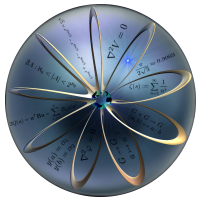

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universal covering space

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Let X be a topological space. A *universal covering space* is a covering space $\tilde{X}$ of X which is connected and simply connected.

If X is based, with basepoint x, then a *based cover* of X is cover of X which is also a based space with a basepoint x' such that the covering is a map of based spaces. Note that any cover can be made into a based cover by choosing a basepoint from the pre-images of x.

The universal covering space has the following universal property: If $\pi : (\tilde{X}, x_0) \to (X, x)$ is a based universal cover, then for any connected based cover $\pi' : (X', x') \to (X, x)$, there is a unique covering map $\pi'' : (\tilde{X}, x_0) \to (X', x')$ such that $\pi = \pi' \circ \pi''$.

Clearly, if a universal covering exists, it is unique up to unique isomorphism. But not every topological space has a universal cover. In fact X has a universal cover if and only if it is semi-locally simply connected (for example, if it is a locally finite CW-complex or a manifold).