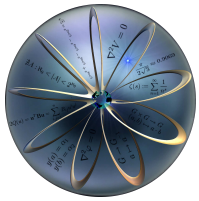

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example of a semilocally simply connected space which is not locally simply connected

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Let HR be the Hawaiian rings, and define X to be the cone over HR. Then, X is connected, locally connected, and semilocally simply connected, but *not* locally simply connected.

Too see this, let $p \in HR$ be the point to which the circles converge in HR, and represent X as $HR \times [0,1]/HR \times \{0\}$. Then, every small enough neighborhood of $q := (p,1) \in X$ fails to be simply connected. However, since X is a cone, it is contractible, so all loops (in particular, loops in a neighborhood of q) can be contracted to a point within X.