ex7_sol

June 20, 2018

1 Exercise 7 - AutoEncoders

This exercise is based on https://github.com/leriomaggio/deep-learning-keras-tensorflow and https://blog.keras.io/building-autoencoders-in-keras.html

"Autoencoding" is a data compression algorithm where the compression and decompression functions are data-specific, lossy, and learned automatically from examples rather than engineered by a human. Additionally, in almost all contexts where the term "autoencoder" is used, the compression and decompression functions are implemented with neural networks.

The aim of an autoencoder is to learn a representation (encoding) for a set of data.. typically for the purpose of dimensionality reduction or feature learning.

To build an autoencoder, you need three things: an encoding function, a decoding function, and a distance function between the amount of information loss between the compressed representation of your data and the decompressed representation (i.e. a "loss" function)

Autoencoders are not a true unsupervised learning technique (which would imply a different learning process altogether), they are a self-supervised technique, a specific instance of supervised learning where the targets are generated from the input data. In order to get self-supervised models to learn interesting features, you have to come up with an interesting synthetic target and loss function, and that's where problems arise: merely learning to reconstruct your input in minute detail might not be the right choice here.

1.1 AutoEncoder and the MNIST dataset

We will use the MNIST dataset in order to train a few simple autoencoder.

1.2 Single fully-connected neural layer Autoencoder

1.2.1 Building the Model with *Keras Functional API*

The Keras functional API is the way to go for defining complex models, such as multi-output models, directed acyclic graphs, or models with shared layers. All the Functional API relies on the fact that each `keras.Layer` object is a *callable* object! More details can be found here: https://keras.io/getting-started/functional-api-guide/

Let's start with the simplest possible model:

```
In [1]: from keras.layers import Input, Dense
        from keras.models import Model
        import numpy as np
```

```
# this is the size of our encoded representations
encoding_dim = 32  # 32 floats -> compression of factor 24.5, assuming the input is 784

# this is our input placeholder
input_img = Input(shape=(784,))
# "encoded" is the encoded representation of the input
encoded = Dense(encoding_dim, activation='relu')(input_img)
# "decoded" is the lossy reconstruction of the input
decoded = Dense(784, activation='sigmoid')(encoded)

# this model maps an input to its reconstruction
autoencoder = Model(input_img, decoded)
```

```
/media/nackenho/Data/programs/ML/anaconda2/lib/python2.7/site-packages/h5py/__init__.py:34: Futu
  from ._conv import register_converters as _register_converters
Using TensorFlow backend.
```

Let's also create a separate encoder model:

```
In [2]: # this model maps an input to its encoded representation
        encoder = Model(input_img, encoded)
```

As well as the decoder model:

```
In [3]: # create a placeholder for an encoded (32-dimensional) input
        encoded_input = Input(shape=(encoding_dim,))
        # retrieve the last layer of the autoencoder model
        decoder_layer = autoencoder.layers[-1]
        # create the decoder model
        decoder = Model(encoded_input, decoder_layer(encoded_input))
```

First, we'll configure our autoencoder model to use a per-pixel binary crossentropy loss, and the Adadelta optimizer:

```
In [4]: autoencoder.compile(optimizer='adadelta', loss='binary_crossentropy')
        autoencoder.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_1 (InputLayer)         (None, 784)               0
_________________________________________________________________
dense_1 (Dense)              (None, 32)                25120
_________________________________________________________________
dense_2 (Dense)              (None, 784)               25872
=================================================================
Total params: 50,992
Trainable params: 50,992
Non-trainable params: 0
_________________________________________________________________
```

1.3 Data preparation for Dense-Layered Autoencoder

Let's prepare our input data. We're using MNIST digits, we won't need the labels (since we're only interested in encoding/decoding the input images).

```
In [5]: import numpy as np
        from keras.datasets import mnist
        #Load
        (x_train, y_train), (x_test, y_test) = mnist.load_data()

        #Flattening a bit more elegant
        x_train = x_train.reshape((len(x_train), np.prod(x_train.shape[1:])))
        x_test = x_test.reshape((len(x_test), np.prod(x_test.shape[1:])))

        #Shortcut for scaling today ;-)
        x_train = x_train.astype('float32') / 255.
        x_test = x_test.astype('float32') / 255.


        print x_train.shape
        print x_test.shape
```

```
(60000, 784)
(10000, 784)
```

Split Training and Validation Data

```
In [6]: from sklearn.model_selection import train_test_split

        x_train, x_val, y_train, y_val = train_test_split(x_train, y_train, test_size=0.3, rando
```

1.4 Training the Autoencoder

Now let's train our autoencoder:

```
In [7]: #note: x_train, x_train and x_val, x_val :)
        history = autoencoder.fit(x_train, x_train,
                        epochs=50,
                        batch_size=256,
                        shuffle=True,
                        validation_data=(x_val, x_val))
```

```
Train on 42000 samples, validate on 18000 samples
Epoch 1/50
42000/42000 [==============================] - 3s 65us/step - loss: 0.3892 - val_loss: 0.2764
Epoch 2/50
42000/42000 [==============================] - 3s 67us/step - loss: 0.2719 - val_loss: 0.2662
Epoch 3/50
```

```
42000/42000 [==============================] - 2s 58us/step - loss: 0.2603 - val_loss: 0.2523
Epoch 4/50
42000/42000 [==============================] - 3s 65us/step - loss: 0.2458 - val_loss: 0.2381
Epoch 5/50
42000/42000 [==============================] - 3s 73us/step - loss: 0.2318 - val_loss: 0.2244
Epoch 6/50
42000/42000 [==============================] - 4s 84us/step - loss: 0.2192 - val_loss: 0.2129
Epoch 7/50
42000/42000 [==============================] - 3s 81us/step - loss: 0.2090 - val_loss: 0.2038
Epoch 8/50
42000/42000 [==============================] - 3s 76us/step - loss: 0.2007 - val_loss: 0.1964
Epoch 9/50
42000/42000 [==============================] - 4s 85us/step - loss: 0.1938 - val_loss: 0.1899
Epoch 10/50
42000/42000 [==============================] - 3s 73us/step - loss: 0.1880 - val_loss: 0.1846
Epoch 11/50
42000/42000 [==============================] - 3s 71us/step - loss: 0.1830 - val_loss: 0.1799
Epoch 12/50
42000/42000 [==============================] - 3s 65us/step - loss: 0.1787 - val_loss: 0.1760
Epoch 13/50
42000/42000 [==============================] - 3s 65us/step - loss: 0.1748 - val_loss: 0.1724
Epoch 14/50
42000/42000 [==============================] - 3s 63us/step - loss: 0.1713 - val_loss: 0.1689
Epoch 15/50
42000/42000 [==============================] - 3s 60us/step - loss: 0.1679 - val_loss: 0.1656
Epoch 16/50
42000/42000 [==============================] - 3s 66us/step - loss: 0.1647 - val_loss: 0.1624
Epoch 17/50
42000/42000 [==============================] - 3s 80us/step - loss: 0.1616 - val_loss: 0.1595
Epoch 18/50
42000/42000 [==============================] - 4s 84us/step - loss: 0.1588 - val_loss: 0.1569
Epoch 19/50
42000/42000 [==============================] - 3s 74us/step - loss: 0.1561 - val_loss: 0.1541
Epoch 20/50
42000/42000 [==============================] - 3s 83us/step - loss: 0.1535 - val_loss: 0.1518
Epoch 21/50
42000/42000 [==============================] - 3s 76us/step - loss: 0.1511 - val_loss: 0.1494
Epoch 22/50
42000/42000 [==============================] - 3s 73us/step - loss: 0.1489 - val_loss: 0.1473
Epoch 23/50
42000/42000 [==============================] - 3s 69us/step - loss: 0.1467 - val_loss: 0.1452
Epoch 24/50
42000/42000 [==============================] - 3s 69us/step - loss: 0.1447 - val_loss: 0.1432
Epoch 25/50
42000/42000 [==============================] - 3s 71us/step - loss: 0.1428 - val_loss: 0.1414
Epoch 26/50
42000/42000 [==============================] - 3s 65us/step - loss: 0.1409 - val_loss: 0.1396
Epoch 27/50
```

```
42000/42000 [==============================] - 3s 67us/step - loss: 0.1392 - val_loss: 0.1379
Epoch 28/50
42000/42000 [==============================] - 3s 70us/step - loss: 0.1375 - val_loss: 0.1363
Epoch 29/50
42000/42000 [==============================] - 3s 79us/step - loss: 0.1359 - val_loss: 0.1348
Epoch 30/50
42000/42000 [==============================] - 3s 71us/step - loss: 0.1344 - val_loss: 0.1333
Epoch 31/50
42000/42000 [==============================] - 3s 67us/step - loss: 0.1330 - val_loss: 0.1319
Epoch 32/50
42000/42000 [==============================] - 3s 72us/step - loss: 0.1316 - val_loss: 0.1307
Epoch 33/50
42000/42000 [==============================] - 3s 65us/step - loss: 0.1303 - val_loss: 0.1293
Epoch 34/50
42000/42000 [==============================] - 3s 64us/step - loss: 0.1290 - val_loss: 0.1281
Epoch 35/50
42000/42000 [==============================] - 3s 68us/step - loss: 0.1277 - val_loss: 0.1270
Epoch 36/50
42000/42000 [==============================] - 3s 73us/step - loss: 0.1265 - val_loss: 0.1257
Epoch 37/50
42000/42000 [==============================] - 3s 73us/step - loss: 0.1254 - val_loss: 0.1246
Epoch 38/50
42000/42000 [==============================] - 3s 65us/step - loss: 0.1242 - val_loss: 0.1234
Epoch 39/50
42000/42000 [==============================] - 3s 73us/step - loss: 0.1231 - val_loss: 0.1224
Epoch 40/50
42000/42000 [==============================] - 3s 67us/step - loss: 0.1221 - val_loss: 0.1213
Epoch 41/50
42000/42000 [==============================] - 3s 69us/step - loss: 0.1211 - val_loss: 0.1204
Epoch 42/50
42000/42000 [==============================] - 3s 70us/step - loss: 0.1201 - val_loss: 0.1195
Epoch 43/50
42000/42000 [==============================] - 3s 67us/step - loss: 0.1192 - val_loss: 0.1185
Epoch 44/50
42000/42000 [==============================] - 3s 73us/step - loss: 0.1183 - val_loss: 0.1176
Epoch 45/50
42000/42000 [==============================] - 4s 102us/step - loss: 0.1174 - val_loss: 0.1168
Epoch 46/50
42000/42000 [==============================] - 4s 98us/step - loss: 0.1166 - val_loss: 0.1160
Epoch 47/50
42000/42000 [==============================] - 4s 87us/step - loss: 0.1158 - val_loss: 0.1153
Epoch 48/50
42000/42000 [==============================] - 3s 81us/step - loss: 0.1151 - val_loss: 0.1146
Epoch 49/50
42000/42000 [==============================] - 3s 74us/step - loss: 0.1144 - val_loss: 0.1139
Epoch 50/50
42000/42000 [==============================] - 3s 73us/step - loss: 0.1137 - val_loss: 0.1132
```

