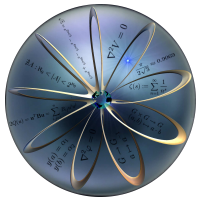

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Alexandroff space is T1 if and only if it is discrete

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Proposition. Let X be an Alexandroff space. Then X is T_1 if and only if X is discrete.

Proof. „⇐" It is easy to see, that every discrete space is Alexandroff and T_1.

„⇒" Recall that topological space is T_1 if and only if every subset is equal to the intersection of all its open neighbourhoods. So let $x \in X$. Then the intersection of all open neighbourhoods $\{x\}^o$ of x is equal to $\{x\}$. But since X is Alexandroff, then $\{x\}^o = \{x\}$ is open and thus points are open. Therefore X is discrete. □