ex7_sol

June 13, 2019

1 Exercise 7 - AutoEncoders

This exercise is based on https://github.com/leriomaggio/deep-learning-keras-tensorflow and `https://blog.keras.io/building-autoencoders-in-keras.html`

"Autoencoding" is a data compression algorithm where the compression and decompression functions are data-specific, lossy, and learned automatically from examples rather than engineered by a human. Additionally, in almost all contexts where the term "autoencoder" is used, the compression and decompression functions are implemented with neural networks.

The aim of an autoencoder is to learn a representation (encoding) for a set of data.. typically for the purpose of dimensionality reduction or feature learning.

To build an autoencoder, you need three things: an encoding function, a decoding function, and a distance function between the amount of information loss between the compressed representation of your data and the decompressed representation (i.e. a "loss" function)

Autoencoders are not a true unsupervised learning technique (which would imply a different learning process altogether), they are a self-supervised technique, a specific instance of supervised learning where the targets are generated from the input data. In order to get self-supervised models to learn interesting features, you have to come up with an interesting synthetic target and loss function, and that's where problems arise: merely learning to reconstruct your input in minute detail might not be the right choice here.

1.1 AutoEncoder and the MNIST dataset

We will use the MNIST dataset in order to train a few simple autoencoder.

1.2 Single fully-connected neural layer Autoencoder

1.2.1 Building the Model with *Keras Functional API*

The Keras functional API is the way to go for defining complex models, such as multi-output models, directed acyclic graphs, or models with shared layers. All the Functional API relies on the fact that each `keras.Layer` object is a *callable* object! More details can be found here: https://keras.io/getting-started/functional-api-guide/

Let's start with the simplest possible model:

```
In [1]: from keras.layers import Input, Dense
        from keras.models import Model
        import numpy as np
        # this is the size of our encoded representations
        encoding_dim = 32  # 32 floats -> compression of factor 24.5, assuming the input is 78

        # this is our input placeholder
        input_img = Input(shape=(784,))
        # "encoded" is the encoded representation of the input
        encoded = Dense(encoding_dim, activation='relu')(input_img)
        # "decoded" is the lossy reconstruction of the input
        decoded = Dense(784, activation='sigmoid')(encoded)

        # this model maps an input to its reconstruction
        autoencoder = Model(input_img, decoded)
```

```
Using TensorFlow backend.


WARNING:tensorflow:From /home/nackenho/miniconda/envs/TUDortmundMLSeminar/lib/python2.7/site-pa
Instructions for updating:
Colocations handled automatically by placer.
```

Let's also create a separate encoder model:

```
In [2]: # this model maps an input to its encoded representation
        encoder = Model(input_img, encoded)
```

As well as the decoder model:

```
In [3]: # create a placeholder for an encoded (32-dimensional) input
        encoded_input = Input(shape=(encoding_dim,))
        # retrieve the last layer of the autoencoder model
        decoder_layer = autoencoder.layers[-1]
        # create the decoder model
        decoder = Model(encoded_input, decoder_layer(encoded_input))
```

First, we'll configure our autoencoder model to use a per-pixel binary crossentropy loss, and the Adadelta optimizer:

```
In [4]: autoencoder.compile(optimizer='adadelta', loss='binary_crossentropy')
        autoencoder.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_1 (InputLayer)         (None, 784)               0
_________________________________________________________________
dense_1 (Dense)              (None, 32)                25120
_________________________________________________________________
dense_2 (Dense)              (None, 784)               25872
```

```
=================================================================
Total params: 50,992
Trainable params: 50,992
Non-trainable params: 0
_________________________________________________________________
```

1.3 Data preparation for Dense-Layered Autoencoder

Let's prepare our input data. We're using MNIST digits, we won't need the labels (since we're only interested in encoding/decoding the input images).

```
In [5]: import numpy as np
        from keras.datasets import mnist
        #Load
        (x_train, y_train), (x_test, y_test) = mnist.load_data()

        #Flattening a bit more elegant
        x_train = x_train.reshape((len(x_train), np.prod(x_train.shape[1:])))
        x_test = x_test.reshape((len(x_test), np.prod(x_test.shape[1:])))

        #Shortcut for scaling today ;-)
        x_train = x_train.astype('float32') / 255.
        x_test = x_test.astype('float32') / 255.


        print x_train.shape
        print x_test.shape
```

```
(60000, 784)
(10000, 784)
```

Split Training and Validation Data

```
In [6]: from sklearn.model_selection import train_test_split

        x_train, x_val, y_train, y_val = train_test_split(x_train, y_train, test_size=0.3, ranc
```

