University of Feng Chia

Power system lab

AC Power Flows, Generalized OPF of L-index

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1 Notation

n_b, n_g, n_l	number of buses, generators, branches .
v_n, θ_n	voltage magnitude and angle at bus n .
Y_{bus}	$n_b \times n_b$ system bus admittance matrix.
Y_{LL}, Y_{LG}	the submatrices of system admittance Y_N with respect to load buses(by subscript L) and generator buses(by subscript G) .
Z_{LL}	inverse matrix of Y_{LL} .
F_{LG}	$-Z_{LL} \times Y_{LG}$
α_L, α_G	the sets of load node and gen node.
L_n	L-index of node n.
t	the object.

2 Introduction

We will be looking at complex functions of the real valued vector

$$X = \begin{bmatrix} \theta \\ V \\ P_g \\ Q_g \\ t \end{bmatrix} \tag{1}$$

For a complex scalar function $f : \mathbb{R}^n \to \mathbb{C}$ of a real vector $x = [x_1, x_2, \ldots, x_n]^T$ we use the following notation for the first derivatives (transpose of the gradient)

$$f_x = \frac{\partial f}{\partial x} = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2,} \quad \cdots \frac{\partial f}{, \partial x_n}\right] \tag{2}$$

The matrix of second partial derivatives, the Hessian of f, is

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial x}(\frac{\partial f}{\partial x})^T = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \cdots & \frac{\partial f}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_1 \partial x_n} & \cdots & \frac{\partial f}{\partial x_n^2} \end{bmatrix} \tag{3}$$

3 Gradient

Let

$$H_l = \left| 1 - \frac{\sum_{g \in \alpha_G} F_{lg} V_g}{V_l} \right| - t < 0$$

Let

$$Z_l = \frac{\sum_{g \in \alpha_G} F_{lg} v_g e^{\theta_g}}{v_l e^{\theta_l}}$$

$$\begin{aligned} F_l &= \sqrt{(1-Z)\overline{(1-Z)}} \\ &= \sqrt{1 - Z - \overline{Z} + Z\overline{Z}} \end{aligned}$$

$$\therefore \frac{\partial H}{\partial x} = \frac{-\frac{\partial Z}{\partial x} - \overline{\frac{\partial Z}{\partial x}} + \frac{\partial Z\overline{Z}}{\partial x}}{2F}$$

$$\frac{\partial^2 H}{\partial x^2} = \frac{2F(-\frac{\partial^2 Z}{\partial x^2} - \overline{\frac{\partial^2 Z}{\partial x^2}} + \frac{\partial^2 Z\overline{Z}}{\partial x^2})}{4F^2}$$

$$\frac{-2\frac{\partial F}{\partial x}(-\frac{\partial Z}{\partial x} - \overline{\frac{\partial Z}{\partial x}} + \frac{\partial Z\overline{Z}}{\partial x})}{4F^2}$$

$$\frac{\partial Z_l}{\partial \theta_g} = \frac{\overline{\partial \overline{Z_l}}}{\partial \theta_g} = j\frac{F_{lg} v_g e^{j\theta_g - j\theta_l}}{v_l} \tag{4}$$

$$\frac{\partial Z_l}{\partial v_g} = \frac{\overline{\partial \overline{Z_l}}}{\partial v_g} = \frac{F_{lg} e^{j\theta_g - j\theta_l}}{v_l} \tag{5}$$

$$\frac{\partial Z_l}{\partial \theta_l} = \frac{\overline{\partial \overline{Z_l}}}{\partial \theta_l} = -j\frac{\sum_{g\in\alpha_G} F_{lg} v_g e^{j\theta_g - j\theta_l}}{v_l} \tag{6}$$

$$\frac{\partial Z_l}{\partial v_l} = \frac{\overline{\partial \overline{Z_l}}}{\partial v_l} = -\frac{\sum_{g\in\alpha_G} F_{lg} v_g e^{j\theta_g - j\theta_l}}{v_l^2} \tag{7}$$

