1 Intro

This paper is aimed to study magnetic dynamics of the synthetic antiferromagnets under the influence of spin-polarized current and external magnetic field.

The first step in this direction is to consider the system in details:

- to give a definition of the synthetic antiferromagnetic system and to describe the main components of it;
- to describe assumptions about intrinsic structure of the components;

The second step is to consider the linear approximation of the dynamic equations. Here we expect to get the analytical expression of the critical current and the eigenfrequencies of the system.

At the next step we expect to write dynamic equations in the convenient form to use it in numerical experiment. During the numerical experiment we expect to confirm our analytical results.

2 Model

Let's consider a magnetic system which consists of two subsystems and the transition region.

The first subsystem is consists of the ferromagnetic films separated by the nonmagnetic layers. The width of the separators determines the intensity and the type of the interaction between magnetic films. We will consider the antiferromagnetic type of the interaction which means that the vectors of magnetization of the neighboring magnetic layers are aligned oppositely. This subsystem we will call the polarizer.

The structure of the second subsystem is identical to the first one. We will call it the analyzer. The main difference between the polarizer and the analyzer is that the magnetization of the former is fixed by some external reason and the magnetization of the latter is free.

The two subsystems are arranged one after the other in the plane of magnetic films, so that against each magnetic layer of analyzer is (напротив каждого магнитного слоя анализатора находиться феромагнитный слой поляризатораб намагниченностя которых направлена противоположно)

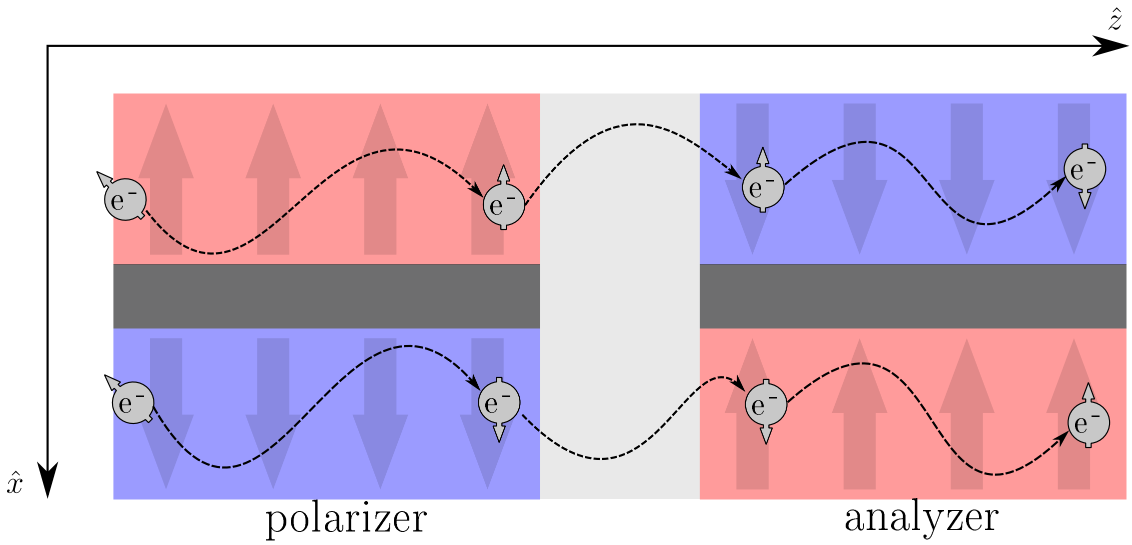

Рис. 1: Schema of the model

These two subsystems are separated by the transition region, the magnetization of which is relatively small. This region we will call the interface. The width of the interface is determined by two factors: it should be sufficiently large to prevent direct exchange interaction between subsystems; also it should be sufficiently small to provide electrons transitions between subsystems in the ballistic mode.

The electron current density flowing through the polarizer along z axis gains the spin-polarization which is aligned parallel to the magnetization of the appropriate magnetic layer, so the polarization is changed periodically along x axis due to periodicity of the structure. The appropriate influence of the current to the magnetic structure is neglected due to the magnetization fixing which was mentioned earlier. Thus, we obtain spin-polarized current which flows into the analyzer through the interface.