1.4 Training the Autoencoder

Now let's train our autoencoder:

```
In [7]: #note: x_train, x_train and x_val, x_val :)
        history = autoencoder.fit(x_train, x_train,
                        epochs=50,
                        batch_size=256,
                        shuffle=True,
                        validation_data=(x_val, x_val))
```

```
WARNING:tensorflow:From /home/nackenho/miniconda/envs/TUDortmundMLSeminar/lib/python2.7/site-pa
Instructions for updating:
Use tf.cast instead.
Train on 42000 samples, validate on 18000 samples
Epoch 1/50
42000/42000 [==============================] - 3s 75us/step - loss: 0.3970 - val_loss: 0.2765
Epoch 2/50
42000/42000 [==============================] - 2s 55us/step - loss: 0.2720 - val_loss: 0.2662
Epoch 3/50
42000/42000 [==============================] - 2s 56us/step - loss: 0.2606 - val_loss: 0.2526
Epoch 4/50
42000/42000 [==============================] - 2s 50us/step - loss: 0.2452 - val_loss: 0.2366
Epoch 5/50
42000/42000 [==============================] - 2s 50us/step - loss: 0.2303 - val_loss: 0.2231
Epoch 6/50
42000/42000 [==============================] - 2s 52us/step - loss: 0.2183 - val_loss: 0.2125
Epoch 7/50
42000/42000 [==============================] - 2s 59us/step - loss: 0.2090 - val_loss: 0.2043
Epoch 8/50
42000/42000 [==============================] - 2s 55us/step - loss: 0.2013 - val_loss: 0.1971
Epoch 9/50
42000/42000 [==============================] - 2s 59us/step - loss: 0.1945 - val_loss: 0.1906
Epoch 10/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1885 - val_loss: 0.1850
Epoch 11/50
42000/42000 [==============================] - 4s 89us/step - loss: 0.1832 - val_loss: 0.1801
Epoch 12/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1786 - val_loss: 0.1758
Epoch 13/50
42000/42000 [==============================] - 3s 63us/step - loss: 0.1746 - val_loss: 0.1722
Epoch 14/50
42000/42000 [==============================] - 2s 56us/step - loss: 0.1711 - val_loss: 0.1688
Epoch 15/50
42000/42000 [==============================] - 2s 55us/step - loss: 0.1679 - val_loss: 0.1657
Epoch 16/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1649 - val_loss: 0.1629
Epoch 17/50
42000/42000 [==============================] - 2s 50us/step - loss: 0.1622 - val_loss: 0.1602
Epoch 18/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1596 - val_loss: 0.1579
Epoch 19/50
42000/42000 [==============================] - 2s 53us/step - loss: 0.1572 - val_loss: 0.1555
Epoch 20/50
42000/42000 [==============================] - 2s 55us/step - loss: 0.1549 - val_loss: 0.1533
Epoch 21/50
42000/42000 [==============================] - 2s 52us/step - loss: 0.1527 - val_loss: 0.1511
Epoch 22/50
42000/42000 [==============================] - 2s 53us/step - loss: 0.1506 - val_loss: 0.1491
```

```
Epoch 23/50
42000/42000 [==============================] - 2s 53us/step - loss: 0.1486 - val_loss: 0.1472
Epoch 24/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1467 - val_loss: 0.1452
Epoch 25/50
42000/42000 [==============================] - 2s 55us/step - loss: 0.1448 - val_loss: 0.1434
Epoch 26/50
42000/42000 [==============================] - 2s 53us/step - loss: 0.1430 - val_loss: 0.1416
Epoch 27/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1412 - val_loss: 0.1399
Epoch 28/50
42000/42000 [==============================] - 2s 52us/step - loss: 0.1395 - val_loss: 0.1383
Epoch 29/50
42000/42000 [==============================] - 2s 56us/step - loss: 0.1378 - val_loss: 0.1367
Epoch 30/50
42000/42000 [==============================] - 2s 52us/step - loss: 0.1362 - val_loss: 0.1350
Epoch 31/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1347 - val_loss: 0.1336
Epoch 32/50
42000/42000 [==============================] - 2s 53us/step - loss: 0.1332 - val_loss: 0.1324
Epoch 33/50
42000/42000 [==============================] - 2s 55us/step - loss: 0.1317 - val_loss: 0.1306
Epoch 34/50
42000/42000 [==============================] - 2s 55us/step - loss: 0.1303 - val_loss: 0.1293
Epoch 35/50
42000/42000 [==============================] - 2s 57us/step - loss: 0.1290 - val_loss: 0.1280
Epoch 36/50
42000/42000 [==============================] - 2s 55us/step - loss: 0.1277 - val_loss: 0.1270
Epoch 37/50
42000/42000 [==============================] - 2s 56us/step - loss: 0.1265 - val_loss: 0.1256
Epoch 38/50
42000/42000 [==============================] - 2s 56us/step - loss: 0.1253 - val_loss: 0.1245
Epoch 39/50
42000/42000 [==============================] - 2s 55us/step - loss: 0.1242 - val_loss: 0.1234
Epoch 40/50
42000/42000 [==============================] - 2s 56us/step - loss: 0.1231 - val_loss: 0.1223
Epoch 41/50
42000/42000 [==============================] - 2s 53us/step - loss: 0.1221 - val_loss: 0.1213
Epoch 42/50
42000/42000 [==============================] - 2s 55us/step - loss: 0.1211 - val_loss: 0.1203
Epoch 43/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1201 - val_loss: 0.1194
Epoch 44/50
42000/42000 [==============================] - 2s 55us/step - loss: 0.1192 - val_loss: 0.1186
Epoch 45/50
42000/42000 [==============================] - 2s 53us/step - loss: 0.1183 - val_loss: 0.1178
Epoch 46/50
42000/42000 [==============================] - 2s 56us/step - loss: 0.1175 - val_loss: 0.1169
```

```
Epoch 47/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1168 - val_loss: 0.1162
Epoch 48/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1160 - val_loss: 0.1155
Epoch 49/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1153 - val_loss: 0.1148
Epoch 50/50
42000/42000 [==============================] - 2s 54us/step - loss: 0.1147 - val_loss: 0.1142
```

