

Question 1:

$$\text{In[ ]:= } \mathbf{a[t\_]} = 2 + 2 \cos[t]$$

$$\text{Out[ ]:= } 2 + 2 \cos[t]$$

$$\text{In[ ]:= } \mathbf{b[t\_]} = 2 + 2 \sin[t]$$

$$\text{Out[ ]:= } 2 + 2 \sin[t]$$

$$\text{In[ ]:= } \mathbf{c[t\_]} = 2 - \cos[t/2]$$

$$\text{Out[ ]:= } 2 - \cos\left[\frac{t}{2}\right]$$

$$\text{In[ ]:= } \mathbf{d[t\_]} = 2 - \sin[t/2]$$

$$\text{Out[ ]:= } 2 - \sin\left[\frac{t}{2}\right]$$

$$\text{In[ ]:= } \mathbf{e[t\_]} = 1 + \cos[t] / 12$$

$$\text{Out[ ]:= } 1 + \frac{\cos[t]}{12}$$

$$\text{In[ ]:= } \mathbf{f[t\_]} = 3 + \sin[t] / 12$$

$$\text{Out[ ]:= } 3 + \frac{\sin[t]}{12}$$

$$\text{In[ ]:= } \mathbf{g[t\_]} = 3 + \cos[t] / 12$$

$$\text{Out[ ]:= } 3 + \frac{\cos[t]}{12}$$

$$\text{In[ ]:= } \mathbf{h[t\_]} = 2 + \sin[t] / 8$$

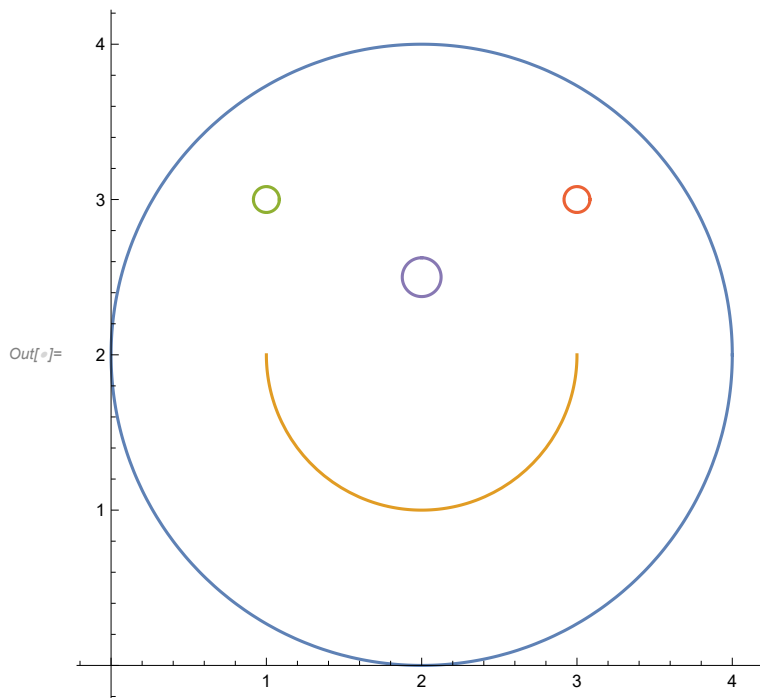
$$\text{Out[ ]:= } 2 + \frac{\sin[t]}{8}$$

$$\text{In[ ]:= } \mathbf{i[t\_]} = 2.5 + \cos[t] / 8$$

$$\text{Out[ ]:= } 2.5 + \frac{\cos[t]}{8}$$

Note: h[t] and i[t] are the extra nose I gave him for fun.

```
In[ ]:= ParametricPlot[
  {{a[t], b[t]}, {c[t], d[t]}, {e[t], f[t]}, {g[t], f[t]}, {h[t], i[t]}}, {t, 0, 2 π}]
```



```
In[ ]:= Clear[a, b, c, d, e, f, g, h, i]
```

### Question 2:

```
In[ ]:= x[t_] = Sin[π * t]
```

```
Out[ ]:= Sin[π t]
```

```
In[ ]:= y[t_] = t^2 + t
```

```
Out[ ]:= t + t^2
```

```
In[ ]:= x[1]
```

```
Out[ ]:= 0
```

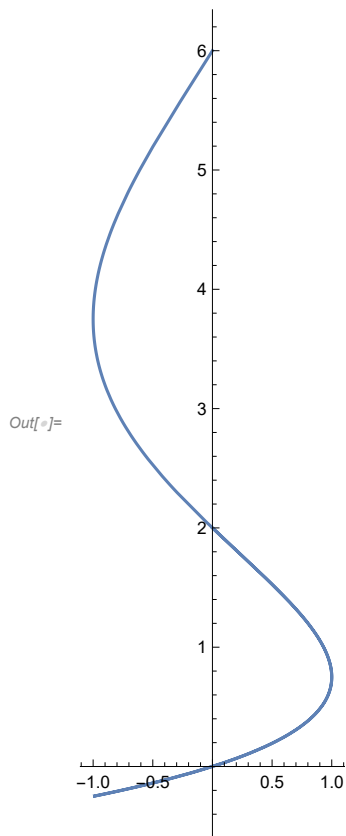
```
In[ ]:= y[1]
```

```
Out[ ]:= 2
```

```
In[ ]:= m[t_] =  $\frac{y'[t]}{x'[t]}$ 
```

```
Out[ ]:=  $\frac{(1 + 2 t) \operatorname{Sec}[\pi t]}{\pi}$ 
```

