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asymptotically stable

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Defines	Lyapunov stable

Let (X, d) be a metric space and $f\colon X \to X$ a continuous function. A point $x \in X$ is said to be *Lyapunov stable* if for each $\epsilon > 0$ there is $\delta > 0$ such that for all $n \in \mathbb{N}$ and all $y \in X$ such that $d(x, y) < \delta$, we have $d(f^n(x), f^n(y)) < \epsilon$.

We say that x is asymptotically stable if it belongs to the interior of its stable set, i.e. if there is $\delta > 0$ such that $\lim_{n\to\infty} d(f^n(x), f^n(y)) = 0$ whenever $d(x, y) < \delta$.

In a similar way, if $\varphi\colon X \times \mathbb{R} \to X$ is a flow, a point $x \in X$ is said to be Lyapunov stable if for each $\epsilon > 0$ there is $\delta > 0$ such that, whenever $d(x, y) < \delta$, we have $d(\varphi(x, t), \varphi(y, t)) < \epsilon$ for each $t \geq 0$; and x is called asymptotically stable if there is a neighborhood U of x such that $\lim_{t\to\infty} d(\varphi(x, t), \varphi(y, t)) = 0$ for each $y \in U$.