categories of Polish groups and Polish spaces

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Defines	metrizable topological groups

0.1 Introduction

Definition 0.1. Let us recall that a *Polish space* is a separable, completely metrizable topological space, and that Polish groups G_P are metrizable (topological) groups whose topology is Polish, and thus they admit a compatible metric d which is left-invariant; (a topological group G_T is *metrizable* iff G_T is Hausdorff, and the identity e of G_T has a countable neighborhood basis).

Remark 0.1. Polish spaces can be classified up to a (Borel) isomorphism according to the following provable `http://planetmath.org/PolishSpacesUpToBorelIsomorphism`results:

- "*All uncountable Polish spaces are Borel isomorphic to* $\mathbb{R}$ *equipped with the standard topology;*"

  This also implies that all uncountable Polish space have the cardinality of the continuum.

- "*Two Polish spaces are Borel isomorphic if and only if they have the same cardinality.*"

Furthermore, the subcategory of Polish spaces that are Borel isomorphic is, in fact, a Borel groupoid.

0.2 Category of Polish groups

Definition 0.2. The *category of Polish groups* $\mathcal{P}$ has, as its objects, all Polish groups G_P and, as its morphisms the group homomorphisms g_P between Polish groups, compatible with the *Polish topology* Π on G_P.

Remark 0.2. $\mathcal{P}$ is obviously a subcategory of $\mathcal{T}_{grp}$ the category of topological groups; moreover, $\mathcal{T}_{grp}$ is a subcategory of $\mathcal{T}_{\mathbb{G}}$ -the category of topological groupoids and topological groupoid homomorphisms.