

# **MIET2499 STRESS ANALYSIS**

## **MINI-PROJECT # 2**

### **BRIDGE DESIGN**

**Date: May 17, 2022**

**Course/Section: MIET2499 Stress Analysis**

**Lecturer: Mr.Trung Nguyen**

**Team Members: Do Le Tri Dung - s3852872**

**Nguyen Quoc Hung - s3752578**

**C**

# Table of Contents

<b>Introduction &amp; Theory</b>	<b>2</b>
<b>Design Proposal and Analysis</b>	<b>3</b>
Design Overview	4
Design Load Cases	7
A. Scenario 1: distributed load at the end	7
Maximum Stress Analysis	7
Deflection Analysis	8
B. Scenario 1: distributed load at the middle	9
Maximum Stress Analysis	9
Deflection Analysis	10
<b>Conclusion</b>	<b>11</b>
<b>References</b>	<b>11</b>

# I. Introduction & Theory

a.

In this project, our group is required to design a bridge that can sustain the weight of 2 Pasco cars without deflecting more than 5mm. The fundamental support of the bridge was simple, one of its ends is fixed to the Pasco track and the second end has no reaction force to be strengthened. The purpose of this project is for students to utilize the principles of Stress Analysis to increase the durability of the second end of the bridge and fulfill the requirements. In addition, we need to determine all the loads acting on the bridge to find the support force needed.

b. For this project to be well calculated, we will need at least these 3 theoretical principles to be considered:

The first principle we need to calculate is the bending stress of the beam where the load acts on a particular point and produces stress inside the beam. This result is used to find the stress value that the beam experienced. Therefore, it serves the design process to make the bridge more sustainable.

- The bending stress happens on the bridge:

The second principle is the longitudinal shear stress where the shear force acting on the beam causes the surface areas to stress. This result is also required to analyze the stress in the parallel direction of the beam surface.

- Longitudinal shear stress:

The final fundamental principle is the deflection of the beam where different types of loads acting on the beam surface cause the bending moment that deflects the beam at an angle from the original coordinates.

- Beam deflection:

Each of the principles above has the same value of  $I$  that is relevant to all the equations, Therefore, after finding all of the maximum stress and deflection, we can determine a suitable  $I$  value to sustain the loads and design qualified support to minimize the deflection of the bridge.

There are some different equations that support the calculation:

- Shear diagram's equation:

- Moment diagram's equation:

- Safety factor:
- Elastic curve equation:

c.

During the project, there might have been some errors due to the theoretical calculations and actual environment differences. Additionally, errors from the design might also come from the design sketching phase or installation of each part of the bridge. For example, we might glue each part in error dimension or the completed product might be unsatisfactory due to the manual assembly. Furthermore, errors can occur if we use the wrong equations or round up the result values, in some cases, the material condition affected by temperature or time can also be a factor to cause an error.

d. There are some requirements from the project that can't change:

- The total mass of the load ( 2 Pasco carts) is 8 kg, which means:
- The maximum deflection of the bridge must be smaller than 5 mm.
- The maximum allowable material is  $3000 \text{ cm}^2$ .
- The thickness of each Acrylic part is 3 mm.
- The bridge span is 0.5 m: .
- The safety factor is 1.5: .

On the other hand, the moment of inertia, maximum bending and shear stress, and maximum deflection of the beam are the parameters that can be moderated by calculation and mostly depend on the design qualification.

e.

## II. Design Proposal and Analysis

## Design Overview

After applying relevant theories and optimal solutions for the problem, the shape of the bridge is considered to be an I beam which is appropriate to utilise formulas and design.

Specifically, the design will include 1 upper deck which is provided at the beginning of the project, 1 lower deck with the same dimension as the upper one to reinforce the bridge, 5 supports to increase the load endurance, and 2 guard rails to prevent the vehicles from deviation.

In this design, the upper deck, lower deck, and 5 supports will be designed with the same length to create a strong and tough block. Besides, the I shape is an optimal choice to have high moments of inertia that will diminish the bending stress and deflection, which is the purpose of the project [https://northern-weldarc.com/why-are-i-beams-used-in-steel-construction/].

The pictures below illustrate the bridge in the normal and exploded view and its fully dimensioned orthographic drawing:

