net

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Defines	subnet
Defines	Moore-Smith convergence
Defines	cluster point

Let X be a set. A *net* is a map from a directed set to X. In other words, it is a pair (A, γ) where A is a directed set and γ is a map from A to X. If $a \in A$ then $\gamma(a)$ is normally written x_a, and then the net is written $(x_a)_{a \in A}$, or simply (x_a) if the direct set A is understood.

Now suppose X is a topological space, A is a directed set, and $(x_a)_{a \in A}$ is a net. Let $x \in X$. Then (x_a) is said to *converge* to x if whenever U is an open neighbourhood of x, there is some $b \in A$ such that $x_a \in U$ whenever $a \geq b$.

Similarly, x is said to be an *accumulation point* (or *cluster point*) of (x_a) if whenever U is an open neighbourhood of x and $b \in A$ there is $a \in A$ such that $a \geq b$ and $x_a \in U$.

Nets are sometimes called *Moore–Smith sequences*, in which case convergence of nets may be called *Moore–Smith convergence*.

If B is another directed set, and $\delta \colon B \to A$ is an increasing map such that $\delta(B)$ is cofinal in A, then the pair $(B, \gamma \circ \delta)$ is said to be a *subnet* of (A, γ). Alternatively, a subnet of a net $(x_\alpha)_{\alpha \in A}$ is sometimes defined to be a net $(x_{\alpha_\beta})_{\beta \in B}$ such that for each $\alpha_0 \in A$ there exists a $\beta_0 \in B$ such that $\alpha_\beta \geq \alpha_0$ for all $\beta \geq \beta_0$.

Nets are a generalisation of `http://planetmath.org/Sequence`sequences, and in many respects they work better in arbitrary topological spaces than sequences do. For example:

- If X is Hausdorff then any net in X converges to at most one point.
- If Y is a subspace of X then $x \in \overline{Y}$ if and only if there is a net in Y converging to x.
- if X' is another topological space and $f \colon X \to X'$ is a map, then f is continuous at x if and only if whenever (x_a) is a net converging to x, $(f(x_a))$ is a net converging to $f(x)$.
- X is compact if and only if every net has a convergent subnet.