uniform space

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Defines	uniform structure
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A *uniform structure* (or *uniformity*) on a set X is a non empty set $\mathcal{U}$ of subsets of $X \times X$ which satisfies the following axioms:

1. Every subset of $X \times X$ which contains a set of $\mathcal{U}$ belongs to $\mathcal{U}$.

2. Every finite intersection of sets of $\mathcal{U}$ belongs to $\mathcal{U}$.

3. Every set of $\mathcal{U}$ is a reflexive relation on X (i.e. contains the diagonal).

4. If V belongs to $\mathcal{U}$, then $V' = \{(y, x) : (x, y) \in V\}$ belongs to $\mathcal{U}$.

5. If V belongs to $\mathcal{U}$, then exists V' in $\mathcal{U}$ such that, whenever $(x, y), (y, z) \in V'$, then $(x, z) \in V$ (i.e. $V' \circ V' \subseteq V$).

The sets of $\mathcal{U}$ are called *entourages* or *vicinities*. The set X together with the uniform structure $\mathcal{U}$ is called a *uniform space*.

If V is an entourage, then for any $(x, y) \in V$ we say that x and y are V*-close*.

Every uniform space can be considered a topological space with a natural topology induced by uniform structure. The uniformity, however, provides in general a richer structure, which formalize the concept of relative closeness: in a uniform space we can say that x is close to y as z is to w, which makes no sense in a topological space. It follows that uniform spaces are the most natural environment for uniformly continuous functions and Cauchy sequences, in which these concepts are naturally involved.

Examples of uniform spaces are metric spaces, topological groups, and topological vector spaces.