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Cauchy sequence

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A sequence x_0, x_1, x_2, \dots in a metric space (X, d) is a *Cauchy sequence* if, for every real number $\epsilon > 0$, there exists a natural number N such that $d(x_n, x_m) < \epsilon$ whenever $n, m > N$.

Likewise, a sequence v_0, v_1, v_2, \dots in a topological vector space V is a *Cauchy sequence* if and only if for every neighborhood U of $\mathbf{0}$, there exists a natural number N such that $v_n - v_m \in U$ for all $n, m > N$. These two definitions are equivalent when the topology of V is induced by a metric.