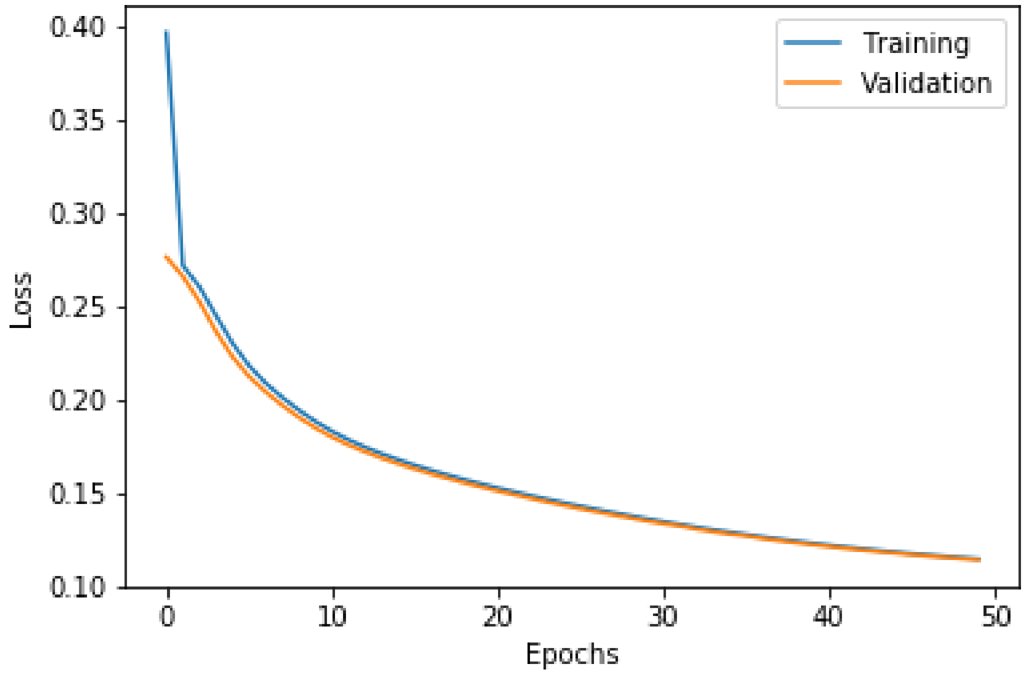
Let's plot the loss function to see if the training is stable

```
In [8]: print(history.history.keys())
```

```
['loss', 'val_loss']
```

```
In [9]: %matplotlib inline
        from matplotlib import pyplot as plt
        def plot_history(network_history):
            plt.figure()
            plt.xlabel('Epochs')
            plt.ylabel('Loss')
            plt.plot(network_history.history['loss'])
            plt.plot(network_history.history['val_loss'])
            plt.legend(['Training', 'Validation'])
```

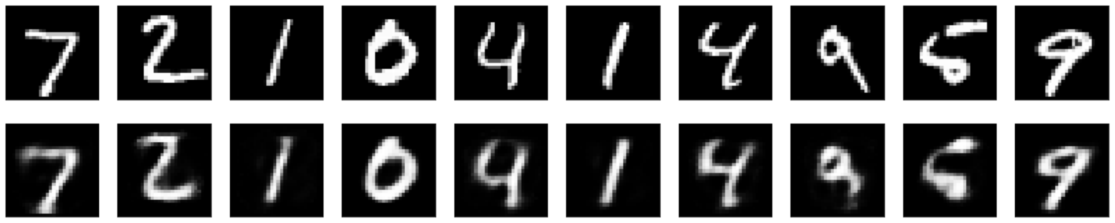
```
In [10]: plot_history(history)
```



Well, not yet done, but we don't have more time....

