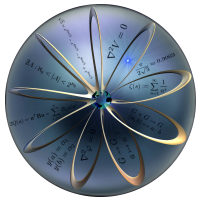

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a space is T1 if and only if every subset A is the intersection of all open sets containing A

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Say we have X, a T^1-space, and A, a subset of X. We aim to show that the intersection of all open sets containing A equals A. By de Morgan's laws, that would be true if the complement of A, A^c, equalled the union of all closed sets in A^c. Let's call this union of closed sets C.

Each set that makes up C is contained by A^c, so $C \subset A^c$. If we could show $A^c \subset C$, we'd be done.

`http://planetmath.org/ASpaceIsT1IfAndOnlyIfEverySingletonIsClosed`Since X is T^1, each singleton in A^c is closed. Their union, a subset of C, contains A^c, so we're through.

Now suppose we know that in some topological space X, any subset A of X is the intersection of all open sets containing A. Given $x \neq y$, we're looking for an open set containing x but not y, to show that X is T^1.

$$\{x\} = \bigcap_{\substack{U \text{ open} \\ U \ni x}} U$$

by hypothesis. If all open sets containing x contained y, y would be in the intersection; since y isn't in the intersection, X must be T^1.