proof of inverse function theorem (topological spaces)

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We only have to prove that whenever $A \subset X$ is an open set, then also $B = (f^{-1})^{-1}(A) = f(A) \subset Y$ is open (f is an open mapping). Equivalently it is enough to prove that $B' = Y \setminus B$ is closed.

Since f is bijective we have $B' = Y \setminus B = f(X \setminus A)$

As $A' = X \setminus A$ is closed and since X is compact A' is compact too (this and the following are well know properties of compact spaces). Moreover being f continuous we know that also $B' = f(A')$ is compact. Finally since Y is Hausdorff then B' is closed.