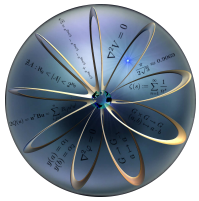

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noncommutative topology

Canonical name	NoncommutativeTopology
Date of creation	2013-03-22 17:40:18
Last modified on	2013-03-22 17:40:18
Owner	asteroid (17536)
Last modified by	asteroid (17536)
Numerical id	14
Author	asteroid (17536)
Entry type	Topic
Classification	msc 54A99
Classification	msc 46L85
Classification	msc 46L05
Related topic	GelfandTransform
Related topic	NoncommutativeGeometry
Defines	noncommutative topology dictionary
Defines	Noncommutative Geometry

1 Noncommutative Topology

Noncommutative topology is basically the theory of http://planetmath.org/CAlgebraC^*-algebras. But why the name noncommutative topology then?

It turns out that commutative C^*-algebras and http://planetmath.org/LocallyCompactHaus compact Hausdorff spaces are one and the same "thing" (this will be explained further ahead). Every commutative C^*-algebra corresponds to a locally compact Hausdorff space and vice-versa and there is a correspondence between topological properties of spaces and C^*-algebraic properties (see the noncommutative topology dictionary below).

The C^*-algebraic properties and concepts are of course present in noncommutative C^*-algebras too. Thus, although noncommutative C^*-algebras cannot be associated with "standard" topological spaces, all the topological/C^* concepts are present. For this reason, this of mathematics was given the name "noncommutative topology".

In this , noncommutative topology can be seen as "topology, but without spaces".

2 The Commutative Case

Given a locally compact Hausdorff space X, all of its topological properties are encoded in $C_0(X)$, the algebra of complex-valued continuous functions in X that vanish at . Notice that $C_0(X)$ is a commutative C^*-algebra.

Conversely, given a commutative C^*-algebra $\mathcal{A}$, the Gelfand transform provides an isomorphism between $\mathcal{A}$ and $C_0(X)$, for a suitable locally compact Hausdorff space X.

Furthermore, there is an http://planetmath.org/EquivalenceOfCategoriesequivalence between the category of locally compact Hausdorff spaces and the category of commutative C^*-algebras. This is the content of the Gelfand-Naimark theorem.

This equivalence of categories is one of the reasons for saying that locally compact Hausdorff spaces and commutative C^*-algebras are the same thing. The other reason is the correspondence between topological and C^*-algebraic properties, present in the following dictionary.

3 Noncommutative Topology Dictionary

We only provide a short list of easy-to-state concepts. Some correspondences of properties are technical and could not be easily stated here. Some of them originate new of "noncommutative mathematics", such as noncommutative measure theory.

Topological properties and concepts	C^***-a**
topological space	C^*-al
proper map	*-hor
homeomorphism	*-isor
open subset	ideal
closed subset	http
compact space	algeb
compactification	uniti
one-point compactification	http
http://planetmath.org/StoneVCechCompactificationStone-Cech compactification	uniti
second countable	separ
connected	proje
connected components and topological sums	proje
complement of singleton	maxi
Radon measure	

3.1 Remarks:

1. Noncommutative topology can be considered as part of http://aux.planetphysics.us/files/ Algebraic Topology (NAAT).

2.A specialized form of noncommutative topology is generally known as http://planetmath.org/NoncommutativeGeometry*Noncommutative Geometry* and has been introduced and developed by Professor Alain Connes (Field Medialist in 1982 and Crafoord Prize in 2001).