continuity is preserved when codomain is extended

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Theorem 1. *Suppose X, Y are topological space and let $Z \subseteq Y$ be equipped with the subspace topology. If*

$$f \colon X \to Z$$

is continuous, then

$$f \colon X \to Y$$

is continuous.

Proof. Let $U \subseteq Y$ be an open set. Then

$$
\begin{aligned}
f^{-1}(U) &= \{x \in X : f(x) \in U\} \\
&= \{x \in X : f(x) \in U \cap Z\} \\
&= f^{-1}(U \cap Z).
\end{aligned}
$$

Since $U \cap Z$ is open in Z, $f^{-1}(U)$ is open in X. $\square$