ex5_sol

May 23, 2018

1 Exercise 5 - Convolutional Neural Networks and the MNIST dataset

This exercise is based on https://github.com/leriomaggio/deep-learning-keras-tensorflow

We want to solve the same multinomial classification problem as in last weeks exercise 4 using the MNIST dataset, but this time we want to use a convolutional neural network for it.

Before we start, we define a few useful functions, which we used in exercise 4:

```
In [1]: ####################################################################################

        import matplotlib.pyplot as plt
        %matplotlib inline

        def plot_history(network_history):
            plt.figure()
            plt.xlabel('Epochs')
            plt.ylabel('Loss')
            plt.plot(network_history.history['loss'])
            plt.plot(network_history.history['val_loss'])
            plt.legend(['Training', 'Validation'])

            plt.figure()
            plt.xlabel('Epochs')
            plt.ylabel('Accuracy')
            plt.plot(network_history.history['acc'])
            plt.plot(network_history.history['val_acc'])
            plt.legend(['Training', 'Validation'], loc='lower right')
            plt.show()

        ####################################################################################

        import itertools
        def plot_confusion_matrix(cm, classes,
                                  normalize=False,
                                  title='Confusion matrix',
                                  cmap=plt.cm.Blues):
            """
            This function prints and plots the confusion matrix.
```

```
    Normalization can be applied by setting `normalize=True`.
    """
    plt.imshow(cm, interpolation='nearest', cmap=cmap)
    plt.title(title)
    plt.colorbar()
    tick_marks = np.arange(len(classes))
    plt.xticks(tick_marks, classes, rotation=45)
    plt.yticks(tick_marks, classes)

    if normalize:
        cm = cm.astype('float') / cm.sum(axis=1)[:, np.newaxis]

    thresh = cm.max() / 2.
    for i, j in itertools.product(range(cm.shape[0]), range(cm.shape[1])):
        plt.text(j, i, cm[i, j],
                 horizontalalignment="center",
                 color="white" if cm[i, j] > thresh else "black")

    plt.tight_layout()
    plt.ylabel('True label')
    plt.xlabel('Predicted label')

###################################################################################
import matplotlib.cm as cm
def display_errors(errors_index,img_errors,pred_errors, obs_errors):
    """ This function shows 6 images with their predicted and real labels"""
    n = 0
    nrows = 2
    ncols = 3
    fig, ax = plt.subplots(nrows,ncols,sharex=True,sharey=True)
    for row in range(nrows):
        for col in range(ncols):
            error = errors_index[n]
            ax[row,col].imshow((img_errors[error]).reshape((28,28)), cmap=cm.Greys, inte
            ax[row,col].set_title("Predicted label :{}\nTrue label :{}".format(pred_erro
            n += 1

###################################################################################
```

1.1 Data Preparation

1.1.1 Very Important:

When dealing with images & convolutions, you need to handle the `image_data_format` properly, i.e. is the channel given first or last. The channel axis is an additional dimension of the input data used to access different views of the date, e.g. red/green/blue of a color image, left or right of a stereo sound file)

```
In [2]: from keras import backend as K
```

```
Using TensorFlow backend.
```

```
---------------------------------------------------------------------------

RuntimeError                              Traceback (most recent call last)

RuntimeError: module compiled against API version 0xc but this version of numpy is 0xb



---------------------------------------------------------------------------

RuntimeError                              Traceback (most recent call last)

RuntimeError: module compiled against API version 0xc but this version of numpy is 0xb
```

In [3]:
```
img_rows, img_cols = 28, 28

if K.image_data_format() == 'channels_first':
    shape_ord = (1, img_rows, img_cols)
else:  # channel_last
    shape_ord = (img_rows, img_cols, 1)
```

1.1.2 Task 1: Data preprocessing

- Load the mnist data of the keras datasets
- Scale the design matrix to values between 0 and 1
- Convert the design matrix to the expected (60000, 28, 28, 1) shape
- Convert the target vector to one-hot-vectors for the 10 classes
- Split the training data into 70% training and 30% validation data sets

1.1.3 Loading Data

In [4]:
```
#Import the required libraries
import numpy as np
from keras.utils import np_utils

np.random.seed(1338)  # for reproducibilty!!

from keras.datasets import mnist

(X_train, y_train), (X_test, y_test) = mnist.load_data()
```

1.1.4 Preprocess and Normalise Data

Convert the data to float, scale it with a MinMaxScaler and add the channel dimension

```
In [5]: X_train.shape
```

```
Out[5]: (60000, 28, 28)
```

```
In [6]: X_train = X_train.astype('float32')
        X_test = X_test.astype('float32')

        from sklearn.preprocessing import MinMaxScaler
        scaler = MinMaxScaler(feature_range=(0, 1))

        X_train = scaler.fit_transform(X_train.reshape(X_train.shape[0],img_rows*img_cols))
        X_test = scaler.transform(X_test.reshape(X_test.shape[0],img_rows*img_cols))

        X_train = X_train.reshape(X_train.shape[0],img_rows,img_cols)
        X_test = X_test.reshape(X_test.shape[0],img_rows,img_cols)
```

```
In [7]: X_train.shape
```

```
Out[7]: (60000, 28, 28)
```

```
In [8]: X_train = X_train.reshape((X_train.shape[0],) + shape_ord)
        X_test = X_test.reshape((X_test.shape[0],) + shape_ord)
```

```
In [9]: X_train.shape
```

```
Out[9]: (60000, 28, 28, 1)
```

1.1.5 Convert target vector

```
In [10]: from keras.utils import np_utils
         num_classes = 10
         Y_train = np_utils.to_categorical(y_train, num_classes)
         Y_test = np_utils.to_categorical(y_test, num_classes)
         Y_test[0]
```

```
Out[10]: array([ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  1.,  0.,  0.], dtype=float32)
```

1.1.6 Split Training and Validation Data

```
In [11]: from sklearn.model_selection import train_test_split

         X_train, X_val, Y_train, Y_val = train_test_split(X_train, Y_train, test_size=0.3, rand
```

1.2 A simple convolutional neural network

Convolution2D

```
from keras.layers.convolutional import Conv2D
```

```
Conv2D(filters, kernel_size, strides=(1, 1), padding='valid',
```

```
data_format=None, dilation_rate=(1, 1), activation=None,
use_bias=True, kernel_initializer='glorot_uniform',
bias_initializer='zeros', kernel_regularizer=None,
bias_regularizer=None, activity_regularizer=None,
kernel_constraint=None, bias_constraint=None)
```

Arguments: filters: Integer, the dimensionality of the output space (i.e. the number output of filters in the convolution).

kernel_size: An integer or tuple/list of 2 integers, specifying the width and height of the 2D convolution window. Can be a single integer to specify the same value for all spatial dimensions.

strides: An integer or tuple/list of 2 integers, specifying the strides of the convolution along the width and height. Can be a single integer to specify the same value for all spatial dimensions. Specifying any stride value != 1 is incompatible with specifying any dilation_rate value != 1.

padding: one of "valid" or "same" (case-insensitive).

data_format: A string, one of channels_last (default) or channels_first. The ordering of the dimensions in the inputs. channels_last corresponds to inputs with shape (batch, height, width, channels) while channels_first corresponds to inputs with shape (batch, channels, height, width). It defaults to the image_data_format value found in your Keras config file at ~/.keras/keras.json. If you never set it, then it will be "channels_last".

dilation_rate: an integer or tuple/list of 2 integers, specifying the dilation rate to use for dilated convolution. Can be a single integer to specify the same value for all spatial dimensions. Currently, specifying any dilation_rate value != 1 is incompatible with specifying any stride value != 1.

activation: Activation function to use (see activations). If you don't specify anything, no activation is applied (ie. "linear" activation: a(x) = x).

use_bias: Boolean, whether the layer uses a bias vector.

kernel_initializer: Initializer for the kernel weights matrix (see initializers).

bias_initializer: Initializer for the bias vector (see initializers).

kernel_regularizer: Regularizer function applied to the kernel weights matrix (see regularizer).

bias_regularizer: Regularizer function applied to the bias vector (see regularizer).

activity_regularizer: Regularizer function applied to the output of the layer (its "activation"). (see regularizer).

kernel_constraint: Constraint function applied to the kernel matrix (see constraints).

bias_constraint: Constraint function applied to the bias vector (see constraints).

