Statistical Inference course project - simulation exercise

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Overview

This is the first part of the project for the statistical inference class. It is aimed at exploring inference and doing some simple inferential data analysis using simulated data. We'll investigate the exponential distribution and compare it with the Central Limit Theorem.
The CLT states that **the averages are approximately normal with distributions centered at the population mean with standard deviation equal to the standard error of the mean**.

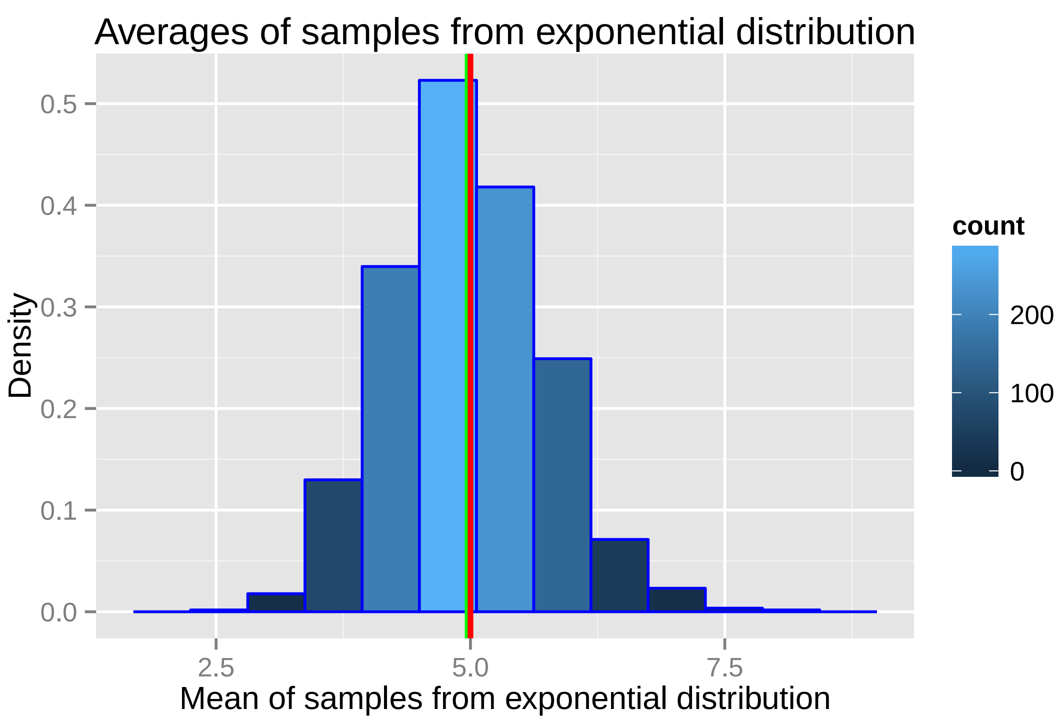
Simulations

The exponential distribution can be simulated in R with `rexp(n, lambda)` where lambda is the rate parameter. The mean of exponential distribution is $\frac{1}{\lambda}$ and the standard deviation is also $\frac{1}{\lambda}$.

We perform 1000 simulations for which we calculate the average of 40 random generated numbers from exponential distribution with lambda = 0.2.

Simulation results

1. Sample Mean versus Theoretical Mean



Theoretical mean of the simultated distribution of averages of 40 numbers from exponential distribution is the same as the mean of the original exponential distribution: 1/lambda:

Theoretical mean = 5
Sample mean = 4.971972

From the simulation we can see that the sample mean is almost identical to the theoretical mean. On the histogram there are green and red vertical lines which represent the sample mean and the theoretical mean accordingly.