1.5 Testing the Autoencoder

```
In [11]: # encode and decode some digits
         # note that we take them from the *test* set
         encoded_imgs = encoder.predict(x_test)
         decoded_imgs = decoder.predict(encoded_imgs)

         n = 10  # how many digits we will display
         plt.figure(figsize=(20, 4))
         for i in range(n):
             # display original
             ax = plt.subplot(2, n, i + 1)
             plt.imshow(x_test[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)

             # display reconstruction
             ax = plt.subplot(2, n, i + 1 + n)
             plt.imshow(decoded_imgs[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
         plt.show()
```



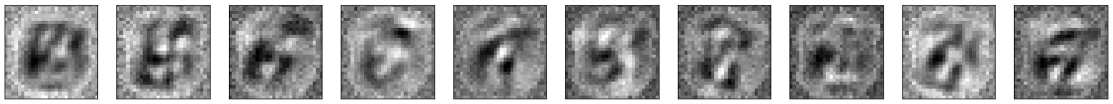
The top row is the original digits, and the bottom row is the reconstructed digits. We are losing quite a bit of detail with this basic approach.

1.6 Sample generation with Autoencoder

Can we actually generate numbers using the trained decoder, if we just draw random numbers for the encoded image?

```
In [12]: encoded_imgs = np.random.rand(10,32)
         decoded_imgs = decoder.predict(encoded_imgs)
```

```
n = 10
plt.figure(figsize=(20, 4))
for i in range(n):
    # generation
    ax = plt.subplot(2, n, i + 1 + n)
    plt.imshow(decoded_imgs[i].reshape(28, 28))
    plt.gray()
    ax.get_xaxis().set_visible(False)
    ax.get_yaxis().set_visible(False)
plt.show()
```



Although you might see some structures which look similar to numbers in the middle, using the decoder as a generator for handwritten digits clearly doesn't work.

We could now study deeper structures, feel free to try this at home, but the result will only improve slightly. Another way to constrain the representations to be compact is to add a sparsity contraint on the activity of the hidden representations, so fewer units would "fire" at a given time. In Keras, this can be done by adding the L1 norm regularizer as an `activity_regularizer` to our Dense layer.

1.7 Convolutional AutoEncoder

Since our inputs are images, it makes sense to use convolutional neural networks (`convnets`) as encoders and decoders. In practical settings, autoencoders applied to images are always convolutional autoencoders --they simply perform much better.

The encoder will consist in a stack of `Conv2D` and `MaxPooling2D` layers (max pooling being used for spatial down-sampling), while the decoder will consist in a stack of `Conv2D` and `UpSampling2D` layers.

```
In [13]: from keras.layers import Input, Dense, Conv2D, MaxPooling2D, UpSampling2D
         from keras.models import Model
         from keras import backend as K

         input_img = Input(shape=(28, 28, 1))  # adapt this if using `channels_first` image da

         x = Conv2D(16, (3, 3), activation='relu', padding='same')(input_img)
         x = MaxPooling2D((2, 2), padding='same')(x)
         x = Conv2D(8, (3, 3), activation='relu', padding='same')(x)
         x = MaxPooling2D((2, 2), padding='same')(x)
         x = Conv2D(8, (3, 3), activation='relu', padding='same')(x)
```

```
encoded = MaxPooling2D((2, 2), padding='same')(x)

# at this point the representation is (4, 4, 8) i.e. 128-dimensional

x = Conv2D(8, (3, 3), activation='relu', padding='same')(encoded)
x = UpSampling2D((2, 2))(x)
x = Conv2D(8, (3, 3), activation='relu', padding='same')(x)
x = UpSampling2D((2, 2))(x)
x = Conv2D(16, (3, 3), activation='relu')(x)
x = UpSampling2D((2, 2))(x)
decoded = Conv2D(1, (3, 3), activation='sigmoid', padding='same')(x)

conv_autoencoder = Model(input_img, decoded)
conv_autoencoder.compile(optimizer='adadelta', loss='binary_crossentropy')
conv_autoencoder.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_3 (InputLayer)         (None, 28, 28, 1)         0
_________________________________________________________________
conv2d_1 (Conv2D)            (None, 28, 28, 16)        160
_________________________________________________________________
max_pooling2d_1 (MaxPooling2 (None, 14, 14, 16)        0
_________________________________________________________________
conv2d_2 (Conv2D)            (None, 14, 14, 8)         1160
_________________________________________________________________
max_pooling2d_2 (MaxPooling2 (None, 7, 7, 8)           0
_________________________________________________________________
conv2d_3 (Conv2D)            (None, 7, 7, 8)           584
_________________________________________________________________
max_pooling2d_3 (MaxPooling2 (None, 4, 4, 8)           0
_________________________________________________________________
conv2d_4 (Conv2D)            (None, 4, 4, 8)           584
_________________________________________________________________
up_sampling2d_1 (UpSampling2 (None, 8, 8, 8)           0
_________________________________________________________________
conv2d_5 (Conv2D)            (None, 8, 8, 8)           584
_________________________________________________________________
up_sampling2d_2 (UpSampling2 (None, 16, 16, 8)         0
_________________________________________________________________
conv2d_6 (Conv2D)            (None, 14, 14, 16)        1168
_________________________________________________________________
up_sampling2d_3 (UpSampling2 (None, 28, 28, 16)        0
_________________________________________________________________
conv2d_7 (Conv2D)            (None, 28, 28, 1)         145
=================================================================
Total params: 4,385
```

```
Trainable params: 4,385
Non-trainable params: 0
_________________________________________________________________
```