In[ ]:= **p1 = ParametricPlot[{x[t], y[t]}, {t, -2, 2}]**



In[ ]:= **m[1]**

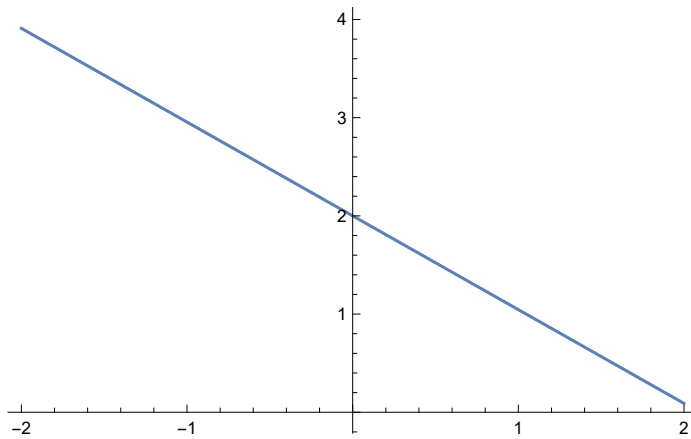
Out[ ]:=  $-\frac{3}{\pi}$

In[ ]:= **Solve** $\left[y - y[1] == -\frac{3}{\pi} (x - x[1]), y\right]$

Out[ ]:=  $\left\{\left\{y \rightarrow \frac{2\pi - 3x}{\pi}\right\}\right\}$

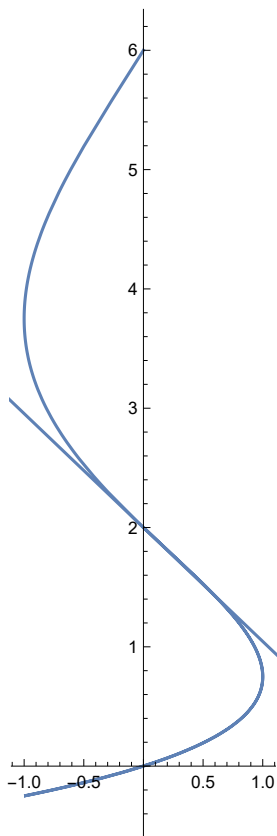
```
In[ ]:= p2 = Plot[ $\frac{2\pi - 3x}{\pi}$ , {x, -2, 2}]
```

```
Out[ ]:=
```



```
In[ ]:= Show[p1, p2]
```

```
Out[ ]:=
```



```
In[ ]:= Clear[x, y, m]
```

Question 3:

```
In[ ]:= x[t_] = Cos[t] + Log[Tan[ $\frac{1}{2}$  * t]]
```

```
Out[ ]:= Cos[t] + Log[Tan[ $\frac{t}{2}$ ]]
```

```
In[ ]:= x[ $\pi / 4$ ]
```

```
Out[ ]:=  $\frac{1}{\sqrt{2}}$  + Log[Tan[ $\frac{\pi}{8}$ ]]
```

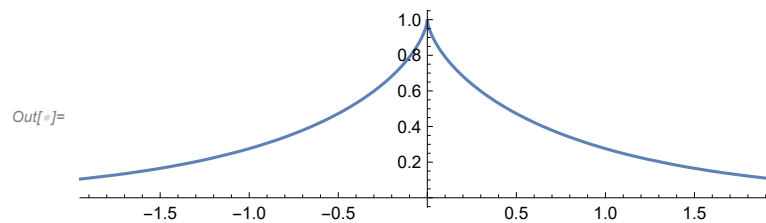
```
In[ ]:= y[t_] = Sin[t]
```

```
Out[ ]:= Sin[t]
```

```
In[ ]:= y[ $\pi / 2$ ]
```

```
Out[ ]:= 1
```

```
In[ ]:= ParametricPlot[{x[t], y[t]}, {t, 0, 2  $\pi$ }]
```



```
In[ ]:= L[t_] =  $\int_{\pi/4}^{3\pi/4} \sqrt{(x'[t])^2 + (y'[t])^2} dt$ 
```

```
Out[ ]:= Log[2]
```

```
In[ ]:= N[%]
```

```
Out[ ]:= 0.693147
```

```
In[46]:= Clear[x, y, L]
```

Question 4:

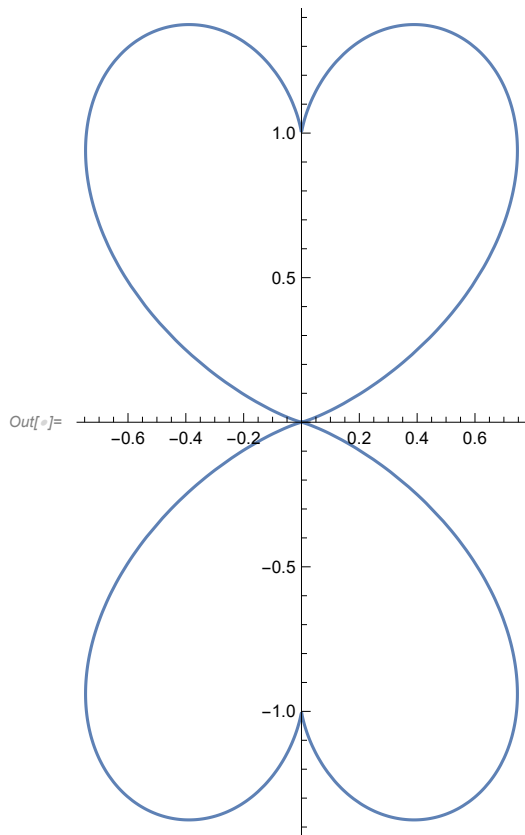
```
In[ ]:= r[ $\theta$ _] = Abs[Tan[ $\theta$ ]] ^ Abs[Cot[ $\theta$ ]]
```

```
Out[ ]:= Abs[Tan[ $\theta$ ]] ^ Abs[Cot[ $\theta$ ]]
```

