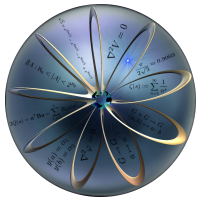

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clopen subset

Canonical name	ClopenSubset
Date of creation	2013-03-22 13:25:29
Last modified on	2013-03-22 13:25:29
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	14
Author	mathcam (2727)
Entry type	Definition
Classification	msc 54D05
Synonym	clopen set
Synonym	clopen
Synonym	closed and open
Related topic	IdentityTheorem

A subset of a topological space X is called *clopen* if it is both open and closed.

Theorem 1. *The clopen subsets form a Boolean algebra under the operation of union, intersection and complement. In other words:*

- *X and $\emptyset$ are clopen,*
- *the complement of a clopen set is clopen,*
- *finite unions and intersections of clopen sets are clopen.*

Proof. The first follows by the definition of a topology, the second by noting that complements of open sets are closed, and vice versa, and the third by noting that this property holds for both open and closed sets. □

One application of clopen sets is that they can be used to describe connectness. In particular, a topological space is connected if and only if its only clopen subsets are itself and the empty set.

If a space has finitely many connected components then each connected component is clopen. This may not be the case if there are infinitely many components, as the case of the rational numbers demonstrates.