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local homeomorphisms between real numbers

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Proposition. Let I be an open interval and $f : I \to \mathbb{R}$ be a continuous map. Then f is a local homeomorphism if and only if f is a homeomorphism onto image.

Proof. ,,$\Leftarrow$" If f is a homeomorphism onto image, then (in particular) f is monotonic and continuous, thus $f(I)$ is open in $\mathbb{R}$ (please, see `http://planetmath.org/InjectiveMapBetweenRealNumbersIsAHomeomorphism`this entry for more details). It is easy to see that therefore f is a local homeomorphism.

,,$\Rightarrow$" Assume that f is not a homeomorphism onto image. It is well known, that this implies that f is not injective (please, see `http://planetmath.org/InjectiveMap` entry for more details). Let $x, y \in I$ be such that $x < y$ and $f(x) = f(y)$. Then there exists $c \in I$ such that $x < c < y$ and c is a local maximum of f. Thus (since f is a Darboux function) for any $\varepsilon > 0$ there are points $x_\varepsilon, y_\varepsilon \in (c - \varepsilon, c + \varepsilon)$ such that $f(x_\varepsilon) = f(y_\varepsilon)$. This obviously implies that f cannot be locally inverted around c. $\square$