point and a compact set in a Hausdorff space have disjoint open neighborhoods.

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Theorem. *Let X be a Hausdorff space, let A be a compact non-empty set in X, and let y a point in the complement of A. Then there exist disjoint open sets U and V in X such that $A \subset U$ and $y \in V$.*

Proof. First we use the fact that X is a Hausdorff space. Thus, for all $x \in A$ there exist disjoint open sets U_x and V_x such that $x \in U_x$ and $y \in V_x$. Then $\{U_x\}_{x \in A}$ is an open cover for A. Using `http://planetmath.org/YIsCompactIfAndOnlyIfEveryOpe` characterization of compactness, it follows that there exist a finite set $A_0 \subset A$ such that $\{U_x\}_{x \in A_0}$ is a finite open cover for A. Let us define

$$U = \bigcup_{x \in A_0} U_x, \qquad V = \bigcap_{x \in A_0} V_x.$$

Next we show that these sets satisfy the given conditions for U and V. First, it is clear that U and V are open. We also have that $A \subset U$ and $y \in V$. To see that U and V are disjoint, suppose $z \in U$. Then $z \in U_x$ for some $x \in A_0$. Since U_x and V_x are disjoint, z can not be in V_x, and consequently z can not be in V. $\square$

The above result and proof follows [**?**] (Chapter 5, Theorem 7) or [**?**] (page 27).

References

[1] J.L. Kelley, *General Topology*, D. van Nostrand Company, Inc., 1955.

[2] I.M. Singer, J.A.Thorpe, *Lecture Notes on Elementary Topology and Geometry*, Springer-Verlag, 1967.