proof that components of open sets in a locally connected space are open

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Theorem. *A topological space X is locally connected if and only if each component of an open set is open.*

Proof. First, suppose that X is locally connected and that U is an open set of X. Let $p \in C$, where C is a component of U. Since X is locally connected there is an open connected set, say V with $p \in V \subset U$. Since C is a component of U it must be that $V \subset C$. Hence, C is open. For the converse, suppose that each component of each open set is open. Let $p \in X$. Let U be an open set containing p. Let C be the component of U which contains p. Then C is open and connected, so X is locally connected. $\square$

As a corollary, we have that the components of a locally connected space are both open and closed.