finite intersection property

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Defines	finite intersection property

A collection $\mathcal{A} = \{A_\alpha\}_{\alpha \in I}$ of subsets of a set X is said to have the *finite intersection property*, abbreviated f.i.p., if every finite subcollection $\{A_1, A_2, \ldots, A_n\}$ of $\mathcal{A}$ satisifes $\bigcap_{i=1}^{n} A_i \neq \emptyset$.

The finite intersection property is most often used to give the following http://planetmath.org/node/3769equivalent condition for the http://planetmath.org/node/5 of a topological space (a proof of which may be found http://planetmath.org/node/4181here):

Proposition. *A topological space X is compact if and only if for every collection $\mathcal{C} = \{C_\alpha\}_{\alpha \in J}$ of closed subsets of X having the finite intersection property, $\bigcap_{\alpha \in J} C_\alpha \neq \emptyset$.*

An important special case of the preceding is that in which $\mathcal{C}$ is a countable collection of non-empty nested sets, i.e., when we have

$$C_1 \supset C_2 \supset C_3 \supset \cdots.$$

In this case, $\mathcal{C}$ automatically has the finite intersection property, and if each C_i is a closed subset of a compact topological space, then, by the proposition, $\bigcap_{i=1}^{\infty} C_i \neq \emptyset$.

The f.i.p. characterization of may be used to prove a general result on the uncountability of certain compact Hausdorff spaces, and is also used in a proof of Tychonoff's Theorem.

References

[1] J. Munkres, *Topology*, 2nd ed. Prentice Hall, 1975.