In [12]:
```
from keras.models import Sequential
from keras.layers.core import Dense, Dropout, Activation, Flatten

from keras.layers.convolutional import Conv2D
from keras.layers.pooling import MaxPooling2D
```

1.2.1 Model Definition

In [13]:
```
# -- Initializing the values for the convolution neural network

nb_epoch = 10  # kept very low! Please increase if you can use a GPU

batch_size = 256
# number of convolutional filters to use
```

```
nb_filters = 32
# size of pooling area for max pooling
nb_pool = 2
# convolution kernel size
nb_conv = 3
```

In [14]:
```
model = Sequential()

model.add(Conv2D(nb_filters, kernel_size=(nb_conv, nb_conv), padding='valid', activatio
                 input_shape=shape_ord))  # note: the very first layer **must** always
model.add(Flatten())
model.add(Dense(10, activation='softmax'))

model.compile(loss='categorical_crossentropy', optimizer='adam', metrics=['accuracy'])

model.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_1 (Conv2D)            (None, 26, 26, 32)        320
_________________________________________________________________
flatten_1 (Flatten)          (None, 21632)             0
_________________________________________________________________
dense_1 (Dense)              (None, 10)                216330
=================================================================
Total params: 216,650
Trainable params: 216,650
Non-trainable params: 0
_________________________________________________________________
```

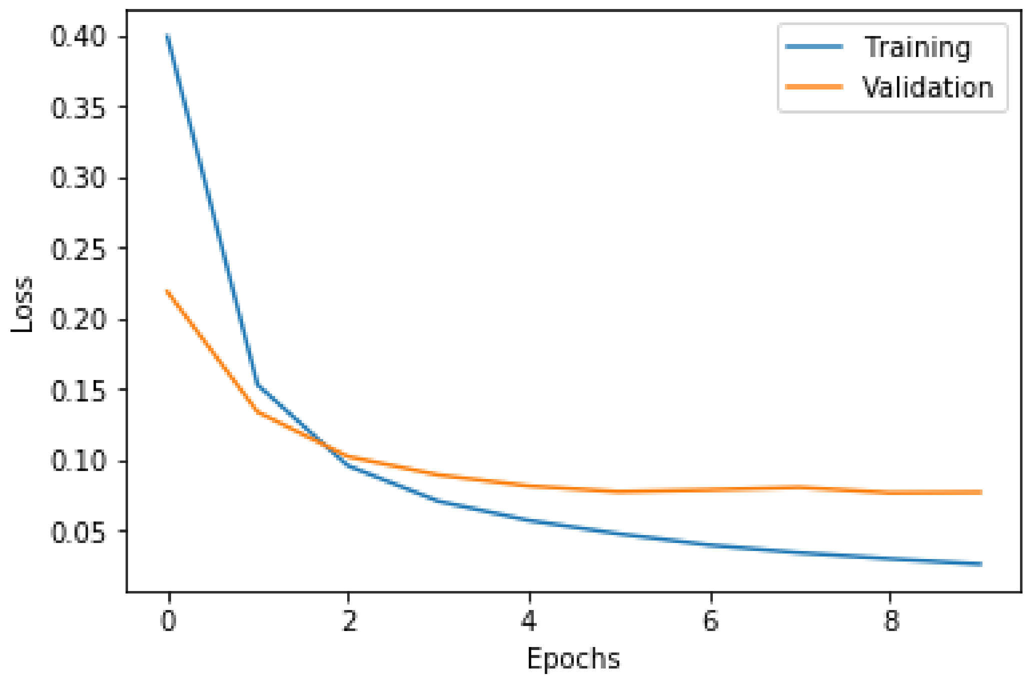
1.2.2 Training

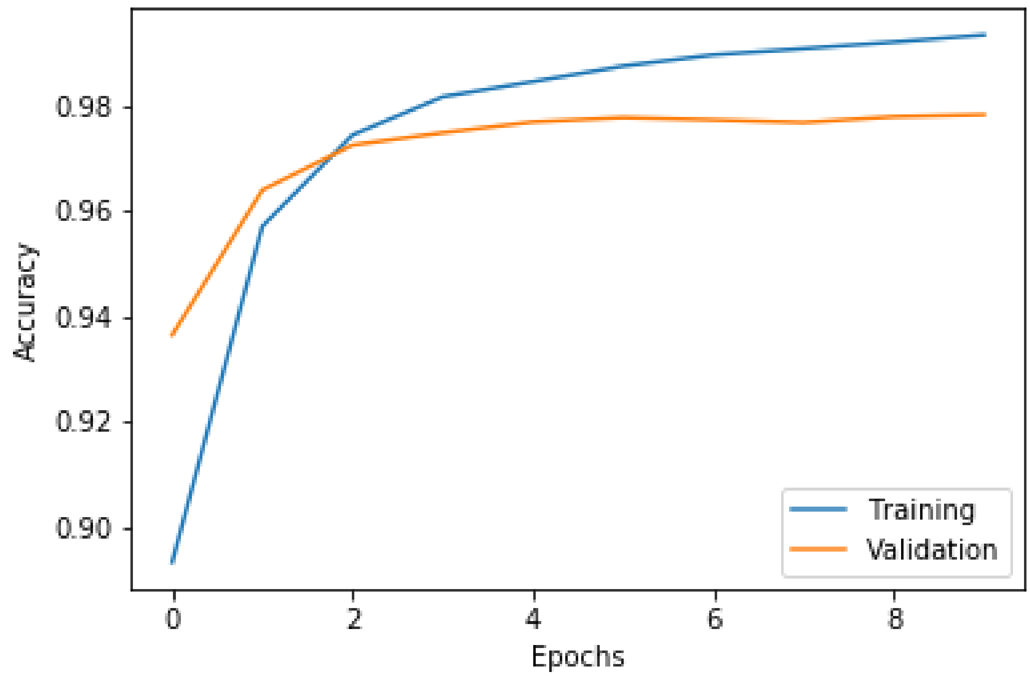
In [15]:
```
hist = model.fit(X_train, Y_train, batch_size=batch_size,
                 epochs=nb_epoch, verbose=1,
                 validation_data=(X_val, Y_val))
```

```
Train on 42000 samples, validate on 18000 samples
Epoch 1/10
42000/42000 [==============================] - 12s 291us/step - loss: 0.3995 - acc: 0.8932 - val
Epoch 2/10
42000/42000 [==============================] - 23s 552us/step - loss: 0.1529 - acc: 0.9570 - val
Epoch 3/10
42000/42000 [==============================] - 19s 456us/step - loss: 0.0958 - acc: 0.9744 - val
Epoch 4/10
42000/42000 [==============================] - 19s 444us/step - loss: 0.0706 - acc: 0.9816 - val
Epoch 5/10
42000/42000 [==============================] - 19s 441us/step - loss: 0.0570 - acc: 0.9845 - val
```

```
Epoch 6/10
42000/42000 [==============================] - 22s 516us/step - loss: 0.0474 - acc: 0.9874 - val
Epoch 7/10
42000/42000 [==============================] - 23s 553us/step - loss: 0.0395 - acc: 0.9895 - val
Epoch 8/10
42000/42000 [==============================] - 18s 433us/step - loss: 0.0340 - acc: 0.9907 - val
Epoch 9/10
42000/42000 [==============================] - 21s 505us/step - loss: 0.0298 - acc: 0.9920 - val
Epoch 10/10
42000/42000 [==============================] - 16s 372us/step - loss: 0.0260 - acc: 0.9933 - val
```

In [16]:
```
plot_history(hist)
```





1.2.3 Task 2: Evaluating - Define an `evaluate` function, with the following properties:

- It takes X_test and Y_test as arguments
- It calculates the loss, the accuracy and the classification report
- It plots the probability of being a zero for true zeros (red) and non-zeros (blue)
- It computes and plots the confusion matrix
- It plots the image, the prediction and the true value for the top 6 errors
- It plots image and predictions for the first 15 examples

