



Math for the people, by the people.

product of metric spaces

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Theorem 1. *Let (X_i, ϱ_i) be a metric space for each $i = 1, 2, \dots$, where the diameter of X_i using ϱ_i is less than $1/i$. Then the product topology for the space $\prod_{i=1}^{\infty} X_i$ is given by the metric*

$$\varrho(x, y) = \sum_{i=1}^{\infty} \frac{1}{2^i} \varrho_i(x_i, y_i).$$

Hence, a countable product of metrizable topological spaces is again metrizable.

Since the convergence in the product topology is the pointwise convergence, the same is true for the metric space with the above metric.