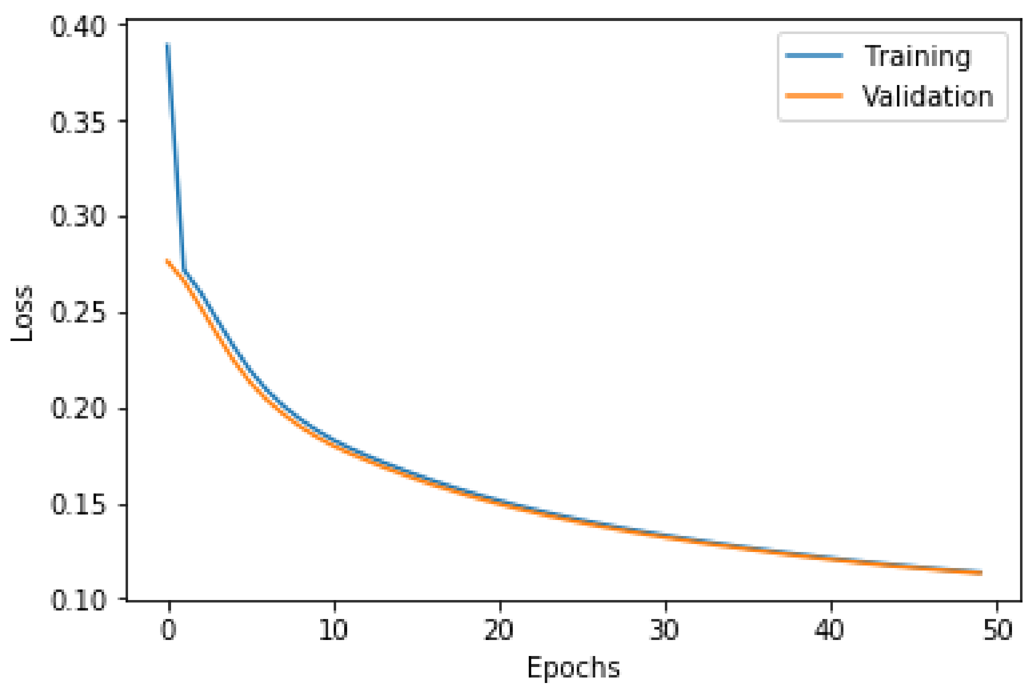
Let's plot the loss function to see if the training is stable

```
In [8]: print(history.history.keys())
```

```
['loss', 'val_loss']
```

```
In [9]: %matplotlib inline
        from matplotlib import pyplot as plt
        def plot_history(network_history):
            plt.figure()
            plt.xlabel('Epochs')
            plt.ylabel('Loss')
            plt.plot(network_history.history['loss'])
            plt.plot(network_history.history['val_loss'])
            plt.legend(['Training', 'Validation'])
```

```
In [10]: plot_history(history)
```

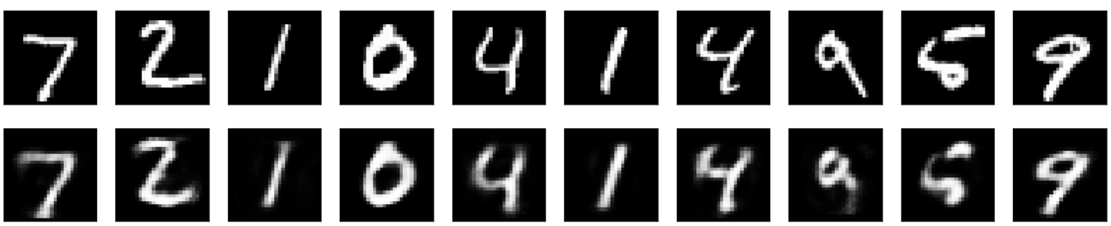


Well, not yet done, but we don't have more time....

1.5 Testing the Autoencoder

```
In [11]: # encode and decode some digitsdecode some digits
         # note that we take them from the *test* set
         encoded_imgs = encoder.predict(x_test)
```

```python
decoded_imgs = decoder.predict(encoded_imgs)

n = 10  # how many digits we will display
plt.figure(figsize=(20, 4))
for i in range(n):
    # display original
    ax = plt.subplot(2, n, i + 1)
    plt.imshow(x_test[i].reshape(28, 28))
    plt.gray()
    ax.get_xaxis().set_visible(False)
    ax.get_yaxis().set_visible(False)

    # display reconstruction
    ax = plt.subplot(2, n, i + 1 + n)
    plt.imshow(decoded_imgs[i].reshape(28, 28))
    plt.gray()
    ax.get_xaxis().set_visible(False)
    ax.get_yaxis().set_visible(False)
plt.show()
```



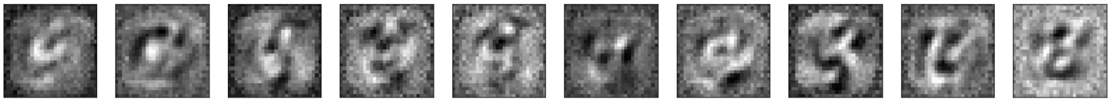
The top row is the original digits, and the bottom row is the reconstructed digits. We are losing quite a bit of detail with this basic approach.

1.6 Sample generation with Autoencoder

Can we actually generate numbers using the trained decoder, if we just draw random numbers for the encoded image?

```python
In [12]: encoded_imgs = np.random.rand(10,32)
         decoded_imgs = decoder.predict(encoded_imgs)

         n = 10
         plt.figure(figsize=(20, 4))
         for i in range(n):
             # generation
             ax = plt.subplot(2, n, i + 1 + n)
             plt.imshow(decoded_imgs[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
```

```
        ax.get_yaxis().set_visible(False)
    plt.show()
```



Although you might see some structures which look similar to numbers in the middle, using the decoder as a generator for handwritten digits clearly doesn't work.

We could now study deeper structures, feel free to try this at home, but the result will only improve slightly. Another way to constrain the representations to be compact is to add a sparsity contraint on the activity of the hidden representations, so fewer units would "fire" at a given time. In Keras, this can be done by adding the L1 norm regularizer as an `activity_regularizer` to our Dense layer.

