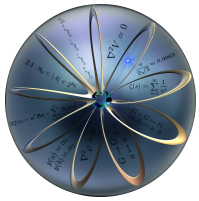

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## sober space

| | |
|---|---|
| Canonical name | SoberSpace |
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| Entry type | Definition |
| Classification | msc 54E99 |
| Defines | irreducible set |

Let $X$ be a topological space. A subset $A$ of $X$ is said to be *irreducible* if whenever $A \subseteq B \cup C$ with $B, C$ closed, we have $A \subseteq B$ or $A \subseteq C$. Any singleton and its closure are irreducible. More generally, the closure of an irreducible set is irreducible.

A topological space $X$ is called a *sober space* if every irreducible closed subset is the closure of some *unique* point in $X$.

**Remarks**.

- For any sober space, the closure of a point determines the point. In other words, $\mathrm{cl}(x) = \mathrm{cl}(y)$ implies $x = y$.
- A space is sober iff the closure of every irreducible set is the closure of a unique point.
- Any sober space is T0.
- Any Hausdorff space is sober.
- A closed subspace of a sober space is sober.
- Any product of sober spaces is sober.