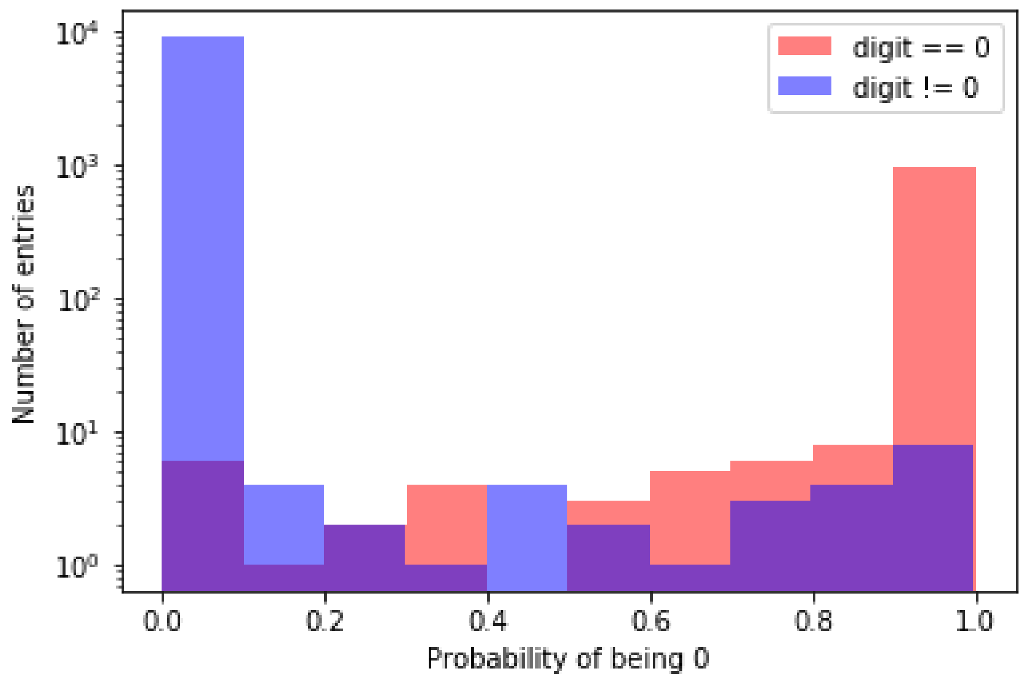
```
In [17]: from sklearn.metrics import confusion_matrix,classification_report

         def evaluate(X_test, Y_test):

             ##Evaluate loss and metrics
             loss, accuracy = model.evaluate(X_test, Y_test, verbose=0)
             print('Test Loss:', loss)
             print('Test Accuracy:', accuracy)
             # Predict the values from the test dataset
             Y_pred = model.predict(X_test)
             # Convert predictions classes to one hot vectors
             Y_cls = np.argmax(Y_pred, axis = 1)
             # Convert validation observations to one hot vectors
             Y_true = np.argmax(Y_test, axis = 1)
             print 'Classification Report:\n', classification_report(Y_true,Y_cls)
```

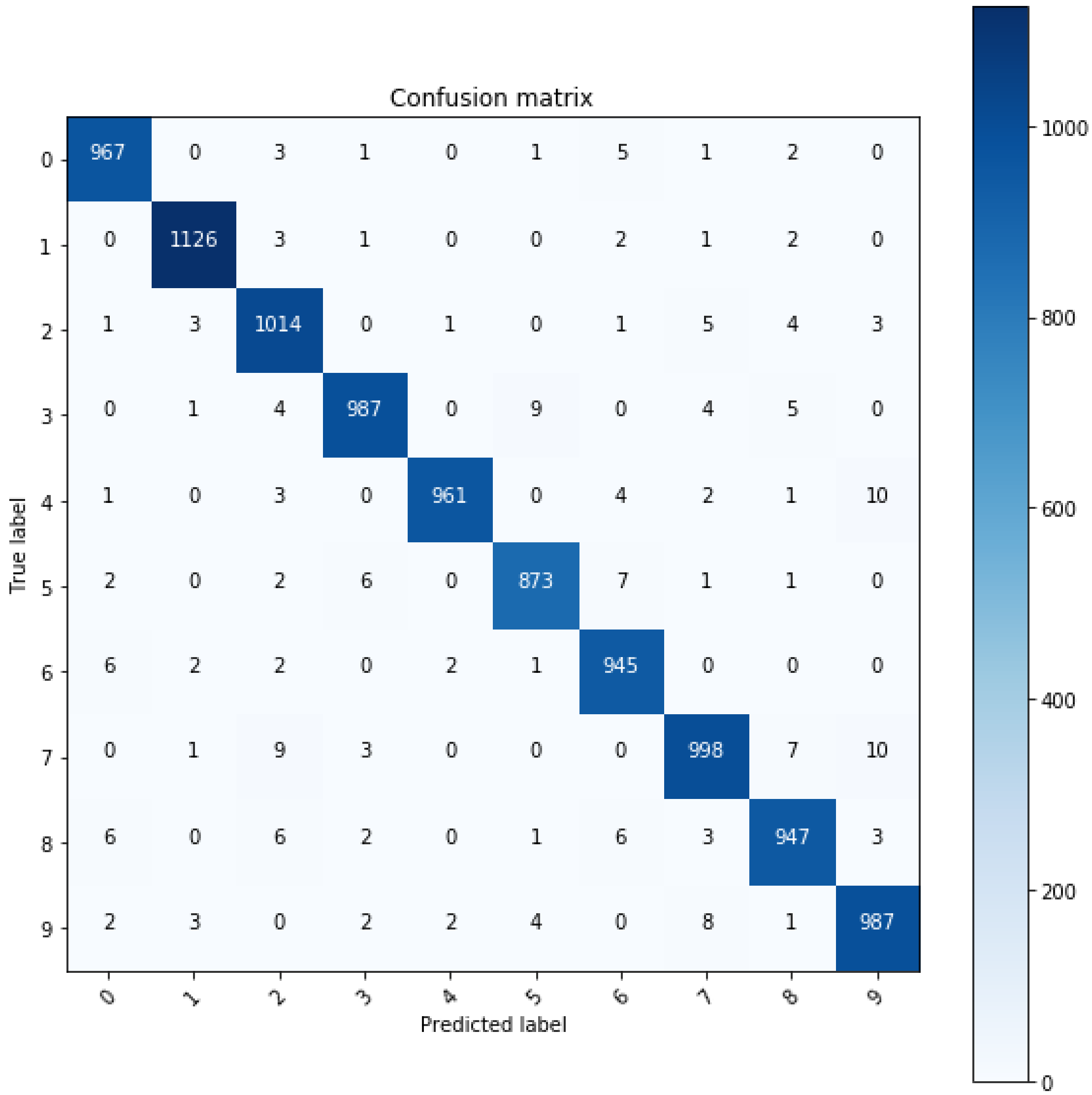
```
        ## Plot 0 probability
        label=0
        Y_pred_prob = Y_pred[:,label]
        plt.hist(Y_pred_prob[Y_true == label], alpha=0.5, color='red', bins=10, log = True)
        plt.hist(Y_pred_prob[Y_true != label], alpha=0.5, color='blue', bins=10, log = True
        plt.legend(['digit == 0', 'digit != 0'], loc='upper right')
        plt.xlabel('Probability of being 0')
        plt.ylabel('Number of entries')
        plt.show()

        # compute the confusion matrix
        confusion_mtx = confusion_matrix(Y_true, Y_cls)
        # plot the confusion matrix
        plt.figure(figsize=(8,8))
        plot_confusion_matrix(confusion_mtx, classes = range(10))

        #Plot largest errors
        errors = (Y_cls - Y_true != 0)
        Y_cls_errors = Y_cls[errors]
        Y_pred_errors = Y_pred[errors]
        Y_true_errors = Y_true[errors]
        X_test_errors = X_test[errors]
        # Probabilities of the wrong predicted numbers
        Y_pred_errors_prob = np.max(Y_pred_errors,axis = 1)
        # Predicted probabilities of the true values in the error set
        true_prob_errors = np.diagonal(np.take(Y_pred_errors, Y_true_errors, axis=1))
        # Difference between the probability of the predicted label and the true label
        delta_pred_true_errors = Y_pred_errors_prob - true_prob_errors
        # Sorted list of the delta prob errors
        sorted_dela_errors = np.argsort(delta_pred_true_errors)
        # Top 6 errors
        most_important_errors = sorted_dela_errors[-6:]
        # Show the top 6 errors
        display_errors(most_important_errors, X_test_errors, Y_cls_errors, Y_true_errors)

        ##Plot predictions
        slice = 15
        predicted = model.predict(X_test[:slice]).argmax(-1)
        plt.figure(figsize=(16,8))
        for i in range(slice):
            plt.subplot(1, slice, i+1)
            plt.imshow(X_test[i].reshape(28,28), interpolation='nearest')
            plt.text(0, 0, predicted[i], color='black',
                     bbox=dict(facecolor='white', alpha=1))
            plt.axis('off')
```