2. Sample Variance versus Theoretical Variance

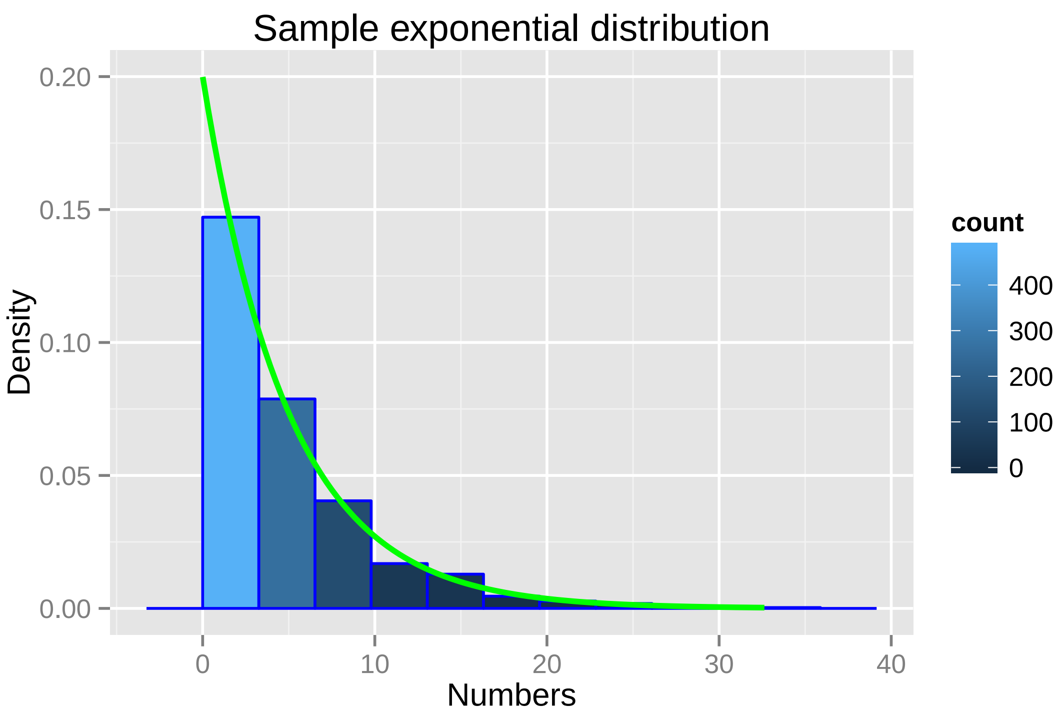
Theoretical variance of the distribution of averages of samples from exponential distribution is $\frac{1}{\lambda^2 n}$:

Theoretical variance = $\frac{1}{0.2^2 * 40}$ **= 0.625**
Sample variance = 0.5954369

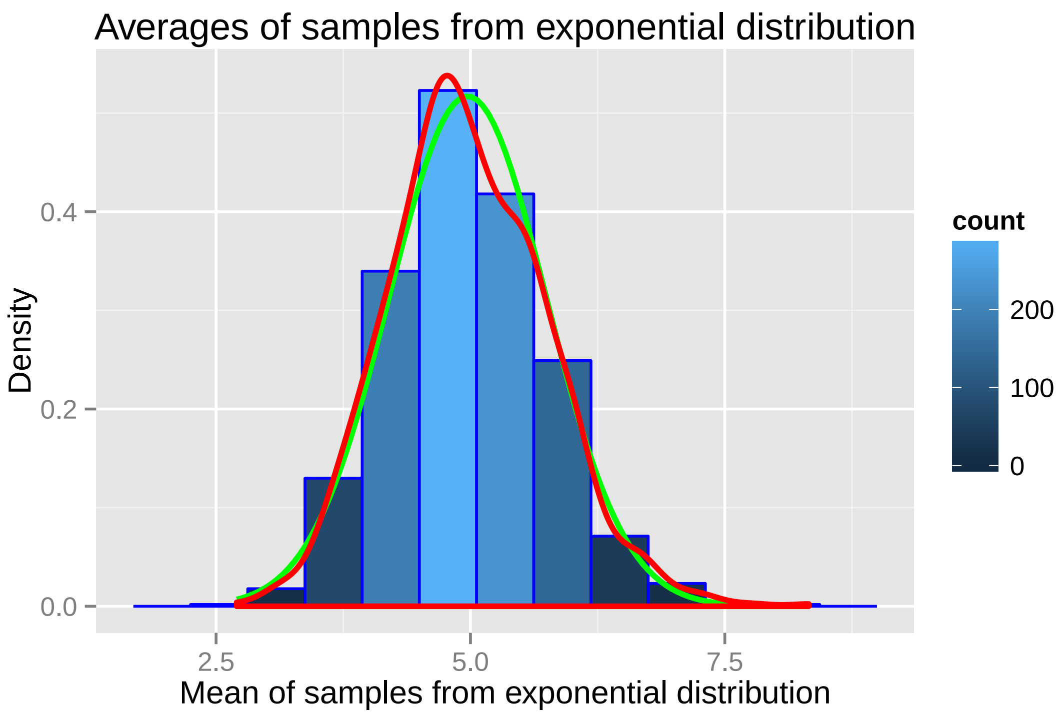
The sample variance of the simulated distribution of averages can be used to get the approximation of the variance of the original exponential distribution: $Var_{exp} = Var_{sim} n = 0.5954369 * 40 = 23.8174762$ which is close to theoretical value $\frac{1}{\lambda^2} = 25$