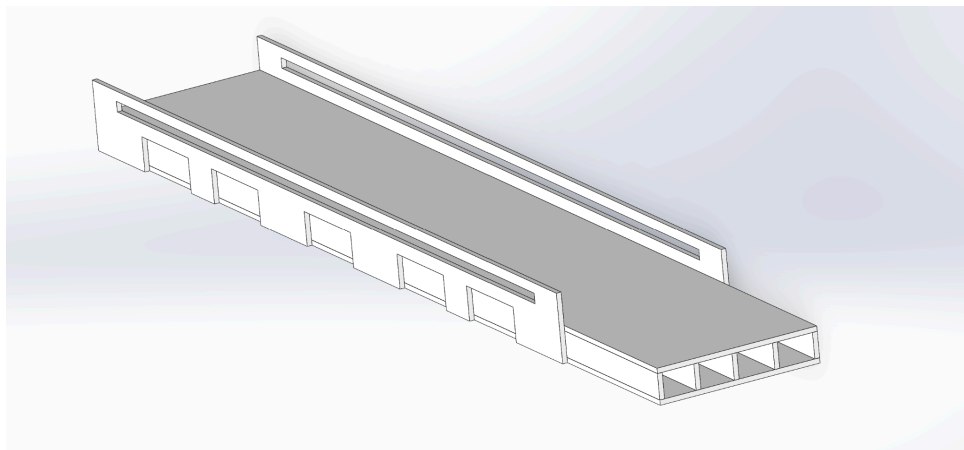


Figure: The complete bridge

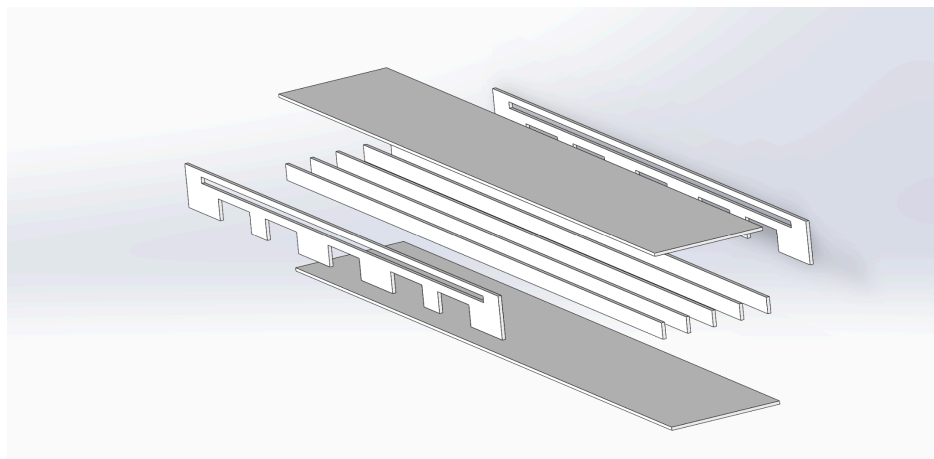


Figure: The bridge in the exploded view

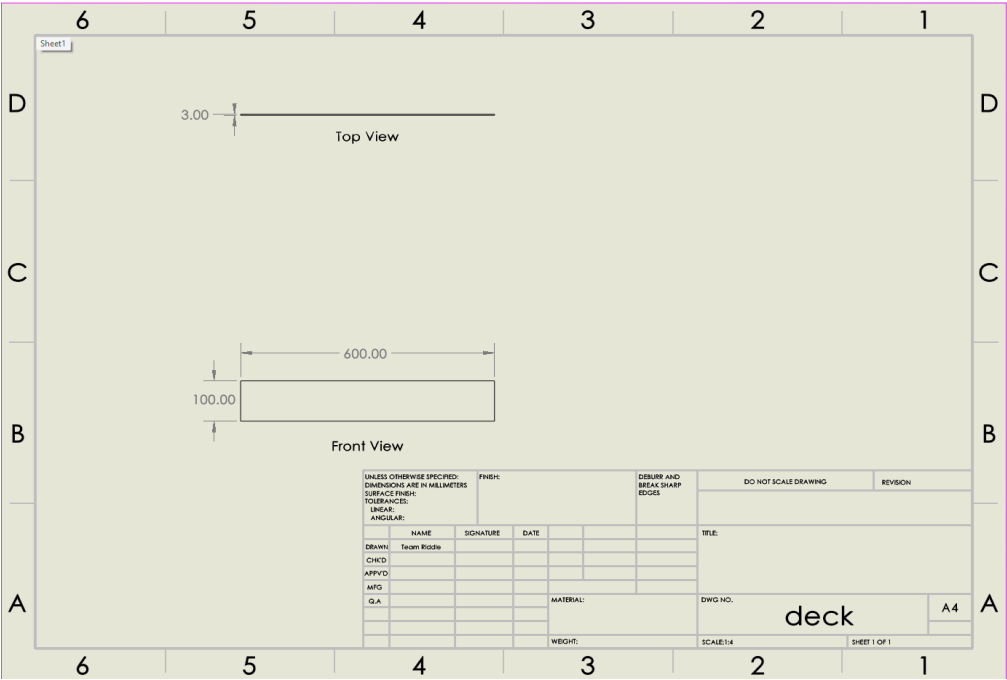


Figure: Drawing of the deck