1.7 Convolutional AutoEncoder

Since our inputs are images, it makes sense to use convolutional neural networks (`convnets`) as encoders and decoders. In practical settings, autoencoders applied to images are always convolutional autoencoders --they simply perform much better.

The encoder will consist in a stack of `Conv2D` and `MaxPooling2D` layers (max pooling being used for spatial down-sampling), while the decoder will consist in a stack of `Conv2D` and `UpSampling2D` layers.

```
In [13]: from keras.layers import Input, Dense, Conv2D, MaxPooling2D, UpSampling2D
         from keras.models import Model
         from keras import backend as K

         input_img = Input(shape=(28, 28, 1))  # adapt this if using `channels_first` image data

         x = Conv2D(16, (3, 3), activation='relu', padding='same')(input_img)
         x = MaxPooling2D((2, 2), padding='same')(x)
         x = Conv2D(8, (3, 3), activation='relu', padding='same')(x)
         x = MaxPooling2D((2, 2), padding='same')(x)
         x = Conv2D(8, (3, 3), activation='relu', padding='same')(x)
         encoded = MaxPooling2D((2, 2), padding='same')(x)

         # at this point the representation is (4, 4, 8) i.e. 128-dimensional

         x = Conv2D(8, (3, 3), activation='relu', padding='same')(encoded)
         x = UpSampling2D((2, 2))(x)
         x = Conv2D(8, (3, 3), activation='relu', padding='same')(x)
         x = UpSampling2D((2, 2))(x)
         x = Conv2D(16, (3, 3), activation='relu')(x)
```

```
x = UpSampling2D((2, 2))(x)
decoded = Conv2D(1, (3, 3), activation='sigmoid', padding='same')(x)

conv_autoencoder = Model(input_img, decoded)
conv_autoencoder.compile(optimizer='adadelta', loss='binary_crossentropy')
conv_autoencoder.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_3 (InputLayer)         (None, 28, 28, 1)         0
_________________________________________________________________
conv2d_1 (Conv2D)            (None, 28, 28, 16)        160
_________________________________________________________________
max_pooling2d_1 (MaxPooling2 (None, 14, 14, 16)        0
_________________________________________________________________
conv2d_2 (Conv2D)            (None, 14, 14, 8)         1160
_________________________________________________________________
max_pooling2d_2 (MaxPooling2 (None, 7, 7, 8)           0
_________________________________________________________________
conv2d_3 (Conv2D)            (None, 7, 7, 8)           584
_________________________________________________________________
max_pooling2d_3 (MaxPooling2 (None, 4, 4, 8)           0
_________________________________________________________________
conv2d_4 (Conv2D)            (None, 4, 4, 8)           584
_________________________________________________________________
up_sampling2d_1 (UpSampling2 (None, 8, 8, 8)           0
_________________________________________________________________
conv2d_5 (Conv2D)            (None, 8, 8, 8)           584
_________________________________________________________________
up_sampling2d_2 (UpSampling2 (None, 16, 16, 8)         0
_________________________________________________________________
conv2d_6 (Conv2D)            (None, 14, 14, 16)        1168
_________________________________________________________________
up_sampling2d_3 (UpSampling2 (None, 28, 28, 16)        0
_________________________________________________________________
conv2d_7 (Conv2D)            (None, 28, 28, 1)         145
=================================================================
Total params: 4,385
Trainable params: 4,385
Non-trainable params: 0
_________________________________________________________________
```

1.8 Task 1: Train and evaluate the Convolutional Autoencoder

- Prepare the data for the Convolutional Autoencoder
- Train the Convolutional Autoencoder and plot the training and validation loss

- Test the Convolutional Autoencoder by plotting 10 of the decoded predictions next to the original images
- Plot the encoded representations in order to visualize how the digits are encoded

1.9 Data Preparation for Convolutional Autoencoder

```
In [14]: from keras import backend as K

         if K.image_data_format() == 'channels_last':
             shape_ord = (28, 28, 1)
         else:
             shape_ord = (1, 28, 28)

         #Load
         (x_train, y_train), (x_test, y_test) = mnist.load_data()

         #Scale
         x_train = x_train.astype('float32') / 255.
         x_test = x_test.astype('float32') / 255.

         #Shape for CNN
         x_train = np.reshape(x_train, ((x_train.shape[0],) + shape_ord))
         x_test = np.reshape(x_test, ((x_test.shape[0],) + shape_ord))

         #Split into validation
         x_train, x_val, y_train, y_val = train_test_split(x_train, y_train, test_size=0.3, rand
```

```
In [15]: x_train.shape
```

```
Out[15]: (42000, 28, 28, 1)
```

1.10 Training the Convolutional Autoencoder

Let's train this model. For the sake of demonstrating how to visualize the results of a model during training, we will be using the TensorFlow backend and the TensorBoard callback.

First, let's open up a terminal and start a TensorBoard server that will read logs stored at /tmp/autoencoder.

tensorboard --logdir=/tmp/autoencoder

Then let's train our model. In the callbacks list we pass an instance of the TensorBoard callback. After every epoch, this callback will write logs to /tmp/autoencoder, which can be read by our TensorBoard server.

```
In [16]: from keras.callbacks import TensorBoard

         history = conv_autoencoder.fit(x_train, x_train,
                         epochs=50,
                         batch_size=256,
                         shuffle=True,
```

```
                        validation_data=(x_test, x_test),
                        callbacks=[TensorBoard(log_dir='/tmp/autoencoder')])
```

```
Train on 42000 samples, validate on 10000 samples
Epoch 1/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.2658 - val_loss: 0.2163
Epoch 2/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1949 - val_loss: 0.1834
Epoch 3/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1769 - val_loss: 0.1738
Epoch 4/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1662 - val_loss: 0.1632
Epoch 5/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1577 - val_loss: 0.1488
Epoch 6/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1505 - val_loss: 0.1493
Epoch 7/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1455 - val_loss: 0.1398
Epoch 8/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1407 - val_loss: 0.1558
Epoch 9/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1369 - val_loss: 0.1337
Epoch 10/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1344 - val_loss: 0.1350
Epoch 11/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1323 - val_loss: 0.1284
Epoch 12/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1303 - val_loss: 0.1384
Epoch 13/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1283 - val_loss: 0.1284
Epoch 14/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1268 - val_loss: 0.1258
Epoch 15/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1252 - val_loss: 0.1280
Epoch 16/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1243 - val_loss: 0.1223
Epoch 17/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1232 - val_loss: 0.1202
Epoch 18/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1221 - val_loss: 0.1213
Epoch 19/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1210 - val_loss: 0.1156
Epoch 20/50
42000/42000 [==============================] - 42s 998us/step - loss: 0.1204 - val_loss: 0.1279
Epoch 21/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1194 - val_loss: 0.1165
Epoch 22/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1189 - val_loss: 0.1153
```

```
Epoch 23/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1178 - val_loss: 0.1144
Epoch 24/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1170 - val_loss: 0.1206
Epoch 25/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1163 - val_loss: 0.1194
Epoch 26/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1158 - val_loss: 0.1169
Epoch 27/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1150 - val_loss: 0.1149
Epoch 28/50
42000/42000 [==============================] - 46s 1ms/step - loss: 0.1145 - val_loss: 0.1106
Epoch 29/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1138 - val_loss: 0.1173
Epoch 30/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1136 - val_loss: 0.1096
Epoch 31/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1129 - val_loss: 0.1153
Epoch 32/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1125 - val_loss: 0.1080
Epoch 33/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1118 - val_loss: 0.1120
Epoch 34/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1113 - val_loss: 0.1096
Epoch 35/50
42000/42000 [==============================] - 42s 1ms/step - loss: 0.1108 - val_loss: 0.1132
Epoch 36/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1105 - val_loss: 0.1136
Epoch 37/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1100 - val_loss: 0.1128
Epoch 38/50
42000/42000 [==============================] - 45s 1ms/step - loss: 0.1096 - val_loss: 0.1096
Epoch 39/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1092 - val_loss: 0.1106
Epoch 40/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1090 - val_loss: 0.1083
Epoch 41/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1086 - val_loss: 0.1042
Epoch 42/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1078 - val_loss: 0.1066
Epoch 43/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1073 - val_loss: 0.1061
Epoch 44/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1074 - val_loss: 0.1044
Epoch 45/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1067 - val_loss: 0.1078
Epoch 46/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1065 - val_loss: 0.1063
```

```
Epoch 47/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1068 - val_loss: 0.1067
Epoch 48/50
42000/42000 [==============================] - 46s 1ms/step - loss: 0.1063 - val_loss: 0.1039
Epoch 49/50
42000/42000 [==============================] - 43s 1ms/step - loss: 0.1058 - val_loss: 0.1065
Epoch 50/50
42000/42000 [==============================] - 44s 1ms/step - loss: 0.1053 - val_loss: 0.1089
```

