support of function

Canonical name	SupportOfFunction
Date of creation	2013-03-22 13:46:10
Last modified on	2013-03-22 13:46:10
Owner	matte (1858)
Last modified by	matte (1858)
Numerical id	16
Author	matte (1858)
Entry type	Definition
Classification	msc 54-00
Synonym	support
Synonym	carrier
Related topic	ZeroOfAFunction
Related topic	ApplicationsOfUrysohnsLemmaToLocallyCompactHausdorffSpaces

Definition Suppose X is a topological space, and $f\colon X \to \mathbb{C}$ is a function. Then the *support* of f (written as $\operatorname{supp} f$), is the set

$$\operatorname{supp} f = \overline{\{x \in X \mid f(x) \neq 0\}}.$$

In other words, $\operatorname{supp} f$ is the closure of the set where f does not vanish.

Properties

Let $f\colon X \to \mathbb{C}$ be a function.

1. $\operatorname{supp} f$ is closed.
2. If $x \notin \operatorname{supp} f$, then $f(x) = 0$.
3. If $\operatorname{supp} f = \emptyset$, then $f = 0$.
4. If $\chi\colon X \to \mathbb{C}$ is such that $\chi = 1$ on $\operatorname{supp} f$, then $f = \chi f$.
5. If $f, g\colon X \to \mathbb{C}$ are functions, then we have
$$\begin{aligned} \operatorname{supp}(fg) &\subset \operatorname{supp} f \cap \operatorname{supp} g, \\ \operatorname{supp}(f+g) &\subset \operatorname{supp} f \cup \operatorname{supp} g. \end{aligned}$$
6. If Y is another topological space, and $\Psi\colon Y \to X$ is a homeomorphism, then
$$\operatorname{supp}(f \circ \Psi) = \Psi^{-1}(\operatorname{supp} f).$$