example of pseudometric space

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Defines	trivial pseudometric

Let $X = \mathbb{R}^2$ and consider the function $d : X \times X$ to the non-negative real numbers given by

$$d((x_1, x_2), (y_1, y_2)) = |x_1 - y_1|.$$

Then $d(x, x) = |x_1 - x_1| = 0$, $d(x, y) = |x_1 - y_1| = |y_1 - x_1| = d(y, z)$ and the triangle inequality follows from the triangle inequality on $\mathbb{R}^1$, so (X, d) satisfies the defining conditions of a pseudometric space.

Note, however, that this is not an example of a metric space, since we can have two distinct points that are distance 0 from each other, e.g.

$$d((2, 3), (2, 5)) = |2 - 2| = 0.$$

Other examples:

- Let X be a set, $x_0 \in X$, and let $F(X)$ be functions $X \to R$. Then $d(f, g) = |f(x_0) - g(x_0)|$ is a pseudometric on $F(X)$ [**?**].
- If X is a vector space and p is a seminorm over X, then $d(x, y) = p(x-y)$ is a pseudometric on X.
- The *trivial pseudometric* $d(x, y) = 0$ for all $x, y \in X$ is a pseudometric.

References

[1] S. Willard, *General Topology*, Addison-Wesley, Publishing Company, 1970.