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boundary of a closed set is nowhere dense

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Let A be closed. In general, the boundary of a set is closed. So it suffices to show that ∂A has empty interior.

Let $U \subset \partial A$ be open. Since $\partial A \subset \overline{A} = A$, this implies that $U \subset A$. Since $\text{int}(A)$ is the largest open subset of A , we must have $U \subset \text{int}(A)$. Therefore $U \subset \partial A \cap \text{int}(A)$. But $\partial A \cap \text{int}(A) = (\overline{A} - \text{int}(A)) \cap \text{int}(A) = \emptyset$, so $U = \emptyset$.