In[ ]:= **PolarPlot**[r[ $\theta$ ], { $\theta$ ,  $-\pi$ ,  $\pi$ }]

General: 0.000128228<sup>7798.59</sup> is too small to represent as a normalized machine number; precision may be lost.

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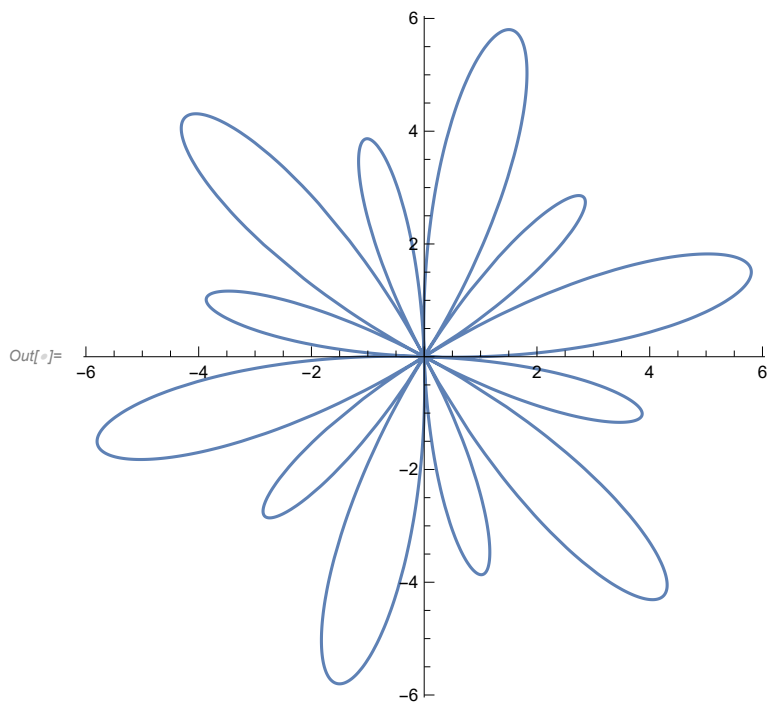
In[ ]:= **Clear**[r]

Question 5:

In[ ]:= **r**[ $\theta$ \_] = **1 + 5 Sin**[**6 \*  $\theta$** ]

Out[ ]:= **1 + 5 Sin**[**6  $\theta$** ]

In[ ]:= **PolarPlot**[r[θ], {θ, 0, 2 π}]



In[ ]:= **A**[θ\_] = 1 / 2 \*  $\int_0^{2\pi} (r[\theta])^2 d\theta$

Out[ ]:=  $\frac{27\pi}{2}$

In[ ]:= **Clear**[r, A]

Question 6:

In[ ]:= **p1** = **Normal**[**Series**[x \* **Exp**[1] ^ -x, {x, -1, 10}]]

Out[ ]:= 
$$-e + 2e(1+x) - \frac{3}{2}e(1+x)^2 + \frac{2}{3}e(1+x)^3 - \frac{5}{24}e(1+x)^4 + \frac{1}{20}e(1+x)^5 - \frac{7}{720}e(1+x)^6 + \frac{1}{630}e(1+x)^7 - \frac{e(1+x)^8}{4480} + \frac{e(1+x)^9}{36288} - \frac{11e(1+x)^{10}}{3628800}$$

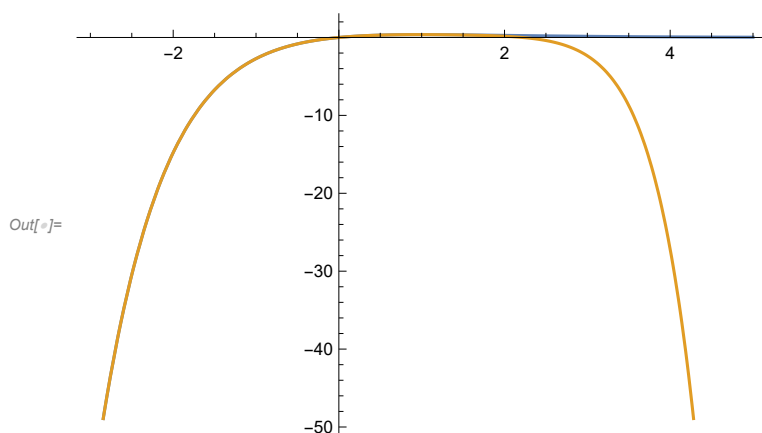
In[ ]:= **f**[x\_] = x \* **Exp**[1] ^ -x

