University of Feng Chia

Power system lab

AC Power Flows, Generalized OPF of Reactive Power Cost

Author:
林逸松

Supervisor:

September 12, 2016

1 Notation

n_b, n_g, n_l	number of buses, generators, branches .
v_n, θ_n	voltage magnitude and angle at bus n .
Y_{bus}	$n_b \times n_b$ system bus admittance matrix.

2 Introduction

We will be looking at complex functions of the real valued vector

$$X = \begin{bmatrix} \theta \\ V \\ P_g \\ Q_g \end{bmatrix} \tag{1}$$

For a complex scalar function $f : \mathbb{R}^n \to \mathbb{C}$ of a real vector $x = [x_1, x_2, \ldots, x_n]^T$ we use the following notation for the first derivatives (transpose of the gradient)

$$f_x = \frac{\partial f}{\partial x} = \left[\frac{\partial f}{\partial x_1,} \frac{\partial f}{\partial x_2,} \quad \cdots \frac{\partial f}{,\partial x_n} \right] \tag{2}$$

The matrix of second partial derivatives, the Hessian of f, is

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial x}\right)^T = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \cdots & \frac{\partial f}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_1 \partial x_n} & \cdots & \frac{\partial f}{\partial x_n^2} \end{bmatrix} \tag{3}$$

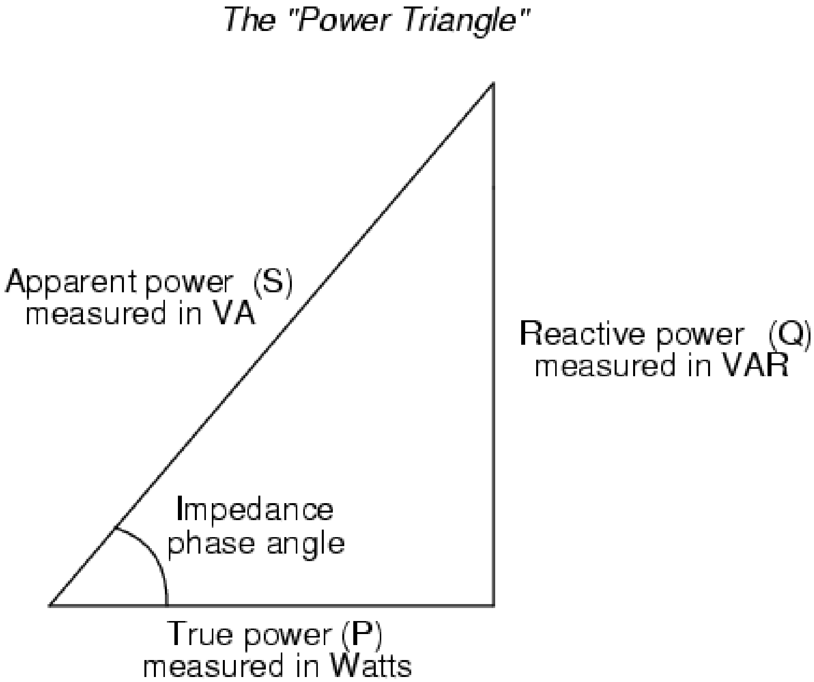

Figure 1: $S^2 = P^2 + Q^2$

Although data is usually given in terms of $\$/MW$,as nominal operation of a generator is always preferred ,the capital in term of capacity $\$/MVA$ can be derived from the following equation:

$$\$/MVA = \$/MW * pf \tag{4}$$

where, pf is nominal power factor of the generator. Therefore,the capital cost in terms of reactive power can also be specified:

$$\$/MVar = \$/MVA * sin\Theta = \$/MVA * sin(cos^{-1}pf) \tag{5}$$

3 Gradient

$$f = a_Q \times Q_g^2 + b_Q \times Q_g + c_Q \tag{6}$$

$$a_Q = a_P \times (\frac{P_g \times Q_g}{P_g^2 + Q_g^2})^2 \tag{7}$$

$$b_Q = b_P \times \frac{P_g \times Q_g}{P_g^2 + Q_g^2} \tag{8}$$

$$c_Q = c_P \tag{9}$$

$$\begin{aligned} \frac{\partial f}{\partial P_g} =& 2a_P \times (\frac{P_g Q_g^4}{(P_g^2 + Q_g^2)^2} - \frac{2P_g^3 Q_g^4}{(P_g^2 + Q_g^2)^3}) \\ &+ b_P \times (\frac{Q_g^2}{P_g^2 + Q_g^2} - \frac{2P_g^2 Q_g^2}{(P_g^2 + Q_g^2)^2}) \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{\partial f}{\partial Q_g} =& 4a_P \times (\frac{P_g^2 Q_g^3}{(P_g^2 + Q_g^2)^2} - \frac{P_g^2 Q_g^5}{(P_g^2 + Q_g^2)^3}) \\ &+ 2b_P \times (\frac{P_g Q_g}{P_g^2 + Q_g^2} - \frac{P_g Q_g^3}{(P_g^2 + Q_g^2)^2}) \end{aligned} \tag{11}$$

4 Hessian

$$\frac{\partial^2 f}{\partial P_g^2} = 2a_P \times (\frac{P_g^4}{(P_g^2+Q_g^2)^2} - \frac{10P_g^2Q_g^4}{(P_g^2+Q_g^2)^3} + \frac{12P_g^4Q_g^4}{(P_g^2+Q_g^2)^4}) + 2b_P \times (\frac{4P_g^3Q_g^2}{(P_g^2+Q_g^2)^3} - \frac{3P_gQ_g^2}{(P_g^2+Q_g^2)^2}) \tag{12}$$

$$\frac{\partial^2 f}{\partial P_g \partial Q_g} = 8a_P \times (\frac{P_gQ_g^3}{(P_g^2+Q_g^2)^2} - \frac{P_gQ_g^5}{(P_g^2+Q_g^2)^3} - \frac{2P_g^3Q_g^3}{(P_g^2+Q_g^2)^3} + \frac{3P_g^3Q_g^5}{(P_g^2+Q_g^2)^4}) + 2b_P \times (\frac{Q_g}{P_g^2+Q_g^2} - \frac{P_g^3}{(P_g^2+Q_g^2)^2} - \frac{2P_g^2Q_g}{(P_g^2+Q_g^2)^2} + \frac{4P_g^2Q_g^3}{(P_g^2+Q_g^2)^3}) \tag{13}$$

$$\frac{\partial^2 f}{\partial Q_g^2} = 12a_P \times (\frac{P_g^2Q_g^2}{(P_g^2+Q_g^2)^2} - \frac{3P_g^2Q_g^4}{(P_g^2+Q_g^2)^3} + \frac{2P_g^2Q_g^6}{(P_g^2+Q_g^2)^4}) + 2b_P \times (\frac{P_g}{P_g^2+Q_g^2} - \frac{5P_gQ_g^2}{(P_g^2+Q_g^2)^2} + \frac{4P_gQ_g^4}{(P_g^2+Q_g^2)^3}) \tag{14}$$