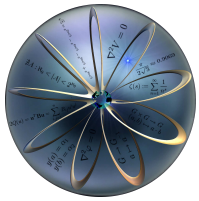

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a complete subspace of a metric space is closed

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Let X be a metric space, and let Y be a complete subspace of X. Then Y is closed.

Proof

Let $x \in \overline{Y}$ be a point in the closure of Y. Then by the definition of closure, from each ball $B(x, \frac{1}{n})$ centered in x, we can select a point $y_n \in Y$. This is clearly a Cauchy sequence in Y, and its limit is x, hence by the completeness of Y, $x \in Y$ and thus $Y = \overline{Y}$.