Eugene DUPAS
97 210

Final Project: Pricing Derivatives with VBA

1. Introduction

This project simulates and compares two hedging strategies for a short position in a European call option, using Monte Carlo simulations and VBA-implemented models in Excel. The strategies are:

Delta Hedging: dynamic adjustment of the portfolio according to the option's delta.
Stop-Loss Hedging: buying/selling shares when the stock price crosses a specified threshold.

Inputs (Value if not precises):
- Spot Price (49€), Strike (50€), Maturity (20/52), Risk-Free Rate (5%), Volatility (20%), Trend (13%), Number of Simulations (1000), Number of Rebalancing Steps (150), Stop-Loss Tolerance (1%).
- The short call position involves 100,000 shares and is sold for €300,000.

Outputs:
- Mean Hedging Cost, Standard Deviation, Max/Min Hedging Costs, turnover, Probability of Profit.

Launch:
- Launch **Delta Hedging Simulation**: Click on **Button 5** in the spreadsheet **"Monte Carlo delta hedge"**
- Launch **Stop Loss Hedging Simulation**: Click on **Button 2** in the spreadsheet **"Monte Carlo Stop loss hedge"**
- **Strategies Comparisons** in the spreadsheet **"Strategies Comparaison"**

2. Dynamic Delta Hedge

The code simulates multiple stock price trajectories using a Geometric Brownian Motion model, recalculating the option's delta at each rebalancing point. The share inventory is continuously adjusted to match the target delta derived from the Black-Scholes formula, ensuring a dynamic and theoretically accurate hedge. Cash flows are carefully tracked, including proceeds from share trades and accrued interest from borrowing or lending. At maturity, the program handles the final position by delivering the required number of shares if the option is in the money or liquidating the remaining inventory if it is out of the money.

Sensibility to the time intervale between two portfolio adjustments

Number of Steps	Average Hedging Costs (€)	Standard dev. of Hedging Costs (€)	Max Hedging Costs (€)	Min Hedging Costs (€)	Turnover (Nb of Shares)	Probability of a gain of money
10	- 56 544	62 525	302 984	- 253 138	108 114	83,30%
50	- 60 582	30 361	106 678	- 144 235	231 653	96,90%
150	- 60 978	17 089	21 911	- 150 764	380 497	99,90%
500	- 61 487	9 556	- 21 976	- 102 085	719 581	100,00%

As the number of rebalancing steps increases from 10 to 500, the average hedging cost improves significantly, moving from -€56,544 to -€61,487, and the probability of a gain rises from 83.3% to nearly 100%. The standard deviation of hedging costs also drops sharply, reflecting more stable outcomes with frequent adjustments. However, this comes at the cost of a much higher turnover, indicating increased trading activity and potential transaction costs.

An average hedging cost of –€60,000, for an option sold at €300,000, is consistent with the Black-Scholes price of approximately €240,000 (€240,000 = €300,000 – €60,000). This makes sense because the Black-Scholes price represents the theoretical cost of a perfectly hedged position, achievable only with continuous rebalancing at an infinite frequency.

3. Stop Loss Hedge

The Stop-Loss Hedging code simulates stock price trajectories using Geometric Brownian Motion and applies predefined thresholds for action. At the start, the code checks if the stock price is below the strike; if so, no shares are bought, otherwise, 100,000 shares are acquired. As the simulation progresses, the stock price is updated at each rebalancing step, and decisions are made based on whether the price crosses a threshold above K+I or below K−I. The share inventory is adjusted accordingly: buying 100,000 shares if the upper threshold is crossed and selling them if the price drops below the lower threshold. Cash flows are tracked throughout the simulation, including proceeds from share transactions and accumulated interest from financing. At maturity, the code manages the final position: if the option is in the money, shares are delivered and the payoff is settled; if out of the money, any remaining inventory is liquidated.