$$\begin{aligned}\frac{\partial Z_l \overline{Z}_l}{\partial \theta_g} = {} & j\frac{F_{lg} v_g e^{j\theta_g} \overline{\sum_{k\in\alpha_G, k\neq g} F_{lk} v_k e^{j\theta_k}}}{v_l^2} \\ & - j\frac{\overline{F_{lg} v_g e^{j\theta_g}} \sum_{k\in\alpha_G, k\neq g} F_{lk} v_k e^{j\theta_k}}{v_l^2}\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial Z_l \overline{Z}_l}{\partial v_g} = {} & 2\frac{F_{lg}\overline{F_{lg}} v_g}{v_l^2} \\ & + \frac{F_{lg} e^{j\theta_g} \overline{\sum_{k\in\alpha_G, k\neq g} F_{lk} v_k e^{j\theta_k}}}{v_l^2} \\ & + \frac{\overline{F_{lg} e^{j\theta_g}} \sum_{k\in\alpha_G, k\neq g} F_{lk} v_k e^{j\theta_k}}{v_l^2}\end{aligned} \tag{9}$$

$$\frac{\partial Z_l \overline{Z}_l}{\partial \theta_l} = 0 \tag{10}$$

$$\frac{\partial Z_l \overline{Z}_l}{\partial v_l} = -2\frac{\sum_{g\in\alpha_G} F_{lg} v_g e^{j\theta_g} \overline{\sum_{g\in\alpha_G} F_{lg} v_g e^{j\theta_g}}}{v_l^3} \tag{11}$$

4 Hessian

$$\frac{\partial^2 Z_l}{\partial \theta_g^2} = \overline{\frac{\partial \overline{Z_l^2}}{\partial \theta_g^2}} = -\frac{F_{lg} v_g e^{j\theta_g - j\theta_l}}{v_l} \tag{12}$$

$$\frac{\partial^2 Z_l}{\partial \theta_g \partial v_g} = \overline{\frac{\partial \overline{Z_l^2}}{\partial \theta_g \partial v_g}} = j\frac{F_{lg} e^{j\theta_g - j\theta_l}}{v_l} \tag{13}$$

$$\frac{\partial^2 Z_l}{\partial \theta_g \partial \theta_l} = \overline{\frac{\partial \overline{Z_l^2}}{\partial \theta_g \partial \theta_l}} = \frac{F_{lg} v_g e^{j\theta_g - j\theta_l}}{v_l} \tag{14}$$

$$\frac{\partial^2 Z_l}{\partial \theta_g \partial v_l} = \overline{\frac{\partial \overline{Z_l^2}}{\partial \theta_g \partial v_l}} = -j\frac{F_{lg} v_g e^{j\theta_g - j\theta_l}}{v_l^2} \tag{15}$$

$$\frac{\partial^2 Z_l}{\partial v_g \theta_g} = \overline{\frac{\partial^2 \overline{Z_l}}{\partial v_g \theta_g}} = (13)$$

$$\frac{\partial^2 Z_l}{\partial v_g^2} = \overline{\frac{\partial^2 \overline{Z_l}}{\partial v_g^2}} = 0 \tag{16}$$

$$\frac{\partial^2 Z_l}{\partial v_g \theta_l} = \overline{\frac{\partial^2 \overline{Z_l}}{\partial v_g \theta_l}} = -j\frac{F_{lg} e^{j\theta_g - j\theta_l}}{v_l} \tag{17}$$

$$\frac{\partial^2 Z_l}{\partial v_g v_l} = \overline{\frac{\partial^2 \overline{Z_l}}{\partial v_g v_l}} = -\frac{F_{lg} e^{j\theta_g - j\theta_l}}{v_l^2} \tag{18}$$

$$\frac{\partial^2 Z_l}{\partial \theta_l \partial \theta_g} = \overline{\frac{\partial \overline{Z_l^2}}{\partial \theta_l \partial \theta_g}} = (14)$$

