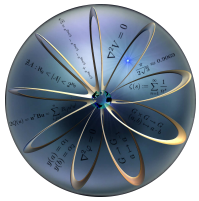

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suspension

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Related topic	SuspensionIsomorphism
Defines	suspension
Defines	reduced suspension
Defines	based suspension
Defines	unreduced suspension
Defines	unbased suspension

1 The unreduced suspension

Given a topological space X, the *suspension* of X, often denoted by SX, is defined to be the quotient space $X \times [0,1]/\sim$, where $(x,0) \sim (y,0)$ and $(x,1) \sim (y,1)$ for any $x, y \in X$.

Given a continuous map $f : X \to Y$, there is a map $Sf : SX \to SY$ defined by $Sf([x,t]) := [f(x),t]$. This makes S into a functor from the category of topological spaces into itself.

Note that SX is homeomorphic to the join $X \star S^0$, where S^0 is a discrete space with two points.

The space SX is sometimes called the *unreduced*, *unbased* or *free* suspension of X, to distinguish it from the reduced suspension described below.

2 The reduced suspension

If (X, x_0) is a based topological space, the *reduced suspension* of X, often denoted ΣX (or $\Sigma_{x_0} X$ when the basepoint needs to be explicit), is defined to be the quotient space $X \times [0,1]/(X \times \{0\} \cup X \times \{1\} \cup \{x_0\} \times [0,1]$. Setting the basepoint of ΣX to be the equivalence class of $(x_0, 0)$, the reduced suspension is a functor from the category of based topological spaces into itself.

An important property of this functor is that it is a left adjoint to the functor Ω taking a (based) space X to its loop space ΩX. In other words, $\mathrm{Maps}_* (\Sigma X, Y) \cong \mathrm{Maps}_* (X, \Omega Y)$ naturally, where $\mathrm{Maps}_* (X, Y)$ stands for continuous maps which preserve basepoints.

The reduced suspension is also known as the *based* suspension.