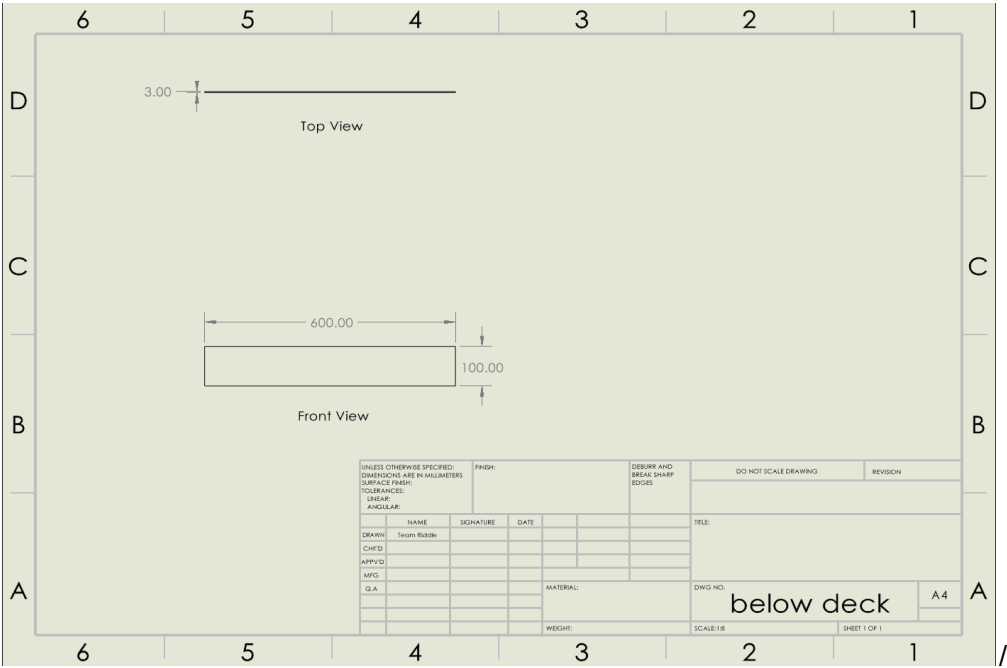


Figure: Drawing of the below deck

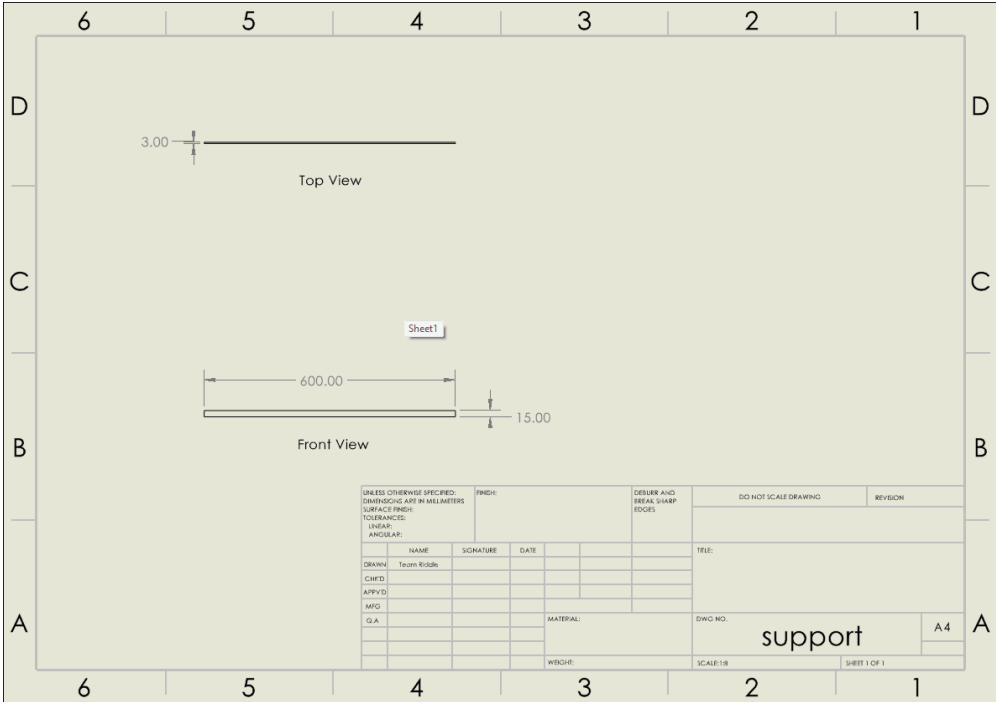


Figure: Drawing of the support

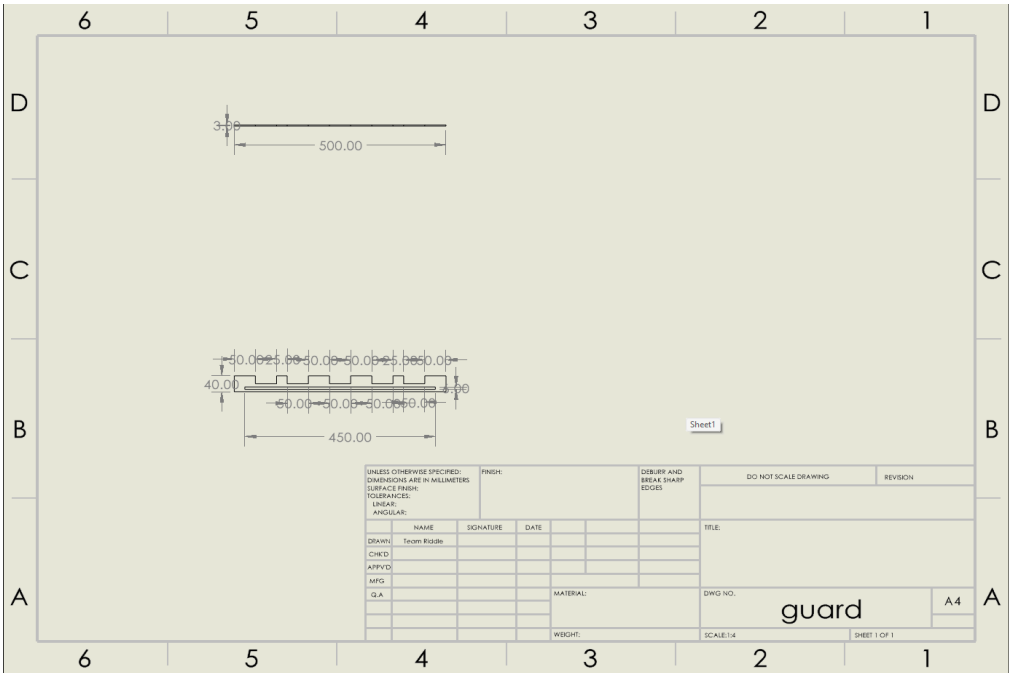


Figure: Drawing of the guard

Bill of Materials:

Part Names	Description	Material Usage	Dimension (mm)	QTY.
------------	-------------	----------------	----------------	------

Deck	The upper surface of the bridge which is provided at the beginning of the project	$1242 \text{ cm}^2$ PMMA	Length: 600 Width: 100 Height: 3	1
Below deck	Another deck under the provided bridge deck to increase the endurance	$1242 \text{ cm}^2$ PMMA	Length: 600 Width: 100 Height: 3	1
Support	Increase the endurance	$216.9 \text{ cm}^2$ PMMA	Length: 600 Width: 15 Height: 3	5
Guard Rail	To prevent the car from deviation	$311.76 \text{ cm}^2$ PMMA	Length: 500 Width: 40 Height: 3	2

## Design Load Cases

### A. Scenario 1: distributed load at the end

#### a. Maximum Stress Analysis

Maximum allowable tensile stress =  $48/1.5 = 32 \text{ MPa}$

Maximum allowable compressive stress =  $117.2 / 1.5 = 78.1 \text{ MPa}$

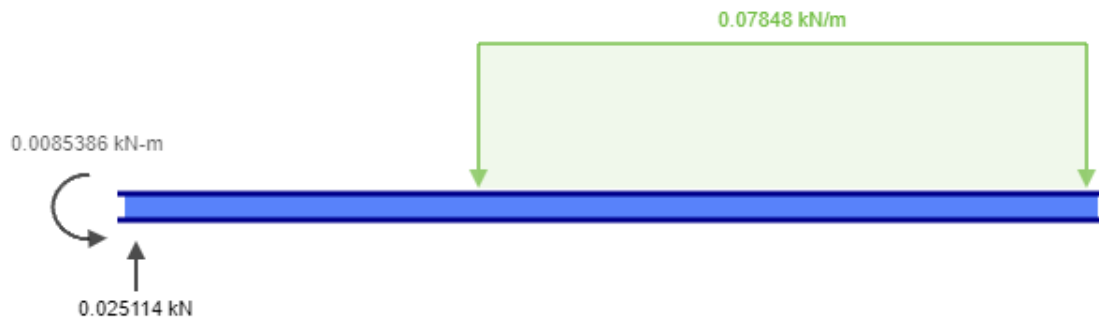
The second moment of inertia:

$$I = 2[1/12(100)(3)^3 + (100)(3)(12)^2] + 5[1/12(3)(15)^3] = 91068.75 \text{ mm}^4$$

The first moment of inertia:

$$F_{eq} = w.L = 8 \times 9.81 \times 0.32 = 25.11 \text{ N}$$





Equilibrium equations at point A:

$$F_y = 0 = A_y - F_{eq} \Rightarrow A_y = 25.11 \text{ N}$$

$$M_a = 0 = M_a - F_{eq} \cdot (0.5 - 0.32/2) \Rightarrow M_a = 8.5374 \text{ Nm}$$

