University of Feng Chia

Power system lab

AC Power Flows, Generalized OPF of $\sum L^2$

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1 Notation

n_b, n_g, n_l	number of buses, generators, branches .
v_i, θ_i	voltage magnitude and angle at bus i .
V_i	complex bus voltage at bus i, $\lvert v_i \rvert e^{j\theta_i}$.
Y_{bus}	$n_b \times n_b$ system bus admittance matrix.
Y_{LL}, Y_{LG}	the submatrices of system admittance Y_N with respect to load buses(by subscript L) and generator buses(by subscript G) .
Z_{LL}	inverse matrix of Y_{LL} .
F_{LG}	$-Z_{LL} \times Y_{LG}$
α_L, α_G	the sets of load node and gen node.
L_n	L-index of node n.

2 Introduction

We will be looking at complex functions of the real valued vector

$$X = \begin{bmatrix} \theta \\ V \\ P_g \\ Q_g \end{bmatrix} \tag{1}$$

For a complex scalar function $f : \mathbb{R}^n \to \mathbb{C}$ of a real vector $x = [x_1, x_2, \ldots, x_n]^T$ we use the following notation for the first derivatives (transpose of the gradient)

$$f_x = \frac{\partial f}{\partial x} = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \quad \cdots \frac{\partial f}{, \partial x_n} \right] \tag{2}$$

The matrix of second partial derivatives, the Hessian of f, is

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial x}\right)^T = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \cdots & \frac{\partial f}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_1 \partial x_n} & \cdots & \frac{\partial f}{\partial x_n^2} \end{bmatrix} \tag{3}$$

3 Gradient

Let

$$Z_l = \frac{\sum_{g \in \alpha_G} F_{lg} V_g}{V_l} \tag{4}$$

$$F = \sum_{l \in \alpha_L} L_l^2 = \sum_{l \in \alpha_L} \left| 1 - \frac{\sum_{i \in \alpha_g} F_{lg} V_g}{V_l} \right|^2 \tag{5}$$

$$= \sum_{l \in \alpha_L} (1 - Z_l - \overline{Z_l} + Z_l \overline{Z}_l) \tag{6}$$

$$\frac{\partial Z_l}{\partial \theta_g} = \frac{j F_{lg} v_g e^{j(\theta_g - \theta_l)}}{v_l} \tag{7}$$

$$\frac{\partial \overline{Z_l}}{\partial \theta_g} = \frac{-j \overline{F_{lg}} v_g e^{-j(\theta_g - \theta_l)}}{v_l} \tag{8}$$

$$\frac{\partial Z_l}{\partial v_g} = \frac{F_{lg} e^{j(\theta_g - \theta_l)}}{v_l} \tag{9}$$

$$\frac{\partial \overline{Z_l}}{\partial v_g} = \frac{\overline{F_{lg}} e^{-j(\theta_g - \theta_l)}}{v_l} \tag{10}$$

$$\frac{\partial Z_l}{\partial \theta_l} = \frac{-j F_{lg} v_g e^{j(\theta_g - \theta_l)}}{v_l} \tag{11}$$

$$\frac{\partial \overline{Z_l}}{\partial \theta_l} = \frac{j \overline{F_{lg}} v_g e^{-j(\theta_g - \theta_l)}}{v_l} \tag{12}$$

$$\frac{\partial Z_l}{\partial v_l} = \frac{-F_{lg} v_g e^{j(\theta_g - \theta_l)}}{v_l^2} \tag{13}$$

$$\frac{\partial \overline{Z_l}}{\partial v_l} = \frac{-\overline{F_{lg}} v_g e^{-j(\theta_g - \theta_l)}}{v_l^2} \tag{14}$$