Further, we will consider the influence of this current on the magnetic states of the analyzer.

Flowing through the bulk of the analyzer the moving electrons scatter on the magnetic sublattice. This can be effectively described as a torque acting on the analyzers magnetization.

For the sake of the simple analytical describing of the analyzer let's divide it into two sublattices. Each sublattice contains all magnetic layers with the same direction of the magnetization. Here we assume that only two opposite directions is allowed in equilibrium state.

So, we can consider each sublattice as a simple ferromagnetic system, and apply standard approach to depict the influence of spin-polarized current on it.

$$\dot{\mathbf{M}}_k = -\gamma \left[\mathbf{M}_k \times \mathbf{H}_k\right] + \frac{\alpha_G}{M_0} \left[\mathbf{M}_k \times \dot{\mathbf{M}}_k\right] + \frac{\sigma I}{M_0} \left[\mathbf{M}_k \times \left[\mathbf{M}_k \times \hat{\mathbf{p}}_k\right]\right]. \tag{1}$$

Here k denotes sublattice. In our case $k = 1, 2$. $\mathbf{M}_k$ is a vector of magnetization which is defined as a magnetic moment of the unit volume of the appropriate sublattice. $\hat{\mathbf{p}}$ is a current polarization vector – unit vector, directed along the polarization of the current in sufficient sublattice. $\mathbf{H}_k = -\partial W / \partial \mathbf{M}_k$ is an effective magnetic field acting on k-th sublattice, W is a density of the free energy of the analyzer. It is important to mention that it contains the term which describe the interaction between sublattices.

As a first touch, we consider isotropic ferromagnetic material. Therefor, the density of the free energy can be considered as following:

$$W = J_{\text{int}} \left(\mathbf{M}_1 \cdot \mathbf{M}_2\right) - \frac{1}{2} K \left(M_{1z}^2 + M_{2z}^2\right). \tag{2}$$

This quantity achieves it minimum value when the magnetizations of the sublattices directs oppositely to each other and along the z axis. The former statement says that we deal with antiferromagnetic interaction between sublattices, and the latter statement defines preferred direction in the structure of the ferromagnetic material. According to this assumption $p_k = (0, 0, (-1)^{k+1})$.

3 Analytics

3.1 Linear approximation

According to the model which was described in the section 2 the equilibrium states of the magnetization vector in each sublattice directed along z axis:

$$\mathbf{M}_k^0 = \big(0, 0, (-1)^{k+1} M_0\big)$$

where M_0 – saturation value of the magnetization.

Let's assume that the magnetization vector is slightly deviated from the equilibrium states by the value which is much smaller then M_0. This excited vector can be written as a sum of two perpendicular vectors:

$$\mathbf{M}_k = \mathbf{m}_k + \mathbf{M}_{kz},$$

where $\mathbf{m}_k$ is a small vector which lies on xy plane.

The equations of motion (1) leave the magnitude of the magnetization vector constant, so $|\mathbf{M}_k| = M_0$. In this case, the magnitude of the $\mathbf{M}_{kz}$ can be approximated as following:

$$\begin{aligned}
\mathbf{M}_{kz} &= \mathbf{M}_k - \mathbf{m}_k; \\
M_{kz}^2 &= M_k^2 - 2\mathbf{M}_k \cdot \mathbf{m}_k + m_k^2 \\
&= M_k^2 - m_k^2 = M_0^2 - m_k^2; \\
|\mathbf{M}_{kz}| &= \sqrt{M_0^2 - m_k^2} = M_0 \sqrt{1 - \frac{m_k^2}{M_0^2}}
\end{aligned}$$

The expending of the last expression in the Taylor series gives $M_{kz} \approx M_0 \left(1 - \frac{1}{2}\frac{m_i^2}{M_0^2}\right)$. Hence, in linear approximation the magnetization vector can be considered as following:

$$\mathbf{M}_k = \big(m_{kx}, m_{kx}, (-1)^{k+1} M_0\big) \tag{3}$$

In our case the current flowing through the bulk is a source of the external amount of energy which manifesting in the deviations of the magnetization vector from it equilibrium state. The internal damping dissipate this energy. So, there is a value of the current when this two processes compensate each other. This value is called critical current. Let's find it analytical expression.

