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properties of the closure operator

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Suppose X is a topological space, and let \overline{A} be the closure of A in X . Then the following properties hold:

1. $\overline{A} = A \cup A'$ where A' is the derived set of A .
2. $A \subseteq \overline{A}$, and $A = \overline{A}$ if and only if A is closed
3. $\overline{A} = \emptyset$ if and only if $A = \emptyset$.
4. If Y is another topological space, then $f: X \rightarrow Y$ is a continuous map, if and only if $\overline{f(A)} \subseteq f(\overline{A})$ for all $A \subseteq X$. If f is also a homeomorphism, then $f(\overline{A}) = \overline{f(A)}$.