compact-open topology

Canonical name	CompactopenTopology
Date of creation	2013-03-22 13:25:26
Last modified on	2013-03-22 13:25:26
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Last modified by	antonio (1116)
Numerical id	8
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Entry type	Definition
Classification	msc 54-00
Synonym	topology of compact convergence
Related topic	UniformConvergence

Let X and Y be topological spaces, and let $C(X,Y)$ be the set of continuous maps from X to Y. Given a compact subspace K of X and an open set U in Y, let

$$\mathcal{U}_{K,U} := \{f \in C(X,Y) : \ f(x) \in U \text{ whenever } x \in K\}.$$

Define the *compact-open topology* on $C(X,Y)$ to be the topology generated by the subbasis

$$\{\mathcal{U}_{K,U} : \ K \subset X \text{ compact}, \quad U \subset Y \text{ open}\}.$$

If Y is a uniform space (for example, if Y is a metric space), then this is the topology of uniform convergence on compact sets. That is, a sequence (f_n) converges to f in the compact-open topology if and only if for every compact subspace K of X, (f_n) converges to f uniformly on K. If in addition X is a compact space, then this is the topology of uniform convergence.