4 Hessian

$$\frac{\partial^2 Z_l}{\partial \theta_g^2} = \frac{-F_{lg} v_g e^{j(\theta_g - \theta_l)}}{v_l} \tag{15}$$

$$\frac{\partial^2 \overline{Z_l}}{\partial \theta_g^2} = \frac{-\overline{F_{lg}} v_g e^{-j(\theta_g - \theta_l)}}{v_l} \tag{16}$$

$$\frac{\partial^2 Z_l}{\partial \theta_g \partial v_g} = \frac{j F_{lg} e^{j(\theta_g - \theta_l)}}{v_l} \tag{17}$$

$$\frac{\partial^2 \overline{Z_l}}{\partial \theta_g \partial v_g} = \frac{-j \overline{F_{lg}} e^{-j(\theta_g - \theta_l)}}{v_l} \tag{18}$$

$$\frac{\partial^2 Z_l}{\partial \theta_g \partial \theta_l} = \frac{F_{lg} v_g e^{j(\theta_g - \theta_l)}}{v_l} \tag{19}$$

$$\frac{\partial^2 \overline{Z_l}}{\partial \theta_g \partial \theta_l} = \frac{\overline{F_{lg}} v_g e^{-j(\theta_g - \theta_l)}}{v_l} \tag{20}$$

$$\frac{\partial^2 Z_l}{\partial \theta_g \partial v_l} = \frac{-j F_{lg} v_g e^{j(\theta_g - \theta_l)}}{v_l^2} \tag{21}$$

$$\frac{\partial^2 \overline{Z_l}}{\partial \theta_g \partial v_l} = \frac{j \overline{F_{lg}} v_g e^{-j(\theta_g - \theta_l)}}{v_l^2} \tag{22}$$

$$\frac{\partial^2 Z_l}{\partial v_g^2} = 0 \tag{23}$$

$$\frac{\partial^2 \overline{Z_l}}{\partial v_g^2} = 0 \tag{24}$$

$$\frac{\partial^2 Z_l}{\partial v_g \partial \theta_l} = \frac{-jF_{lg}e^{j(\theta_g-\theta_l)}}{v_l} \tag{25}$$

$$\frac{\partial^2 \overline{Z_l}}{\partial v_g \partial \theta_l} = \frac{j\overline{F_{lg}}e^{j(\theta_g-\theta_l)}}{v_l} \tag{26}$$

$$\frac{\partial^2 Z_l}{\partial v_g \partial v_l} = \frac{-F_{lg}e^{j(\theta_g-\theta_l)}}{v_l^2} \tag{27}$$

$$\frac{\partial^2 \overline{Z_l}}{\partial v_g \partial v_l} = \frac{-\overline{F_{lg}}e^{-j(\theta_g-\theta_l)}}{v_l^2} \tag{28}$$

$$\frac{\partial^2 Z_l}{\partial \theta_l^2} = \frac{-F_{lg}v_g e^{j(\theta_g-\theta_l)}}{v_l} \tag{29}$$

$$\frac{\partial^2 \overline{Z_l}}{\partial \theta_l^2} = \frac{-\overline{F_{lg}}v_g e^{-j(\theta_g-\theta_l)}}{v_l} \tag{30}$$

$$\frac{\partial^2 Z_l}{\partial \theta_l \partial v_l} = \frac{jF_{lg}v_g e^{j(\theta_g-\theta_l)}}{v_l^2} \tag{31}$$

$$\frac{\partial^2 \overline{Z_l}}{\partial \theta_l \partial v_l} = \frac{-j\overline{F_{lg}}v_g e^{-j(\theta_g-\theta_l)}}{v_l^2} \tag{32}$$

$$\frac{\partial^2 Z_l}{\partial v_l^2} = \frac{2F_{lg}v_g e^{j(\theta_g-\theta_l)}}{v_l^3} \tag{33}$$

$$\frac{\partial^2 \overline{Z_l}}{\partial v_l^2} = \frac{2\overline{F_{lg}}v_g e^{-j(\theta_g-\theta_l)}}{v_l^3} \tag{34}$$