Alexandroff space

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Topological space X is called *Alexandroff* if the intersection of every family of open sets is open.

Of course every finite topological space is Alexandroff, but there are also bigger Alexandroff spaces. For example let $\mathbb{R}$ denote the set of real numbers and let $\tau = \{[a, \infty) \mid a \in \mathbb{R}\} \cup \{(b, \infty) \mid b \in \mathbb{R}\}$. Then τ is a topology on $\mathbb{R}$ and $(\mathbb{R}, \tau)$ is an Alexandroff space.

If X is an Alexandroff space and $A \subseteq X$, then we may talk about smallest open neighbourhood of A. Indeed, let

$$A^o = \bigcap \{U \subseteq X \mid U \text{ is open and } A \text{ is contained in } U\}.$$

Then A^o is open.