$$\frac{\partial^2 Z_l}{\partial \theta_l \partial v_g} = \overline{\frac{\partial \overline{Z_l^2}}{\partial \theta_l \partial v_g}} = (17)$$

$$\frac{\partial^2 Z_l}{\partial \theta_l^2} = \overline{\frac{\partial \overline{Z_l^2}}{\partial \theta_l^2}} = -\frac{\sum_{g \in \alpha_G} F_{lg} v_g e^{j\theta_g - j\theta_l}}{v_l} \tag{19}$$

$$\begin{aligned}
\frac{\partial^2 Z_l}{\partial \theta_l \partial v_l} &= \frac{\overline{\partial \overline{Z_l^2}}}{\partial \theta_l \partial v_l} &&= j \frac{\sum_{g \in \alpha_G} F_{lg} v_g e^{j\theta_g - j\theta_l}}{v_l^2} && (20) \\
\frac{\partial^2 Z_l}{\partial v_l \partial \theta_g} &= \frac{\overline{\partial \overline{Z_l^2}}}{\partial v_l \partial \theta_g} &&= (15) \\
\frac{\partial^2 Z_l}{\partial v_l \partial v_g} &= \frac{\overline{\partial \overline{Z_l^2}}}{\partial v_l \partial v_g} &&= (18) \\
\frac{\partial^2 Z_l}{\partial v_l \partial \theta_l} &= \frac{\overline{\partial \overline{Z_l^2}}}{\partial v_l \partial \theta_l} &&= (20) \\
\frac{\partial^2 Z_l}{\partial v_l^2} &= \frac{\overline{\partial \overline{Z_l^2}}}{\partial v_l^2} &&= 2 \frac{\sum_{g \in \alpha_G} F_{lg} v_g e^{j\theta_g - j\theta_l}}{v_l^3} && (21)
\end{aligned}$$

$$\frac{\partial^2 Z_l\overline{Z}_l}{\partial\theta_g^2} = -\frac{F_{lg}v_g e^{j\theta_g}\overline{\sum_{k\in\alpha_G,k\neq g} F_{lg}v_k e^{j\theta_k}}}{v_l^2} - \frac{\overline{F_{lg}v_g e^{j\theta_g}}\sum_{k\in\alpha_G,k\neq g} F_{lg}v_k e^{j\theta_k}}{v_l^2} \tag{22}$$

$$\frac{\partial^2 Z_l\overline{Z}_l}{\partial\theta_g\partial\theta_k} = \frac{F_{lg}v_g e^{j\theta_g}\overline{\sum_{k\in\alpha_G,k\neq g} F_{lg}v_k e^{j\theta_k}}}{v_l^2} + \frac{\overline{F_{lg}v_g e^{j\theta_g}}\sum_{k\in\alpha_G,k\neq g} F_{lg}v_k e^{j\theta_k}}{v_l^2} \tag{23}$$

$$\frac{\partial^2 Z_l\overline{Z}_l}{\partial\theta_g\partial v_g} = j\frac{F_{lg}e^{j\theta_g}\overline{\sum_{k\in\alpha_G,k\neq g} F_{lg}v_k e^{j\theta_k}}}{v_l^2} - j\frac{\overline{F_{lg}e^{j\theta_g}}\sum_{k\in\alpha_G,k\neq g} F_{lg}v_k e^{j\theta_k}}{v_l^2} \tag{24}$$

$$\frac{\partial^2 Z_l\overline{Z}_l}{\partial\theta_g\partial v_k} = j\frac{F_{lg}v_g e^{j\theta_g}\overline{F_{lk}e^{j\theta_k}}}{v_l^2} - j\frac{\overline{F_{lg}v_g e^{j\theta_g}}F_{lk}e^{j\theta_k}}{v_l^2} \tag{25}$$

