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examples of nowhere dense sets

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Note that $\mathbb{Z}$ is nowhere dense in $\mathbb{R}$ under the usual topology: $\operatorname{int}\overline{\mathbb{Z}} = \operatorname{int}\mathbb{Z} = \emptyset$. Similarly, $\frac{1}{n}\mathbb{Z}$ is nowhere dense for every $n \in \mathbb{Z}$ with $n > 0$.

This result provides an alternative way to prove that $\mathbb{Q}$ is meager in $\mathbb{R}$ under the usual topology, since $\mathbb{Q} = \bigcup\limits_{n \in \mathbb{Z} \text{ and } n > 0} \frac{1}{n}\mathbb{Z}$.