characterization of $T2$ spaces

Canonical name	CharacterizationOfT2Spaces
Date of creation	2013-03-22 14:41:47
Last modified on	2013-03-22 14:41:47
Owner	matte (1858)
Last modified by	matte (1858)
Numerical id	7
Author	matte (1858)
Entry type	Theorem
Classification	msc 54D10
Related topic	LocallyCompactHausdorffSpace

Proposition 1. *[?, ?] Suppose X is a topological space. Then X is a* ***http://planetmath.org/T2Space**T_2 space if and only if for all $x \in X$, we have*

$$\{x\} = \bigcap \{A \mid A \subseteq X \textit{ closed}, \exists \textit{ open set } U \textit{ such that } x \in U \subseteq A\}. \quad (1)$$

Proof. By manipulating the definition using de Morgan's laws, the claim can be rewritten as

$$\{x\}^{\complement} = \bigcup \{V \mid V \subseteq X \text{ open}, \exists \text{ open set } U \text{ such that } x \in U \subseteq V^{\complement}\}.$$

Suppose $y \in \{x\}^{\complement}$. As X is a T_2 space, there are open sets U, V such that $x \in U, y \in V$, and $U \cap V = \emptyset$. Thus, the inclusion from left to right holds. On the other hand, suppose $y \in V$ for some open V such that $\{x\} \subseteq V^{\complement}$. Then

$$y \in V \subseteq \{x\}^{\complement}$$

and the claim follows. □

Notes

If we adopt the notation that a neighborhood of x is any set containing an open set containing x, then the equation **??** can be written as

$$\{x\} = \bigcap \{A \mid A \subseteq X \text{ is a closed neighborhood of } x\}.$$

References

[1] L.A. Steen, J.A.Seebach, Jr., *Counterexamples in topology*, Holt, Rinehart and Winston, Inc., 1970.

[2] N. Bourbaki, *General Topology, Part 1*, Addison-Wesley Publishing Company, 1966.