are not homeomorphisms.

Let $Y = Z = X$ as a sets but topology on Y is antidiscrete and on Z is discrete. Then obviously identity mappings $\mathrm{id} : X \to Y$ and $\mathrm{id} : Z \to X$ are continuous, but since X is neither discrete nor antidiscrete, these mappings are not homeomorphisms.

Example 2. Consider the function $f : [0, 1) \to S^1$ (here S^1 denotes the unit circle in a complex plane) defined by the formula $f(t) = e^{2\pi i t}$. It is easy to see that f is a continuous bijection, but f is not a homeomorphism (because $[0, 1)$ is not compact).