star refinement

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Entry type	Definition
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Defines	star
Defines	star refine
Defines	barycentric refinement

Let X be a set and $\mathscr{C} = \{C_i \mid i \in I\}$ be a cover of X (we assume C_i and X are all subsets of some universe). Let $A \subseteq X$. The *star* of A (with respect to the cover $\mathscr{C}$) is defined as

$$\star(A, \mathscr{C}) := \bigcup \{C_i \in \mathscr{C} \mid C_i \cap A \neq \varnothing\}.$$

When A is a singleton, we write $\star(x, \mathscr{C}) = \star(\{x\}, \mathscr{C})$.

Properties of $\star$

1. $A \subseteq \star(A, \mathscr{C})$.
2. If $A \subseteq B$, then $\star(A, \mathscr{C}) \subseteq \star(B, \mathscr{C})$.
3. For any cover $\mathscr{C}$ of X, the sets $\mathscr{C}^\star := \{\star(C_i, \mathscr{C}) \mid C_i \in \mathscr{C}\}$ and $\mathscr{C}^b := \{\star(x, \mathscr{C}) \mid x \in X\}$ are both covers of X.
4. $\mathscr{C} \preceq \mathscr{C}^b \preceq \mathscr{C}^\star$ ($\preceq$ denotes cover refinement).

Definitions. Let $\mathscr{C}, \mathscr{D}$ be two covers of X. If $\mathscr{C}^\star \preceq \mathscr{D}$, then we say that $\mathscr{C}$ is a *star refinement* of $\mathscr{D}$, denoted by $\mathscr{C} \preceq^\star \mathscr{D}$. If $\mathscr{C}^b \preceq \mathscr{D}$, then we say that $\mathscr{C}$ is a *barycentric refinement* of $\mathscr{D}$, denoted by $\mathscr{C} \preceq^b \mathscr{D}$.

Remark. By property 4 above, it is easy to see that $\mathscr{C} \preceq^\star \mathscr{D} \Rightarrow \mathscr{C} \preceq^b \mathscr{D} \Rightarrow \mathscr{C} \preceq \mathscr{D}$.

References

[1] S. Willard, *General Topology*, Addison-Wesley, Publishing Company, 1970.