3. Distribution

In this section we'll compare the original exponential distribution with the distribution of averages that we simulated before.
Here is the histogram representing an exponential distribution with the density line:

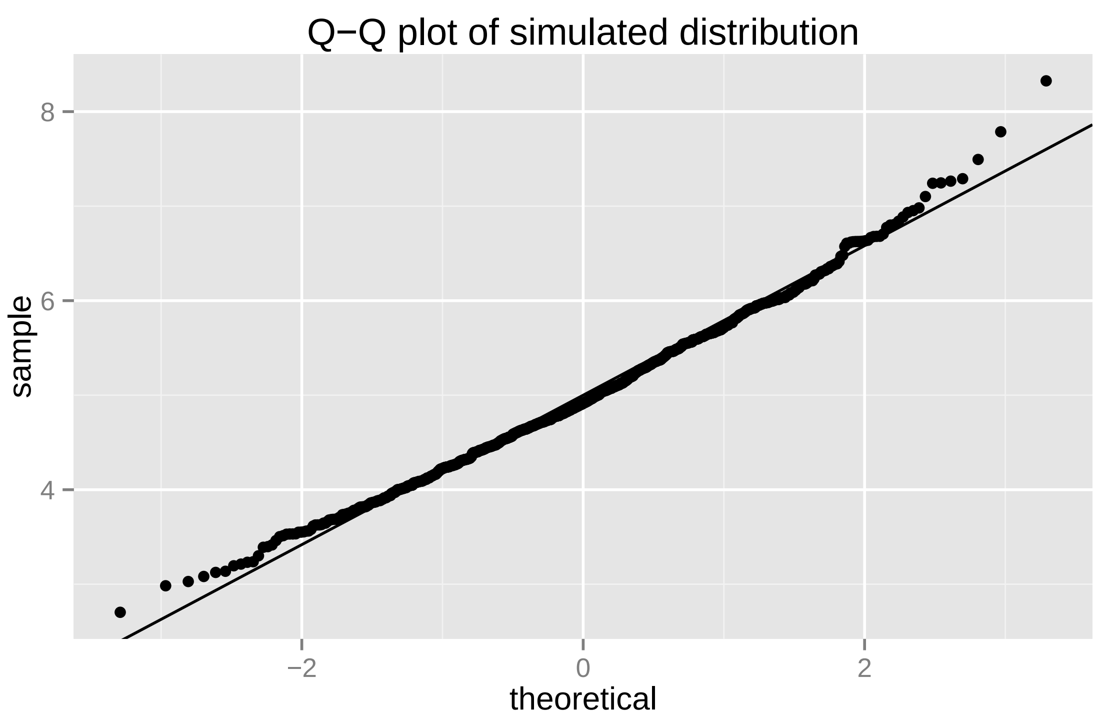


Below is the histogram of the same distribution of averages that was simulated before and used in previous sections but this time density lines are added (theoretical and actual).



