condensation point

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Let X be a topological space and $A \subset X$. A point $x \in X$ is called a *condensation point* of A if every open neighbourhood of x contains *uncountably* many points of A.

For example, if $X = \mathbb{R}$ and A any subset, then any accumulation point of A is automatically a condensation point. But if $X = \mathbb{Q}$ and A any subset, then A does not have any condensation points at all.

We have further classifications of *condensation point* where the topological space is an ordered field. Namely,

1. *unilateral condensation point:* x is a condensation point of A and there is a positive ϵ with either $(x-\epsilon, x) \cap A$ countable or $(x, x+\epsilon) \cap A$ countable.

2. *bilateral condensation point:* For all $\epsilon > 0$, we have both $(x-\epsilon, x) \cap A$ and $(x, x+\epsilon) \cap A$ uncountable.

   If κ is any cardinal (i.e. an ordinal which is the least among all ordinals of the same cardinality as itself), then a κ-condensation point can be defined similarly.