1.8 Task 1: Train and evaluate the Convolutional Autoencoder

- Prepare the data for the Convolutional Autoencoder
- Train the Convolutional Autoencoder and plot the training and validation loss
- Test the Convolutional Autoencoder by plotting 10 of the decoded predictions next to the original images
- Plot the encoded representations in order to visualize how the digits are encoded

1.9 Data Preparation for Convolutional Autoencoder

```
In [14]: from keras import backend as K

         if K.image_data_format() == 'channels_last':
             shape_ord = (28, 28, 1)
         else:
             shape_ord = (1, 28, 28)

         #Load
         (x_train, y_train), (x_test, y_test) = mnist.load_data()

         #Scale
         x_train = x_train.astype('float32') / 255.
         x_test = x_test.astype('float32') / 255.

         #Shape for CNN
         x_train = np.reshape(x_train, ((x_train.shape[0],) + shape_ord))
         x_test = np.reshape(x_test, ((x_test.shape[0],) + shape_ord))

         #Split into validation
         x_train, x_val, y_train, y_val = train_test_split(x_train, y_train, test_size=0.3, ra
```

```
In [15]: x_train.shape

Out[15]: (42000, 28, 28, 1)
```

1.10 Training the Convolutional Autoencoder

Let's train this model. For the sake of demonstrating how to visualize the results of a model during training, we will be using the TensorFlow backend and the TensorBoard callback.

First, let's open up a terminal and start a TensorBoard server that will read logs stored at `/tmp/autoencoder`.

