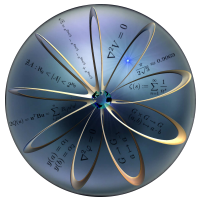

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classification of topological properties according to behaviour under mapping

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Defines	hereditarily
Defines	weakly hereditary
Defines	continuous
Defines	open
Defines	closed invariant

Topological properties may be classified by their behaviour with respect to mappings. The basis of such a classification is the following question: Given two topological spaces X and Y and a continuous map $f\colon X \to Y$, can one infer that one of the spaces has a certain topological property from the fact that the other space has this property?

A trivial case of this question may be disposed of. If f is a homeomorphism, then the spaces X and Y cannot be distinguished using only the techniques of topology, and hence both spaces will have exactly the same topological properties.

To obtain a non-trivial classification, we must consider more general maps. Since every map may be expressed as the composition of an inclusion and a surjection, it is natural to consider the cases where f is an inclusion and where it is a surjection.

In the case of an inclusion, we can define the following classifications:

A property of a topological space is called **hereditary** if it is the case that whenever a space has that property, every subspace of that space also has the same property.

A property of a topological space is called **weakly hereditary** if it is the case that whenever a space has that property, every *closed* subspace of that space also has the same property.

In the case of a surjection, we can define the following classifications:

A property of a topological space is called **continuous** if it is the case that, whenever a space has this property, the images of this space under all continuous mapping also have the same property.

A property of a topological space is called **open** if it is the case that, whenever a space has this property, the images of this space under all open continuous mappings also have the same property.

A property of a topological space is called **closed invariant** if it is the case that, whenever a space has this property, the images of this space under all closed continuous mapping also have the same property.