By substituting the expression for magnetization vector (3) into the equations of motion (1) and neglecting the terms of the *second order of smallness* (Here we assume that the small quantity is one of the vectors $\mathbf{m}_k$ or it component) we obtain a system of equations for deviation terms:

$$\begin{aligned}
\dot{m}_{1x} &= -\sigma I m_{1x} - \gamma M_0 (J_{\text{int}} + K) m_{1y} - \gamma M_0 J_{\text{int}} m_{2y} - \alpha_G \dot{m}_{1y} \\
\dot{m}_{1y} &= \gamma M_0 (J_{\text{int}} + K) m_{1x} - \sigma I m_{1y} + \gamma M_0 J_{\text{int}} m_{2x} + \alpha_G \dot{m}_{1x} \\
\dot{m}_{2x} &= \gamma M_0 J_{\text{int}} m_{1y} - \sigma I m_{2x} + \gamma M_0 (J_{\text{int}} + K) m_{2y} + \alpha_G \dot{m}_{2y} \\
\dot{m}_{2y} &= -\gamma M_0 J_{\text{int}} m_{1x} - \gamma M_0 (J_{\text{int}} + K) m_{2x} - \sigma I m_{2y} - \alpha_G \dot{m}_{2x}
\end{aligned}$$

The solutions of this system of equations we will seek in the exponent form $m_i \sim e^{i\omega t}$. In this case the eigenvalues of the frequency is following:

$$\begin{aligned}
\omega &= i\frac{\sigma I + \gamma M_0 (K + J_{\text{int}})\alpha_G}{\alpha_G^2 + 1} \\
&\pm \frac{\sqrt{(\gamma M_0 (K + J_{\text{int}}) - \alpha_G \sigma I)^2 - (\gamma M_0 J_{\text{int}})^2 (\alpha_G^2 + 1)}}{\alpha_G^2 + 1}
\end{aligned} \tag{4}$$

This expression is complex value. The real part of it is responsible for the frequency of precession of magnetization vector around the preferred direction in the ferromagnetic layer. The imaginary part is responsible for either increasing or decreasing of the amplitude of precession. It is determined by the ratio between amount of pumping and damping energy.

Let's consider the situation when there is no friction in the system ($\alpha_G = 0$). In this case the eigenfrequencies of such system is determined by the expression:

$$\omega_{\text{nofric}} = i\sigma I \pm \gamma M_0 \sqrt{K\,(K + 2J_{\text{int}})}.$$

The real part of this frequency increase with either increasing of antiferromagnetic interaction between the layers (J_{int}) or increasing of *preferred direction quality*.

The imaginary part depends only on current intensity. Depending on the direction of the current flow it can play role of the effective damping or the effective pumping (it determined by the sign of I).

In the general case the situation is more complicated. According to the expression 4 nonzero friction leads to the dependency of the real part of frequency both on the coefficient of damping and the current intensity. The pure frictional terms tends to reduce the frequency, however the term which contains current intensity can either enhance or reduce the frequency depending on it sign.

As for imaginary part, here situation is similar. The damping tends to turn the magnetization in the equilibrium position and minimize energy. The influence of current can be either dumping style (work in the same direction with damping, and as a result increasing effective damping in the system) or pumping style, when it work in opposite direction with damping, hence it work as a source of energy, so tends to increase deviations of magnetization from it equilibrium position.

When the current is working in pumping style it is possible curious situation when the current compensates energy losses in the system from the friction. In this case, the intensity of the current is called critical current. This possible when the imaginary part in the (4) is nullified. Hence the critical current is determined by the following expression:

$$I_{\mathrm{cr}} = -\alpha_G \frac{\gamma M_0 \left(K + J_{\mathrm{int}}\right)}{\sigma}. \tag{5}$$

As we can see, in our notation this value is negative.

It is important to mention that the anisotropy and antiferromagnetic constants determine the scale of energy in the system. According to the expression for critical current, the damping effects is proportional to sum of these two, so it proportional to the amount of energy in the system. It is also important to mention that critical current is inversely proportional to the σ. This parameter is influenced by the range of polarization of the current, the ability of the system to perceive the spin current, and it independent from the K and J_{int}.