Out[ ]:=  $e^{-x} x$

In[ ]:= **f**[2]

Out[ ]:=  $\frac{2}{e^2}$

In[ ]:= **Plot**[{f[x], p1}, {x, -3, 5}]



In[ ]:= **f**[3]

Out[ ]:=  $\frac{3}{e^3}$

In[ ]:= **N**[%]

Out[ ]:= 0.149361

In[ ]:= **p1**

Out[ ]:= 
$$-e + 2e(1+x) - \frac{3}{2}e(1+x)^2 + \frac{2}{3}e(1+x)^3 - \frac{5}{24}e(1+x)^4 + \frac{1}{20}e(1+x)^5 - \frac{7}{720}e(1+x)^6 + \frac{1}{630}e(1+x)^7 - \frac{e(1+x)^8}{4480} + \frac{e(1+x)^9}{36288} - \frac{11e(1+x)^{10}}{3628800}$$

In[ ]:= **g**[x\_] = 
$$-e + 2e(1+x) - \frac{3}{2}e(1+x)^2 + \frac{2}{3}e(1+x)^3 - \frac{5}{24}e(1+x)^4 + \frac{1}{20}e(1+x)^5 - \frac{7}{720}e(1+x)^6 + \frac{1}{630}e(1+x)^7 - \frac{e(1+x)^8}{4480} + \frac{e(1+x)^9}{36288} - \frac{11e(1+x)^{10}}{3628800}$$

Out[ ]:= 
$$-e + 2e(1+x) - \frac{3}{2}e(1+x)^2 + \frac{2}{3}e(1+x)^3 - \frac{5}{24}e(1+x)^4 + \frac{1}{20}e(1+x)^5 - \frac{7}{720}e(1+x)^6 + \frac{1}{630}e(1+x)^7 - \frac{e(1+x)^8}{4480} + \frac{e(1+x)^9}{36288} - \frac{11e(1+x)^{10}}{3628800}$$

In[ ]:= **g**[3]

Out[ ]:= 
$$-\frac{1753e}{2025}$$

In[ ]:= **N**[%]

Out[ ]:= -2.35316

In[ ]:= **Clear**[f, g, p1]



## Question 7:

```
In[ ]:= a = {-1, 4, 8}
```

```
Out[ ]:= {-1, 4, 8}
```

```
In[ ]:= b = {12, 1, 2}
```

```
Out[ ]:= {12, 1, 2}
```

```
In[ ]:= s = (a.b) / Norm[a]
```

```
Out[ ]:=  $\frac{8}{9}$ 
```

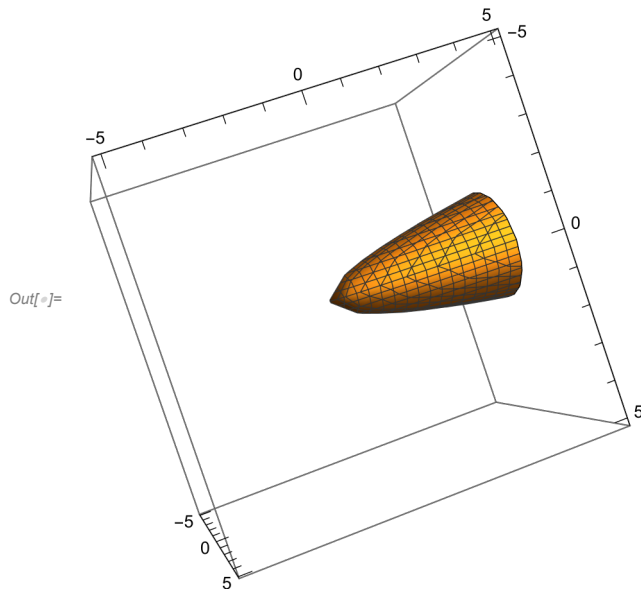
```
In[ ]:= v = s (a / Norm[a])
```

```
Out[ ]:=  $\left\{-\frac{8}{81}, \frac{32}{81}, \frac{64}{81}\right\}$ 
```

```
In[ ]:= Clear[a, b, s, v]
```

## Question 8:

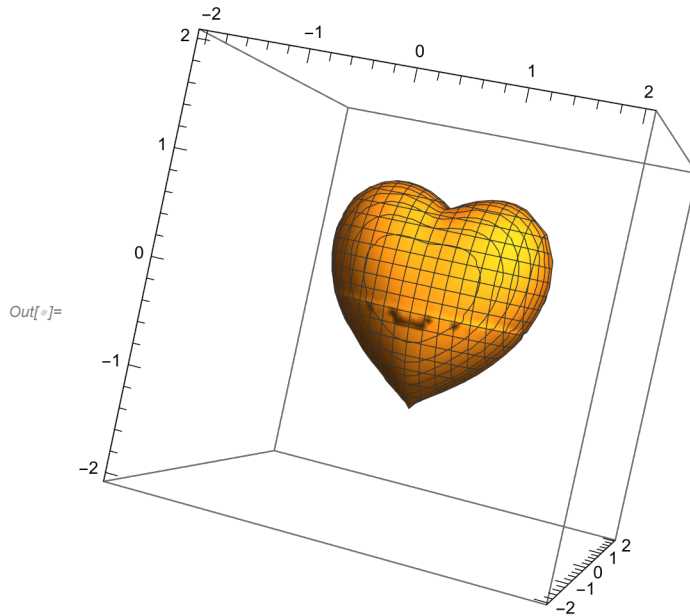
```
In[ ]:= ContourPlot3D[y == 2 * x^2 + z^2, {x, -5, 5}, {y, -5, 5}, {z, -5, 5}]
```



