hyperbolic metric space

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Defines	δ hyperbolic

Let $\delta \geq 0$. A metric space (X, d) is δ *hyperbolic* if, for any figure ABC in X that is a geodesic triangle with respect to d and for every $P \in \overline{AB}$, there exists a point $Q \in \overline{AC} \cup \overline{BC}$ such that $d(P, Q) \leq \delta$.

A *hyperbolic metric space* is a metric space that is δ hyperbolic for some $\delta \geq 0$.

Although a metric space is hyperbolic if it is δ hyperbolic for some $\delta \geq 0$, one usually tries to find the smallest value of δ for which a hyperbolic metric space (X, d) is δ hyperbolic.

A example of a hyperbolic metric space is the real line under the usual metric. Given any three points $A, B, C \in \mathbb{R}$, we always have that $\overline{AB} \subseteq \overline{AC} \cup \overline{BC}$. Thus, for any $P \in \overline{AB}$, we can take $Q = P$. Therefore, the real line is 0 hyperbolic. reasoning can be used to show that every real tree is 0 hyperbolic.