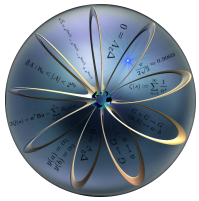

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isomorphism

Canonical name	Isomorphism
Date of creation	2013-03-22 12:19:20
Last modified on	2013-03-22 12:19:20
Owner	djao (24)
Last modified by	djao (24)
Numerical id	6
Author	djao (24)
Entry type	Definition
Classification	msc 54A05
Classification	msc 15A04
Classification	msc 13A99
Classification	msc 20A05
Classification	msc 18A05
Defines	isomorphic
Defines	automorphism

A morphism $f : A \longrightarrow B$ in a category is an *isomorphism* if there exists a morphism $f^{-1} : B \longrightarrow A$ which is its inverse. The objects A and B are *isomorphic* if there is an isomorphism between them.

A morphism which is both an isomorphism and an endomorphism is called an *automorphism*. The set of automorphisms of an object A is denoted $\mathrm{Aut}(A)$.

Examples:

- In the category of sets and functions, a function $f : A \longrightarrow B$ is an isomorphism if and only if it is bijective.
- In the category of groups and group homomorphisms (or rings and ring homomorphisms), a homomorphism $\phi : G \longrightarrow H$ is an isomorphism if it has an inverse map $\phi^{-1} : H \longrightarrow G$ which is also a homomorphism.
- In the category of vector spaces and linear transformations, a linear transformation is an isomorphism if and only if it is an invertible linear transformation.
- In the category of topological spaces and continuous maps, a continuous map is an isomorphism if and only if it is a homeomorphism.