tensorboard --logdir=/tmp/autoencoder

Then let's train our model. In the callbacks list we pass an instance of the TensorBoard callback. After every epoch, this callback will write logs to /tmp/autoencoder, which can be read by our TensorBoard server.

```
In [16]: from keras.callbacks import TensorBoard

         history = conv_autoencoder.fit(x_train, x_train,
                         epochs=50,
                         batch_size=256,
                         shuffle=True,
                         validation_data=(x_test, x_test),
                         callbacks=[TensorBoard(log_dir='/tmp/autoencoder')])
```

```
Train on 42000 samples, validate on 10000 samples
Epoch 1/50
42000/42000 [==============================] - 16s 377us/step - loss: 0.2832 - val_loss: 0.203
Epoch 2/50
42000/42000 [==============================] - 15s 368us/step - loss: 0.1869 - val_loss: 0.176
Epoch 3/50
42000/42000 [==============================] - 25s 590us/step - loss: 0.1702 - val_loss: 0.163
Epoch 4/50
42000/42000 [==============================] - 15s 350us/step - loss: 0.1602 - val_loss: 0.164
Epoch 5/50
42000/42000 [==============================] - 14s 330us/step - loss: 0.1529 - val_loss: 0.181
Epoch 6/50
42000/42000 [==============================] - 13s 315us/step - loss: 0.1467 - val_loss: 0.143
Epoch 7/50
42000/42000 [==============================] - 13s 311us/step - loss: 0.1426 - val_loss: 0.140
Epoch 8/50
42000/42000 [==============================] - 13s 309us/step - loss: 0.1393 - val_loss: 0.133
Epoch 9/50
42000/42000 [==============================] - 13s 305us/step - loss: 0.1366 - val_loss: 0.131
Epoch 10/50
42000/42000 [==============================] - 13s 311us/step - loss: 0.1341 - val_loss: 0.138
Epoch 11/50
42000/42000 [==============================] - 13s 312us/step - loss: 0.1321 - val_loss: 0.132
Epoch 12/50
42000/42000 [==============================] - 13s 307us/step - loss: 0.1303 - val_loss: 0.131
Epoch 13/50
42000/42000 [==============================] - 13s 317us/step - loss: 0.1281 - val_loss: 0.122
Epoch 14/50
42000/42000 [==============================] - 13s 315us/step - loss: 0.1269 - val_loss: 0.127
Epoch 15/50
42000/42000 [==============================] - 13s 312us/step - loss: 0.1256 - val_loss: 0.127
Epoch 16/50
42000/42000 [==============================] - 14s 330us/step - loss: 0.1243 - val_loss: 0.129
Epoch 17/50
42000/42000 [==============================] - 13s 319us/step - loss: 0.1230 - val_loss: 0.121
```

```
Epoch 18/50
42000/42000 [==============================] - 13s 308us/step - loss: 0.1219 - val_loss: 0.1316
Epoch 19/50
42000/42000 [==============================] - 13s 312us/step - loss: 0.1211 - val_loss: 0.1186
Epoch 20/50
42000/42000 [==============================] - 13s 312us/step - loss: 0.1200 - val_loss: 0.1186
Epoch 21/50
42000/42000 [==============================] - 13s 315us/step - loss: 0.1189 - val_loss: 0.1176
Epoch 22/50
42000/42000 [==============================] - 13s 310us/step - loss: 0.1185 - val_loss: 0.1147
Epoch 23/50
42000/42000 [==============================] - 13s 309us/step - loss: 0.1174 - val_loss: 0.1161
Epoch 24/50
42000/42000 [==============================] - 13s 310us/step - loss: 0.1166 - val_loss: 0.1243
Epoch 25/50
42000/42000 [==============================] - 13s 316us/step - loss: 0.1161 - val_loss: 0.1146
Epoch 26/50
42000/42000 [==============================] - 13s 315us/step - loss: 0.1156 - val_loss: 0.1204
Epoch 27/50
42000/42000 [==============================] - 13s 314us/step - loss: 0.1150 - val_loss: 0.1153
Epoch 28/50
42000/42000 [==============================] - 13s 311us/step - loss: 0.1147 - val_loss: 0.1164
Epoch 29/50
42000/42000 [==============================] - 13s 318us/step - loss: 0.1138 - val_loss: 0.1120
Epoch 30/50
42000/42000 [==============================] - 14s 328us/step - loss: 0.1136 - val_loss: 0.1163
Epoch 31/50
42000/42000 [==============================] - 13s 316us/step - loss: 0.1134 - val_loss: 0.1137
Epoch 32/50
42000/42000 [==============================] - 13s 311us/step - loss: 0.1126 - val_loss: 0.1083
Epoch 33/50
42000/42000 [==============================] - 13s 316us/step - loss: 0.1116 - val_loss: 0.1151
Epoch 34/50
42000/42000 [==============================] - 13s 317us/step - loss: 0.1116 - val_loss: 0.1078
Epoch 35/50
42000/42000 [==============================] - 13s 321us/step - loss: 0.1113 - val_loss: 0.1103
Epoch 36/50
42000/42000 [==============================] - 13s 317us/step - loss: 0.1110 - val_loss: 0.1085
Epoch 37/50
42000/42000 [==============================] - 13s 314us/step - loss: 0.1107 - val_loss: 0.1084
Epoch 38/50
42000/42000 [==============================] - 14s 323us/step - loss: 0.1102 - val_loss: 0.1101
Epoch 39/50
42000/42000 [==============================] - 13s 318us/step - loss: 0.1102 - val_loss: 0.1069
Epoch 40/50
42000/42000 [==============================] - 14s 322us/step - loss: 0.1096 - val_loss: 0.1090
Epoch 41/50
42000/42000 [==============================] - 13s 313us/step - loss: 0.1093 - val_loss: 0.1053
```

```
Epoch 42/50
42000/42000 [==============================] - 13s 315us/step - loss: 0.1091 - val_loss: 0.107
Epoch 43/50
42000/42000 [==============================] - 13s 314us/step - loss: 0.1086 - val_loss: 0.107
Epoch 44/50
42000/42000 [==============================] - 13s 321us/step - loss: 0.1087 - val_loss: 0.113
Epoch 45/50
42000/42000 [==============================] - 13s 321us/step - loss: 0.1083 - val_loss: 0.110
Epoch 46/50
42000/42000 [==============================] - 13s 313us/step - loss: 0.1081 - val_loss: 0.107
Epoch 47/50
42000/42000 [==============================] - 14s 323us/step - loss: 0.1079 - val_loss: 0.106
Epoch 48/50
42000/42000 [==============================] - 13s 315us/step - loss: 0.1074 - val_loss: 0.104
Epoch 49/50
42000/42000 [==============================] - 13s 317us/step - loss: 0.1074 - val_loss: 0.108
Epoch 50/50
42000/42000 [==============================] - 13s 321us/step - loss: 0.1072 - val_loss: 0.107
```