Elliptic Paraboloid

## Question 9:

```
In[ ]:= ContourPlot3D[(x^(2) + 9/4 (y^(2)) + z^(2) - 1)^(3) - (x^2) (z^3) - 9/80 (y^2) (z^3) == 0,
{x, -2, 2}, {y, -2, 2}, {z, -2, 2}]
```



Question 10:

```
In[ ]:= r[t_] = {Log[t + 1], t * Cos[2 * t], 2^t}
```

```
Out[ ]:= {Log[1 + t], t Cos[2 t], 2^t}
```

```
In[ ]:= a = r'[t]
```

```
Out[ ]:= {1/(1 + t), Cos[2 t] - 2 t Sin[2 t], 2^t Log[2]}
```

```
In[ ]:= b = r''[t]
```

```
Out[ ]:= {-1/(1 + t)^2, -4 t Cos[2 t] - 4 Sin[2 t], 2^t Log[2]^2}
```

```
In[ ]:= Cross[a, b]
```

```
Out[ ]:= {2^(2+t) t Cos[2 t] Log[2] + 2^t Cos[2 t] Log[2]^2 + 2^(2+t) Log[2] Sin[2 t] - 2^(1+t) t Log[2]^2 Sin[2 t],
- (2^t Log[2]) / (1 + t)^2 - (2^t Log[2]^2) / (1 + t), (Cos[2 t]) / (1 + t)^2 - (4 t Cos[2 t]) / (1 + t) - (2 t Sin[2 t]) / (1 + t)^2 - (4 Sin[2 t]) / (1 + t)}
```

In[ ]:= **a / Norm[a]**

$$\text{Out[ ]} = \left\{ \frac{1}{(1+t) \sqrt{\frac{1}{\text{Abs}[1+t]^2} + \text{Abs}[\cos[2t] - 2t \sin[2t]]^2 + 2^{2\text{Re}[t]} \log[2]^2}}, \right. \\ \frac{\cos[2t] - 2t \sin[2t]}{\sqrt{\frac{1}{\text{Abs}[1+t]^2} + \text{Abs}[\cos[2t] - 2t \sin[2t]]^2 + 2^{2\text{Re}[t]} \log[2]^2}}, \\ \left. \frac{2^t \log[2]}{\sqrt{\frac{1}{\text{Abs}[1+t]^2} + \text{Abs}[\cos[2t] - 2t \sin[2t]]^2 + 2^{2\text{Re}[t]} \log[2]^2}} \right\}$$

In[ ]:= **Clear[a, r]**

Question 11:

In[13]:= **r[t\_] = {t, Exp[1]^(-t), t \* Exp[1]^(-t)}**

Out[13]=  $\{t, e^{-t}, e^{-t} t\}$

In[14]:= **r'[t]**

Out[14]=  $\{1, -e^{-t}, e^{-t} - e^{-t} t\}$

In[15]:=  $\int_1^3 \sqrt{(r'[t])^2} dt$

Out[15]=  $\left\{2, \frac{-1 + e^2}{e^3}, \frac{-3 + e^2}{e^3}\right\}$

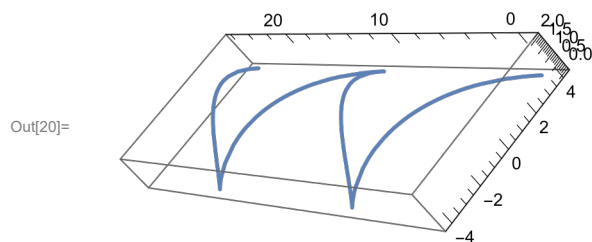
In[17]:= **N[%]**

$\{2., 0.31809, 0.21851\}$

In[18]:= **Clear[r]**

Question 12:

In[20]:= **ParametricPlot3D[{x = t - Sin[t], y = 1 - Cos[t], z = 4 \* Cos[t/2]}, {t, 0, 8π}]**



In[21]:= **r[t\_] = {t - Sin[t], 1 - Cos[t], 4 \* Cos[t / 2]}**

Out[21]=  $\left\{t - \sin[t], 1 - \cos[t], 4 \cos\left[\frac{t}{2}\right]\right\}$

In[22]:= **(Norm[Cross[r'[t], r''[t]]) / ((Norm[r'[t]])^3)**

Out[22]= 
$$\frac{\sqrt{\left(\text{Abs}\left[2 \cos[t] \sin\left[\frac{t}{2}\right] - \cos\left[\frac{t}{2}\right] \sin[t]\right]^2 + \text{Abs}\left[\cos\left[\frac{t}{2}\right] - \cos\left[\frac{t}{2}\right] \cos[t] - 2 \sin\left[\frac{t}{2}\right] \sin[t]\right]^2 + \text{Abs}\left[\cos[t] - \cos[t]^2 - \sin[t]^2\right]^2\right)}}{\left(\text{Abs}\left[1 - \cos[t]\right]^2 + 4 \text{Abs}\left[\sin\left[\frac{t}{2}\right]\right]^2 + \text{Abs}\left[\sin[t]\right]^2\right)^{3/2}}$$

