Hwk#9

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This homework is to practice finding the maximum likelihood estimates for a Poisson regression.

High-dimensional NewtonRaphson (HDNR)function

```
newtonraphson <- function(ftn, x0, tol = 1e-9, max.iter = 100) {
  x <- x0 # x0: the initial value
  fx <- ftn(x)
  iter <- 0
  while ((max(abs(fx[[1]])) > tol) & (iter < max.iter)) {
    x <- x - solve(fx[[2]]) %*% fx[[1]]
    fx <- ftn(x)
    iter <- iter + 1
  }
  if (max(abs(fx[[1]])) > tol) {
    cat('Algorithm failed to converge\n')
    return(NULL)
  } else { # max(abs(fx[[1]])) <= tol
    cat("Algorithm converged\n")
    return(x)
  }
}
```

#Ex20-4 preparing data

```
#Preparing data
rate<- read.csv("Data/rate.csv", header = T)
rate$Age.f <- factor(rate$Age)
head(rate)
```

```
##   Age      PY Death sex Age.f
## 1   1 1299868    55   m     1
## 2   2 1240595    49   m     2
## 3   3 1045453    38   m     3
## 4   4  795776    26   m     4
## 5   5  645991    19   m     5
## 6   6  599729    17   m     6
```

#Ex20-4 (Part 1) using Newton-Raphson method to find the MLE of the regression coefficients of the Poisson regression

```
#constructing design matrix for X
X <- model.matrix(~Age.f+ ifelse(rate$sex=='m', 1,0),rate)
colnames(X)[13] <- "Sex"
head(X)
```

```
##   (Intercept) Age.f2 Age.f3 Age.f4 Age.f5 Age.f6 Age.f7 Age.f8 Age.f9 Age.f10
## 1           1      0      0      0      0      0      0      0      0       0
## 2           1      1      0      0      0      0      0      0      0       0
## 3           1      0      1      0      0      0      0      0      0       0
## 4           1      0      0      1      0      0      0      0      0       0
## 5           1      0      0      0      1      0      0      0      0       0
## 6           1      0      0      0      0      1      0      0      0       0
##   Age.f11 Age.f12 Sex
## 1       0       0   1
## 2       0       0   1
## 3       0       0   1
## 4       0       0   1
## 5       0       0   1
## 6       0       0   1
```

```
dim(X)
```

```
## [1] 24 13
```

```
Y <- rate$Death # preparing column vector for Y

ftn <- function(betacoeff) {
  mu <- exp(X%*%betacoeff+log(rate$PY/100000))
  gradient <- t(X)%*%(Y-mu)
  hessian <- -t(X)%*%diag(c(mu), length(Y))%*%X
  loglike <- sum(-mu+Y*log(mu)-log(factorial(Y)))
  return(list(gradient, hessian, loglike)) #preparing function for high-dimensionalNR
}

newtonraphson(ftn,c(0,0,0,0,0,0,0,0,0,0,0,0,0)) # running HDNR to find intercept and first 12 regressio
```

```
## Algorithm converged
```

```
##                    [,1]
## (Intercept)  0.94453534
## Age.f2      -0.11158760
## Age.f3      -0.19307396
## Age.f4      -0.39994535
## Age.f5      -0.57528753
## Age.f6      -0.40114660
## Age.f7      -0.31142104
## Age.f8       0.04452828
## Age.f9       0.07301330
## Age.f10      0.17111769
## Age.f11      0.15256371
## Age.f12     -0.27115727
## Sex          0.56064525
```

```r
glm(Death~Age.f+sex, offset=log(PY/100000), data=rate, family=poisson)
```

```
##
## Call:  glm(formula = Death ~ Age.f + sex, family = poisson, data = rate,
##     offset = log(PY/1e+05))
##
## Coefficients:
## (Intercept)       Age.f2       Age.f3       Age.f4       Age.f5       Age.f6
##     0.94454     -0.11159     -0.19307     -0.39995     -0.57529     -0.40115
##      Age.f7       Age.f8       Age.f9      Age.f10      Age.f11      Age.f12
##    -0.31142      0.04453      0.07301      0.17112      0.15256     -0.27116
##        sexm
##     0.56065
##
## Degrees of Freedom: 23 Total (i.e. Null);  11 Residual
## Null Deviance:       62.98
## Residual Deviance: 8.558     AIC: 144.8
```

