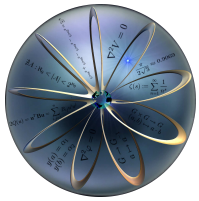

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quasimetric space

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Defines	quasimetric
Defines	quasi-metric

A *quasimetric space* (X, d) is a set X together with a non-negative real-valued function $d : X \times X \longrightarrow \mathbb{R}$ (called a *quasimetric*) such that, for every $x, y, z \in X$,

- $d(x, y) \geq 0$ with equality if and only if $x = y$.
- $d(x, z) \leq d(x, y) + d(y, z)$

In other words, a quasimetric space is a generalization of a metric space in which we drop the requirement that, for two points x and y, the "distance" between x and y is the same as the "distance" between y and x (i.e. the symmetry axiom of metric spaces).

Some properties:

- If (X, d) is a quasimetric space, we can form a metric space (X, d') where d' is defined for all $x, y \in X$ by

$$d'(x, y) = \frac{1}{2}(d(x, y) + d(y, x)).$$

- Every metric space is trivially a quasimetric space.
- A quasimetric that is (i.e. $d(x, y) = d(y, x)$ for all $x, y \in X$ is a metric.

References

[1] L.A. Steen, J.A.Seebach, Jr., *Counterexamples in topology*, Holt, Rinehart and Winston, Inc., 1970.

[2] Z. Shen, *Lectures of Finsler geometry*, World Sientific, 2001.