



Math for the people, by the people.

join

Canonical name	Join1
Date of creation	2013-03-22 13:25:40
Last modified on	2013-03-22 13:25:40
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	7
Author	mathcam (2727)
Entry type	Definition
Classification	msc 54B99
Related topic	Cone
Related topic	Suspension
Defines	join

Given two topological spaces  $X$  and  $Y$ , their *join*, denoted by  $X \star Y$ , is defined to be the quotient space

$$X \star Y := X \times [0, 1] \times Y / \sim,$$

where the equivalence relation  $\sim$  is generated by

$$\begin{aligned} (x, 0, y_1) &\sim (x, 0, y_2) \quad \text{for any } x \in X, y_1, y_2 \in Y, \text{ and} \\ (x_1, 1, y) &\sim (x_2, 1, y) \quad \text{for any } y \in Y, x_1, x_2 \in X. \end{aligned}$$

Intuitively,  $X \star Y$  is formed by taking the disjoint union of the two spaces and attaching a line segment joining every point in  $X$  to every point in  $Y$ .

Some examples:

- The join of a space  $X$  with a one-point space is called the *cone* of  $X$ .
- The join of the spheres  $S^n$  and  $S^m$  is the sphere  $S^{n+m+1}$ .