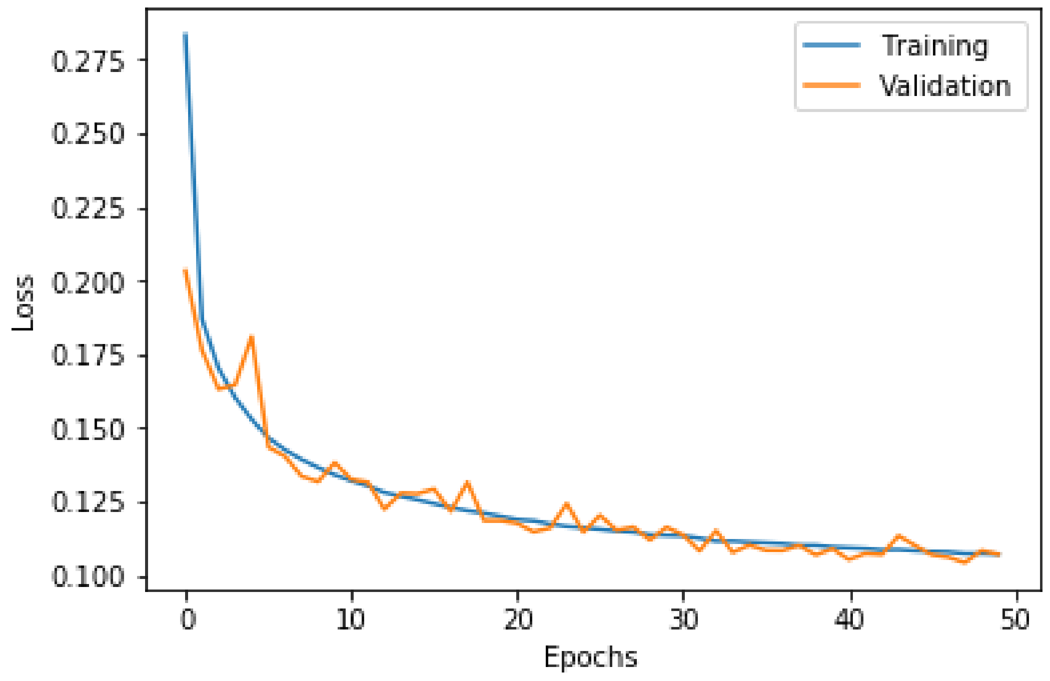
This allows us to monitor training in the TensorBoard web interface (by navighating to http://0.0.0.0:6006

Tensorboard is pretty cool, you can also have a look at the graphs to see a representation of you model. Using tensorboard is quite useful to understand if your model is working an visualize a few features of your model right in the beginning of your training. This way you know immediately if it is working or not and don't need to wait until the training is done. More information can be found here:

https://www.tensorflow.org/programmers_guide/summaries_and_tensorboard

Let's plot the loss here as well:

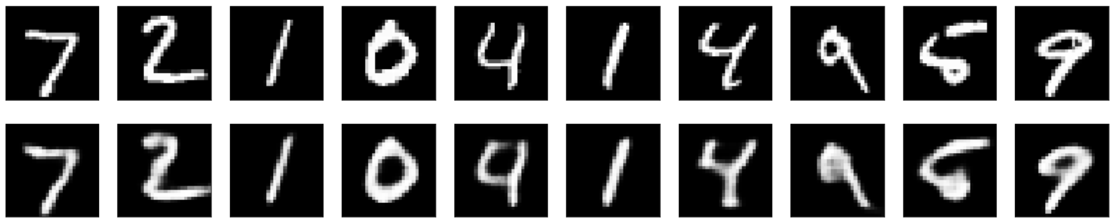
```
In [17]: plot_history(history)
```



The model converges to a loss, which is significantly better than our previous models (this is in large part due to the higher entropic capacity of the encoded representation, 128 dimensions vs. 32 previously). Let's take a look at the reconstructed digits:

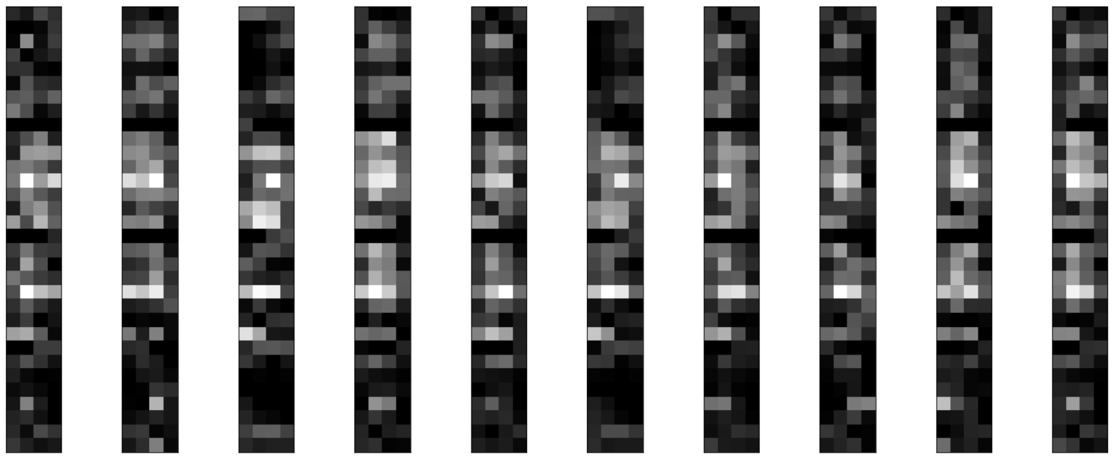
1.11 Testing the Convolutional Autoencoder

```
In [18]: decoded_imgs = conv_autoencoder.predict(x_test)

         n = 10
         plt.figure(figsize=(20, 4))
         for i in range(n):
             # display original
             ax = plt.subplot(2, n, i+1)
             plt.imshow(x_test[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)

             # display reconstruction
             ax = plt.subplot(2, n, i + n + 1)
             plt.imshow(decoded_imgs[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
         plt.show()
```



