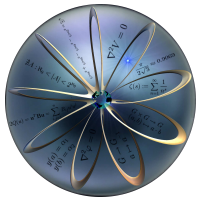

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closure axioms

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A *closure operator* on a set X is an operator which assigns a set A^c to each subset A of X, and such that the following (Kuratowski's closure axioms) hold for any subsets A and B of X:

1. $\emptyset^c = \emptyset$;
2. $A \subset A^c$;
3. $(A^c)^c = A^c$;
4. $(A \cup B)^c = A^c \cup B^c$.

The following theorem due to Kuratowski says that a closure operator characterizes a unique topology on X:

Theorem. Let c be a closure operator on X, and let $\mathcal{T} = \{X - A : A \subseteq X,\ A^c = A\}$. Then $\mathcal{T}$ is a topology on X, and A^c is the $\mathcal{T}$-closure of A for each subset A of X.