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**compact Hausdorff space is extremally disconnected if its function algebra is a bounded complete lattice**

Canonical name	CompactHausdorffSpaceIsExtremallyDisconnectedIfItsFunctionAlgebraIsABo
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Synonym	sufficient condition for a compact Hausdorf space to be extremally disconnect

Let  $X$  be a compact Hausdorff space and  $C(X)$  the algebra of continuous functions  $X \rightarrow \mathbb{C}$ . Recall that  $C(X)$  is a vector lattice with the usual <http://planetmath.org/PartialOrder> order:  $f \leq g \iff g - f$  takes positive (or zero) values.

**Theorem -** If every subset of  $C(X)$  that is bounded from above has a least upper bound (i.e.  $C(X)$  is a bounded complete lattice), then  $X$  is extremally disconnected.