composition with coercive function

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Theorem 1. *Suppose X, Y, Z are topological spaces, $f\colon X \to Y$ is a bijective proper map, and $g\colon Y \to Z$ is a coercive map. Then $g \circ f\colon X \to Z$ is a coercive map.*

Proof. Let $J \subseteq Z$ be a compact set. As g is coercive, there is a compact set $K \subseteq Y$ such that

$$g(Y \setminus K) \subseteq Z \setminus J.$$

Let $I = f^{-1}(K)$, and since f is a proper map I is compact. Thus

$$(g \circ f)(X \setminus I) = g(Y \setminus K) \subseteq Z \setminus J$$

and $g \circ f$ is coercive. □