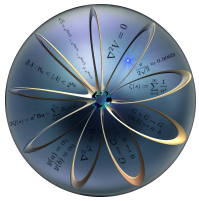

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graph topology

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A graph (V, E) is identified by its vertices $V = \{v_1, v_2, \ldots\}$ and its edges $E = \{\{v_i, v_j\}, \{v_k, v_l\}, \ldots\}$. A graph also admits a natural topology, called the *graph topology*, by identifying every edge $\{v_i, v_j\}$ with the unit interval $I = [0, 1]$ and gluing them together at coincident vertices.

This construction can be easily realized in the framework of simplicial complexes. We can form a simplicial complex $G = \{\{v\} \mid v \in V\} \cup E$. And the desired topological realization of the graph is just the geometric realization $|G|$ of G.

Viewing a graph as a topological space has several advantages:

- The notion of graph isomorphism becomes that of simplicial (or cell) `http://planetmath.org/CWComplex`complex isomorphism.
- The notion of a connected graph coincides with topological `http://planetmath.org/Connect`
- A connected graph is a tree if and only if its fundamental group is trivial.

Remark: A graph is/can be regarded as a one-dimensional CW-complex.