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Urysohn’s lemma

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Defines	Urysohn function
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A *normal space* is a topological space X such that whenever A and B are disjoint closed subsets of X , then there are disjoint open subsets U and V of X such that $A \subseteq U$ and $B \subseteq V$.

(Note that some authors include T_1 in the definition, which is equivalent to requiring the space to be Hausdorff.)

Urysohn's Lemma states that X is normal if and only if whenever A and B are disjoint closed subsets of X , then there is a continuous function $f: X \rightarrow [0, 1]$ such that $f(A) \subseteq \{0\}$ and $f(B) \subseteq \{1\}$. (Any such function is called an *Urysohn function*.)

A corollary of Urysohn's Lemma is that normal <http://planetmath.org/T1Space> T_1 spaces are completely regular.