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separable space

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Defines	separable

Definition

A topological space is said to be *separable* if it has a countable dense subset.

Properties

All second-countable spaces are separable. A metric space is separable if and only if it is second-countable.

A continuous image of a separable space is separable.

An open subset of a separable space is separable (in the subspace topology).

A <http://planetmath.org/ProductTopologyproduct> of 2^{\aleph_0} or fewer separable spaces is separable. This is a special case of the Hewitt-Marczewski-Pondiczery Theorem.

A Hilbert space is separable if and only if it has a countable orthonormal basis.