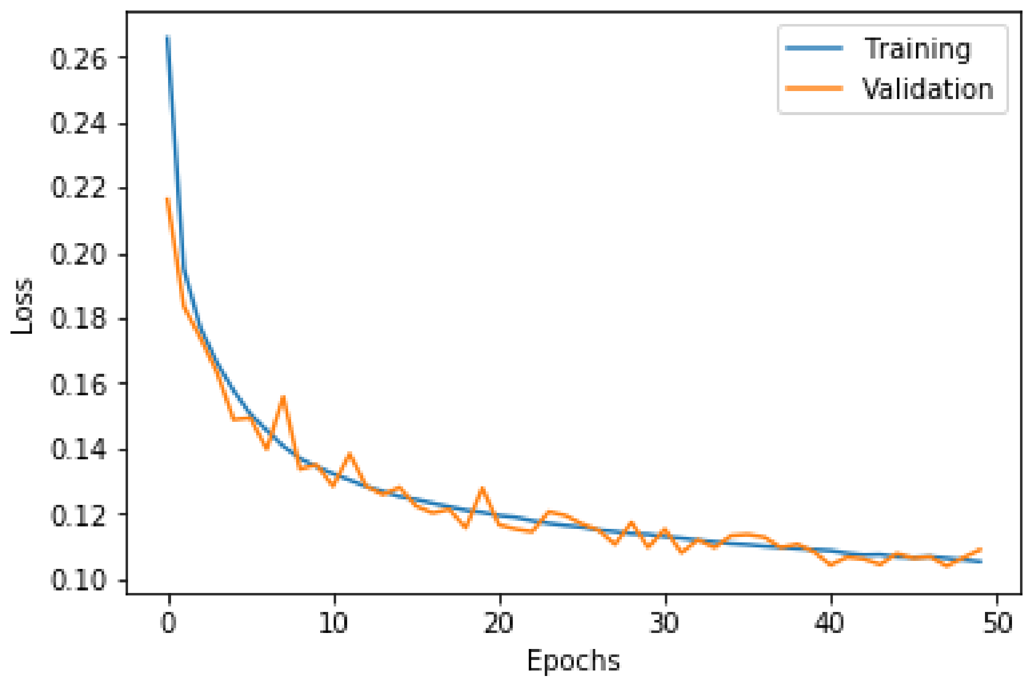
This allows us to monitor training in the TensorBoard web interface (by navighating to http://0.0.0.0:6006

Tensorboard is pretty cool, you can also have a look at the graphs to see a representation of you model. Using tensorboard is quite useful to understand if your model is working an visualize a few features of your model right in the beginning of your training. This way you know immediately if it is working or not and don't need to wait until the training is done. More information can be found here:

https://www.tensorflow.org/programmers_guide/summaries_and_tensorboard

Let's plot the loss here as well:

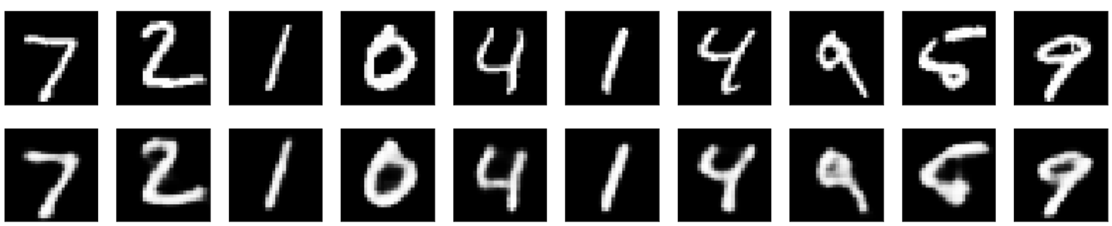
```
In [17]: plot_history(history)
```



The model converges to a loss, which is significantly better than our previous models (this is in large part due to the higher entropic capacity of the encoded representation, 128 dimensions vs. 32 previously). Let's take a look at the reconstructed digits:

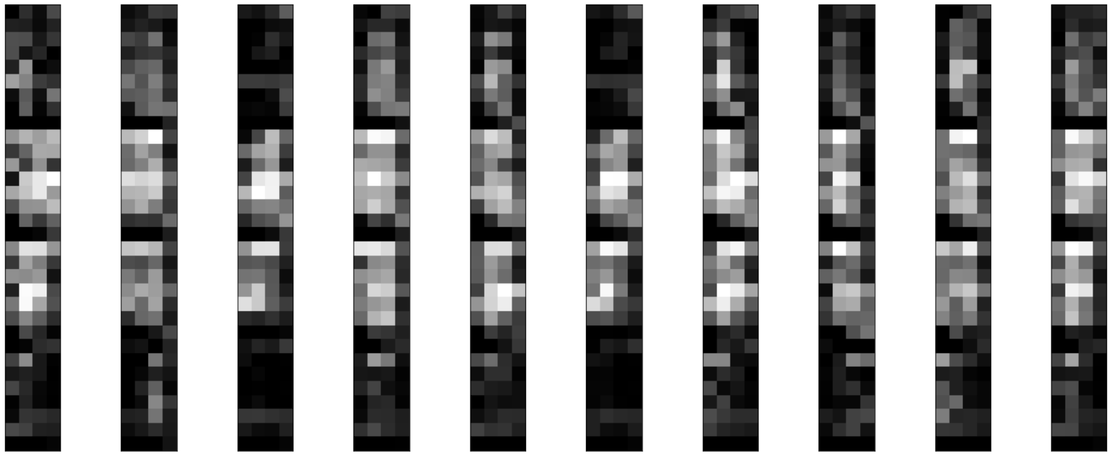
1.11 Testing the Convolutional Autoencoder

In [18]:
```
decoded_imgs = conv_autoencoder.predict(x_test)

n = 10
plt.figure(figsize=(20, 4))
for i in range(n):
    # display original
    ax = plt.subplot(2, n, i+1)
    plt.imshow(x_test[i].reshape(28, 28))
    plt.gray()
    ax.get_xaxis().set_visible(False)
    ax.get_yaxis().set_visible(False)

    # display reconstruction
    ax = plt.subplot(2, n, i + n + 1)
    plt.imshow(decoded_imgs[i].reshape(28, 28))
    plt.gray()
    ax.get_xaxis().set_visible(False)
    ax.get_yaxis().set_visible(False)
plt.show()
```



We can also have a look at the 128-dimensional encoded representations. These representations are 8x4x4, so we reshape them to 4x32 in order to be able to display them as grayscale images.

In [19]:
```
conv_encoder = Model(input_img, encoded)
encoded_imgs = conv_encoder.predict(x_test)

n = 10
plt.figure(figsize=(20, 8))
for i in range(n):
    ax = plt.subplot(1, n, i+1)
    plt.imshow(encoded_imgs[i].reshape(4, 4 * 8).T)
    plt.gray()
    ax.get_xaxis().set_visible(False)
    ax.get_yaxis().set_visible(False)
plt.show()
```



1.12 Application to Image Denoising

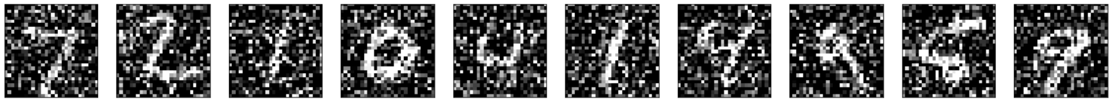
Let's put our convolutional autoencoder to work on an image denoising problem. It's simple: we will train the autoencoder to map noisy digits images to clean digits images.

Here's how we will generate synthetic noisy digits: we just apply a gaussian noise matrix and clip the images between 0 and 1.

```
In [20]: noise_factor = 0.5
         x_train_noisy = x_train + noise_factor * np.random.normal(loc=0.0, scale=1.0, size=x_tr
         x_test_noisy = x_test + noise_factor * np.random.normal(loc=0.0, scale=1.0, size=x_test

         x_train_noisy = np.clip(x_train_noisy, 0., 1.)
         x_test_noisy = np.clip(x_test_noisy, 0., 1.)
```

Here's how the noisy digits look like:

```
In [21]: n = 10
         plt.figure(figsize=(20, 2))
         for i in range(n):
             ax = plt.subplot(1, n, i+1)
             plt.imshow(x_test_noisy[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
         plt.show()
```



If you squint you can still recognize them, but barely.