Green line is drawn using theoretical mean and standard deviation. Red line is the density of the actual simulated data. These curves are the visual evidence that the simulated collection of averages has normal distribution.

There is another way to show that the distribution is normal using a Q-Q plot. Here is how it looks like:



All the points lie very close to the line which is drawn using theoretical mean and standard deviation of the distribution of averages. This plot also confirms that the simulated distribution of averages is normal.

Appendix - source code

Initialization code

```
lambda <- 0.2
n <- 40
nsim <- 1000
```

Simulation code

```
# initializing variable for averages of samples from exponential distribution
simulated <- NULL

set.seed(12345)

for(i in 1:nsim)
    simulated <- c(simulated, mean(rexp(n, lambda)))
```

1. Sample Mean versus Theoretical Mean code

```
# common parameter for histograms - number of bins
nbins = 10
# range is needed to calculate the width of the bin of a histogram
simulated.range <- max(simulated) - min(simulated)

# theoretical mean is the same both for the original exponential distribution
# and for simulated distribution of averages.
theory.mean <- 1/lambda
# calculate the smaple mean of the simulated distribution of averages
simulated.mean <- mean(simulated)

ggplot(data = NULL, aes(simulated)) +
    geom_histogram(binwidth = simulated.range/nbins,
                   col = "blue",
                   aes(y = ..density..,
                       fill=..count..)) +
    # sample mean will be shown as a vertical green line on the histogram.
    geom_vline(aes(xintercept = simulated.mean),
               color = "green",
               size = 1) +
    # theoretical mean will be shown as a vertical red line on the histogram.
    geom_vline(aes(xintercept = theory.mean),
               color = "red",
               size = 1) +
    labs(title = "Averages of samples from exponential distribution",
         x = "Mean of samples from exponential distribution",
         y = "Density")
```

2. Sample Variance versus Theoretical Variance code

```r
# theoretical value of the standard deviation of the sample distribution of
# averages is 1/lambda * 1/sqrt(n)
theory.sd <- 1/lambda * 1/sqrt(n)
theory.var <- theory.sd^2

# standard deviation and variance of the simulated distribution of averages
simulated.sd = sd(simulated)
simulated.var = simulated.sd^2
```

3. Distribution code

Exponential distribution sample:

```r
ssize <- 1000
expsample <- rexp(ssize, lambda)

expsample.range <- max(expsample) - min(expsample)

ggplot(data = NULL, aes(expsample)) +
    geom_histogram(binwidth = expsample.range/nbins,
                   col = "blue",
                   aes(y = ..density..,
                       fill=..count..)) +
    # draw density line of theoretical exponential distribution with lambda
    stat_function(fun = dexp,
                  color = "green",
                  size = 1,
                  args = list(rate = lambda)) +
    labs(title = "Sample exponential distribution",
         x = "Sample of exponential distribution",
         y = "Density")
```

Normal distribution of averages:

```r
ggplot(data = NULL, aes(simulated)) +
    geom_histogram(binwidth = simulated.range/nbins,
                   col = "blue",
                   aes(y = ..density..,
                       fill = ..count..)) +
    # draw a normal distribution curve using theoretical mean and sd.
    stat_function(fun = dnorm,
                  color = "green",
                  size = 1,
                  args = list(mean = simulated.mean,
                              sd = simulated.sd)) +
    # draw the actual density curve.
    geom_density(size = 1,
                 color = "red") +
    labs(title = "Averages of samples from exponential distribution",
```

```
         x = "Mean of samples from exponential distribution",
         y = "Density")
```

Q-Q plot:

```
# Q-Q plot to confirm data normality
qplot(sample = simulated) +
    geom_abline(intercept = theory.mean,
                slope = theory.sd) +
    labs(title = "Q-Q plot of simulated distribution")
```