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Borel groupoid

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Related topic	BorelGSpace
Related topic	CategoryOfBorelGroupoids
Defines	Borel function
Defines	analytic groupoid
Defines	set of composable pairs
Defines	$Grp(2)$
Defines	analytic (Borel) groupoid
Defines	analytic Borel space
Defines	product structure

0.1 Definitions

- **a. Borel function**

  Definition 0.1. A function $f_B : (X; \mathcal{B}) \to (X; \mathcal{C})$ of `http://planetmath.org/BorelSpace`Borel spaces is defined to be a *Borel function* if the inverse image of every Borel set under f_B^{-1} is also a Borel set.

- **b. Borel groupoid**

  Definition 0.2. Let $\mathbb{G}$ be a groupoid and $\mathbb{G}^{(2)}$ a subset of $\mathbb{G} \times \mathbb{G}$– the set of its composable pairs. A *Borel groupoid* is defined as a groupoid $\mathbb{G}_B$ such that $\mathbb{G}_B^{(2)}$ is a Borel set in the product structure on $\mathbb{G}_B \times \mathbb{G}_B$, and also with functions $(x, y) \mapsto xy$ from $\mathbb{G}_B^{(2)}$ to $\mathbb{G}_B$, and $x \mapsto x^{-1}$ from $\mathbb{G}_B$ to $\mathbb{G}_B$ defined such that they are all `http://planetmath.org/MeasurableFunctions`(measurable) Borel functions (ref. [**?**]).

0.1.1 Analytic Borel space

$\mathbb{G}_B$ becomes an `http://planetmath.org/LocallyCompactGroupoids`analytic groupoid if its Borel structure is `http://planetmath.org/Analytic`analytic.

A Borel space $(X; \mathcal{B})$ is called *analytic* if it is countably separated, and also if it is the image of a Borel function from a standard Borel space.

References

[1] M.R. Buneci. 2006., `http://www.utgjiu.ro/math/mbuneci/preprint/p0024.pdf`Groupoid C*-Algebras., *Surveys in Mathematics and its Applications*, Volume 1, p.75 .