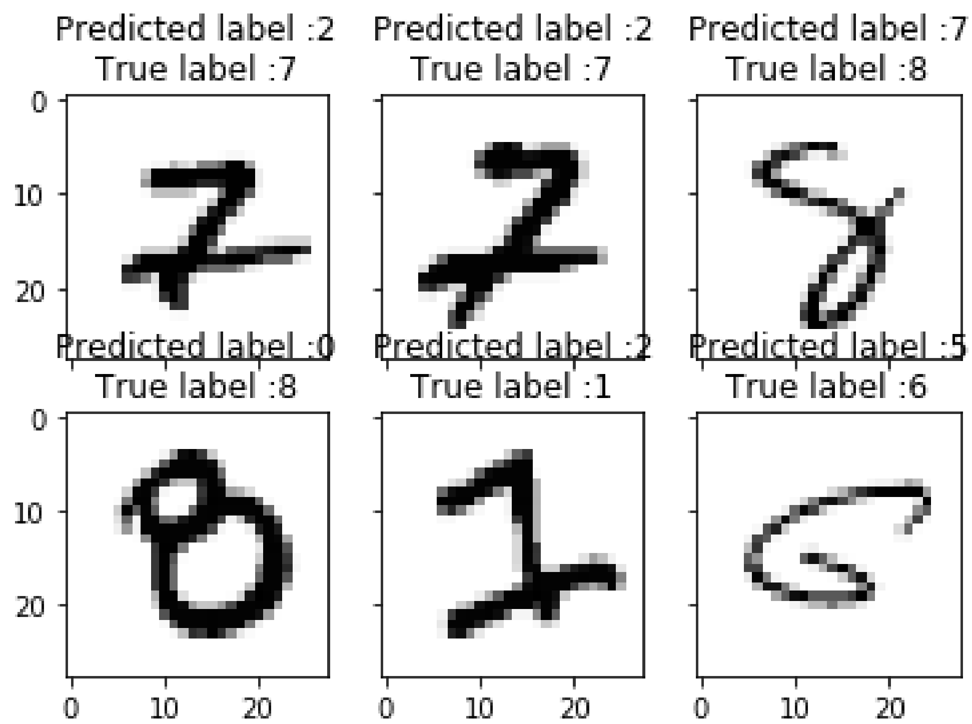
```
In [18]: evaluate(X_test, Y_test)
```

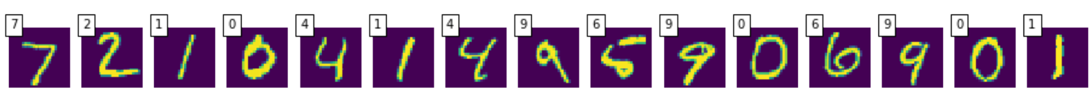
```
('Test Loss:', 0.065599114515818652)
```

```
('Test Accuracy:', 0.98050000000000004)
Classification Report:
             precision    recall  f1-score   support

          0       0.98      0.99      0.98       980
          1       0.99      0.99      0.99      1135
          2       0.97      0.98      0.98      1032
          3       0.99      0.98      0.98      1010
          4       0.99      0.98      0.99       982
          5       0.98      0.98      0.98       892
          6       0.97      0.99      0.98       958
          7       0.98      0.97      0.97      1028
          8       0.98      0.97      0.97       974
          9       0.97      0.98      0.98      1009

avg / total       0.98      0.98      0.98     10000
```




Confusion matrix
True label
Predicted label
0 1 2 3 4 5 6 7 8 9
967 0 3 1 0 1 5 1 2 0
0 1126 3 1 0 0 2 1 2 0
1 3 1014 0 1 0 1 5 4 3
0 1 4 987 0 9 0 4 5 0
1 0 3 0 961 0 4 2 1 10
2 0 2 6 0 873 7 1 1 0
6 2 2 0 2 1 945 0 0 0
0 1 9 3 0 0 0 998 7 10
6 0 6 2 0 1 6 3 947 3
2 3 0 2 2 4 0 8 1 987
1000
800
600
400
200
0





1.3 Adding more Dense Layers and Dropout

1.3.1 Task 3: Adding additional classification layers

- Add a dense layer between the flatten layer and the output layer
- Add a 25% dropout layer before the flatten layer
- Add a 50% dropout layer between the two dense layers
- Build the model, train the NN, plot the loss and accuracy evolution
- Evaluate the new model

In [19]:
```
model = Sequential()

model.add(Conv2D(nb_filters, kernel_size=(nb_conv, nb_conv), padding='valid', activatio
                 input_shape=shape_ord))
model.add(Dropout(0.25))
model.add(Flatten())
model.add(Dense(128, activation='relu'))
```

```
model.add(Dropout(0.5))
model.add(Dense(10, activation='softmax'))

model.compile(loss='categorical_crossentropy',
              optimizer='adam',
              metrics=['accuracy'])

model.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_2 (Conv2D)            (None, 26, 26, 32)        320
_________________________________________________________________
dropout_1 (Dropout)          (None, 26, 26, 32)        0
_________________________________________________________________
flatten_2 (Flatten)          (None, 21632)             0
_________________________________________________________________
dense_2 (Dense)              (None, 128)               2769024
_________________________________________________________________
dropout_2 (Dropout)          (None, 128)               0
_________________________________________________________________
dense_3 (Dense)              (None, 10)                1290
=================================================================
Total params: 2,770,634
Trainable params: 2,770,634
Non-trainable params: 0
_________________________________________________________________
```

