differential entropy

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Let $(X, \mathfrak{B}, \mu)$ be a probability space, and let $f \in L^p(X, \mathfrak{B}, \mu)$, $||f||_p = 1$ be a function. The *differential entropy* $h(f)$ is defined as

$$h(f) \equiv -\int_X |f|^p \log |f|^p \; d\mu \tag{1}$$

Differential entropy is the continuous version of the Shannon entropy, $H[\mathbf{p}] = -\sum_i p_i \log p_i$. Consider first u_a, the uniform 1-dimensional distribution on $(0, a)$. The differential entropy is

$$h(u_a) = -\int_0^a \frac{1}{a} \log \frac{1}{a} \; d\mu = \log a. \tag{2}$$

Next consider probability distributions such as the function

$$g = \frac{1}{2\pi\sigma} e^{-\frac{(t-\mu)^2}{2\sigma^2}}, \tag{3}$$

the 1-dimensional Gaussian. This pdf has differential entropy

$$h(g) = -\int_{\mathbb{R}} g \log g \; dt = \frac{1}{2} \log 2\pi e \sigma^2. \tag{4}$$

For a general n-dimensional `http://planetmath.org/JointNormalDistribution`Gaussian $\mathcal{N}_n(\mu, \mathbf{K})$ with mean vector μ and covariance matrix $\mathbf{K}$, $K_{ij} = \operatorname{cov}(x_i, x_j)$, we have

$$h(\mathcal{N}_n(\mu, \mathbf{K})) = \frac{1}{2} \log(2\pi e)^n |\mathbf{K}| \tag{5}$$

where $|\mathbf{K}| = \det \mathbf{K}$.