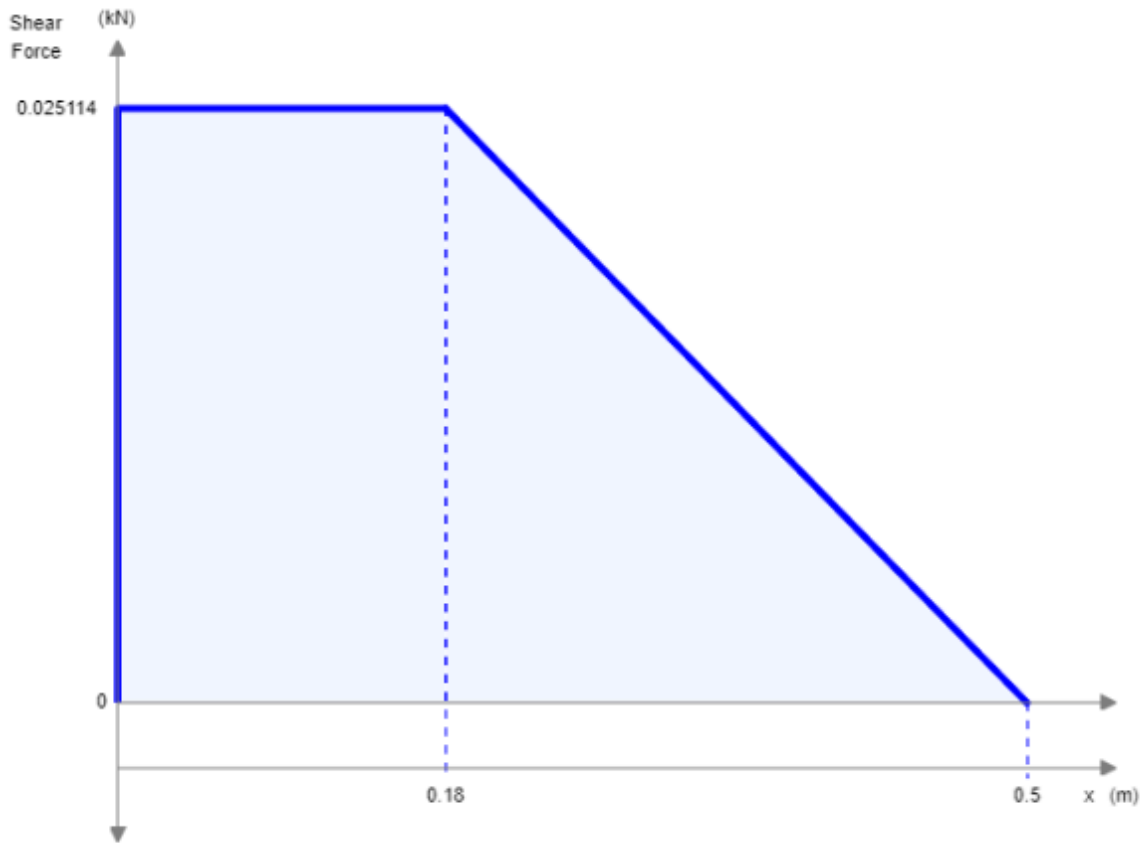


Figure: Shear diagram

Maximum shear force is  $25.1 \text{ N}$

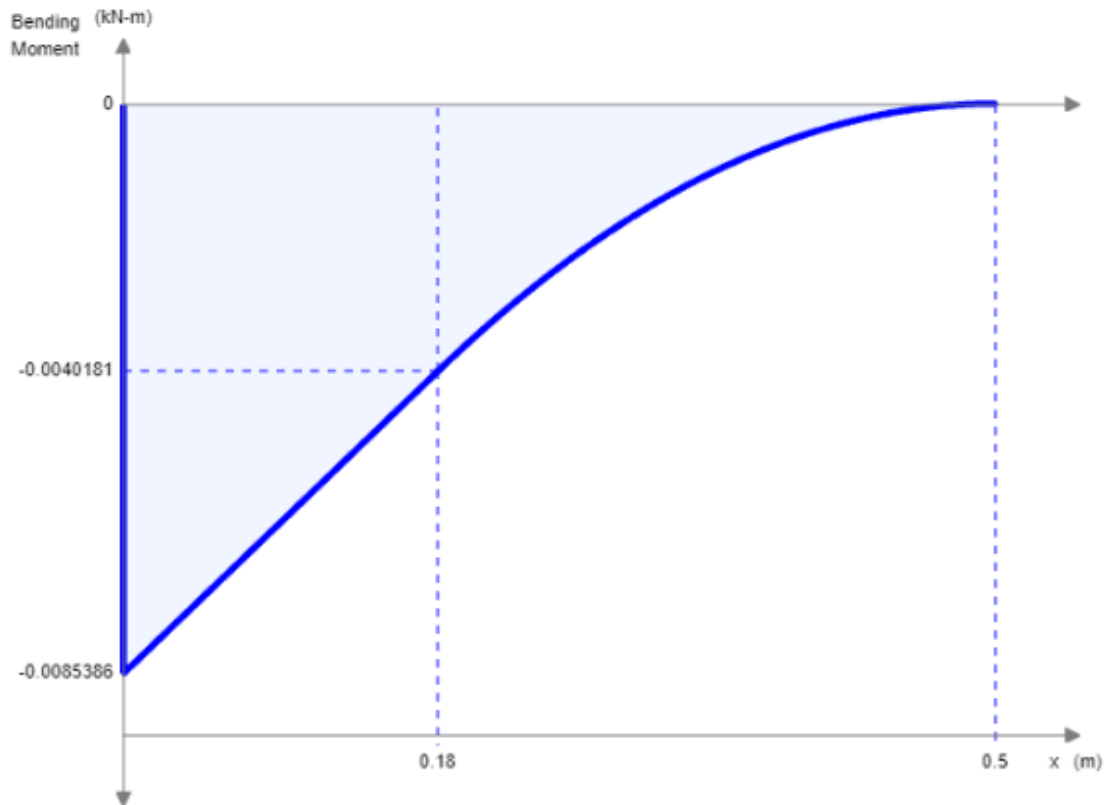


Figure: Moment diagram

Maximum moment is -8.5 Nm

Maximum tensile bending stress =  $M.y / I = [8.5 \cdot (0.021/2)] / 9.1069 \cdot 10^{-8} = 980026 \text{ Pa} = 1 \text{ MPa}$

The maximum tensile bending stress in this case is smaller than the safety factor for maximum tensile bending stress ( $1 \text{ MPa} < 32 \text{ MPa}$ )

Maximum compressive bending stress = 1 MPa

The maximum compressive bending stress in this case is smaller than the safety factor for maximum compressive bending stress ( $1 \text{ MPa} < 78.1 \text{ MPa}$ )

Maximum longitudinal shear stress =  $VQ/It$

The location of maximum shear stress and bending moment in this case is at the location 0mm and the fixed support is bearing that stress

## b. Deflection Analysis

The maximum radius of curvature of the bridge

$$\frac{1}{\rho} = \frac{M}{EI} = 3000 \cdot 10^6 \cdot 9 \cdot 1069 \cdot 10^{-8} / 8.5 = 32.142 \text{ m}$$

$$\text{Slope at end} = P \cdot a^2 / 2EI = 25.1 \times 340^2 / 2 \times 3000 \times 91068 = 0.0053 \text{ rad} = 0.03 \text{ degree}$$

$$\text{Maximum deflection} = P \cdot a^2 / 6 \cdot EI \cdot (3L - a) = 25.1(340)^2 / 6 \cdot 3000 \cdot 91068 \cdot (3 \cdot 500 - 340) = 2.05 \text{ mm} [1]$$

