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separated

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Definition Suppose A and B are subsets of a topological space X . Then A and B are **separated** provided that

$$\begin{aligned}\overline{A} \cap B &= \emptyset, \\ A \cap \overline{B} &= \emptyset,\end{aligned}$$

where \overline{A} is the <http://planetmath.org/Closure> closure operator in X .

Properties

1. If A, B are separated in X , and $f: X \rightarrow Y$ is a homeomorphism, then $f(A)$ and $f(B)$ are separated in Y .

Examples

1. On \mathbb{R} , the intervals $(0, 1)$ and $(1, 2)$ are separated.
2. If $d(x, y) \geq r + s$, then the open balls $B_r(x)$ and $B_s(y)$ are separated <http://planetmath.org/WhenAreBallsSeparated>(proof.).
3. If A is a clopen set, then A and A^c are separated. This follows since $\overline{S} = S$ when S is a closed set.

Remarks

The above definition follows [?]. In [?], separated sets are called **strongly disjoint** sets.

References

- [1] J.L. Kelley, *General Topology*, D. van Nostrand Company, Inc., 1955.
- [2] G.J. Jameson, *Topology and Normed Spaces*, Chapman and Hall, 1974.