spherical metric

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Suppose that $\hat{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$ is the extended complex plane (the Riemann sphere).

Definition. Suppose $\gamma\colon [0,1] \to \hat{\mathbb{C}}$ is a path in $\hat{\mathbb{C}}$. The *spherical length* of γ is defined as

$$\ell(\gamma) := 2\int_\gamma \frac{|dz|}{1+|z|^2} = 2\int_0^1 \frac{|\gamma'(t)|}{1+|\gamma(t)|^2}dt.$$

Definition. Let $z_1, z_2 \in \hat{\mathbb{C}}$, and let Γ be the set of all paths in $\hat{\mathbb{C}}$ from z_1 to z_2, then the distance from z_1 to z_2 in the *spherical metric* is defined as

$$\sigma(z_1, z_2) := \inf_{\gamma\in\Gamma} \ell(\gamma).$$

More intuitivelly this is the shortest distance to travel from z_1 to z_2 if we think of these points as being on the Riemann sphere, and we can only travel on the Riemann sphere itself (we cannot “drill” a straight line from z_1 to z_2).

References

[1] Theodore B. Gamelin. . Springer-Verlag, New York, New York, 2001.