Eugene DUPAS
97 210

Sensibility to the time intervale between two portfolio adjustments

Number of Steps	Average Hedging Costs (€)	Standard dev. of Hedging Costs (€)	Max Hedging Costs (€)	Min Hedging Costs (€)	Turnover (Nb of Shares)	Probability of a gain of money
10	- 48 574	217 869	1 010 550	- 305 819	146 300	62,70%
50	- 48 340	194 928	744 487	- 305 824	272 000	64,00%
150	- 48 470	190 093	944 551	- 305 825	383 100	65,40%
500	- 47 310	189 384	732 762	- 305 825	509 800	63,50%

For Stop-Loss hedging, increasing the number of rebalancing steps from 10 to 500 shows minimal improvement in average hedging cost (around -€48,000) and a persistently high standard deviation (~€190,000). The probability of a gain remains low (about 63-65%), even with more frequent monitoring, suggesting that this strategy's effectiveness is relatively insensitive to frequency if no transaction costs because turnover increases a lot with higher frequencies.

Sensibility to the Stop Loss Tolerance

I (%)	Average Hedging Costs (€)	Standard dev. of Hedging Costs (€)	Max Hedging Costs (€)	Min Hedging Costs (€)	Turnover (Nb of Shares)	Probability of a gain of money
0,5	- 48 308	193 299	710 724	- 305 825	384 700	64,50%
1	- 52 454	192 629	617 531	- 305 825	261 200	66,60%
3	- 53 083	193 862	800 728	- 305 825	115 600	69,30%
5	- 42 115	214 683	789 266	- 305 825	76 400	41,80%

Regarding the tolerance (l) parameter, higher tolerances (3-5%) initially improve the average hedging cost (-€53,083 at 3%) and increase the probability of gains (69.3%) but further increases to 5% degrade performance and consistency (probability drops to 41.8%). This highlights the importance of finding an optimal tolerance level for the Stop-Loss approach. Transactions costs are also important to consider because turnover decreases a lot with lower tolerance.

4. Key Results and Comparative Analysis

Strategies Comparison Table

	Delta Hedge & Forget	Dynamic Delta Hedge	Stop Loss Hedge
Average hedging cost (in €)	1 471 495	- 61 962	- 59 689
Standard dev. of hedging cost (in €)	1 339 852	18 055	189 147
Max hedging cost (in €)	3 440 854	75 199	634 494
Min hedging cost (in €)	- 308 102	- 148 928	- 305 825
Turnover (Nb of Shares)	52 160	377 063	37 1875

When comparing the three strategies, Dynamic Delta Hedging stands out with the lowest average hedging cost (-€61,962) and the lowest standard deviation (€18,055), demonstrating its effectiveness and stability. Stop-Loss Hedging achieves a similar average cost (-€59,689) but with a much higher standard deviation (€189,147), reflecting greater risk and variability. Hedge-and-Forget performs the worst, with a positive average cost (€1,471,495) and an extremely wide distribution (€1,339,852 standard deviation), highlighting its exposure to market movements without adjustments. In terms of maximum and minimum costs, Dynamic Delta Hedging and Stop-Loss show much lower extremes compared to Hedge-and-Forget. Turnover is predictably highest for Dynamic Delta Hedging (~377,000 shares) and lowest for Hedge-and-Forget (~52,000 shares), with Stop-Loss showing moderate trading activity (~372,000 shares). Overall, Dynamic Delta Hedging provides the best balance between cost, risk, and trading frequency.

5. Conclusion

This project demonstrates that, among the three strategies tested, dynamic Delta Hedging achieves the best balance between hedging cost, risk control, and trading activity. While Stop-Loss Hedging is simpler to implement, it presents higher variability and a greater risk of significant losses. Hedge-and-Forget, with its minimal adjustments, incurs high average costs and large potential losses due to unhedged price movements.

However, these simulations are based on idealized assumptions, such as perfect liquidity, absence of transaction costs, and the ability to trade continuously or at fixed intervals. In practice, market participants adjust their strategies based on market conditions, operational constraints, and cost considerations. Delta hedging is often performed at regular intervals (daily) rather than continuously, accepting a degree of intra-period risk. While gamma hedging, which compensates for the convexity of the option's value, may theoretically improve hedging accuracy, it is typically only applied when justified by market volatility and cost-benefit considerations.