every σ-compact set is Lindelöf

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Theorem 1. *Every `http://planetmath.org/SigmaCompact`σ-compact set is Lindelöf (every open cover has a countable subcover).*

Proof. Let X be a σ-compact. Let $\mathcal{A}$ be an open cover of X . Since X is σ-compact, it is the union of countable many compact sets,

$$X = \bigcup_{i=0}^{\infty} X_i$$

with X_i compact. Consider the cover $\mathcal{A}_i = \{A \in \mathcal{A} : X_i \cap A \neq \emptyset\}$ of the set X_i. This cover is well defined, it is not empty and covers X_i: for each $x \in X_i$ there is at least one of the open sets $A \in \mathcal{A}$ such that $x \in A$.

Since X_i is compact, the cover $\mathcal{A}_i$ has a finite subcover. Then

$$X_i \subseteq \bigcup_{j=0}^{N_j} A_i^j$$

and thus

$$X \subseteq \bigcup_{i=0}^{\infty} \left(\bigcup_{j=0}^{N_j} A_i^j \right).$$

That is, the set $\left\{A_i^j\right\}$ is a countable subcover of $\mathcal{A}$ that covers X. $\square$