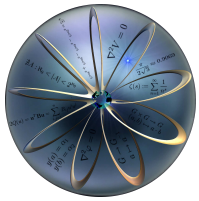

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## vanish at infinity

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|---|---|
| Canonical name | VanishAtInfinity |
| Date of creation | 2013-03-22 17:50:57 |
| Last modified on | 2013-03-22 17:50:57 |
| Owner | asteroid (17536) |
| Last modified by | asteroid (17536) |
| Numerical id | 6 |
| Author | asteroid (17536) |
| Entry type | Definition |
| Classification | msc 54D45 |
| Classification | msc 54C35 |
| Synonym | zero at infinity |
| Synonym | vanishes at infinity |
| Related topic | RegularAtInfinity |
| Related topic | ApplicationsOfUrysohnsLemmaToLocallyCompactHausdorffSpaces |
| Defines | $C_0$ |

Let $X$ be a locally compact space. A function $f : X \longrightarrow \mathbb{C}$ is said to *vanish at infinity* if, for every $\epsilon > 0$, there is a compact set $K \subseteq X$ such that $|f(x)| < \epsilon$ for every $x \in X - K$, where $\| \cdot \|$ denotes the standard `http://planetmath.org/Norm2`norm on $\mathbb{C}$.

If $X$ is non-compact, let $X \cup \{\infty\}$ be the one-point compactification of $X$. The above definition can be rephrased as: The extension of $f$ to $X \cup \{\infty\}$ satisfying $f(\infty) = 0$ is continuous at the point $\infty$.

The set of continuous functions $X \longrightarrow \mathbb{C}$ that vanish at infinity is an algebra over the complex field and is usually denoted by $C_0(X)$.

### 0.0.1 Remarks

- When $X$ is compact, all functions $X \longrightarrow \mathbb{C}$ vanish at infinity. Hence, $C_0(X) = C(X)$.