proof of Tychonoff's theorem

Canonical name	ProofOfTychonoffsTheorem
Date of creation	2013-03-22 17:25:24
Last modified on	2013-03-22 17:25:24
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Last modified by	asteroid (17536)
Numerical id	8
Author	asteroid (17536)
Entry type	Proof
Classification	msc 54D30

This is a proof in of nets. Recall the following facts:

1 - A net $(x_\alpha)_{\alpha\in\mathcal{A}}$ in $\prod_{i\in I} X_i$ converges to $x \in \prod_{i\in I} X_i$ if and only if each coordinate $(x^i_\alpha)_{\alpha\in\mathcal{A}}$ converges to $x^i \in X_i$

2 - A topological space X is compact if and only if every net in X has a convergent subnet.

3 - Every net has a universal subnet.

4 - A `http://planetmath.org/Ultranet`universal net $(x_\alpha)_{\alpha\in\mathcal{A}}$ in a compact space X is convergent. (see this `http://planetmath.org/UniversalNetsInCompactSpacesAr`

We now prove Tychonoff's theorem.

Proof (Tychonoff's theorem) : Let $(x_\alpha)_{\alpha\in\mathcal{A}}$ be a net in $\prod_{i\in I} X_i$.

Using Lemma 3 we can find a subnet $(y_\beta)_{\beta\in\mathcal{B}}$ of $(x_\alpha)_{\alpha\in\mathcal{A}}$.

It is easily seen that each coordinate net $(y^i_\beta)_{\beta\in\mathcal{B}}$ is a net in X_i.

Using Lemma 4 we see that each coordinate net converges, because X_i is compact.

Using Lemma 1 we see that the whole net $(y_\beta)_{\beta\in\mathcal{B}}$ converges in $\prod_{i\in I} X_i$.

We conclude that every net in $\prod_{i\in I} X_i$ has a convergent subnet, so, by Lemma 2, $\prod_{i\in I} X_i$ must be compact. $\square$