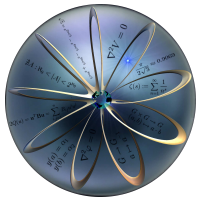

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restriction of a continuous mapping is continuous

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Theorem Suppose X and Y are topological spaces, and suppose $f : X \to Y$ is a continuous function. For a subset $A \subset X$, the `http://planetmath.org/RestrictionOfAFun` of f to A (that is $f|_A$) is a continuous mapping $f|_A : A \to Y$, where A is given the subspace topology from X.

Proof. We need to show that for any open set $V \subset Y$, we can write $(f|_A)^{-1}(V) = A \cap U$ for some set U that is open in X. However, by the properties of the inverse image (see `http://planetmath.org/InverseImage`this page), we have for any open set $V \subset Y$,

$$(f|_A)^{-1}(V) = A \cap f^{-1}(V).$$

Since $f : X \to Y$ is continuous, $f^{-1}(V)$ is open in X, and our claim follows. □