planetmath.org

Math for the people, by the people.

category of Borel spaces

Canonical name	CategoryOfBorelSpaces
Date of creation	2013-03-22 18:25:01
Last modified on	2013-03-22 18:25:01
Owner	bci1 (20947)
Last modified by	bci1 (20947)
Numerical id	16
Author	bci1 (20947)
Entry type	Definition
Classification	msc 54H05
Classification	msc 28A05
Classification	msc 28A12
Classification	msc 28C15
Synonym	category of measure spaces
Related topic	Category
Related topic	BorelSpace
Related topic	BorelGSpace
Related topic	BorelMorphism
Related topic	CategoryOfPointedTopologicalSpaces
Related topic	CategoryOfSets
Related topic	CategoryOfPolishGroups
Related topic	IndexOfCategories
Defines	composition of Borel morphisms
Defines	category of rigid Borel spaces

Definition 0.1. The *category of Borel spaces* $\mathbb{B}$ has, as its objects, all Borel spaces $(X_b; \mathcal{B}(X_b))$, and as its morphisms the Borel morphisms f_b between Borel spaces; the Borel morphism composition is defined so that it preserves the Borel structure determined by the σ-algebra of Borel sets.

Remark 0.1. The *category of (standard) Borel G-spaces* $\mathbb{B}_G$ is defined in a similar manner to $\mathbb{B}$, with the additional condition that Borel G-space morphisms commute with the *Borel actions* $a : G \times X \to X$ defined as `http://planetmath.org/BorelGroupoid`Borel functions (or Borel-measurable maps). Thus, $\mathbb{B}_G$ is a subcategory of $\mathbb{B}$; in its turn, $\mathbb{B}$ is a subcategory of $\mathbb{T}op$–the category of topological spaces and continuous functions.

The *category of rigid Borel spaces* can be defined as above with the additional condition that the only automorphism $f : X_b \to X_b$ (bijection) is the identity $1_{(X_b;\mathcal{B}(X_b))}$.