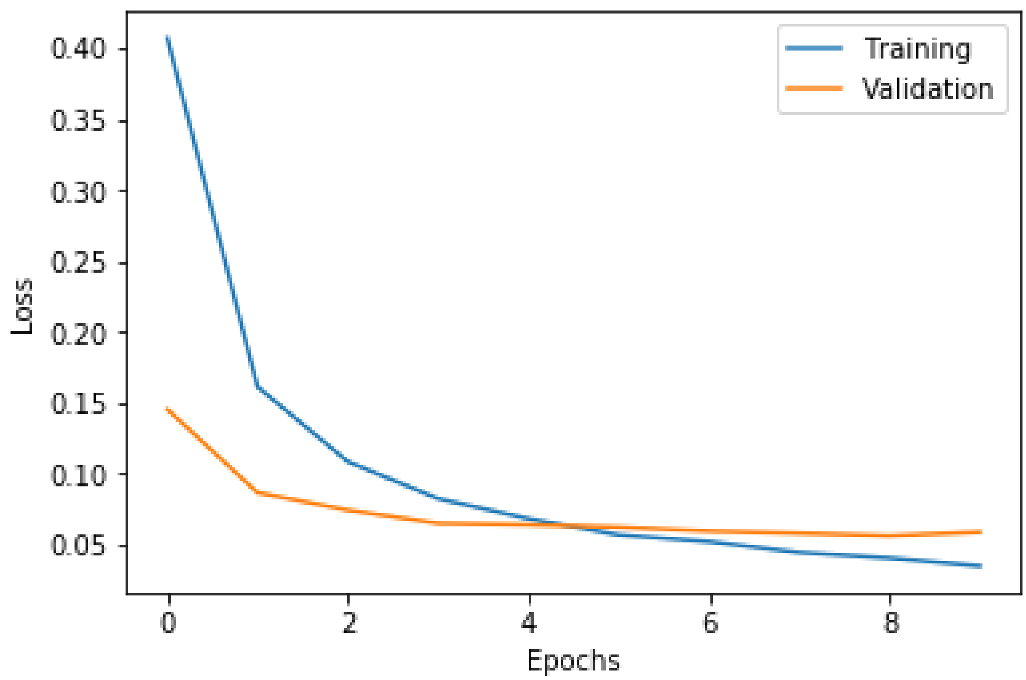
1.3.2 Training

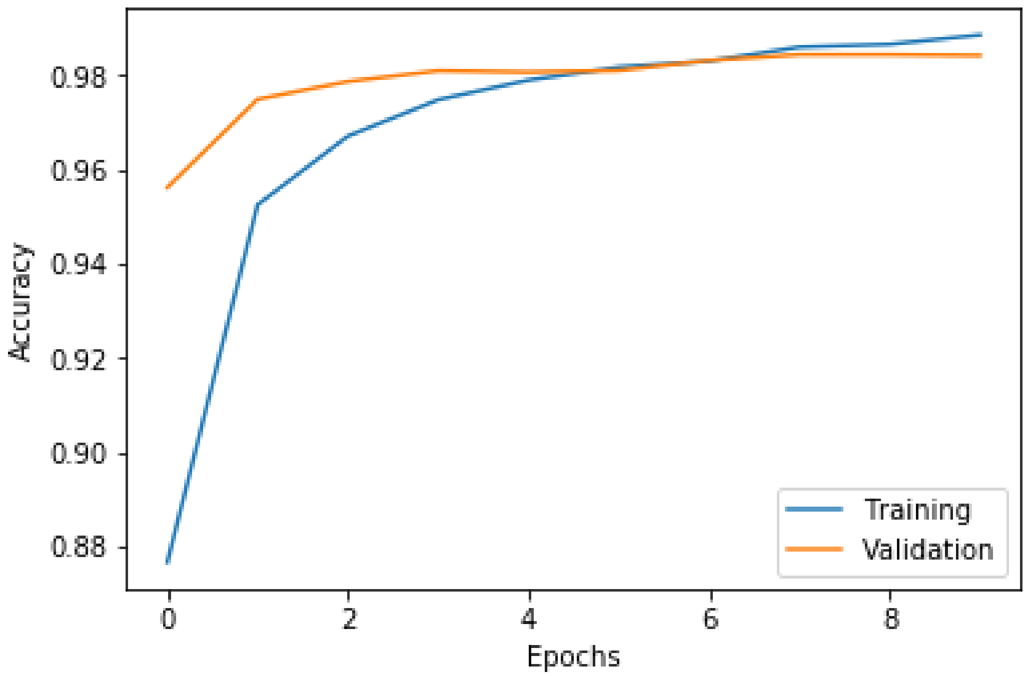
In [20]:
```
hist = model.fit(X_train, Y_train, batch_size=batch_size,
                 epochs=nb_epoch, verbose=1,
                 validation_data=(X_val, Y_val))
```

```
Train on 42000 samples, validate on 18000 samples
Epoch 1/10
42000/42000 [==============================] - 40s 952us/step - loss: 0.4073 - acc: 0.8766 - val
Epoch 2/10
42000/42000 [==============================] - 35s 822us/step - loss: 0.1616 - acc: 0.9524 - val
Epoch 3/10
42000/42000 [==============================] - 36s 860us/step - loss: 0.1088 - acc: 0.9670 - val
Epoch 4/10
42000/42000 [==============================] - 48s 1ms/step - loss: 0.0826 - acc: 0.9747 - val_l
Epoch 5/10
42000/42000 [==============================] - 62s 1ms/step - loss: 0.0685 - acc: 0.9789 - val_l
Epoch 6/10
```

```
42000/42000 [==============================] - 83s 2ms/step - loss: 0.0571 - acc: 0.9816 - val_l
Epoch 7/10
42000/42000 [==============================] - 80s 2ms/step - loss: 0.0524 - acc: 0.9829 - val_l
Epoch 8/10
42000/42000 [==============================] - 88s 2ms/step - loss: 0.0447 - acc: 0.9859 - val_l
Epoch 9/10
42000/42000 [==============================] - 94s 2ms/step - loss: 0.0408 - acc: 0.9865 - val_l
Epoch 10/10
42000/42000 [==============================] - 80s 2ms/step - loss: 0.0352 - acc: 0.9885 - val_l
```