1.13 Question: Can our autoencoder learn to recover the original digits?

Compared to the previous convolutional autoencoder, in order to improve the quality of the reconstructed, we'll use a slightly different model with more filters per layer:

In [22]:
```
input_img = Input(shape=(28, 28, 1))  # adapt this if using `channels_first` image data

x = Conv2D(32, (3, 3), activation='relu', padding='same')(input_img)
x = MaxPooling2D((2, 2), padding='same')(x)
x = Conv2D(32, (3, 3), activation='relu', padding='same')(x)
encoded = MaxPooling2D((2, 2), padding='same')(x)

# at this point the representation is (7, 7, 32)

x = Conv2D(32, (3, 3), activation='relu', padding='same')(encoded)
x = UpSampling2D((2, 2))(x)
x = Conv2D(32, (3, 3), activation='relu', padding='same')(x)
x = UpSampling2D((2, 2))(x)
decoded = Conv2D(1, (3, 3), activation='sigmoid', padding='same')(x)

autoencoder = Model(input_img, decoded)
autoencoder.compile(optimizer='adadelta', loss='binary_crossentropy')
autoencoder.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_4 (InputLayer)         (None, 28, 28, 1)         0
_________________________________________________________________
conv2d_8 (Conv2D)            (None, 28, 28, 32)        320
_________________________________________________________________
max_pooling2d_4 (MaxPooling2 (None, 14, 14, 32)        0
_________________________________________________________________
conv2d_9 (Conv2D)            (None, 14, 14, 32)        9248
_________________________________________________________________
max_pooling2d_5 (MaxPooling2 (None, 7, 7, 32)          0
_________________________________________________________________
conv2d_10 (Conv2D)           (None, 7, 7, 32)          9248
_________________________________________________________________
up_sampling2d_4 (UpSampling2 (None, 14, 14, 32)        0
_________________________________________________________________
conv2d_11 (Conv2D)           (None, 14, 14, 32)        9248
_________________________________________________________________
up_sampling2d_5 (UpSampling2 (None, 28, 28, 32)        0
```

```
_________________________________________________________________
conv2d_12 (Conv2D)           (None, 28, 28, 1)         289
=================================================================
Total params: 28,353
Trainable params: 28,353
Non-trainable params: 0
_________________________________________________________________
```

Let's train the AutoEncoder

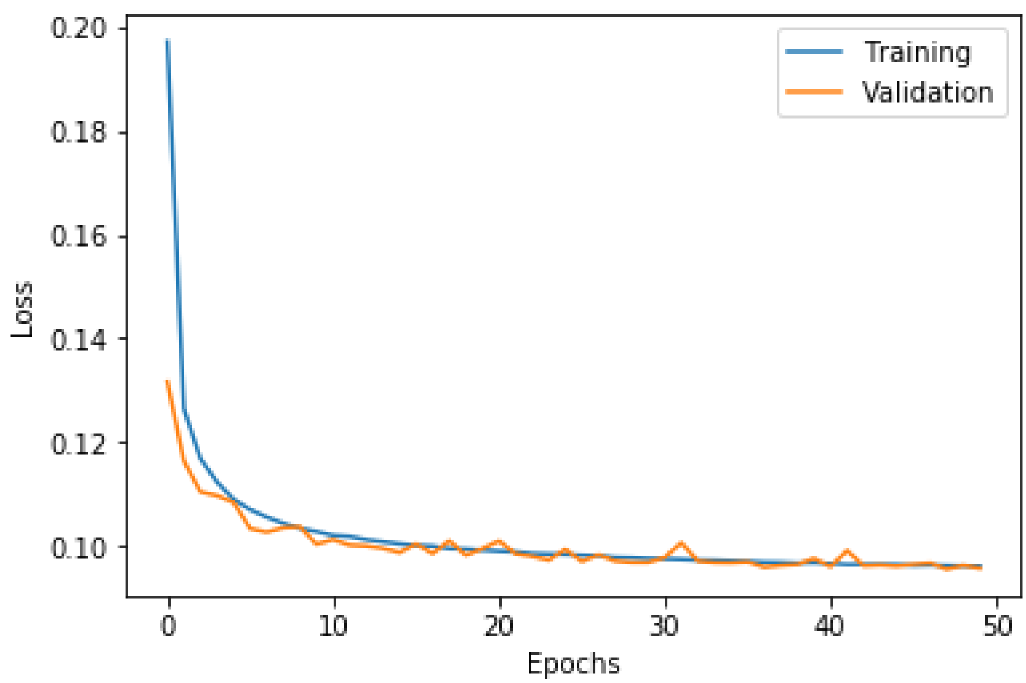
```
In [23]: history = autoencoder.fit(x_train_noisy, x_train,
                         epochs=50,
                         batch_size=128,
                         shuffle=True,
                         validation_data=(x_test_noisy, x_test),
                         callbacks=[TensorBoard(log_dir='/tmp/autoencoder_denoise',
                                               histogram_freq=0, write_graph=False)])
```

```
Train on 42000 samples, validate on 10000 samples
Epoch 1/50
42000/42000 [==============================] - 89s 2ms/step - loss: 0.1972 - val_loss: 0.1317
Epoch 2/50
42000/42000 [==============================] - 96s 2ms/step - loss: 0.1264 - val_loss: 0.1164
Epoch 3/50
42000/42000 [==============================] - 96s 2ms/step - loss: 0.1168 - val_loss: 0.1104
Epoch 4/50
42000/42000 [==============================] - 95s 2ms/step - loss: 0.1123 - val_loss: 0.1097
Epoch 5/50
42000/42000 [==============================] - 94s 2ms/step - loss: 0.1090 - val_loss: 0.1084
Epoch 6/50
42000/42000 [==============================] - 100s 2ms/step - loss: 0.1070 - val_loss: 0.1034
Epoch 7/50
42000/42000 [==============================] - 117s 3ms/step - loss: 0.1056 - val_loss: 0.1027
Epoch 8/50
42000/42000 [==============================] - 142s 3ms/step - loss: 0.1044 - val_loss: 0.1035
Epoch 9/50
42000/42000 [==============================] - 166s 4ms/step - loss: 0.1034 - val_loss: 0.1037
Epoch 10/50
42000/42000 [==============================] - 148s 4ms/step - loss: 0.1028 - val_loss: 0.1003
Epoch 11/50
42000/42000 [==============================] - 104s 2ms/step - loss: 0.1020 - val_loss: 0.1012
Epoch 12/50
42000/42000 [==============================] - 99s 2ms/step - loss: 0.1018 - val_loss: 0.1002
Epoch 13/50
42000/42000 [==============================] - 103s 2ms/step - loss: 0.1012 - val_loss: 0.1000
Epoch 14/50
42000/42000 [==============================] - 98s 2ms/step - loss: 0.1008 - val_loss: 0.0995
```

```
Epoch 15/50
42000/42000 [==============================] - 105s 3ms/step - loss: 0.1004 - val_loss: 0.0988
Epoch 16/50
42000/42000 [==============================] - 99s 2ms/step - loss: 0.1001 - val_loss: 0.1004
Epoch 17/50
42000/42000 [==============================] - 97s 2ms/step - loss: 0.1000 - val_loss: 0.0985
Epoch 18/50
42000/42000 [==============================] - 95s 2ms/step - loss: 0.0995 - val_loss: 0.1010
Epoch 19/50
42000/42000 [==============================] - 99s 2ms/step - loss: 0.0994 - val_loss: 0.0982
Epoch 20/50
42000/42000 [==============================] - 94s 2ms/step - loss: 0.0992 - val_loss: 0.0995
Epoch 21/50
42000/42000 [==============================] - 94s 2ms/step - loss: 0.0990 - val_loss: 0.1010
Epoch 22/50
42000/42000 [==============================] - 107s 3ms/step - loss: 0.0989 - val_loss: 0.0985
Epoch 23/50
42000/42000 [==============================] - 116s 3ms/step - loss: 0.0985 - val_loss: 0.0980
Epoch 24/50
42000/42000 [==============================] - 119s 3ms/step - loss: 0.0984 - val_loss: 0.0973
Epoch 25/50
42000/42000 [==============================] - 102s 2ms/step - loss: 0.0984 - val_loss: 0.0993
Epoch 26/50
42000/42000 [==============================] - 105s 3ms/step - loss: 0.0982 - val_loss: 0.0970
Epoch 27/50
42000/42000 [==============================] - 95s 2ms/step - loss: 0.0981 - val_loss: 0.0983
Epoch 28/50
42000/42000 [==============================] - 94s 2ms/step - loss: 0.0978 - val_loss: 0.0971
Epoch 29/50
42000/42000 [==============================] - 94s 2ms/step - loss: 0.0977 - val_loss: 0.0968
Epoch 30/50
42000/42000 [==============================] - 92s 2ms/step - loss: 0.0976 - val_loss: 0.0968
Epoch 31/50
42000/42000 [==============================] - 95s 2ms/step - loss: 0.0975 - val_loss: 0.0978
Epoch 32/50
42000/42000 [==============================] - 93s 2ms/step - loss: 0.0974 - val_loss: 0.1006
Epoch 33/50
42000/42000 [==============================] - 94s 2ms/step - loss: 0.0973 - val_loss: 0.0971
Epoch 34/50
42000/42000 [==============================] - 100s 2ms/step - loss: 0.0973 - val_loss: 0.0968
Epoch 35/50
42000/42000 [==============================] - 88s 2ms/step - loss: 0.0971 - val_loss: 0.0968
Epoch 36/50
42000/42000 [==============================] - 93s 2ms/step - loss: 0.0970 - val_loss: 0.0969
Epoch 37/50
42000/42000 [==============================] - 93s 2ms/step - loss: 0.0969 - val_loss: 0.0959
Epoch 38/50
42000/42000 [==============================] - 96s 2ms/step - loss: 0.0968 - val_loss: 0.0963
```

