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local homeomorphism

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Definition. Let X and Y be topological spaces. Continuous map $f : X \to Y$ is said to be *locally invertible in* $x \in X$ iff there exist open subsets $U \subseteq X$ and $V \subseteq Y$ such that $x \in U$, $f(x) \in V$ and the restriction

$$f : U \to V$$

is a homeomorphism. If f is locally invertible in every point of X, then f is called a *local homeomorphism*.

Examples. Of course every homeomorphism is a local homeomorphism, but the converse is not true. For example, let $f : \mathbb{C} \to \mathbb{C}$ be an exponential function, i.e. $f(z) = e^z$. Then f is a local homeomorphism, but it is not a homeorphism (indeed, $f(z) = f(z + 2\pi i)$ for any $z \in \mathbb{C}$).

One of the most important theorem of differential calculus (i.e. inverse function theorem) states, that if $f : M \to N$ is a C^1-map between C^1-manifolds such that $T_x f : T_x M \to T_{f(x)} N$ is a linear isomorphism for a given $x \in M$, then f is locally invertible in x (in this case the local inverse is even a C^1-map).