homeomorphisms preserve connected components

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Let X, Y be topological spaces and $X = \bigcup X_i$, $Y = \bigcup Y_j$ be decompositions into connected components.

Proposition. Assume that $f : X \to Y$ is a homeomorphism. Then for any i there exists j such that $f(X_i) = Y_j$.

Proof. Take any i. Because f is continuous $f(X_i)$ is connected, then there exists j such that $f(X_i) \subseteq Y_j$ (because Y_j is a connected component). Now f is a homeomorphism, $f^{-1}(Y_j) \cap X_i \neq \emptyset$, Y_j is connected and X_i is a connected component, so $f^{-1}(Y_j) \subseteq X_i$. Thus $Y_j \subseteq f(X_i)$, which completes the proof. $\square$