$$\frac{\partial^2 Z_l\overline{Z}_l}{\partial\theta_g\partial\theta_l} = 0 \tag{26}$$

$$\frac{\partial^2 Z_l\overline{Z}_l}{\partial\theta_g\partial v_l} = -2j\frac{F_{lg}v_g e^{j\theta_g}\overline{\sum_{k\in\alpha_G,k\neq g} F_{lk}e^{j\theta_k}}}{v_l^3} + 2j\frac{\overline{F_{lg}v_g e^{j\theta_g}}\sum_{k\in\alpha_G,k\neq g} F_{lk}e^{j\theta_k}}{v_l^3} \tag{27}$$

$$\frac{\partial^2 Z_l \overline{Z}_l}{\partial v_g \partial \theta_g} = (24)$$

$$\begin{aligned}\frac{\partial^2 Z_l \overline{Z}_l}{\partial v_g \partial \theta_k} = & -j\frac{F_{lg}e^{j\theta_g}\overline{\sum_{k\in\alpha_G,k\neq g} F_{lk}v_k e^{j\theta_k}}}{v_l^2} \\ & + j\frac{\overline{F_{lg}e^{j\theta_g}}\sum_{k\in\alpha_G,k\neq g} F_{lk}v_k e^{j\theta_k}}{v_l^2}\end{aligned} \tag{28}$$

$$\frac{\partial^2 Z_l \overline{Z}_l}{\partial v_g^2} = 2\frac{F_{lg}\overline{F_{lg}}}{v_l^2} \tag{29}$$

$$\begin{aligned}\frac{\partial^2 Z_l \overline{Z}_l}{\partial v_g \partial v_k} = & \frac{F_{lg}e^{j\theta_g}\overline{F_{lg}e^{j\theta_k}}}{v_l^2} \\ & + \frac{\overline{F_{lg}e^{j\theta_g}}F_{lk}e^{j\theta_k}}{v_l^2}\end{aligned} \tag{30}$$

$$\frac{\partial^2 Z_l \overline{Z}_l}{\partial v_g \partial \theta_l} = 0 \tag{31}$$

$$\begin{aligned}\frac{\partial^2 Z_l \overline{Z}_l}{\partial v_g \partial v_l} = & -4\frac{F_{lg}\overline{F_{lg}}v_g}{v_l^3} \\ & -2\frac{F_{lg}e^{j\theta_g}\overline{\sum_{k\in\alpha_G,k\neq g} F_{lk}v_k e^{j\theta_k}}}{v_l^3} \\ & -2\frac{\overline{F_{lg}e^{j\theta_g}}\sum_{k\in\alpha_G,k\neq g} F_{lk}v_k e^{j\theta_k}}{v_l^3}\end{aligned} \tag{32}$$

$$\frac{\partial^2 Z_l \overline{Z}_l}{\partial \theta_l \partial \theta_g} = (26)$$

$$\frac{\partial^2 Z_l \overline{Z}_l}{\partial \theta_l \partial v_g} = (31)$$

$$\frac{\partial^2 Z_l \overline{Z}_l}{\partial \theta_l^2} = 0 \tag{33}$$

$$\frac{\partial^2 Z_l \overline{Z}_l}{\partial \theta_l \partial v_l} = 0 \tag{34}$$

$$\frac{\partial^2 Z_l \overline{Z}_l}{\partial v_l \partial \theta_g} = (27)$$

$$\frac{\partial^2 Z_l \overline{Z}_l}{\partial v_l \partial v_g} = (32)$$

$$\frac{\partial^2 Z_l \overline{Z}_l}{\partial v_l \partial \theta_l} = (34)$$

$$\frac{\partial^2 Z_l \overline{Z}_l}{\partial v_l^2} = 6\frac{\sum_{g\in\alpha_G} F_{lg} v_g e^{j\theta_g} \overline{\sum_{g\in\alpha_G} F_{lg} v_g e^{j\theta_g}}}{v_l^4} \qquad (35)$$