4 Lagrange description

The magnitude of the magnetization vector stays constant during the motion. As we have shown, in the case of small deviations form the equilibrium state, using of Cartesian coordinate system is straightforward. However it is inconvenient in the case of arbitrary deviations. The symmetry of the system prompts us to consider of the motion of magnetization vector in the spherical coordinate system with fixed length. In this case, however, it is much simpler to obtain the equations of motions from the Lagrangian principle.

In the case that we considered, the Lagrangian function of the system can be written as following:

$$\begin{aligned} \mathcal{L} &= \frac{M_0}{\gamma} \sum_{k=0}^{1} \left[(1 - \cos\theta_k)\, \dot{\phi}_k + \frac{\omega_{FM}}{2} \cos^2\theta_k \right] \\ &- \frac{M_0}{\gamma} \Delta\omega_{\mathrm{int}} \left(\cos\theta_0 \cos\theta_1 + \sin\theta_0 \sin\theta_1 \cos(\phi_0 - \phi_1) \right). \end{aligned} \tag{6}$$

The k index varies between 0 and 1, specifying the appropriate layer.

Here we present two new constants: ω_{FM} and $\Delta\omega_{\mathrm{int}}$. Their meaning is straightforward: ω_{FM} is a ferromagnetic frequency in the absence of external fields and currents; $\Delta\omega_{\mathrm{int}}$ is an additional term to the frequency which determined by the strength of the interlayer interaction. Matching between these constants and those which were presented in (1) and (2) will be given further.

As we deal with non-conservative system, the dissipative function has to be presented:

$$\mathcal{R} = \frac{M_0}{\gamma} \sum_{k=0}^{1} \left[\frac{1}{2} \alpha_G \left(\dot{\theta}_k^2 + \dot{\phi}_k^2 \sin^2\theta_k \right) - \epsilon\sigma I (-1)^k \sin^2\theta_k \dot{\phi}_k \right] \tag{7}$$

The equations of motion are obtained according to the common approach of Lagrangian formalism:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_j} - \frac{\partial \mathcal{L}}{\partial q_j} = -\frac{\partial \mathcal{R}}{\partial \dot{q}_j} \tag{8}$$

By substitution of (6) and (7) into (8) the explicit equations of motion is obtained:

$$\begin{aligned} \sin\theta_k \dot{\phi}_k &= \omega_{\mathrm{FM}} \sin\theta_k \cos\theta_k - \Delta\omega_{\mathrm{int}} \sin\theta_k \cos\theta_{k+1} + \Delta\omega_{\mathrm{int}} \cos\theta_k \sin\theta_{k+1} \cos\Delta\phi + \alpha_{\mathrm{G}} \dot{\theta}_k \\ \dot{\theta}_k &= (-1)^k \left(\epsilon\sigma I \sin\theta_k + \Delta\omega_{\mathrm{int}} \sin\theta_{k+1} \sin\Delta\phi \right) - \alpha_{\mathrm{G}} \sin\theta_k \dot{\phi}_k. \end{aligned} \tag{9}$$

Here $\Delta\phi$ denotes $\phi_0 - \phi_1$.

This system of for equations ($k = 0, 1$) can be rewritten in more convenient form:

$$\begin{aligned} (1 + \alpha_{\mathrm{G}}) \dot{\theta}_k &= (-1)^k \left(\Delta\omega_{\mathrm{int}} \sin\theta_{k+1} \sin\Delta\phi + \epsilon\sigma I \sin\theta_k \right) - \alpha_{\mathrm{G}} \omega_{\mathrm{FM}} \sin\theta_k \cos\theta_k \\ &+ \alpha \Delta\omega_{\mathrm{int}} \left(\sin\theta_k \cos\theta_{k+1} - \cos\theta_k \sin\theta_{k+1} \cos\Delta\phi \right) \\ (1 + \alpha_{\mathrm{G}}) \sin\theta_0 \sin\theta_1 \dot{\Delta\phi} &= (\omega_{\mathrm{FM}} + \Delta\omega_{\mathrm{int}}) \sin\theta_0 \sin\theta_1 (\cos\theta_0 - \cos\theta_1) \end{aligned} \tag{10}$$