generalization of a uniformity

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Related topic	GeneralizationOfAPseudometric
Defines	semi-uniformity
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Defines	quasi-uniform space

Let X be a set. Let $\mathcal{U}$ be a family of subsets of $X \times X$ such that $\mathcal{U}$ is a filter, and that every element of $\mathcal{U}$ contains the diagonal relation Δ (reflexive). Consider the following possible "axioms":

1. for every $U \in \mathcal{U}$, $U^{-1} \in \mathcal{U}$
2. for every $U \in \mathcal{U}$, there is $V \in \mathcal{U}$ such that $V \circ V \in U$,

where U^{-1} is defined as the `http://planetmath.org/OperationsOnRelations`inverse relation of U, and $\circ$ is the `http://planetmath.org/OperationsOnRelations`composition of relations. If $\mathcal{U}$ satisfies Axiom 1, then $\mathcal{U}$ is called a *semi-uniformity*. If $\mathcal{U}$ satisfies Axiom 2, then $\mathcal{U}$ is called a *quasi-uniformity*. The underlying set X equipped with $\mathcal{U}$ is called a *semi-uniform space* or a *quasi-uniform space* according to whether $\mathcal{U}$ is a semi-uniformity or a quasi-uniformity.

A semi-pseudometric space is a semi-uniform space. A quasi-pseudometric space is a quasi-uniform space.

A uniformity is one that satisfies both axioms, which is equivalent to saying that it is both a semi-uniformity and a quasi-uniformity.

References

[1] W. Page, *Topological Uniform Structures*, Wiley, New York 1978.