inequalities for real numbers

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Suppose a is a real number.

1. If $a < 0$ then a is a *negative number*.
2. If $a > 0$ then a is a *positive number*.
3. If $a \leq 0$ then a is a *non-positive number*.
4. If $a \geq 0$ then a is a *non-negative number*.

The first two inequalities are also called **strict inequalities**.
The second two inequalities are also called **loose inequalities**.

Properties

Suppose a and b are real numbers.

1. If $a > b$, then $-a < -b$. If $a < b$, then $-a > -b$.
2. If $a \geq b$, then $-a \leq -b$. If $a \leq b$, then $-a \geq -b$.

Lemma 1. $0 < a$ *iff* $-a < 0$.

Proof. If $0 < a$, then adding $-a$ on both sides of the inequality gives $-a = -a + 0 < -a + a = 0$. This process can also be reversed. $\square$

Lemma 2. *For any $a \in \mathbb{R}$, either $a = 0$ or $0 < a^2$.*

Proof. Suppose $a \neq 0$, then by trichotomy, we have either $0 < a$ or $a < 0$, but not both. If $0 < a$, then $0 = 0 \cdot a < a \cdot a = a^2$. On the other hand, if $-(-a) = a < 0$, then $0 < -a$ by the previous lemma. Then repeating the previous , $0 = 0 \cdot (-a) < (-a)(-a) = a^2$. $\square$

Three direct consequences follow:

Corollary 1. $0 < 1$

Corollary 2. *For any $a \in \mathbb{R}$, $0 < 1 + a^2$.*

Corollary 3. *There is no real solution for x in the equation $1 + x^2 = 0$.*

Inequality for a converging sequence

Suppose $a_0, a_1, \ldots$ is a sequence of real numbers converging to a real number a.

1. If $a_i < b$ or $a_i \leq b$ for some real number b for each i, then $a \leq b$.
2. If $a_i > b$ or $a_i \geq b$ for some real number b for each i, then $a \geq b$.