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## Sierpinski space

Canonical name	SierpinskiSpace
Date of creation	2013-03-22 12:06:26
Last modified on	2013-03-22 12:06:26
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	9
Author	CWoo (3771)
Entry type	Definition
Classification	msc 54G20
Synonym	Sierpiński space
Related topic	T1Space
Related topic	T2Space
Related topic	SeparationAxioms

*Sierpinski space* is the topological space  $X = \{x, y\}$  with the topology given by  $\{X, \{x\}, \emptyset\}$ .

Sierpinski space is <http://planetmath.org/T0T0> but not <http://planetmath.org/T1T1>. It is  $T_0$  because  $\{x\}$  is the open set containing  $x$  but not  $y$ . It is not  $T_1$  because every open set  $U$  containing  $y$  (namely  $X$ ) contains  $x$  (in other words, there is no open set containing  $y$  but not containing  $x$ ).

**Remark.** From the Sierpinski space, one can construct many non- $T_1$   $T_0$  spaces, simply by taking any set  $X$  with at least two elements, and take any non-empty proper subset  $U \subset X$ , and set the topology  $\mathcal{T}$  on  $X$  by  $\mathcal{T} = P(U) \cup \{X\}$ .