interior axioms

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Let S be a set. Then an *interior operator* is a function ${}^\circ \colon \mathcal{P}(S) \to \mathcal{P}(S)$ which satisfies the following properties:

Axiom 1. $S^\circ = S$

Axiom 2. *For all $X \subset S$, one has $X^\circ \subseteq S$.*

Axiom 3. *For all $X \subset S$, one has $(X^\circ)^\circ = X^\circ$.*

Axiom 4. *For all $X, Y \subset S$, one has $(X \cap Y)^\circ = X^\circ \cap Y^\circ$.*

If S is a topological space, then the operator which assigns to each set its interior satisfies these axioms. Conversely, given an interior operator ${}^\circ$ on a set S, the set $\{X^\circ \mid X \subset S\}$ defines a topology on S in which X° is the interior of X for any subset X of S. Thus, specifying an interior operator on a set is equivalent to specifying a topology on that set.

The concepts of interior operator and closure operator are closely related. Given an interior operator ${}^\circ$, one can define a closure operator ^{c} by the condition

$$X^c = ((X')^\circ)'$$

and, given a closure operator ^{c}, one can define an interior operator ${}^\circ$ by the condition

$$X^\circ = ((X')^c)'.$$