continuity of composition of functions

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All functions in this entry are functions from $\mathbb{R}$ to $\mathbb{R}$.

Example 1 Let $f(x) = 1$ for $x \leq 0$ and $f(x) = 0$ for $x > 0$, let $h(x) = 0$ when $x \in \mathbb{C}$ and 1 when x is irrational, and let $g(x) = h(f(x))$. Then $g(x) = 0$ for all $x \in \mathbb{R}$, so the composition of two discontinuous functions can be continuous.

Example 2 If $g(x) = h(f(x))$ is continuous for all functions f, then h is continuous. Simply put $f(x) = x$. Same thing for h and f. If $g(x) = h(f(x))$ is continuous for all functions h, then f is continuous. Simply put $h(x) = x$.

Example 3 Suppose $g(x) = h(f(x))$ is continuous and f is continuous. Then h does not need to be continuous. For a conterexample, put $h(x) = 0$ for all $x \neq 0$, and $h(0) = 1$, and $f(x) = 1 + |x|$. Now $h(f(x)) = 0$ is continuous, but h is not.

Example 4 Suppose $g(x) = h(f(x))$ is continuous and h is continuous. Then f does not need to be continuous. For a counterexample, put $f(x) = 0$ for all $x \neq 0$, and $f(0) = 1$, and $h(x) = 0$ for all x. Now $h(f(x)) = 0$ is continuous, but f is not.