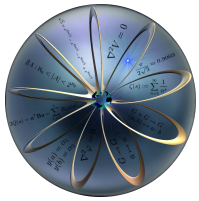

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adjunction space

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Let X and Y be topological spaces, and let A be a subspace of Y. Given a continuous function $f : A \to X$, define the space $Z := X \cup_f Y$ to be the quotient space $X \amalg Y/\sim$, where the symbol $\amalg$ stands for disjoint union and the equivalence relation $\sim$ is generated by

$$y \sim f(y) \quad \text{for all} \quad y \in A.$$

Z is called an *adjunction* of Y to X along f (or along A, if the map f is understood). This construction has the effect of gluing the subspace A of Y to its image in X under f.

Remark 1 Though the definition makes sense for arbitrary A, it is usually assumed that A is a closed subspace of Y. This results in better-behaved adjunction spaces (e.g., the quotient of X by a non-closed set is never Hausdorff).

Remark 2 The adjunction space construction is a special case of the pushout in the category of topological spaces. The two maps being pushed out are f and the inclusion map of A into Y.