# y-homeomorphism

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The **y-homeomorphism** also dubbed **crosscap slide**, is an auto-homeomorphism (or self-homeomorphism) which can be defined only for **non orientable surfaces** whose genus is greater than one.

To define it we take a punctured Klein bottle $K_0 = K \setminus \text{int } D^2$ which can be consider as a pair of closed Möbius bands $M_1, M_2$, one sewed in the other by perforating with a disk (being disjoint from $\partial M_1$) and then identify the boundary of the second with the boundary of that disk, in symbols:

$$K_0 = (M_1 \setminus \text{int } D^2) \cup_\partial M_2$$

where $\partial = \partial D^2 = \partial M_2$. Other way to visualizing that, is by consider $K_0$ as the connected sum of int $M_1$ with a projective plane $\mathbb{R}P^2$.

Now, thinking that the removed disk $D^2$ was located with its center at some point in the core of $M_1$, the second band, $M_2$ will have a pair of points on that part of the core in common with $\partial M_2$.

So, the y-homeomorphism is defined by a isotopy leaving the boundary $\partial M_1$ fixed by sliding the second band $M_2$ one turn around the core of $M_1$ till the original position. The result is an automorphism of $K_0$ which maps $M_2$ into itself but reversing it.

To define this for genus greater than two just consider any other non orientable surface as a connected sum of a Kein bottle plus projective planes.


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2. M. Korkmaz. *Mapping Class Groups of Non-orientable Surfaces*, Geometriae Dedicata 89 (2002), 109-133.

3. W.B.R. Lickorish. *Homeomorphisms of non-orientable two-manifolds*, Math. Proc. Camb. Phil. Soc. 59 (1963), 307-317.