In [21]: `plot_history(hist)`

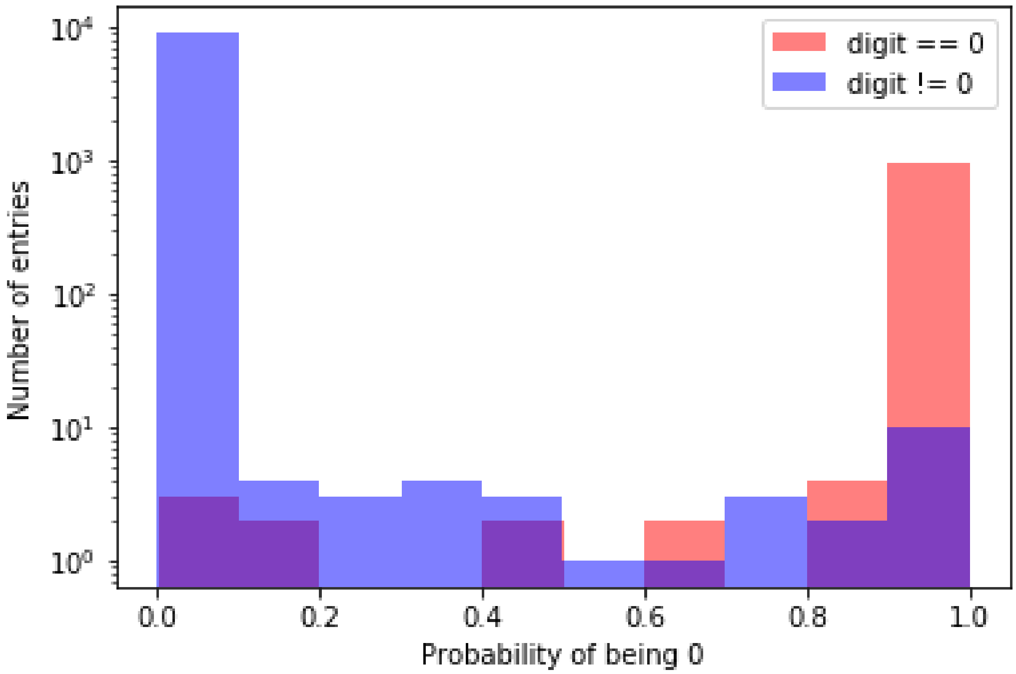


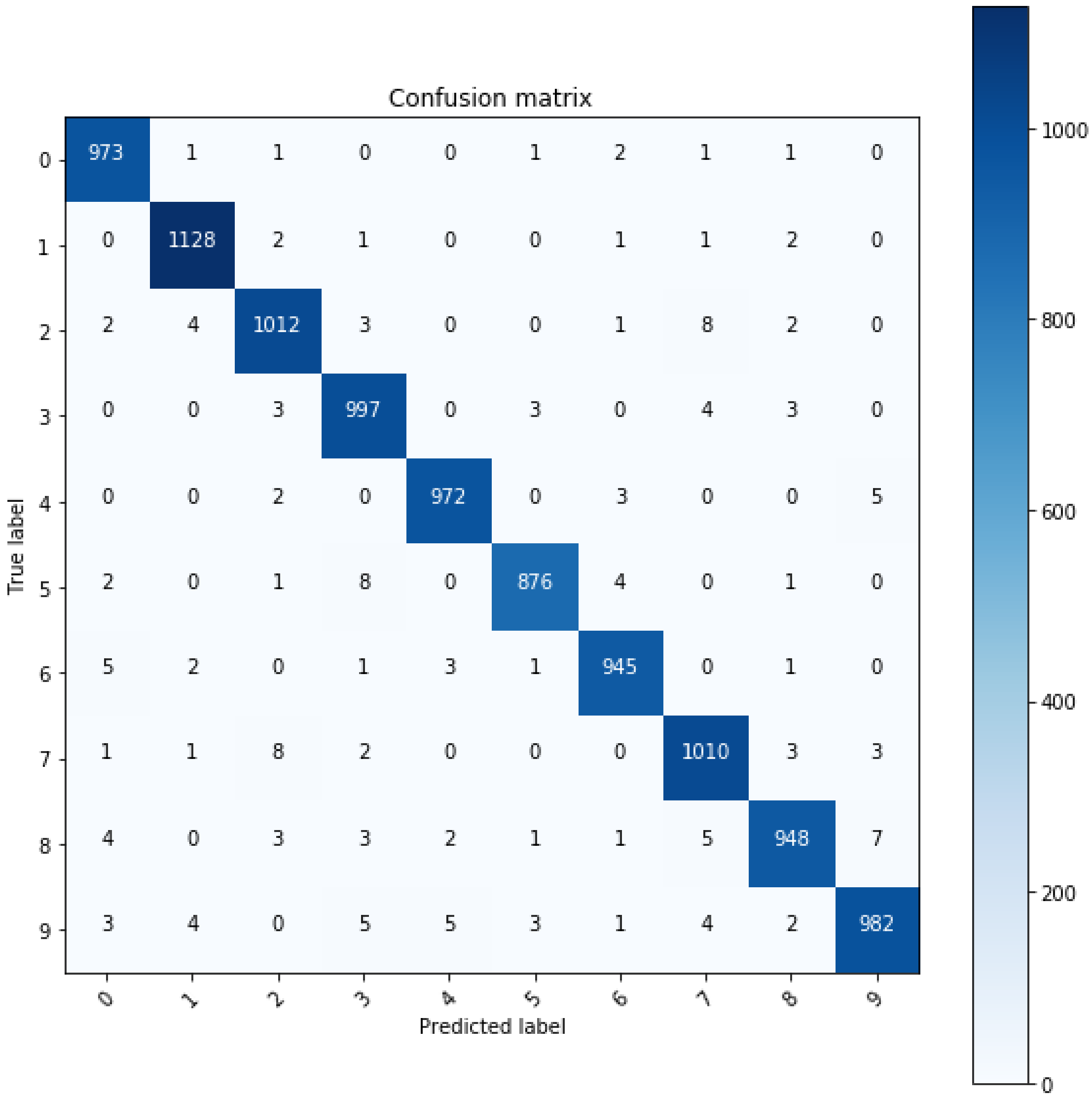


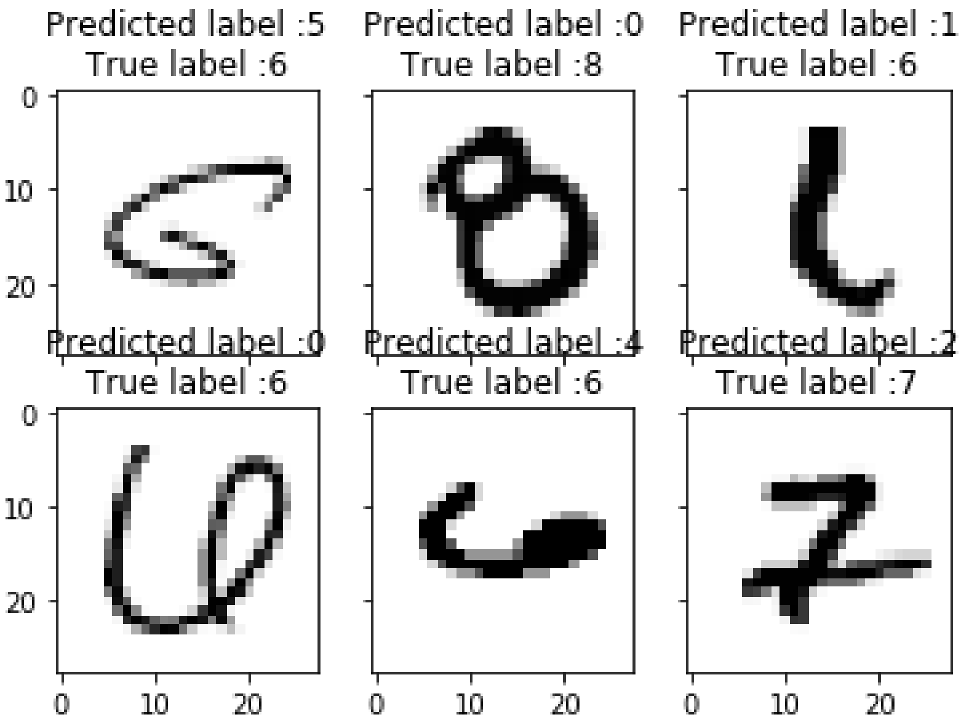
1.3.3 Evaluating

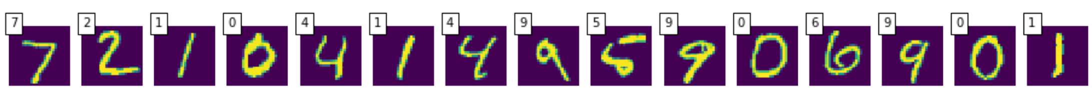
```
In [22]: # Evaluating the model on the test data
         evaluate(X_test, Y_test)
```

```
('Test Loss:', 0.048410624130575161)
('Test Accuracy:', 0.98429999999999995)
Classification Report:
             precision    recall  f1-score   support

          0       0.98      0.99      0.99       980
          1       0.99      0.99      0.99      1135
          2       0.98      0.98      0.98      1032
          3       0.98      0.99      0.98      1010
          4       0.99      0.99      0.99       982
          5       0.99      0.98      0.99       892
          6       0.99      0.99      0.99       958
          7       0.98      0.98      0.98      1028
          8       0.98      0.97      0.98       974
          9       0.98      0.97      0.98      1009

avg / total       0.98      0.98      0.98     10000
```


digit == 0
digit != 0
Number of entries
Probability of being 0
10^4
10^3
10^2
10^1
10^0
0.0
0.2
0.4
0.6
0.8
1.0


Confusion matrix
True label
Predicted label
0
1
2
3
4
5
6
7
8
9
973 1 1 0 0 1 2 1 1 0
0 1128 2 1 0 0 1 1 2 0
2 4 1012 3 0 0 1 8 2 0
0 0 3 997 0 3 0 4 3 0
0 0 2 0 972 0 3 0 0 5
2 0 1 8 0 876 4 0 1 0
5 2 0 1 3 1 945 0 1 0
1 1 8 2 0 0 0 1010 3 3
4 0 3 3 2 1 1 5 948 7
3 4 0 5 5 3 1 4 2 982
1000
800
600
400
200
0





2 Adding an additional convolution layer and a pooling layer

2.0.1 Task 3: Adding additional classification layers

- Add another `Conv2D` layer after the first convolutional layer with 64 filter, 3x3 kernel and `valid` (no) padding
- Add another `MaxPooling2D` layer with a 2x2 pooling size
- Build the model, train the NN, plot the loss and accuracy evolution
- Evaluate the new model

```
In [23]: model = Sequential()

         model.add(Conv2D(nb_filters, kernel_size=(nb_conv, nb_conv), padding='valid', activatio
                          input_shape=shape_ord))
         model.add(Conv2D(64, kernel_size=(nb_conv, nb_conv), padding='valid', activation='relu'
         model.add(MaxPooling2D(pool_size=(2, 2)))
         model.add(Dropout(0.25))
```

```
model.add(Flatten())
model.add(Dense(128, activation='relu'))
model.add(Dropout(0.5))
model.add(Dense(10, activation='softmax'))

model.compile(loss='categorical_crossentropy',
              optimizer='adam',
              metrics=['accuracy'])

model.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_3 (Conv2D)            (None, 26, 26, 32)        320
_________________________________________________________________
conv2d_4 (Conv2D)            (None, 24, 24, 64)        18496
_________________________________________________________________
max_pooling2d_1 (MaxPooling2 (None, 12, 12, 64)        0
_________________________________________________________________
dropout_3 (Dropout)          (None, 12, 12, 64)        0
_________________________________________________________________
flatten_3 (Flatten)          (None, 9216)              0
_________________________________________________________________
dense_4 (Dense)              (None, 128)               1179776
_________________________________________________________________
dropout_4 (Dropout)          (None, 128)               0
_________________________________________________________________
dense_5 (Dense)              (None, 10)                1290
=================================================================
Total params: 1,199,882
Trainable params: 1,199,882
Non-trainable params: 0
_________________________________________________________________
```