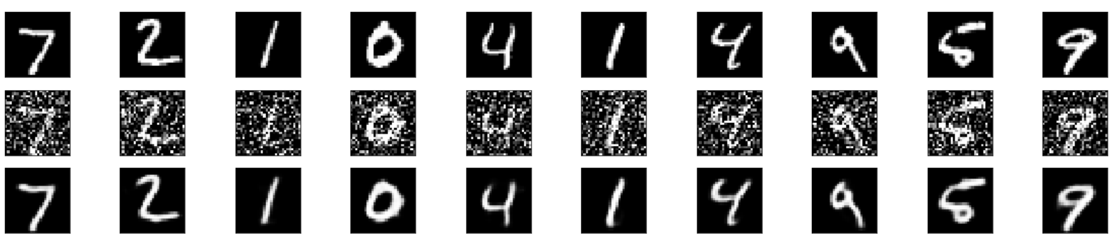
```
Epoch 39/50
42000/42000 [==============================] - 96s 2ms/step - loss: 0.0967 - val_loss: 0.0964
Epoch 40/50
42000/42000 [==============================] - 108s 3ms/step - loss: 0.0968 - val_loss: 0.0976
Epoch 41/50
42000/42000 [==============================] - 97s 2ms/step - loss: 0.0966 - val_loss: 0.0960
Epoch 42/50
42000/42000 [==============================] - 103s 2ms/step - loss: 0.0964 - val_loss: 0.0991
Epoch 43/50
42000/42000 [==============================] - 96s 2ms/step - loss: 0.0965 - val_loss: 0.0962
Epoch 44/50
42000/42000 [==============================] - 104s 2ms/step - loss: 0.0965 - val_loss: 0.0963
Epoch 45/50
42000/42000 [==============================] - 91s 2ms/step - loss: 0.0964 - val_loss: 0.0961
Epoch 46/50
42000/42000 [==============================] - 109s 3ms/step - loss: 0.0963 - val_loss: 0.0965
Epoch 47/50
42000/42000 [==============================] - 96s 2ms/step - loss: 0.0963 - val_loss: 0.0967
Epoch 48/50
42000/42000 [==============================] - 85s 2ms/step - loss: 0.0962 - val_loss: 0.0955
Epoch 49/50
42000/42000 [==============================] - 83s 2ms/step - loss: 0.0962 - val_loss: 0.0963
Epoch 50/50
42000/42000 [==============================] - 99s 2ms/step - loss: 0.0961 - val_loss: 0.0957
```

```
In [24]: plot_history(history)
```



Now let's take a look at the results. Top, the noisy digits fed to the network, and bottom, the digits are reconstructed by the network.

```
In [44]: decoded_imgs = autoencoder.predict(x_test_noisy)

         n = 10
         plt.figure(figsize=(20, 4))
         for i in range(n):
             # display original
             ax = plt.subplot(3, n, i+1)
             plt.imshow(x_test[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
             ax = plt.subplot(3, n, i + n + 1)
             plt.imshow(x_test_noisy[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
             # display reconstruction
             ax = plt.subplot(3, n, i + 2*n + 1)
             plt.imshow(decoded_imgs[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
         plt.show()
```



It seems to work pretty well. If you scale this process to a bigger convnet, you can start building document denoising or audio denoising models.

2 Bonus: Variational AutoEncoder

Reference *https://blog.keras.io/building-autoencoders-in-keras.html* and https://github.com/keras-team/keras/blob/master/examples/variational_autoencoder.py

Variational autoencoders are a slightly more modern and interesting take on autoencoding.

2.0.1 What is a variational autoencoder ?

It's a type of autoencoder with added constraints on the encoded representations being learned.

More precisely, it is an autoencoder that learns a latent variable model for its input data.

So instead of letting your neural network learn an arbitrary function, you are learning the parameters of a probability distribution modeling your data.

If you sample points from this distribution, you can generate new input data samples: a **VAE** is a **"generative model"**.

2.0.2 How does a variational autoencoder work?

First, an encoder network turns the input samples x into two parameters in a latent space, which we will note z_μ and z_{log_σ}.

Then, we randomly sample similar points z from the *latent normal distribution* that is assumed to generate the data, via $z = z_\mu + \exp(z_{log_\sigma}) * \epsilon$, where ϵ is a random normal tensor.

Finally, a decoder network maps these latent space points back to the original input data.

The parameters of the model are trained via two loss functions:

- a **reconstruction loss** forcing the decoded samples to match the initial inputs (just like in our previous autoencoders);
- and the **KL divergence** between the learned latent distribution and the prior distribution, acting as a regularization term.

You could actually get rid of this latter term entirely, although it does help in learning well-formed latent spaces and reducing overfitting to the training data.

2.1 Load MNIST

In [26]:
```
from keras.layers import Lambda, Input, Dense
from keras.models import Model
from keras.datasets import mnist
from keras.losses import binary_crossentropy
from keras.utils import plot_model
from keras import backend as K

import numpy as np
import matplotlib.pyplot as plt
import argparse
import os
```

In [27]:
```
(x_train, y_train), (x_test, y_test) = mnist.load_data()

image_size = x_train.shape[1]
original_dim = image_size * image_size
x_train = np.reshape(x_train, [-1, original_dim])
x_test = np.reshape(x_test, [-1, original_dim])
x_train = x_train.astype('float32') / 255
x_test = x_test.astype('float32') / 255
```

2.2 Encoder Network

First, here's our encoder network, mapping inputs to our latent distribution parameters:

```
In [28]: # network parameters
         input_shape = (original_dim, )
         intermediate_dim = 512
         batch_size = 128
         latent_dim = 2
         epochs = 50
```

```
In [29]: # VAE model = encoder + decoder
         # build encoder model
         inputs = Input(shape=input_shape, name='encoder_input')
         x = Dense(intermediate_dim, activation='relu')(inputs)
         z_mean = Dense(latent_dim, name='z_mean')(x)
         z_log_var = Dense(latent_dim, name='z_log_var')(x)
```

We can use these parameters to sample new similar points from the latent space:

```
In [30]: # reparameterization trick
         # instead of sampling from Q(z|X), sample eps = N(0,I)
         # z = z_mean + sqrt(var)*eps
         def sampling(args):
             """Reparameterization trick by sampling fr an isotropic unit Gaussian.
             # Arguments:
                 args (tensor): mean and log of variance of Q(z|X)
             # Returns:
                 z (tensor): sampled latent vector
             """

             z_mean, z_log_var = args
             batch = K.shape(z_mean)[0]
             dim = K.int_shape(z_mean)[1]
             # by default, random_normal has mean=0 and std=1.0
             epsilon = K.random_normal(shape=(batch, dim))
             return z_mean + K.exp(0.5 * z_log_var) * epsilon

         # use reparameterization trick to push the sampling out as input
         # note that "output_shape" isn't necessary with the TensorFlow backend
         z = Lambda(sampling, output_shape=(latent_dim,), name='z')([z_mean, z_log_var])

         # instantiate encoder model
         encoder = Model(inputs, [z_mean, z_log_var, z], name='encoder')
         encoder.summary()
```

```
__________________________________________________________________________________________________
Layer (type)                    Output Shape         Param #     Connected to
==================================================================================================
```

```
encoder_input (InputLayer)      (None, 784)          0
____________________________________________________________________________________________________
dense_3 (Dense)                 (None, 512)          401920      encoder_input[0][0]
____________________________________________________________________________________________________
z_mean (Dense)                  (None, 2)            1026        dense_3[0][0]
____________________________________________________________________________________________________
z_log_var (Dense)               (None, 2)            1026        dense_3[0][0]
____________________________________________________________________________________________________
z (Lambda)                      (None, 2)            0           z_mean[0][0]
                                                                 z_log_var[0][0]
====================================================================================================
Total params: 403,972
Trainable params: 403,972
Non-trainable params: 0
____________________________________________________________________________________________________
```