#Ex20-4 (Part 2) finding the variance-covariance (VCOV) matrix for the beta coefficients

```
beta <- newtonraphson(ftn,c(0,0,0,0,0,0,0,0,0,0,0,0,0))
```

```
## Algorithm converged
```

```
model <- glm(Death~Age.f+sex, offset=log(PY/100000), data=rate, family=poisson)
solve(-ftn(beta)[[2]])# finding variance-covariance (VCOV)matrix for mle.
```

```
##               (Intercept)       Age.f2        Age.f3        Age.f4
## (Intercept)  0.014509599 -1.084465e-02 -1.086946e-02 -1.089294e-02
## Age.f2      -0.010844653  2.369025e-02  1.086957e-02  1.086941e-02
## Age.f3      -0.010869462  1.086957e-02  2.726301e-02  1.086956e-02
## Age.f4      -0.010892943  1.086941e-02  1.086956e-02  3.718550e-02
## Age.f5      -0.010908767  1.086930e-02  1.086956e-02  1.086982e-02
## Age.f6      -0.010942335  1.086907e-02  1.086956e-02  1.087003e-02
## Age.f7      -0.011051674  1.086832e-02  1.086956e-02  1.087073e-02
## Age.f8      -0.011099002  1.086799e-02  1.086956e-02  1.087104e-02
## Age.f9      -0.011212934  1.086722e-02  1.086956e-02  1.087177e-02
## Age.f10     -0.011420939  1.086579e-02  1.086955e-02  1.087311e-02
## Age.f11     -0.011707698  1.086383e-02  1.086954e-02  1.087495e-02
## Age.f12     -0.012212644  1.086037e-02  1.086953e-02  1.087819e-02
## Sex         -0.005718767 -3.913833e-05 -1.619563e-07  3.672866e-05
##                    Age.f5       Age.f6       Age.f7       Age.f8
## (Intercept) -1.090877e-02 -0.0109423347 -0.0110516743 -0.0110990021
## Age.f2       1.086930e-02  0.0108690672  0.0108683189  0.0108679950
## Age.f3       1.086956e-02  0.0108695632  0.0108695601  0.0108695587
## Age.f4       1.086982e-02  0.0108700326  0.0108707348  0.0108710388
## Age.f5       4.933153e-02  0.0108703489  0.0108715264  0.0108720361
## Age.f6       1.087035e-02  0.0453537786  0.0108732058  0.0108741520
## Age.f7       1.087153e-02  0.0108732058  0.0431367406  0.0108810438
## Age.f8       1.087204e-02  0.0108741520  0.0108810438  0.0358840270
## Age.f9       1.087326e-02  0.0108764297  0.0108867438  0.0108912083
## Age.f10      1.087550e-02  0.0108805880  0.0108971502  0.0109043191
## Age.f11      1.087859e-02  0.0108863207  0.0109114966  0.0109223940
## Age.f12      1.088403e-02  0.0108964153  0.0109367588  0.0109542215
## Sex          6.158833e-05  0.0001143263  0.0002861071  0.0003604625
##                    Age.f9       Age.f10     Age.f11      Age.f12           Sex
## (Intercept) -0.0112129343 -0.0114209388 -0.01170770 -0.012212644 -5.718767e-03
## Age.f2       0.0108672153  0.0108657917  0.01086383  0.010860373 -3.913833e-05
## Age.f3       0.0108695555  0.0108695496  0.01086954  0.010869527 -1.619563e-07
## Age.f4       0.0108717705  0.0108731064  0.01087495  0.010878191  3.672866e-05
## Age.f5       0.0108732631  0.0108755032  0.01087859  0.010884030  6.158833e-05
## Age.f6       0.0108764297  0.0108805880  0.01088632  0.010896415  1.143263e-04
## Age.f7       0.0108867438  0.0108971502  0.01091150  0.010936759  2.861071e-04
## Age.f8       0.0108912083  0.0109043191  0.01092239  0.010954221  3.604625e-04
## Age.f9       0.0431600202  0.0109215770  0.01094863  0.010996260  5.394587e-04
## Age.f10      0.0109215770  0.0509530845  0.01099652  0.011073008  8.662494e-04
## Age.f11      0.0109486274  0.0109965213  0.07772922  0.011178815  1.316770e-03
## Age.f12      0.0109962596  0.0110730079  0.01117881  0.154222269  2.110077e-03
## Sex          0.0005394587  0.0008662494  0.00131677  0.002110077  8.984612e-03
```

```r
vcov(model) #to check if our calculation for VCOV above is correct
```

```
