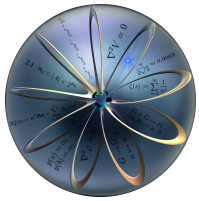

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distance to a set

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Let X be a metric space with a metric d. If A is a non-empty subset of X and $x \in X$, then the *distance from x to A* [**?**] is defined as

$$d(x, A) := \inf_{a \in A} d(x, a).$$

We also write $d(x, A) = d(A, x)$.

Suppose that x, y are points in X, and $A \subset X$ is non-empty. Then we have the following triangle inequality

$$\begin{aligned} d(x, A) &= \inf_{a \in A} d(x, a) \\ &\leq d(x, y) + \inf_{a \in A} d(y, a) \\ &= d(x, y) + d(y, A). \end{aligned}$$

If X is only a pseudo-metric space, then the above definition and triangle-inequality also hold.

References

[1] J.L. Kelley, *General Topology*, D. van Nostrand Company, Inc., 1955.