2.3 Decoder Network

Finally, we can map these sampled latent points back to reconstructed inputs:

In [31]:
```
# build decoder model
latent_inputs = Input(shape=(latent_dim,), name='z_sampling')
x = Dense(intermediate_dim, activation='relu')(latent_inputs)
outputs = Dense(original_dim, activation='sigmoid')(x)

# instantiate decoder model
decoder = Model(latent_inputs, outputs, name='decoder')
decoder.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
z_sampling (InputLayer)      (None, 2)                 0
_________________________________________________________________
dense_4 (Dense)              (None, 512)               1536
_________________________________________________________________
dense_5 (Dense)              (None, 784)               402192
=================================================================
Total params: 403,728
Trainable params: 403,728
Non-trainable params: 0
_________________________________________________________________
```

2.4 Variational AutoEncoder

In [32]:
```
# instantiate VAE model
outputs = decoder(encoder(inputs)[2])
```

```
vae = Model(inputs, outputs, name='vae_mlp')
```

We train the model using the end-to-end model, with a custom loss function: the sum of a reconstruction term, and the KL divergence regularization term.

```
In [33]: reconstruction_loss = binary_crossentropy(inputs,outputs)
         reconstruction_loss *= original_dim
         kl_loss = 1 + z_log_var - K.square(z_mean) - K.exp(z_log_var)
         kl_loss = K.sum(kl_loss, axis=-1)
         kl_loss *= -0.5
         vae_loss = K.mean(reconstruction_loss + kl_loss)
         vae.add_loss(vae_loss)
         vae.compile(optimizer='adam')
         vae.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
encoder_input (InputLayer)   (None, 784)               0
_________________________________________________________________
encoder (Model)              [(None, 2), (None, 2), (N 403972
_________________________________________________________________
decoder (Model)              (None, 784)               403728
=================================================================
Total params: 807,700
Trainable params: 807,700
Non-trainable params: 0
_________________________________________________________________


/media/nackenho/Data/programs/ML/anaconda2/lib/python2.7/site-packages/ipykernel_launcher.py:8:
```

Traing on MNIST Digits

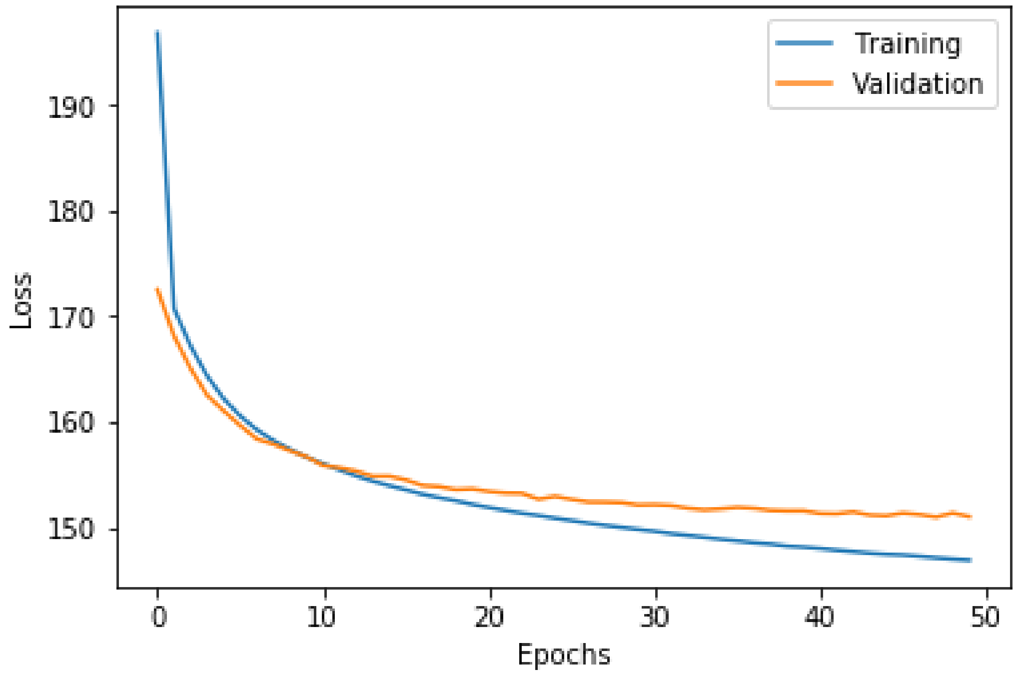
```
In [34]: history = vae.fit(x_train,
                           epochs=epochs,
                           batch_size=batch_size,
                           validation_data=(x_test, None))
```

```
Train on 60000 samples, validate on 10000 samples
Epoch 1/50
60000/60000 [==============================] - 17s 276us/step - loss: 196.8490 - val_loss: 172.5
Epoch 2/50
60000/60000 [==============================] - 13s 210us/step - loss: 170.7229 - val_loss: 168.1
Epoch 3/50
60000/60000 [==============================] - 12s 202us/step - loss: 167.1706 - val_loss: 165.0
Epoch 4/50
```

```
60000/60000 [==============================] - 13s 222us/step - loss: 164.3275 - val_loss: 162.4
Epoch 5/50
60000/60000 [==============================] - 11s 187us/step - loss: 162.1700 - val_loss: 161.0
Epoch 6/50
60000/60000 [==============================] - 13s 212us/step - loss: 160.5360 - val_loss: 159.6
Epoch 7/50
60000/60000 [==============================] - 13s 211us/step - loss: 159.2461 - val_loss: 158.3
Epoch 8/50
60000/60000 [==============================] - 13s 211us/step - loss: 158.2196 - val_loss: 157.8
Epoch 9/50
60000/60000 [==============================] - 15s 245us/step - loss: 157.3817 - val_loss: 157.2
Epoch 10/50
60000/60000 [==============================] - 12s 194us/step - loss: 156.6447 - val_loss: 156.6
Epoch 11/50
60000/60000 [==============================] - 11s 191us/step - loss: 156.0098 - val_loss: 155.8
Epoch 12/50
60000/60000 [==============================] - 13s 220us/step - loss: 155.4347 - val_loss: 155.6
Epoch 13/50
60000/60000 [==============================] - 12s 194us/step - loss: 154.8634 - val_loss: 155.3
Epoch 14/50
60000/60000 [==============================] - 12s 192us/step - loss: 154.3661 - val_loss: 154.8
Epoch 15/50
60000/60000 [==============================] - 11s 177us/step - loss: 153.9315 - val_loss: 154.8
Epoch 16/50
60000/60000 [==============================] - 13s 222us/step - loss: 153.5353 - val_loss: 154.4
Epoch 17/50
60000/60000 [==============================] - 12s 199us/step - loss: 153.1185 - val_loss: 153.9
Epoch 18/50
60000/60000 [==============================] - 12s 195us/step - loss: 152.7968 - val_loss: 153.8
Epoch 19/50
60000/60000 [==============================] - 12s 200us/step - loss: 152.5013 - val_loss: 153.5
Epoch 20/50
60000/60000 [==============================] - 12s 199us/step - loss: 152.1597 - val_loss: 153.6
Epoch 21/50
60000/60000 [==============================] - 11s 182us/step - loss: 151.8627 - val_loss: 153.3
Epoch 22/50
60000/60000 [==============================] - 10s 172us/step - loss: 151.5869 - val_loss: 153.2
Epoch 23/50
60000/60000 [==============================] - 13s 223us/step - loss: 151.3247 - val_loss: 153.2
Epoch 24/50
60000/60000 [==============================] - 13s 224us/step - loss: 151.1015 - val_loss: 152.6
Epoch 25/50
60000/60000 [==============================] - 13s 211us/step - loss: 150.8470 - val_loss: 152.9
Epoch 26/50
60000/60000 [==============================] - 12s 204us/step - loss: 150.6186 - val_loss: 152.6
Epoch 27/50
60000/60000 [==============================] - 15s 245us/step - loss: 150.3777 - val_loss: 152.3
Epoch 28/50
```

```
60000/60000 [==============================] - 13s 209us/step - loss: 150.1684 - val_loss: 152.3
Epoch 29/50
60000/60000 [==============================] - 13s 213us/step - loss: 149.9759 - val_loss: 152.3
Epoch 30/50
60000/60000 [==============================] - 11s 190us/step - loss: 149.7862 - val_loss: 152.1
Epoch 31/50
60000/60000 [==============================] - 13s 218us/step - loss: 149.6037 - val_loss: 152.1
Epoch 32/50
60000/60000 [==============================] - 11s 184us/step - loss: 149.3840 - val_loss: 152.0
Epoch 33/50
60000/60000 [==============================] - 12s 202us/step - loss: 149.2113 - val_loss: 151.7
Epoch 34/50
60000/60000 [==============================] - 13s 224us/step - loss: 149.0337 - val_loss: 151.6
Epoch 35/50
60000/60000 [==============================] - 12s 204us/step - loss: 148.8502 - val_loss: 151.7
Epoch 36/50
60000/60000 [==============================] - 11s 183us/step - loss: 148.6907 - val_loss: 151.8
Epoch 37/50
60000/60000 [==============================] - 13s 217us/step - loss: 148.5186 - val_loss: 151.7
Epoch 38/50
60000/60000 [==============================] - 11s 190us/step - loss: 148.3920 - val_loss: 151.5
Epoch 39/50
60000/60000 [==============================] - 12s 206us/step - loss: 148.1975 - val_loss: 151.5
Epoch 40/50
60000/60000 [==============================] - 12s 200us/step - loss: 148.0993 - val_loss: 151.5
Epoch 41/50
60000/60000 [==============================] - 11s 190us/step - loss: 147.9702 - val_loss: 151.3
Epoch 42/50
60000/60000 [==============================] - 13s 215us/step - loss: 147.8102 - val_loss: 151.2
Epoch 43/50
60000/60000 [==============================] - 11s 182us/step - loss: 147.6717 - val_loss: 151.4
Epoch 44/50
60000/60000 [==============================] - 12s 193us/step - loss: 147.5276 - val_loss: 151.1
Epoch 45/50
60000/60000 [==============================] - 13s 212us/step - loss: 147.4202 - val_loss: 151.0
Epoch 46/50
60000/60000 [==============================] - 11s 183us/step - loss: 147.3526 - val_loss: 151.3
Epoch 47/50
60000/60000 [==============================] - 12s 192us/step - loss: 147.2268 - val_loss: 151.1
Epoch 48/50
60000/60000 [==============================] - 13s 212us/step - loss: 147.0850 - val_loss: 150.9
Epoch 49/50
60000/60000 [==============================] - 11s 183us/step - loss: 146.9646 - val_loss: 151.3
Epoch 50/50
60000/60000 [==============================] - 12s 192us/step - loss: 146.8627 - val_loss: 150.9
```