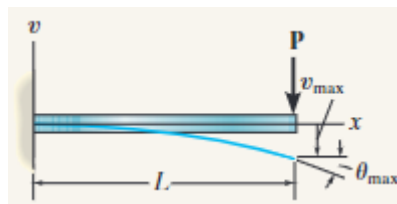
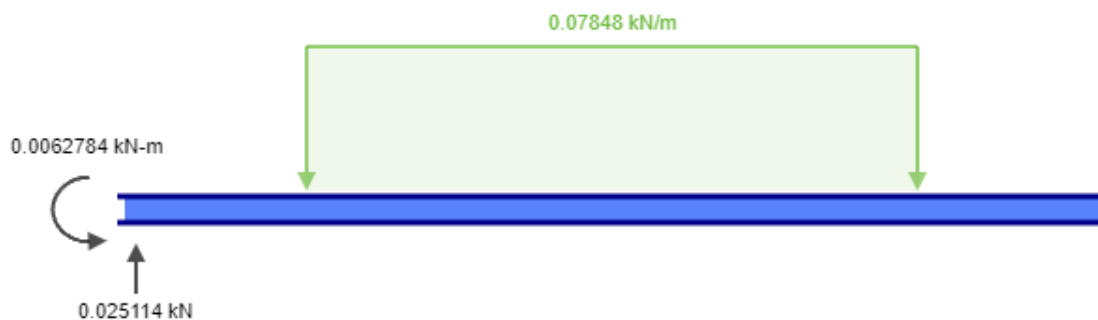


Figure: Illustration for the Elastic Curve of the bridge

The maximum deflection in this case is smaller than the requirement for maximum deflection so the cars can pass ( $2.05 \text{ mm} < 5 \text{ mm}$ ). The maximum deflection will occur at location 500mm or the end of the bridge.

## B. Scenario 1: distributed load at the middle

### a. Maximum Stress Analysis



Equilibrium equations at point A:

$$F_y = 0 = A_y - F_{eq} \Rightarrow A_y = 25.11 \text{ N}$$

$$M_a = 0 = M_a - F_{eq} \cdot 0.25 \Rightarrow M_a = 6.278 \text{ Nm}$$

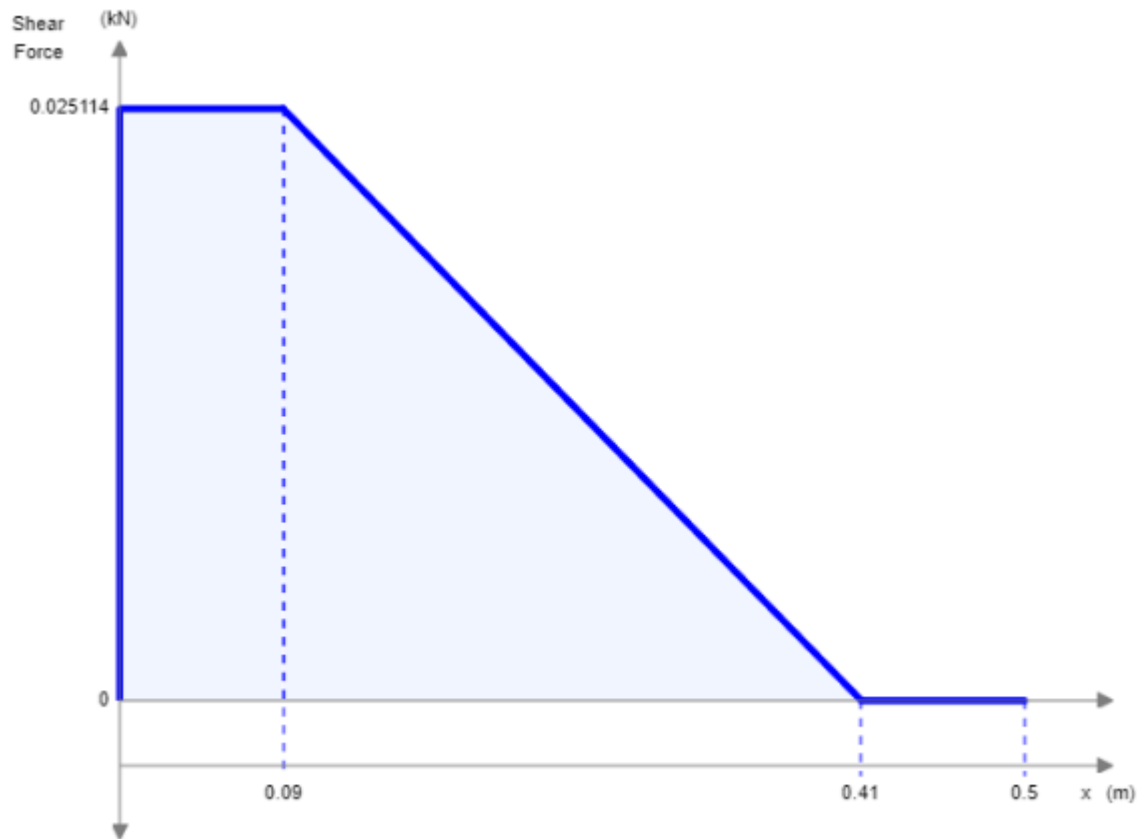


Figure: Shear diagram

Maximum shear force is 25.1 N

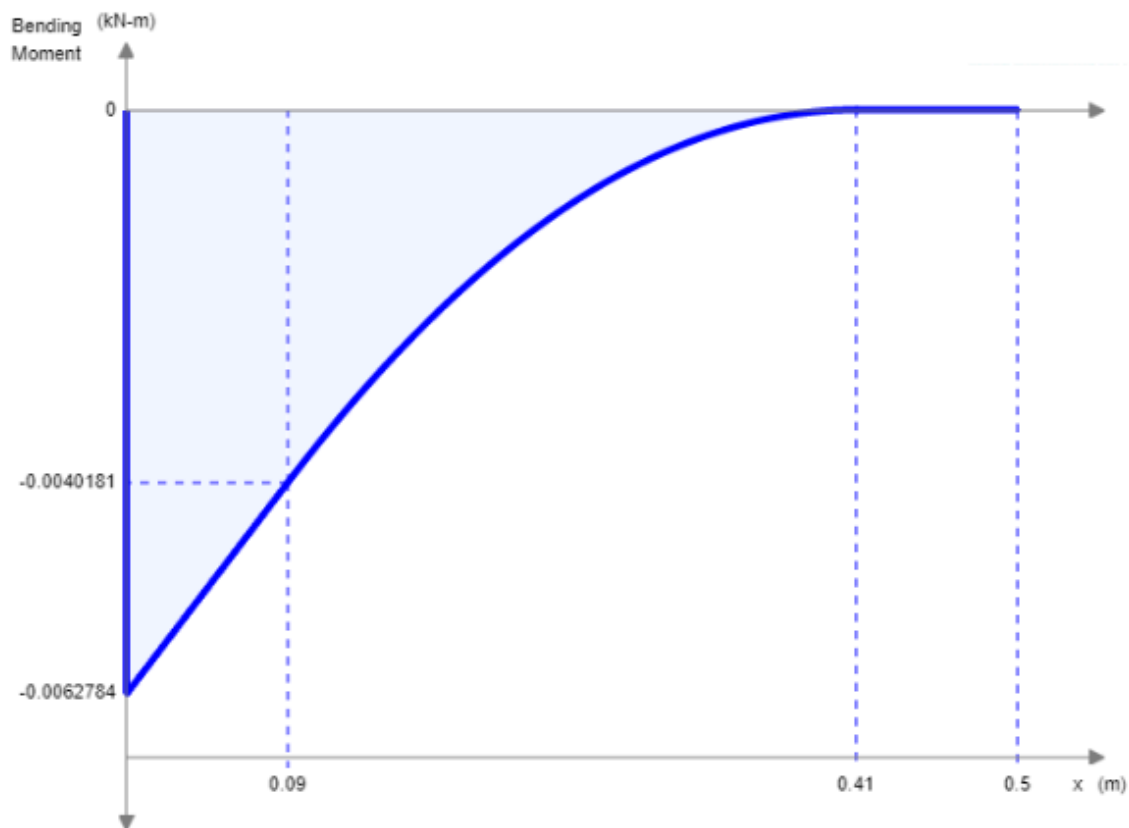


Figure: Moment diagram

Maximum moment is -6.278 Nm

Maximum tensile bending stress =  $M.y / I = [6.278 \cdot (0.021/2)] / 9.1069 \cdot 10^{-8} = 723835 \text{ MPa} = 0.72 \text{ MPa}$

The maximum tensile bending stress in this case is smaller than the safety factor for maximum tensile bending stress ( $0.72 \text{ MPa} < 32 \text{ MPa}$ )

Maximum compressive bending stress =  $0.72 \text{ MPa}$

The maximum compressive bending stress in this case is smaller than the safety factor for maximum compressive bending stress ( $0.72 \text{ MPa} < 78.1 \text{ MPa}$ )

Maximum longitudinal shear stress =  $VQ/It$

The location of maximum shear stress and bending moment in this case is also at the location 0mm and the fixed support is bearing that stress

## b. Deflection Analysis

The maximum radius of curvature of the bridge

$$\frac{1}{\rho} = \frac{M}{EI} = 3000 \cdot 10^6 \cdot 9.1069 \cdot 10^{-8} / 6.278 = 43.52 \text{ m}$$

$$\text{Slope at end} = P \cdot a^2 / 2EI = 25.1 \times 250^2 / 2 \times 3000 \times 9.1068 = 0.0029 \text{ rad} = 0.17 \text{ degree}$$

$$\text{Maximum deflection} = P \cdot a^2 / 6 \cdot EI \cdot (3L - a) = 25.1(250)^2 / 6 \cdot 3000 \cdot 9.1068 \cdot (3.500 - 250) = 1.2 \text{ mm}$$

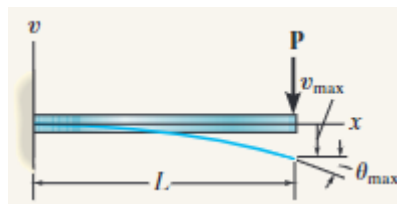


Figure: Illustration for the Elastic Curve of the bridge

The maximum deflection in this case is smaller than the requirement for maximum deflection so the cars can pass ( $1.2 \text{ mm} < 5 \text{ mm}$ ). The maximum deflection will occur at location 500mm or the end of the bridge.

