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injective images of Baire space

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Every uncountable Polish space is, up to a countable subset, an injective image of Baire space  $\mathcal{N}$ .

**Theorem.** *Let  $X$  be an uncountable Polish space. Then, there is a one-to-one and continuous function  $f: \mathcal{N} \rightarrow X$  such that  $X \setminus f(\mathcal{N})$  is countable.*

Although the inverse  $f^{-1}: f(\mathcal{N}) \rightarrow \mathcal{N}$  will not generally be continuous, it is at least Borel measurable. It can be shown that this is true for all one-to-one and continuous functions between Polish spaces, although here it follows directly from <http://planetmath.org/ProofOfInjectiveImagesOfBaireSpace> the construction of  $f$ .