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path

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Entry type	Definition
Classification	msc 54D05
Synonym	pathwise connected
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Synonym	path connected
Related topic	SimplePath
Related topic	DistanceInAGraph
Related topic	LocallyConnected
Related topic	ExampleOfAConnectedSpaceWhichIsNotPathConnected
Related topic	PathConnectnessAsAHomotopyInvariant
Defines	path
Defines	arc
Defines	arcwise connected
Defines	initial point
Defines	terminal point

Let $I = [0, 1] \subset \mathbb{R}$ and let X be a topological space.

A continuous map $f : I \to X$ such that $f(0) = x$ and $f(1) = y$ is called a *path* in X. The point x is called the **initial point** of the path and y is called its **terminal point**. If, in addition, the map is one-to-one, then it is known as an **arc**.

Sometimes, it is convenient to regard two paths or arcs as equivalent if they differ by a reparameterization. That is to say, we regard $f\colon I \to X$ and $g\colon I \to X$ as equivalent if there exists a homeomorphism $h\colon I \to I$ such that $h(0) = 0$ and $h(1) = 1$ and $f = g \circ h$.

If the space X has extra structure, one may choose to restrict the classes of paths and reparameterizations. For example, if X has a differentiable structure, one may consider the class of differentiable paths. Likewise, one can speak of piecewise linear paths, rectifiable paths, and analytic paths in suitable contexts.

The space X is said to be **pathwise connected** if, for every two points $x, y \in X$, there exists a path having x as initial point and y as terminal point. Likewise, the space X is said to be **arcwise connected** if, for every two distinct points $x, y \in X$, there exists an *arc* having x as initial point and y as terminal point.

A pathwise connected space is always a connected space, but a connected space need not be path connected. An arcwise connected space is always a pathwise connected space, but a pathwise connected space need not be arcwise connected. As it turns out, for Hausdorff spaces these two notions coincide — a Hausdorff space is pathwise connected iff it is arcwise connected.