interior

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Defines	exterior

Let A be a subset of a topological space X.

The union of all open sets contained in A is defined to be the *interior* of A. Equivalently, one could define the interior of A to the be the largest open set contained in A.

In this entry we denote the interior of A by $\operatorname{int}(A)$. Another common notation is A°.

The *exterior* of A is defined as the union of all open sets whose intersection with A is empty. That is, the exterior of A is the interior of the complement of A.

The interior of a set enjoys many special properties, some of which are listed below:

1. $\operatorname{int}(A) \subseteq A$
2. $\operatorname{int}(A)$ is open
3. $\operatorname{int}(\operatorname{int}(A)) = \operatorname{int}(A)$
4. $\operatorname{int}(X) = X$
5. $\operatorname{int}(\varnothing) = \varnothing$
6. A is open if and only if $A = \operatorname{int}(A)$
7. $\overline{A^{\complement}} = (\operatorname{int}(A))^{\complement}$
8. $\overline{A}^{\complement} = \operatorname{int}(A^{\complement})$
9. $A \subseteq B$ implies that $\operatorname{int}(A) \subseteq \operatorname{int}(B)$
10. $\operatorname{int}(A) = A \setminus \partial A$, where ∂A is the boundary of A
11. $X = \operatorname{int}(A) \cup \partial A \cup \operatorname{int}(A^{\complement})$

References

[1] S. Willard, *General Topology*, Addison-Wesley Publishing Company, 1970.