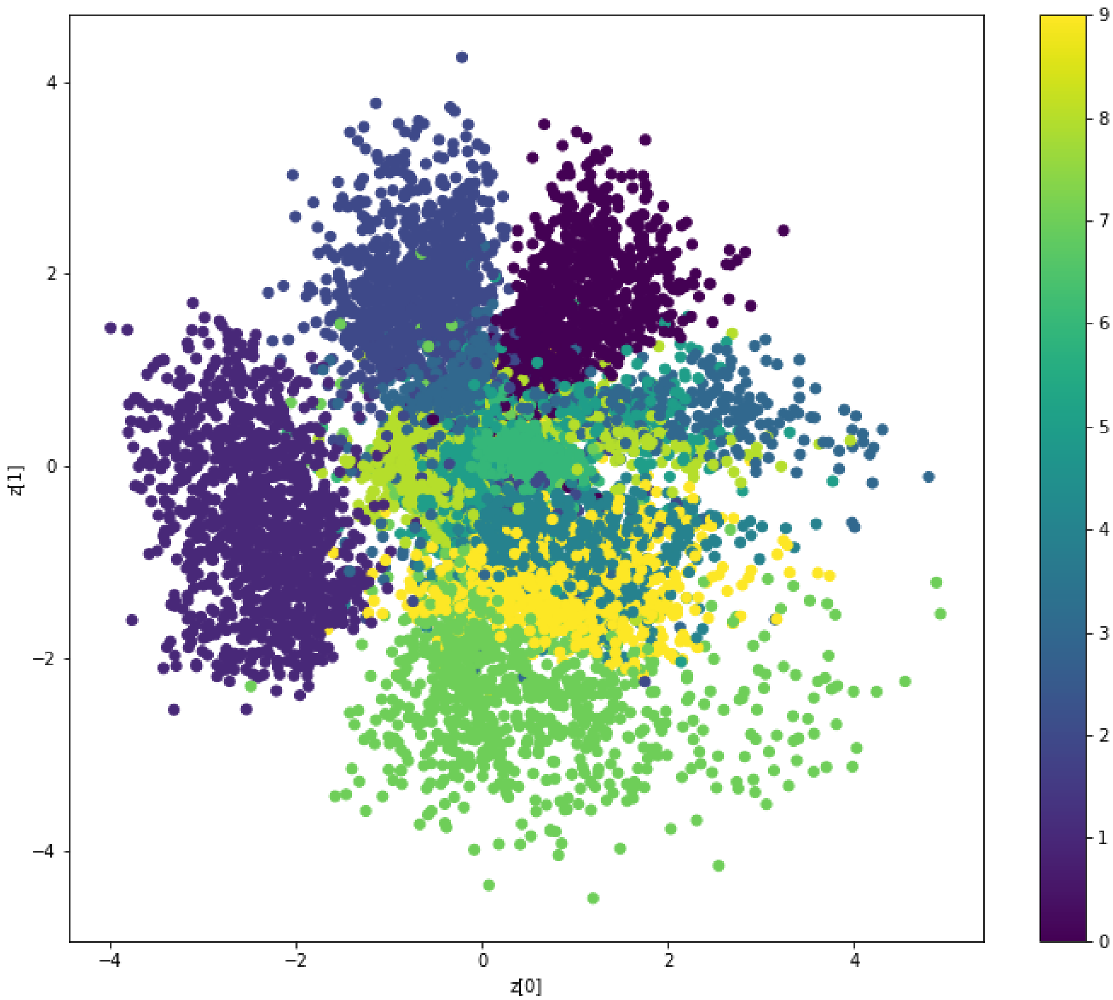
In [35]:
```
plot_history(history)
```



Because our latent space is two-dimensional, there are a few cool visualizations that can be done at this point.

One is to look at the neighborhoods of different classes on the latent 2D plane:

```
In [36]: x_test_encoded = encoder.predict(x_test, batch_size=batch_size)

In [45]: # display a 2D plot of the digit classes in the latent space
         z_mean, _, _ = encoder.predict(x_test,
                                        batch_size=batch_size)
         plt.figure(figsize=(12, 10))
         plt.scatter(z_mean[:, 0], z_mean[:, 1], c=y_test, cmap='viridis')
         plt.colorbar()
         plt.xlabel("z[0]")
         plt.ylabel("z[1]")
         plt.show()
```



Each of these colored clusters is a type of digit. Close clusters are digits that are structurally similar (i.e. digits that share information in the latent space).

Because the VAE is a generative model, we can also use it to generate new digits! Here we will scan the latent plane, sampling latent points at regular intervals, and generating the corresponding digit for each of these points. This gives us a visualization of the latent manifold that "generates" the MNIST digits.

In [38]:
```
# display a 30x30 2D manifold of digits
n = 30
digit_size = 28
figure = np.zeros((digit_size * n, digit_size * n))
# linearly spaced coordinates corresponding to the 2D plot
# of digit classes in the latent space
grid_x = np.linspace(-4, 4, n)
grid_y = np.linspace(-4, 4, n)[::-1]

for i, yi in enumerate(grid_y):
```

```
    for j, xi in enumerate(grid_x):
        z_sample = np.array([[xi, yi]])
        x_decoded = decoder.predict(z_sample)
        digit = x_decoded[0].reshape(digit_size, digit_size)
        figure[i * digit_size: (i + 1) * digit_size,
               j * digit_size: (j + 1) * digit_size] = digit

plt.figure(figsize=(20, 20))
start_range = digit_size // 2
end_range = n * digit_size + start_range + 1
pixel_range = np.arange(start_range, end_range, digit_size)
sample_range_x = np.round(grid_x, 1)
sample_range_y = np.round(grid_y, 1)
plt.xticks(pixel_range, sample_range_x)
plt.yticks(pixel_range, sample_range_y)
plt.xlabel("z[0]")
plt.ylabel("z[1]")
plt.imshow(figure, cmap='Greys_r')
plt.show()
```