In[23]:= **Simplify[%]**

Out[23]= 
$$\sqrt{\frac{\text{Abs}\left[1 - \cos[t]\right]^2 + 4 \text{Abs}\left[\sin\left[\frac{t}{2}\right]\right]^6 + \text{Abs}\left[\sin\left[\frac{t}{2}\right] \sin[t]\right]^2}{\left(\text{Abs}\left[1 - \cos[t]\right]^2 + 4 \text{Abs}\left[\sin\left[\frac{t}{2}\right]\right]^2 + \text{Abs}\left[\sin[t]\right]^2\right)^3}}$$

In[24]:= **Clear[r]**

### Question 13:

In[40]:= **r[t\_] = {Cos[t], Sin[t], Log[Cos[t]]}**

Out[40]=  $\{\cos[t], \sin[t], \log[\cos[t]]\}$

In[41]:= **T[t\_] = (r'[t]) / (Norm[r'[t]])**

Out[41]= 
$$\left\{-\frac{\sin[t]}{\sqrt{\text{Abs}[\cos[t]]^2 + \text{Abs}[\sin[t]]^2 + \text{Abs}[\tan[t]]^2}}, \frac{\cos[t]}{\sqrt{\text{Abs}[\cos[t]]^2 + \text{Abs}[\sin[t]]^2 + \text{Abs}[\tan[t]]^2}}, -\frac{\tan[t]}{\sqrt{\text{Abs}[\cos[t]]^2 + \text{Abs}[\sin[t]]^2 + \text{Abs}[\tan[t]]^2}}\right\}$$

In[29]:= **T[π / 3]**

Out[29]=  $\left\{-\frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2}\right\}$

In[42]:= **n[t\_] = (T'[t]) / (Norm[T'[t]])**

Out[42]= 
$$\left\{-\frac{\cos[t]}{\sqrt{\text{Abs}[\cos[t]]^2 + \text{Abs}[\sin[t]]^2 + \text{Abs}[\tan[t]]^2}} + \frac{(\sin[t] (-2 \text{Abs}[\cos[t]] \sin[t] \text{Abs}'[\cos[t]] + \dots)}{\dots}\right\}$$

$$\begin{aligned}
& \left( 2 \text{Abs}[\text{Sin}[t]] \text{Cos}[t] \text{Abs}'[\text{Sin}[t]] + 2 \text{Abs}[\text{Tan}[t]] \text{Sec}[t]^2 \text{Abs}'[\text{Tan}[t]] \right) / \\
& \left( 2 \left( \text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2 \right)^{3/2} \right) / \\
& \left( \sqrt{\left( \text{Abs} \left[ -\frac{\text{Sin}[t]}{\sqrt{\text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2}} - (\text{Cos}[t] (-2 \text{Abs}[\text{Cos}[t]] \text{Sin}[t] \right. \right. \right. \\
& \quad \left. \left. \text{Abs}'[\text{Cos}[t]] + 2 \text{Abs}[\text{Sin}[t]] \text{Cos}[t] \text{Abs}'[\text{Sin}[t]] + 2 \text{Abs}[\text{Tan}[t]] \text{Sec}[t]^2 \right. \right. \\
& \quad \left. \left. \text{Abs}'[\text{Tan}[t]] \right) \right) / \left( 2 \left( \text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2 \right)^{3/2} \right)^2 + \\
& \quad \left. \text{Abs} \left[ -\frac{\text{Cos}[t]}{\sqrt{\text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2}} + (\text{Sin}[t] (-2 \text{Abs}[\text{Cos}[t]] \text{Sin}[t] \right. \right. \\
& \quad \left. \left. \text{Abs}'[\text{Cos}[t]] + 2 \text{Abs}[\text{Sin}[t]] \text{Cos}[t] \text{Abs}'[\text{Sin}[t]] + 2 \text{Abs}[\text{Tan}[t]] \text{Sec}[t]^2 \right. \right. \\
& \quad \left. \left. \text{Abs}'[\text{Tan}[t]] \right) \right) / \left( 2 \left( \text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2 \right)^{3/2} \right)^2 + \\
& \quad \left. \text{Abs} \left[ -\frac{\text{Sec}[t]^2}{\sqrt{\text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2}} + (\text{Tan}[t] (-2 \text{Abs}[\text{Cos}[t]] \text{Sin}[t] \right. \right. \\
& \quad \left. \left. \text{Abs}'[\text{Cos}[t]] + 2 \text{Abs}[\text{Sin}[t]] \text{Cos}[t] \text{Abs}'[\text{Sin}[t]] + 2 \text{Abs}[\text{Tan}[t]] \text{Sec}[t]^2 \right. \right. \\
& \quad \left. \left. \text{Abs}'[\text{Tan}[t]] \right) \right) / \left( 2 \left( \text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2 \right)^{3/2} \right)^2 \right) \Bigg), \\
& \left( -\frac{\text{Sin}[t]}{\sqrt{\text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2}} - \right. \\
& \quad \left( \text{Cos}[t] (-2 \text{Abs}[\text{Cos}[t]] \text{Sin}[t] \text{Abs}'[\text{Cos}[t]] + \right. \\
& \quad \left. 2 \text{Abs}[\text{Sin}[t]] \text{Cos}[t] \text{Abs}'[\text{Sin}[t]] + 2 \text{Abs}[\text{Tan}[t]] \text{Sec}[t]^2 \text{Abs}'[\text{Tan}[t]] \right) \Bigg) / \\
& \left( 2 \left( \text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2 \right)^{3/2} \right) / \\
& \left( \sqrt{\left( \text{Abs} \left[ -\frac{\text{Sin}[t]}{\sqrt{\text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2}} - \right. \right. \right. \\
& \quad \left( \text{Cos}[t] (-2 \text{Abs}[\text{Cos}[t]] \text{Sin}[t] \text{Abs}'[\text{Cos}[t]] + 2 \text{Abs}[\text{Sin}[t]] \right. \\
& \quad \left. \text{Cos}[t] \text{Abs}'[\text{Sin}[t]] + 2 \text{Abs}[\text{Tan}[t]] \text{Sec}[t]^2 \text{Abs}'[\text{Tan}[t]] \right) \Bigg) / \\
& \quad \left( 2 \left( \text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2 \right)^{3/2} \right)^2 + \\
& \quad \left. \text{Abs} \left[ -\frac{\text{Cos}[t]}{\sqrt{\text{Abs}[\text{Cos}[t]]^2 + \text{Abs}[\text{Sin}[t]]^2 + \text{Abs}[\text{Tan}[t]]^2}} + \right. \right. \\
& \quad \left( \text{Sin}[t] (-2 \text{Abs}[\text{Cos}[t]] \text{Sin}[t] \text{Abs}'[\text{Cos}[t]] + 2 \text{Abs}[\text{Sin}[t]] \right.
\end{aligned}$$



