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a space is T_1 if and only if distinct points are separated

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Theorem 1. *Let X be a topological space. Then X is a T_1 -space if and only if sets $\{x\}$, $\{y\}$ are separated for all distinct $x, y \in X$.*

Proof. Suppose X is a T_1 -space. Then every singleton is closed and if $x, y \in X$ are distinct, then

$$\begin{aligned}\{x\} \cap \overline{\{y\}} &= \{x\} \cap \{y\} = \emptyset, \\ \overline{\{x\}} \cap \{y\} &= \{x\} \cap \{y\} = \emptyset,\end{aligned}$$

and $\{x\}$, $\{y\}$ are separated. On the other hand, suppose that $\{x\} \cap \overline{\{y\}} = \emptyset$ for all $x \neq y$. It follows that $\overline{\{y\}} = \{y\}$, so $\{y\}$ is closed and X is a T_1 -space. \square