Cantor's Intersection Theorem

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Owner	paolini (1187)
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Theorem 1. *Let $K_1 \supset K_2 \supset K_3 \supset \ldots \supset K_n \supset \ldots$ be a sequence of non-empty, compact subsets of a metric space X. Then the intersection $\bigcap_i K_i$ is not empty.*

Proof. Choose a point $x_i \in K_i$ for every $i = 1, 2, \ldots$ Since $x_i \in K_i \subset K_1$ is a sequence in a compact set, by Bolzano-Weierstrass Theorem, there exists a subsequence x_{i_j} which converges to a point $x \in K_1$. Notice, however, that for a fixed index n, the sequence x_{i_j} lies in K_n for all j sufficiently large (namely for all j such that $i_j > n$). So one has $x \in K_n$. Since this is true for every n, the result follows. $\square$