examples of locally compact and not locally compact spaces

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Examples of locally compact spaces include:

- The Euclidean spaces $\mathbb{R}^n$ with the standard topology: their local compactness follows from the Heine-Borel theorem. The http://planetmath.org/Complexcompl plane $\mathbb{C}$ carries the same topology as $\mathbb{R}^2$ and is therefore also locally compact.
- All topological manifolds are locally compact since locally they look like Euclidean space.
- Any closed or open subset of a locally compact space is locally compact. In fact, a subset of a locally compact Hausdorff space X is locally compact if and only if it is the http://planetmath.org/SetDifferencedifference of two closed subsets of X (equivalently: the intersection of an open and a closed subset of X).
- The space of http://planetmath.org/PAdicIntegers$p$-adic rationals is homeomorphic to the Cantor set minus one point, and since the Cantor set is compact as a closed bounded subset of $\mathbb{R}$, we see that the p-adic rationals are locally compact.
- Any discrete space is locally compact, since the singletons can serve as compact neighborhoods.
- The long line is a locally compact topological space.
- If you take any unbounded totally ordered set and equip it with the left order topology (or right order topology), you get a locally compact space. This space, unlike all the others we have looked at, is not Hausdorff.

Examples of spaces which are *not* locally compact include:

- The rational numbers $\mathbb{Q}$ with the standard topology inherited from $\mathbb{R}$: each of its compact subsets has empty interior.
- All infinite-dimensional normed vector spaces: a normed vector space is finite-dimensional if and only if its closed unit ball is compact.
- The subset $X = \{(0,0)\} \cup \{(x,y) \mid x > 0\}$ of $\mathbb{R}^2$: no compact subset of X contains a neighborhood of $(0,0)$.