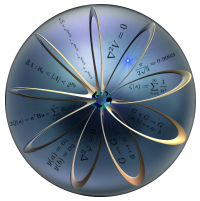

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a space is T1 if and only if every singleton is closed

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Say X is a T_1 topological space. Let's show that $\{x\}$ is closed for every $x \in X$:

The T_1 axiom gives us, for every y distinct from x, an open U_y that contains y but not x. Since we're in a topological space, we can take the union of all these open sets to get a new open set,

$$U = \bigcup_{y \neq x} U_y.$$

$\{x\}$ is the complement of U, closed because U is open: None of the U_y contain x, so U doesn't contain x. But any $y \neq x$ is in U, since $y \in U_y \subset U$. That takes care of that.

Now let's say we have a topological space X in which $\{x\}$ is closed for every $x \in X$. We'd like to show that T_1 holds:

Given $x \neq y$, we want to find an open set that contains x but not y. $\{y\}$ is closed by hypothesis, so its complement is open, and our search is over.