# alternative characterization of ultrafilter

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Let $X$ be a set. A filter $\mathcal{F}$ over $X$ is an ultrafilter if and only if it satisfies the following condition: if $A \coprod B = X$ (see disjoint union), then either $A \in \mathcal{F}$ or $B \in \mathcal{F}$.

This result can be generalized somewhat: a filter $\mathcal{F}$ over $X$ is an ultrafilter if and only if it satisfies the following condition: if $A \cup B = X$ (see union), then either $A \in \mathcal{F}$ or $B \in \mathcal{F}$.

This theorem can be extended to the following two propositions about finite unions:

1. A filter $\mathcal{F}$ over $X$ is an ultrafilter if and only if, whenever $A_1, \ldots, A_n$ are subsets of $X$ such that $\coprod_{i=1}^n A_i = X$ then there exists exactly one $i$ such that $A_i \in \mathcal{F}$.

2. A filter $\mathcal{F}$ over $X$ is an ultrafilter if and only if, whenever $A_1, \ldots, A_n$ are subsets of $X$ such that $\bigcup_{i=1}^n A_i = X$ then there exists an $i$ such that $A_i \in \mathcal{F}$.