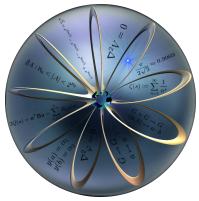

planetmath.org

Math for the people, by the people.

completely Hausdorff

Canonical name	CompletelyHausdorff
Date of creation	2013-03-22 14:16:03
Last modified on	2013-03-22 14:16:03
Owner	PrimeFan (13766)
Last modified by	PrimeFan (13766)
Numerical id	15
Author	PrimeFan (13766)
Entry type	Definition
Classification	msc 54D10
Synonym	completely Hausdorff space
Synonym	$T_{2\frac{1}{2}}$
Synonym	Urysohn space
Related topic	HausdorffSpaceNotCompletelyHausdorff

Definition 1. *[?] Let (X, τ) be a topological space. Suppose that for any two different points $x, y \in X, x \neq y$, we can find two disjoint neighborhoods*

$$U_x, V_y \in \tau, \qquad x \in U_x, y \in Y_y$$

such that their closures are also disjoint:

$$\overline{U_x} \cap \overline{V_y} = \emptyset.$$

Then we say that (X, τ) is a completely Hausdorff space *or a $T_{2\frac{1}{2}}$* space.

Notes

A synonym for functionally Hausdorff space is *Urysohn space* [**?**]. Unfortunately, the definition of completely Hausdorff and $T_{2\frac{1}{2}}$ are not as standard as one would like since. For example, the term completely Hausdorff space is also used to mean a functionally Hausdorff space (e.g. [**?**]). Nevertheless, in the present convention, we have the implication:

$$\text{functionally Hausdorff} \Rightarrow \text{completely Hausdorff} \Rightarrow T_2 = \text{Hausdorff},$$

which suggests why the $T_{2\frac{1}{2}}$ name have been used to denote both completely Hausdorff spaces and functionally Hausdorff spaces.

References

[1] L.A. Steen, J.A.Seebach, Jr., *Counterexamples in topology*, Holt, Rinehart and Winston, Inc., 1970.

[2] S. Willard, *General Topology*, Addison-Wesley Publishing Company, 1970.