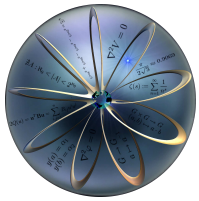

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Stone-Čech compactification

Canonical name	StonevCechCompactification
Date of creation	2013-03-22 14:37:38
Last modified on	2013-03-22 14:37:38
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	10
Author	rspuzio (6075)
Entry type	Definition
Classification	msc 54D30

Stone-Čech compactification is a technique for embedding a Tychonoff topological space in a compact Hausdorff space.

Let X be a Tychonoff space and let C be the space of all continuous functions from X to the closed interval $[0, 1]$. To each element $x \in X$, we may associate the evaluation functional $e_x \colon C \to [0, 1]$ defined by $e_x(f) = f(x)$. In this way, X may be identified with a set of functionals.

The space $[0, 1]^C$ of *all* functionals from C to $[0, 1]$ may be endowed with the Tychonoff product topology. Tychonoff's theorem asserts that, in this topology, $[0, 1]^C$ is a compact Hausdorff space. The closure in this topology of the subset of $[0, 1]^C$ which was identified with X via evaluation functionals is βX, the Stone-Čech compactification of X. Being a closed subset of a compact Hausdorff space, βX is itself a compact Hausdorff space.

This construction has the wonderful property that, for any compact Hausdorff space Y, every continuous function $f \colon X \to Y$ may be extended to a *unique* continuous function $\beta f \colon \beta X \to Y$.