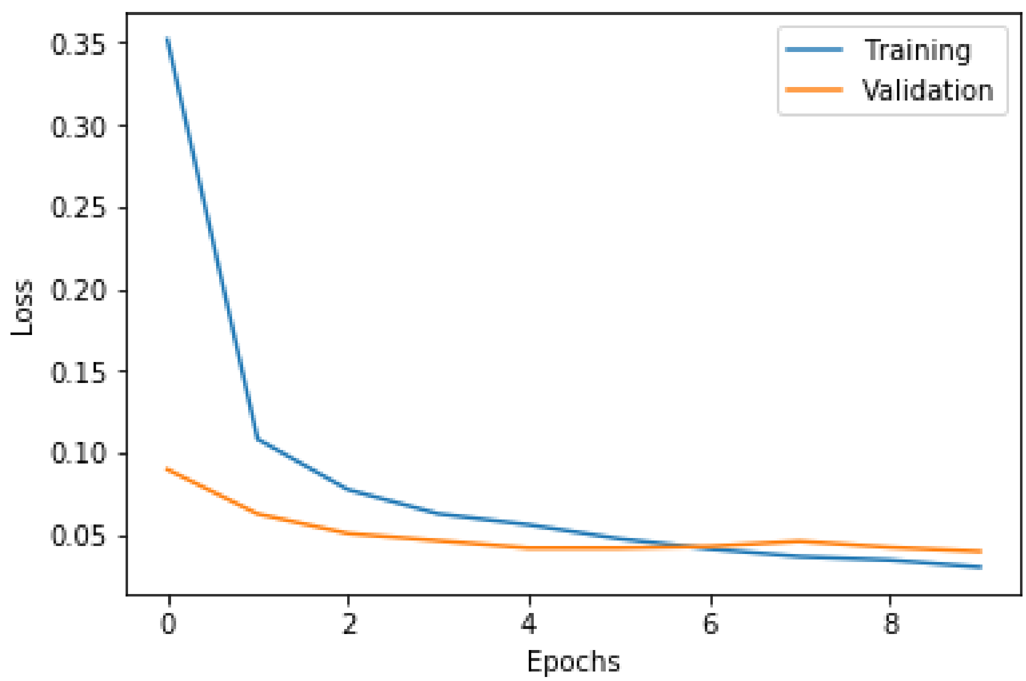
2.0.2 Training

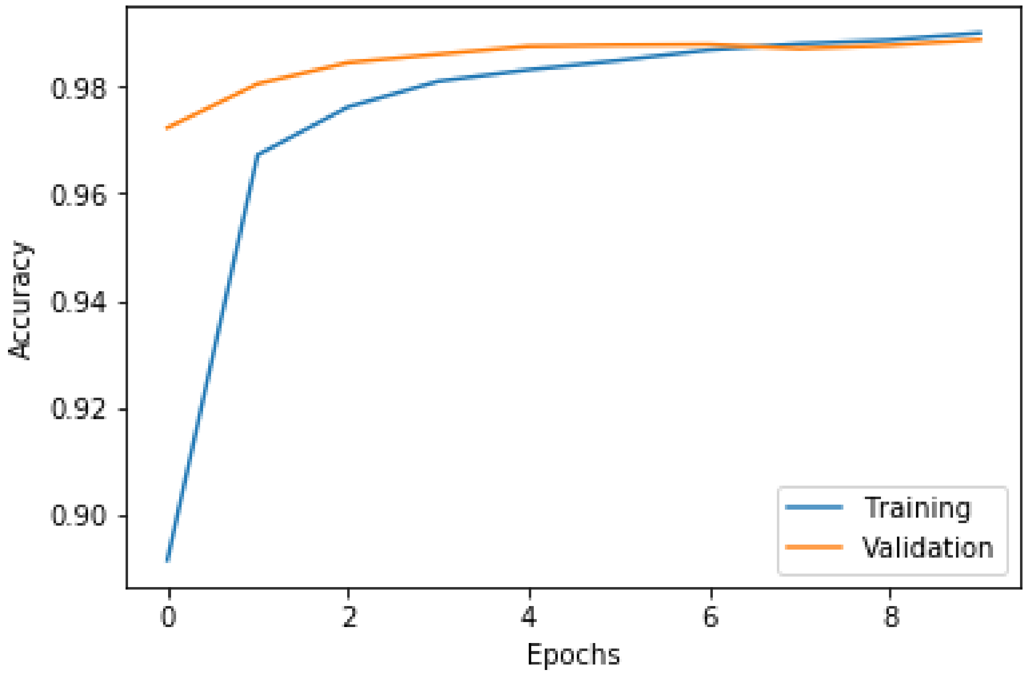
```
In [24]: hist = model.fit(X_train, Y_train, batch_size=batch_size,
                          epochs=nb_epoch, verbose=1,
                          validation_data=(X_val, Y_val))
```

```
Train on 42000 samples, validate on 18000 samples
Epoch 1/10
42000/42000 [==============================] - 254s 6ms/step - loss: 0.3519 - acc: 0.8915 - val_
Epoch 2/10
42000/42000 [==============================] - 260s 6ms/step - loss: 0.1086 - acc: 0.9672 - val_
Epoch 3/10
```

```
42000/42000 [==============================] - 200s 5ms/step - loss: 0.0777 - acc: 0.9762 - val_
Epoch 4/10
42000/42000 [==============================] - 189s 4ms/step - loss: 0.0630 - acc: 0.9810 - val_
Epoch 5/10
42000/42000 [==============================] - 110s 3ms/step - loss: 0.0563 - acc: 0.9831 - val_
Epoch 6/10
42000/42000 [==============================] - 107s 3ms/step - loss: 0.0478 - acc: 0.9848 - val_
Epoch 7/10
42000/42000 [==============================] - 104s 2ms/step - loss: 0.0416 - acc: 0.9868 - val_
Epoch 8/10
42000/42000 [==============================] - 117s 3ms/step - loss: 0.0369 - acc: 0.9880 - val_
Epoch 9/10
42000/42000 [==============================] - 126s 3ms/step - loss: 0.0348 - acc: 0.9886 - val_
Epoch 10/10
42000/42000 [==============================] - 103s 2ms/step - loss: 0.0305 - acc: 0.9900 - val_
```

```
In [25]: plot_history(hist)
```

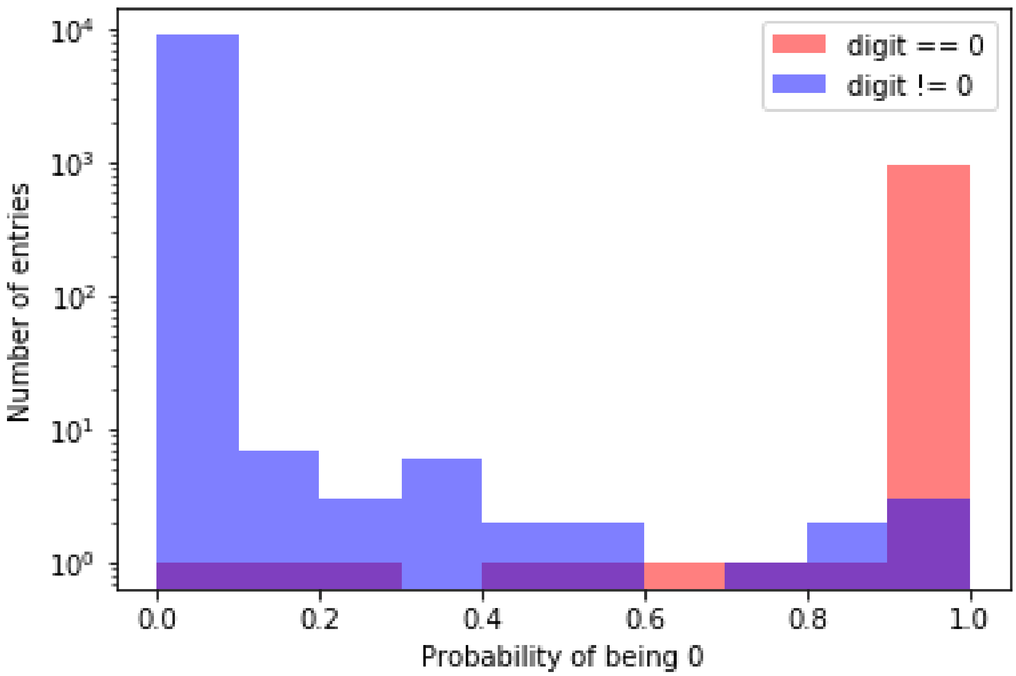




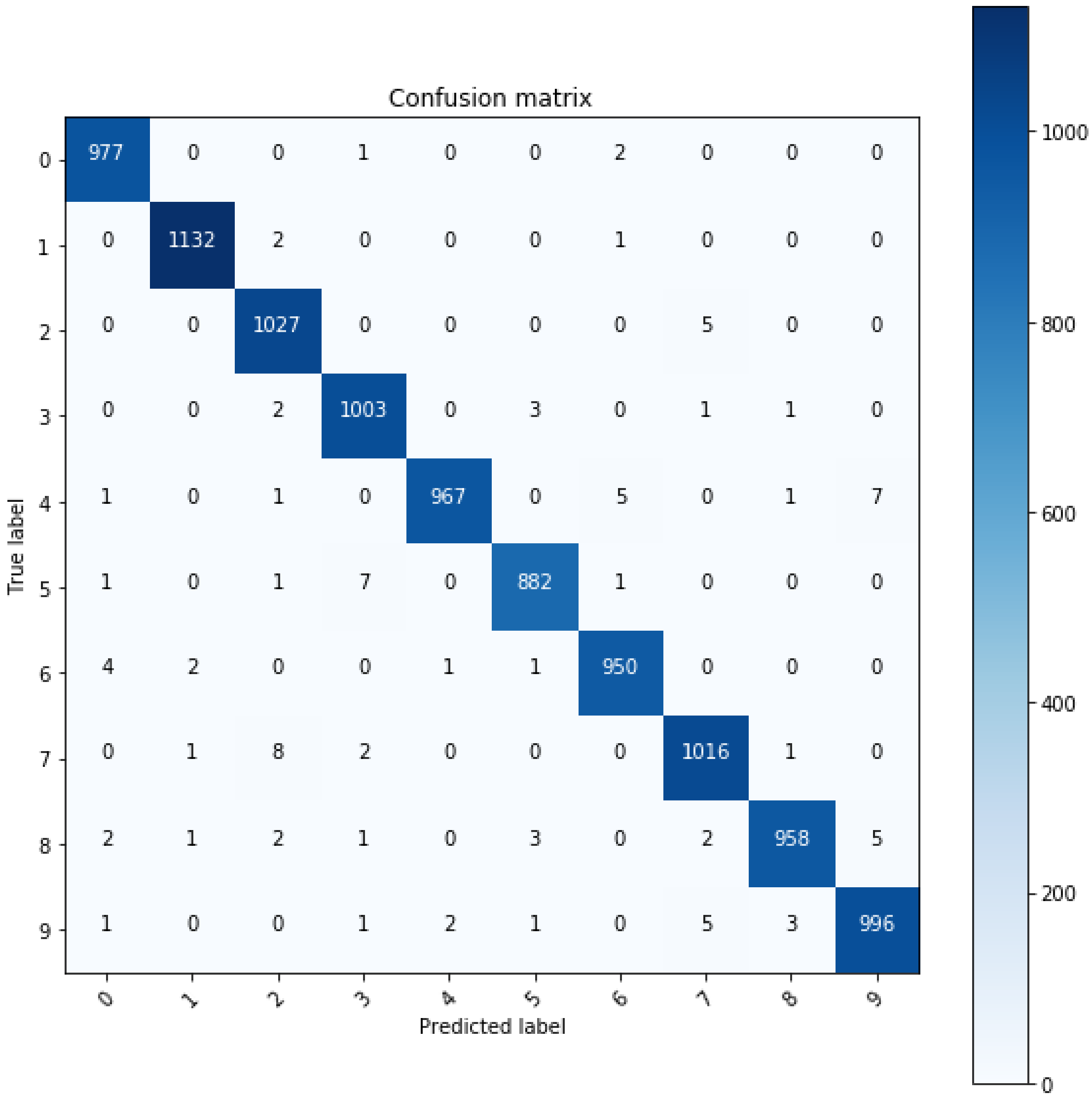
2.0.3 Evaluating

```
In [26]: evaluate(X_test, Y_test)
```