In[32]:= **n**[ $\pi/3$ ]

$$\begin{aligned}
\text{Out[32]} = & \left\{ \left( -\frac{1}{4} + \frac{1}{32} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] + \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] + 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right) / \right. \\
& \left( \sqrt{\left( \text{Abs}\left[-\frac{\sqrt{3}}{4} + \frac{1}{32} \times \left( \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] - \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] - 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right]^2 + \right.} \right. \\
& \left. \left. \text{Abs}\left[-\frac{1}{4} + \frac{1}{32} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] + \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] + 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right]^2 + \right. \right. \\
& \left. \left. \text{Abs}\left[-2 + \frac{1}{16} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] + \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] + 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right]^2 \right) \right), \\
& \left( -\frac{\sqrt{3}}{4} + \frac{1}{32} \times \left( \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] - \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] - 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right) / \\
& \left( \sqrt{\left( \text{Abs}\left[-\frac{\sqrt{3}}{4} + \frac{1}{32} \times \left( \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] - \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] - 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right]^2 + \right.} \right. \\
& \left. \left. \text{Abs}\left[-\frac{1}{4} + \frac{1}{32} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] + \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] + 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right]^2 + \right. \right. \\
& \left. \left. \text{Abs}\left[-2 + \frac{1}{16} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] + \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] + 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right]^2 \right) \right), \\
& \left( -2 + \frac{1}{16} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] + \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] + 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right) / \\
& \left( \sqrt{\left( \text{Abs}\left[-\frac{\sqrt{3}}{4} + \frac{1}{32} \times \left( \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] - \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] - 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right]^2 + \right.} \right. \\
& \left. \left. \text{Abs}\left[-\frac{1}{4} + \frac{1}{32} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] + \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] + 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right]^2 + \right. \right. \\
& \left. \left. \text{Abs}\left[-2 + \frac{1}{16} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{1}{2}\right] + \frac{1}{2} \sqrt{3} \text{Abs}'\left[\frac{\sqrt{3}}{2}\right] + 8 \sqrt{3} \text{Abs}'[\sqrt{3}] \right) \right]^2 \right) \right) \}
\end{aligned}$$

In[33]:= **Simplify[%]**

$$\begin{aligned}
\text{Out[33]} = & \left\{ \left( -16 - 3 \text{Abs}' \left[ \frac{1}{2} \right] + 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 48 \text{Abs}' [\sqrt{3}] \right) / \right. \\
& \left( \sqrt{\left( \text{Abs} \left[ 16 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' [\sqrt{3}] \right]^2 + \right.} \right. \\
& \quad 4 \text{Abs} \left[ 64 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' [\sqrt{3}] \right]^2 + \\
& \quad \left. \left. 3 \text{Abs} \left[ 16 - \text{Abs}' \left[ \frac{1}{2} \right] + \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 16 \text{Abs}' [\sqrt{3}] \right]^2 \right) \right) \right\}, \\
& \left( \sqrt{3} \left( \text{Abs}' \left[ \frac{1}{2} \right] - \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 16 \times (1 + \text{Abs}' [\sqrt{3}]) \right) \right) / \\
& \left( \sqrt{\left( \text{Abs} \left[ 16 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' [\sqrt{3}] \right]^2 + \right.} \right. \\
& \quad 4 \text{Abs} \left[ 64 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' [\sqrt{3}] \right]^2 + \\
& \quad \left. \left. 3 \text{Abs} \left[ 16 - \text{Abs}' \left[ \frac{1}{2} \right] + \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 16 \text{Abs}' [\sqrt{3}] \right]^2 \right) \right) \right\}, \\
& \left( 2 \times \left( -64 - 3 \text{Abs}' \left[ \frac{1}{2} \right] + 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 48 \text{Abs}' [\sqrt{3}] \right) \right) / \\
& \left( \sqrt{\left( \text{Abs} \left[ 16 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' [\sqrt{3}] \right]^2 + \right.} \right. \\
& \quad 4 \text{Abs} \left[ 64 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' [\sqrt{3}] \right]^2 + \\
& \quad \left. \left. 3 \text{Abs} \left[ 16 - \text{Abs}' \left[ \frac{1}{2} \right] + \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 16 \text{Abs}' [\sqrt{3}] \right]^2 \right) \right) \right\}
\end{aligned}$$