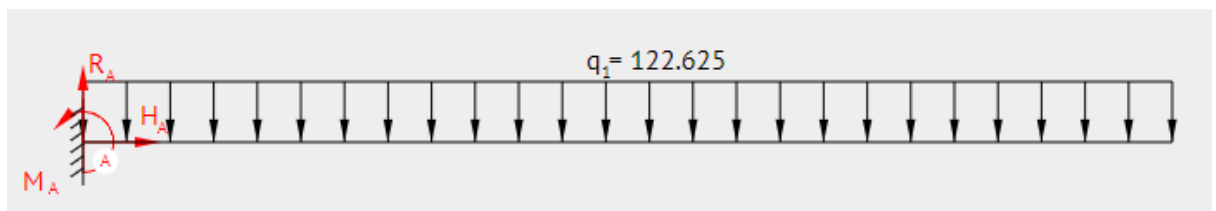
## III. Conclusion

B.

max traffic can be on the bridge at the same time  $500/160 = 3.125$  cars

Combined load of 3.125 cars  $= 4 \times 3.125 = 12.5 \text{ kg}$

$$F_{eq} = w \cdot L = 12.5 \cdot 9.81 \cdot 0.5 = 61.3 \text{ N}$$



Equilibrium equations at point A:

$$F_y = 0 = A_y - F_{eq} \Rightarrow A_y = 61.3 \text{ N}$$

$$M_a = 0 = M_a - F_{eq} \cdot 0.25 \Rightarrow M_a = 15.325 \text{ Nm}$$

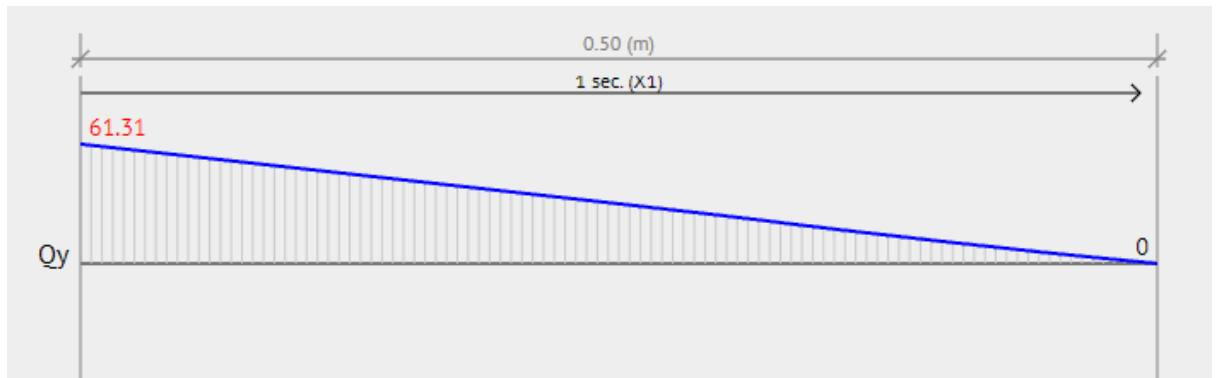


Figure: Shear diagram

Maximum shear force is 61.3 N

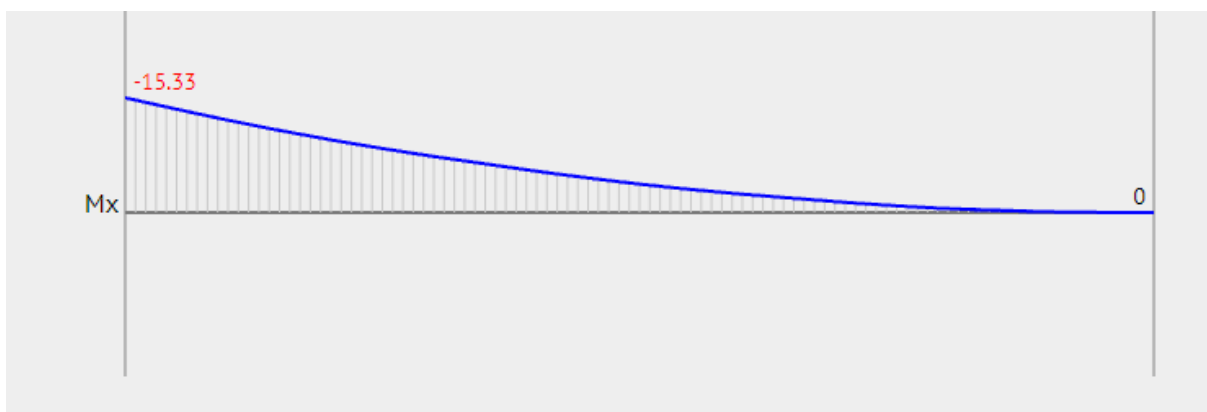


Figure: Moment diagram

Maximum moment is -15.325 Nm

Maximum tensile bending stress =  $M.y / I = [15.325 \cdot (0.021/2)] / 9.1069 \cdot 10^{-8} = 1766929 \text{ Pa} = 1.77 \text{ MPa}$

Maximum compressive bending stress = 1.77 MPa

Maximum longitudinal shear stress =  $VQ/It$

The maximum tensile and compressive bending stress is under the Acrylic safety factor for tensile ( $1.77 \text{ MPa} < 32 \text{ MPa}$ ) and compressive ( $1.77 \text{ MPa} < 78.1 \text{ MPa}$ ) bending stress. The bridge will hold when all the cars travel at once on the bridge

Slope at end =  $w.L^3/6EI = 122.625 \times 500^3 / 6 \times 3000000 \times 91068 = 0.009 \text{ rad} = 0.5 \text{ degree}$

Maximum deflection =  $w.L^4/8EI = 122.625 \times (500)^4 / 8 \times 3000000 \times 91068 = 3.57 \text{ mm} [1]$

The maximum deflection compare to the allowable 5mm deflection for the cars to pass is smaller ( $3.57 \text{ mm} < 5 \text{ mm}$ ). So the traffic can pass through the bridge

## IV. References

[1]<https://mathalino.com/reviewer/strength-materials-mechanics-materials/method-superposition-beam-deflection>