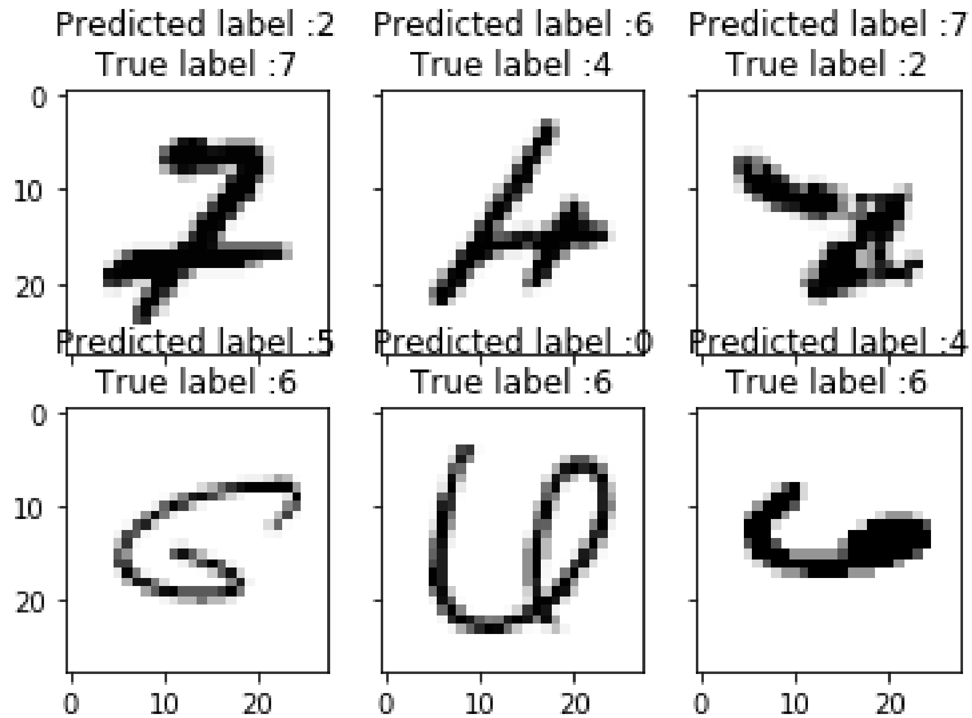
```
('Test Loss:', 0.032512901003119626)
('Test Accuracy:', 0.99080000000000001)
Classification Report:
             precision    recall  f1-score   support

          0       0.99      1.00      0.99       980
          1       1.00      1.00      1.00      1135
          2       0.98      1.00      0.99      1032
          3       0.99      0.99      0.99      1010
          4       1.00      0.98      0.99       982
          5       0.99      0.99      0.99       892
          6       0.99      0.99      0.99       958
          7       0.99      0.99      0.99      1028
          8       0.99      0.98      0.99       974
          9       0.99      0.99      0.99      1009

avg / total       0.99      0.99      0.99     10000
```

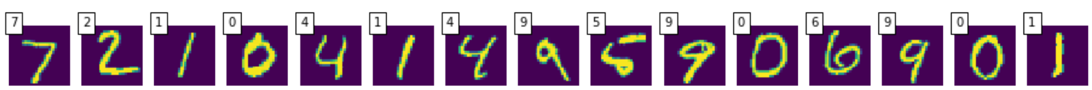


digit == 0
digit != 0
Number of entries
10^4
10^3
10^2
10^1
10^0
0.0
0.2
0.4
0.6
0.8
1.0
Probability of being 0



	0	1	2	3	4	5	6	7	8	9
0	977	0	0	1	0	0	2	0	0	0
1	0	1132	2	0	0	0	1	0	0	0
2	0	0	1027	0	0	0	0	5	0	0
3	0	0	2	1003	0	3	0	1	1	0
4	1	0	1	0	967	0	5	0	1	7
5	1	0	1	7	0	882	1	0	0	0
6	4	2	0	0	1	1	950	0	0	0
7	0	1	8	2	0	0	0	1016	1	0
8	2	1	2	1	0	3	0	2	958	5
9	1	0	0	1	2	1	0	5	3	996





---

3 Bonus: Understanding Convolutional Layers Structure

We will inspect and understand the convolutional layer of our previously defined quite shallow CNN, which contains two [Convolution, Convolution, MaxPooling] stages, and two Dense layers.

3.0.1 Understanding layer shapes

An important feature of Keras layers is that each of them has an `input_shape` attribute, which you can use to visualize the shape of the input tensor, and an `output_shape` attribute, for inspecting the shape of the output tensor.

As we can see, the input shape of the first convolutional layer corresponds to the `input_shape` attribute (which must be specified by the user).

In this case, it is a `28x28` image with three color channels.

Since this convolutional layer has the padding set to same, its output width and height will remain the same, and the number of output channel will be equal to the number of filters learned by the layer, 16.

The following convolutional layer, instead, have the default padding, and therefore reduce width and height by $(k-1)$, where k is the size of the kernel.

MaxPooling layers, instead, reduce width and height of the input tensor, but keep the same number of channels.

Activation layers, of course, don't change the shape.

```
In [27]: for i, layer in enumerate(model.layers):
             print ("Layer", i, "\t", layer.name, "\t\t", layer.input_shape, "\t", layer.output_
```

```
('Layer', 0, '\t', 'conv2d_3', '\t\t', (None, 28, 28, 1), '\t', (None, 26, 26, 32))
('Layer', 1, '\t', 'conv2d_4', '\t\t', (None, 26, 26, 32), '\t', (None, 24, 24, 64))
('Layer', 2, '\t', 'max_pooling2d_1', '\t\t', (None, 24, 24, 64), '\t', (None, 12, 12, 64))
('Layer', 3, '\t', 'dropout_3', '\t\t', (None, 12, 12, 64), '\t', (None, 12, 12, 64))
('Layer', 4, '\t', 'flatten_3', '\t\t', (None, 12, 12, 64), '\t', (None, 9216))
('Layer', 5, '\t', 'dense_4', '\t\t', (None, 9216), '\t', (None, 128))
('Layer', 6, '\t', 'dropout_4', '\t\t', (None, 128), '\t', (None, 128))
('Layer', 7, '\t', 'dense_5', '\t\t', (None, 128), '\t', (None, 10))
```

3.0.2 Understanding weights shape

In the same way, we can visualize the shape of the weights learned by each layer.

In particular, Keras lets you inspect weights by using the get_weights method of a layer object.

This will return a list with two elements, the first one being the **weight tensor** and the second one being the **bias vector**.

In particular:

- **MaxPooling layer** don't have any weight tensor, since they don't have learnable parameters.
- **Convolutional layers**, instead, learn a (n_o, n_i, k, k) weight tensor, where k is the size of the kernel, n_i is the number of channels of the input tensor, and n_o is the number of filters to be learned.

For each of the n_o filters, a bias is also learned.

- **Dense layers** learn a (n_i, n_o) weight tensor, where n_o is the output size and n_i is the input size of the layer. Each of the n_o neurons also has a bias.

```
In [28]: for i, layer in enumerate(model.layers):
             if len(layer.get_weights()) > 0:
                 W, b = layer.get_weights()
                 print("Layer", i, "\t", layer.name, "\t\t", W.shape, "\t", b.shape)
```

```
('Layer', 0, '\t', 'conv2d_3', '\t\t', (3, 3, 1, 32), '\t', (32,))
('Layer', 1, '\t', 'conv2d_4', '\t\t', (3, 3, 32, 64), '\t', (64,))
('Layer', 5, '\t', 'dense_4', '\t\t', (9216, 128), '\t', (128,))
('Layer', 7, '\t', 'dense_5', '\t\t', (128, 10), '\t', (10,))
```