##               (Intercept)        Age.f2        Age.f3        Age.f4
## (Intercept)  0.014509599 -1.084465e-02 -1.086946e-02 -1.089294e-02
## Age.f2      -0.010844653  2.369025e-02  1.086957e-02  1.086941e-02
## Age.f3      -0.010869462  1.086957e-02  2.726301e-02  1.086956e-02
## Age.f4      -0.010892943  1.086941e-02  1.086956e-02  3.718550e-02
## Age.f5      -0.010908767  1.086930e-02  1.086956e-02  1.086982e-02
## Age.f6      -0.010942335  1.086907e-02  1.086956e-02  1.087003e-02
## Age.f7      -0.011051674  1.086832e-02  1.086956e-02  1.087073e-02
## Age.f8      -0.011099002  1.086799e-02  1.086956e-02  1.087104e-02
## Age.f9      -0.011212934  1.086722e-02  1.086956e-02  1.087177e-02
## Age.f10     -0.011420939  1.086579e-02  1.086955e-02  1.087311e-02
## Age.f11     -0.011707698  1.086383e-02  1.086954e-02  1.087495e-02
## Age.f12     -0.012212644  1.086037e-02  1.086953e-02  1.087819e-02
## sexm        -0.005718766 -3.913832e-05 -1.619563e-07  3.672866e-05
##                   Age.f5        Age.f6        Age.f7        Age.f8
## (Intercept) -1.090877e-02 -0.0109423347 -0.0110516743 -0.0110990020
## Age.f2       1.086930e-02  0.0108690672  0.0108683189  0.0108679950
## Age.f3       1.086956e-02  0.0108695631  0.0108695601  0.0108695587
## Age.f4       1.086982e-02  0.0108700326  0.0108707348  0.0108710388
## Age.f5       4.933152e-02  0.0108703489  0.0108715264  0.0108720361
## Age.f6       1.087035e-02  0.0453537786  0.0108732058  0.0108741520
## Age.f7       1.087153e-02  0.0108732058  0.0431367391  0.0108810438
## Age.f8       1.087204e-02  0.0108741520  0.0108810438  0.0358840269
## Age.f9       1.087326e-02  0.0108764296  0.0108867438  0.0108912083
## Age.f10      1.087550e-02  0.0108805880  0.0108971502  0.0109043191
## Age.f11      1.087859e-02  0.0108863207  0.0109114966  0.0109223940
## Age.f12      1.088403e-02  0.0108964153  0.0109367588  0.0109542215
## sexm         6.158832e-05  0.0001143263  0.0002861071  0.0003604625
##                   Age.f9       Age.f10     Age.f11      Age.f12          sexm
## (Intercept) -0.0112129343 -0.0114209388 -0.01170770 -0.012212644 -5.718766e-03
## Age.f2       0.0108672152  0.0108657917  0.01086383  0.010860373 -3.913832e-05
## Age.f3       0.0108695555  0.0108695496  0.01086954  0.010869527 -1.619563e-07
## Age.f4       0.0108717705  0.0108731064  0.01087495  0.010878191  3.672866e-05
## Age.f5       0.0108732631  0.0108755032  0.01087859  0.010884030  6.158832e-05
## Age.f6       0.0108764296  0.0108805880  0.01088632  0.010896415  1.143263e-04
## Age.f7       0.0108867438  0.0108971502  0.01091150  0.010936759  2.861071e-04
## Age.f8       0.0108912083  0.0109043191  0.01092239  0.010954221  3.604625e-04
## Age.f9       0.0431600202  0.0109215770  0.01094863  0.010996260  5.394586e-04
## Age.f10      0.0109215770  0.0509530837  0.01099652  0.011073008  8.662493e-04
## Age.f11      0.0109486274  0.0109965213  0.07772909  0.011178815  1.316770e-03
## Age.f12      0.0109962595  0.0110730078  0.01117881  0.154222064  2.110077e-03
## sexm         0.0005394586  0.0008662493  0.00131677  0.002110077  8.984611e-03
```

#Ex20-4 (Part 3) finding the log likelihood at the beta coefficients

```
ftn1 <- function(betacoeff) {
  mu <- exp(X%*%betacoeff+log(rate$PY/100000))
  gradient <- t(X)%*%(Y-mu)
  hessian <- -t(X)%*%diag(c(mu), length(Y))%*%X
  loglike <- sum(-mu+Y*log(mu)-log(factorial(Y)))
  return(list(gradient, hessian, loglike)) #finding loglikelihood of the betacoeffs
}

ftn1(beta) [[3]] # retrieving the 'loglike' from the list.
```

```
## [1] -59.38966
```

```
logLik(model) #using base R function 'loglike' to check answers.
```

```
## 'log Lik.' -59.38966 (df=13)
```