

## planetmath.org

Math for the people, by the people.

## metric equivalence

Canonical name MetricEquivalence
Date of creation 2013-03-22 19:23:11
Last modified on 2013-03-22 19:23:11

Owner CWoo (3771) Last modified by CWoo (3771)

Numerical id 6

Author CWoo (3771)
Entry type Definition
Classification msc 54E35
Defines equivalent

Let X be a set equipped with two metrics  $\rho$  and  $\sigma$ . We say that  $\rho$  is equivalent to  $\sigma$  (on X) if the identity map on X, is a homeomorphism between the metric topology on X induced by  $\rho$  and the metric topology on X induced by  $\sigma$ .

For example, if  $(X, \rho)$  is a metric space, then the function  $\sigma: X \to \mathbb{R}$  defined by

$$\sigma(x,y) := \frac{\rho(x,y)}{1 + \rho(x,y)}$$

is a metric on X that is equivalent to  $\rho$ . This shows that every metric is equivalent to a bounded metric.