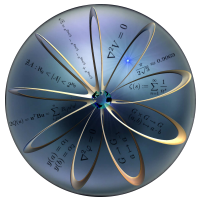

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every filter is contained in an ultrafilter

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Let X be a set and $\mathcal{F}$ be a filter on X. Then there exists an ultrafilter $\mathcal{U}$ on X which is finer than $\mathcal{F}$.

An importance consequnce of this theorem is the existence of free ultrafilters on infinite sets. According to the theorem, there must exist an ultrafilter which is finer than the cofinite filter. Since the cofinite filter is free, every filter finer than it must also be free, and hence there exists a free ultafilter.

Also note that this theorem requires the axiom of choice.