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closed set in a compact space is compact

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*Proof.* Let  $A$  be a closed set in a compact space  $X$ . To show that  $A$  is compact, we show that an arbitrary open cover has a finite subcover. For this purpose, suppose  $\{U_i\}_{i \in I}$  be an arbitrary open cover for  $A$ . Since  $A$  is closed, the complement of  $A$ , which we denote by  $A^c$ , is open. Hence  $A^c$  and  $\{U_i\}_{i \in I}$  together form an open cover for  $X$ . Since  $X$  is compact, this cover has a finite subcover that covers  $X$ . Let  $D$  be this subcover. Either  $A^c$  is part of  $D$  or  $A^c$  is not. In any case,  $D \setminus \{A^c\}$  is a finite open cover for  $A$ , and  $D \setminus \{A^c\}$  is a subcover of  $\{U_i\}_{i \in I}$ . The claim follows.  $\square$