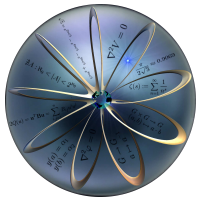

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Hausdorff space

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Synonym	Hausdorff topological space
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Related topic	ASpaceMathnormalXIsHausdorffIfAndOnlyIfDeltaXIsClosed
Related topic	SierpinskiSpace
Related topic	HausdorffSpaceNotCompletelyHausdorff
Related topic	Tychonoff
Related topic	PropertyThatCompactSetsInASpaceAreClosedLies
Defines	Hausdorff
Defines	Hausdorff topology
Defines	T2
Defines	T2 topology
Defines	T2 axiom

A topological space (X, τ) is said to be T_2 (or said to satisfy the T_2 axiom) if given distinct $x, y \in X$, there exist disjoint open sets $U, V \in \tau$ (that is, $U \cap V = \varnothing$) such that $x \in U$ and $y \in V$.

A T_2 space is also known as a *Hausdorff space*. A *Hausdorff topology* for a set X is a topology τ such that (X, τ) is a Hausdorff space.

Properties

The following properties are equivalent:

1. X is a Hausdorff space.
2. The set
$$\Delta = \{(x, y) \in X \times X : x = y\}$$
is closed in the product topology of $X \times X$.
3. For all $x \in X$, we have
$$\{x\} = \bigcap \{A : A \subseteq X \text{ closed}, \exists \text{ open set } U \text{ such that } x \in U \subseteq A\}.$$

Important examples of Hausdorff spaces are metric spaces, manifolds, and topological vector spaces.