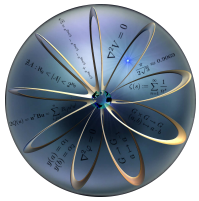

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proof of Dini's theorem

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Without loss of generality we will assume that X is compact and, by replacing f_n with $f - f_n$, that the net converges monotonically to 0.

Let $\epsilon > 0$. For each $x \in X$, we can choose an n_x, such that $f_{n_x}(x) < \epsilon/2$. Since f_{n_x} is continuous, there is an open neighbourhood U_x of x, such that for each $y \in U_x$, we have $f_{n_x}(y) < \epsilon/2$. The open sets U_x cover X, which is compact, so we can choose finitely many $x_1, \ldots, x_k$ such that the U_{x_i} also cover X. Then, if $N \geq n_{x_1}, \ldots, n_{x_k}$, we have $f_n(x) < \epsilon$ for each $n \geq N$ and $x \in X$, since the sequence f_n is monotonically decreasing. Thus, $\{f_n\}$ converges to 0 uniformly on X, which was to be proven.