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paracompact topological space

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Related topic	ExampleOfParacompactTopologicalSpaces
Defines	paracompact
Defines	paracompactness

A topological space  $X$  is said to be *paracompact* if every open cover of  $X$  has a locally finite open refinement.

In more detail, if  $(U_i)_{i \in I}$  is any family of open subsets of  $X$  such that

$$\cup_{i \in I} U_i = X ,$$

then there exists another family  $(V_i)_{i \in I}$  of open sets such that

$$\cup_{i \in I} V_i = X$$

$$V_i \subset U_i \text{ for all } i \in I$$

and any specific  $x \in X$  is in  $V_i$  for only finitely many  $i$ .

Some properties:

- Any metric or metrizable space is paracompact (A. H. Stone).
- Given an open cover of a paracompact space  $X$ , there exists a (continuous) partition of unity on  $X$  subordinate to that cover.
- A paracompact , Hausdorff space is regular.
- A compact or pseudometric space is paracompact.