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a compact set in a Hausdorff space is closed

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Theorem. A compact set in a Hausdorff space is closed.

Proof. Let A be a compact set in a Hausdorff space X . The case when A is empty is trivial, so let us assume that A is non-empty. Using <http://planetmath.org/APointAndACompactSetInAHausdorffSpaceHaveDisjointOpenNeighborhoods> theorem, it follows that each point y in A^c has a neighborhood U_y , which is disjoint to A . (Here, we denote the complement of A by A^c .) We can therefore write

$$A^c = \bigcup_{y \in A^c} U_y.$$

Since an arbitrary union of open sets is open, it follows that A is closed. \square

Note.

The above theorem can, for instance, be found in [?] (page 141), or [?] (Section 2.1, Theorem 2).

References

- [1] J.L. Kelley, *General Topology*, D. van Nostrand Company, Inc., 1955.
- [2] I.M. Singer, J.A.Thorpe, *Lecture Notes on Elementary Topology and Geometry*, Springer-Verlag, 1967.