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regular space

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Definition 1. *A topological space is a regular space if it is both a <http://planetmath.org/TOSpace> space and a <http://planetmath.org/T3Space> T_3 space.*

Example. Consider the set \mathbb{R} with the topology σ generated by the basis

$$\beta = \{U = V - C : V \text{ is open with the standard topology and } C \text{ is (infinite) numerable}\}.$$

Since \mathbb{Q} is numerable and \mathbb{R} open, the set of irrational numbers $\mathbb{R} - \mathbb{Q}$ is open and therefore \mathbb{Q} is closed. It can be shown that $\mathbb{R} - \mathbb{Q}$ is an open set with this topology and \mathbb{Q} is closed.

Take any irrational number x . Any open set V containing all \mathbb{Q} must contain also x , so the regular space property cannot be satisfied. Therefore, (\mathbb{R}, σ) is not a regular space.

Note

In topology, the terminology for separation axioms is not standard. Therefore there are also other meanings of regular. In some references (e.g. [?]) the meanings of regular and T_3 is exchanged. That is, T_3 is a stronger property than regular.

References

- [1] L.A. Steen, J.A. Seebach, Jr., *Counterexamples in topology*, Holt, Rinehart and Winston, Inc., 1970.
- [2] J.L. Kelley, *General Topology*, D. van Nostrand Company, Inc., 1955.