# the category of T0 Alexandroff spaces is equivalent to the category of posets


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Let $\mathcal{AT}$ be the category of all $\mathrm{T}_0$, Alexandroff spaces and continuous maps between them. Furthermore let $\mathcal{POSET}$ be the category of all posets and order preserving maps.

**Theorem.** The categories $\mathcal{AT}$ and $\mathcal{POSET}$ are equivalent.

*Proof.* Consider two functors:

$$T : \mathcal{AT} \to \mathcal{POSET};$$

$$S : \mathcal{POSET} \to \mathcal{AT},$$

such that $T(X, \tau) = (X, \leq)$, where $\leq$ is an induced partial order on an Alexandroff space and $T(f) = f$ for continuous map. Analogously, let $S(X, \leq) = (X, \tau)$, where $\tau$ is an induced Alexandroff topology on a poset and $S(f) = f$ for order preserving maps. One can easily show that $T$ and $S$ are well defined. Furthermore, it is easy to verify that equalities $T \circ S = 1_{\mathcal{POSET}}$ and $S \circ T = 1_{\mathcal{AT}}$ hold, which completes the proof. $\square$

**Remark.** Of course every finite topological space is Alexandroff, thus we have very nice ,,interpretation" of finite $\mathrm{T}_0$ spaces - finite posets (since functors $T$ and $S$ do not change set-theoretic properties of underlying sets such as finitness).