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box topology

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Defines	box product

Let $\{(X_\alpha, \mathcal{T}_\alpha)\}_{\alpha\in A}$ be a family of topological spaces. Let Y denote the generalized Cartesian product of the sets X_α, that is

$$Y = \prod_{\alpha\in A} X_\alpha.$$

Let $\mathcal{B}$ denote the set of all products of open sets of the corresponding spaces, that is

$$\mathcal{B} = \left\{ \prod_{\alpha\in A} U_\alpha \;\middle|\; U_\alpha \in \mathcal{T}_\alpha \text{ for all } \alpha \in A \right\}.$$

Now we can construct the *box product* $(Y, \mathcal{S})$, where $\mathcal{S}$, referred to as the *box topology*, is the topology the base $\mathcal{B}$.

When A is a `http://planetmath.org/Finite`finite set, the box topology coincides with the product topology.

Example

As an example, the box product of two topological spaces $(X_0, \mathcal{T}_0)$ and $(X_1, \mathcal{T}_1)$ is $(X_0 \times X_1, \mathcal{S})$, where the box topology $\mathcal{S}$ (which is the same as the product topology) consists of all sets of the form $\bigcup_{i\in I}(U_i \times V_i)$, where I is some index set and for each $i \in I$ we have $U_i \in \mathcal{T}_0$ and $V_i \in \mathcal{T}_1$.