In[34]:= **B = Cross [T[ $\pi / 3$ ], n[ $\pi / 3$ ]]**

$$\begin{aligned} \text{Out[34]} = & \left\{ - \left( 7 \sqrt{ \left( \text{Abs} \left[ -\frac{\sqrt{3}}{4} + \frac{1}{32} \times \left( \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{1}{2} \right] - \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 8 \sqrt{3} \text{Abs}' \left[ \sqrt{3} \right] \right) \right]^2 + \right. \right. \\ & \left. \left. \text{Abs} \left[ -\frac{1}{4} + \frac{1}{32} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{1}{2} \right] + \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 8 \sqrt{3} \text{Abs}' \left[ \sqrt{3} \right] \right) \right]^2 + \right. \right. \\ & \left. \left. \text{Abs} \left[ -2 + \frac{1}{16} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{1}{2} \right] + \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 8 \sqrt{3} \text{Abs}' \left[ \sqrt{3} \right] \right) \right]^2 \right) \right) \right\}, \\ & - \left( (3 \sqrt{3}) \sqrt{ \left( \text{Abs} \left[ -\frac{\sqrt{3}}{4} + \frac{1}{32} \times \left( \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{1}{2} \right] - \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 8 \sqrt{3} \text{Abs}' \left[ \sqrt{3} \right] \right) \right]^2 + \right. \right. \\ & \left. \left. \text{Abs} \left[ -\frac{1}{4} + \frac{1}{32} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{1}{2} \right] + \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 8 \sqrt{3} \text{Abs}' \left[ \sqrt{3} \right] \right) \right]^2 + \right. \right. \\ & \left. \left. \text{Abs} \left[ -2 + \frac{1}{16} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{1}{2} \right] + \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 8 \sqrt{3} \text{Abs}' \left[ \sqrt{3} \right] \right) \right]^2 \right) \right) \right\}, \\ & 1 \sqrt{ \left( \text{Abs} \left[ -\frac{\sqrt{3}}{4} + \frac{1}{32} \times \left( \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{1}{2} \right] - \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 8 \sqrt{3} \text{Abs}' \left[ \sqrt{3} \right] \right) \right]^2 + \right. \right. \\ & \left. \left. \text{Abs} \left[ -\frac{1}{4} + \frac{1}{32} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{1}{2} \right] + \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 8 \sqrt{3} \text{Abs}' \left[ \sqrt{3} \right] \right) \right]^2 + \right. \right. \\ & \left. \left. \text{Abs} \left[ -2 + \frac{1}{16} \sqrt{3} \left( -\frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{1}{2} \right] + \frac{1}{2} \sqrt{3} \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 8 \sqrt{3} \text{Abs}' \left[ \sqrt{3} \right] \right) \right]^2 \right) \right) \right\} \end{aligned}$$

In[35]:= **Simplify[%]**

$$\begin{aligned} \text{Out[35]} = & \left\{ - \left( 56 \sqrt{\left( \text{Abs} \left[ 16 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' \left[ \sqrt{3} \right] \right]^2 + \right.} \right. \right. \\ & 4 \text{Abs} \left[ 64 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' \left[ \sqrt{3} \right] \right]^2 + \\ & \left. \left. 3 \text{Abs} \left[ 16 - \text{Abs}' \left[ \frac{1}{2} \right] + \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 16 \text{Abs}' \left[ \sqrt{3} \right] \right]^2 \right) \right\}, \\ & - \left( (24 \sqrt{3}) \sqrt{\left( \text{Abs} \left[ 16 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' \left[ \sqrt{3} \right] \right]^2 + \right.} \right. \\ & 4 \text{Abs} \left[ 64 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' \left[ \sqrt{3} \right] \right]^2 + \\ & \left. \left. 3 \text{Abs} \left[ 16 - \text{Abs}' \left[ \frac{1}{2} \right] + \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 16 \text{Abs}' \left[ \sqrt{3} \right] \right]^2 \right) \right\}, \\ & 16 \sqrt{\left( \text{Abs} \left[ 16 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' \left[ \sqrt{3} \right] \right]^2 + \right.} \\ & 4 \text{Abs} \left[ 64 + 3 \text{Abs}' \left[ \frac{1}{2} \right] - 3 \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] - 48 \text{Abs}' \left[ \sqrt{3} \right] \right]^2 + \\ & \left. \left. 3 \text{Abs} \left[ 16 - \text{Abs}' \left[ \frac{1}{2} \right] + \text{Abs}' \left[ \frac{\sqrt{3}}{2} \right] + 16 \text{Abs}' \left[ \sqrt{3} \right] \right]^2 \right) \right\} \end{aligned}$$

In[45]:= **Clear[r, T, n]**