We can also have a look at the 128-dimensional encoded representations. These representations are 8x4x4, so we reshape them to 4x32 in order to be able to display them as grayscale images.

```
In [19]: conv_encoder = Model(input_img, encoded)
         encoded_imgs = conv_encoder.predict(x_test)

         n = 10
         plt.figure(figsize=(20, 8))
         for i in range(n):
             ax = plt.subplot(1, n, i+1)
             plt.imshow(encoded_imgs[i].reshape(4, 4 * 8).T)
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
         plt.show()
```



1.12 Application to Image Denoising

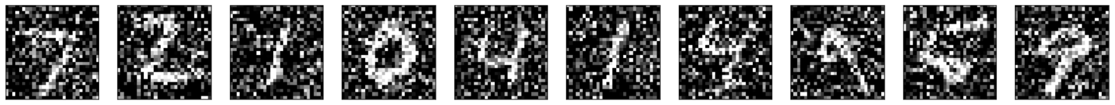
Let's put our convolutional autoencoder to work on an image denoising problem. It's simple: we will train the autoencoder to map noisy digits images to clean digits images.

Here's how we will generate synthetic noisy digits: we just apply a gaussian noise matrix and clip the images between 0 and 1.

```
In [20]: noise_factor = 0.5
         x_train_noisy = x_train + noise_factor * np.random.normal(loc=0.0, scale=1.0, size=x_
         x_test_noisy = x_test + noise_factor * np.random.normal(loc=0.0, scale=1.0, size=x_tes

         x_train_noisy = np.clip(x_train_noisy, 0., 1.)
         x_test_noisy = np.clip(x_test_noisy, 0., 1.)
```

Here's how the noisy digits look like:

```
In [21]: n = 10
         plt.figure(figsize=(20, 2))
         for i in range(n):
             ax = plt.subplot(1, n, i+1)
             plt.imshow(x_test_noisy[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
         plt.show()
```



If you squint you can still recognize them, but barely.

1.13 Question: Can our autoencoder learn to recover the original digits?

Compared to the previous convolutional autoencoder,we'll use a slightly different model with more filters per layer in order to improve the quality of the reconstructed:

```
In [22]: input_img = Input(shape=(28, 28, 1))  # adapt this if using `channels_first` image da

         x = Conv2D(32, (3, 3), activation='relu', padding='same')(input_img)
         x = MaxPooling2D((2, 2), padding='same')(x)
         x = Conv2D(32, (3, 3), activation='relu', padding='same')(x)
         encoded = MaxPooling2D((2, 2), padding='same')(x)

         # at this point the representation is (7, 7, 32)

         x = Conv2D(32, (3, 3), activation='relu', padding='same')(encoded)
         x = UpSampling2D((2, 2))(x)
         x = Conv2D(32, (3, 3), activation='relu', padding='same')(x)
         x = UpSampling2D((2, 2))(x)
         decoded = Conv2D(1, (3, 3), activation='sigmoid', padding='same')(x)
```

```
autoencoder = Model(input_img, decoded)
autoencoder.compile(optimizer='adadelta', loss='binary_crossentropy')
autoencoder.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_4 (InputLayer)         (None, 28, 28, 1)         0
_________________________________________________________________
conv2d_8 (Conv2D)            (None, 28, 28, 32)        320
_________________________________________________________________
max_pooling2d_4 (MaxPooling2 (None, 14, 14, 32)        0
_________________________________________________________________
conv2d_9 (Conv2D)            (None, 14, 14, 32)        9248
_________________________________________________________________
max_pooling2d_5 (MaxPooling2 (None, 7, 7, 32)          0
_________________________________________________________________
conv2d_10 (Conv2D)           (None, 7, 7, 32)          9248
_________________________________________________________________
up_sampling2d_4 (UpSampling2 (None, 14, 14, 32)        0
_________________________________________________________________
conv2d_11 (Conv2D)           (None, 14, 14, 32)        9248
_________________________________________________________________
up_sampling2d_5 (UpSampling2 (None, 28, 28, 32)        0
_________________________________________________________________
conv2d_12 (Conv2D)           (None, 28, 28, 1)         289
=================================================================
Total params: 28,353
Trainable params: 28,353
Non-trainable params: 0
_________________________________________________________________
```

Let's train the AutoEncoder

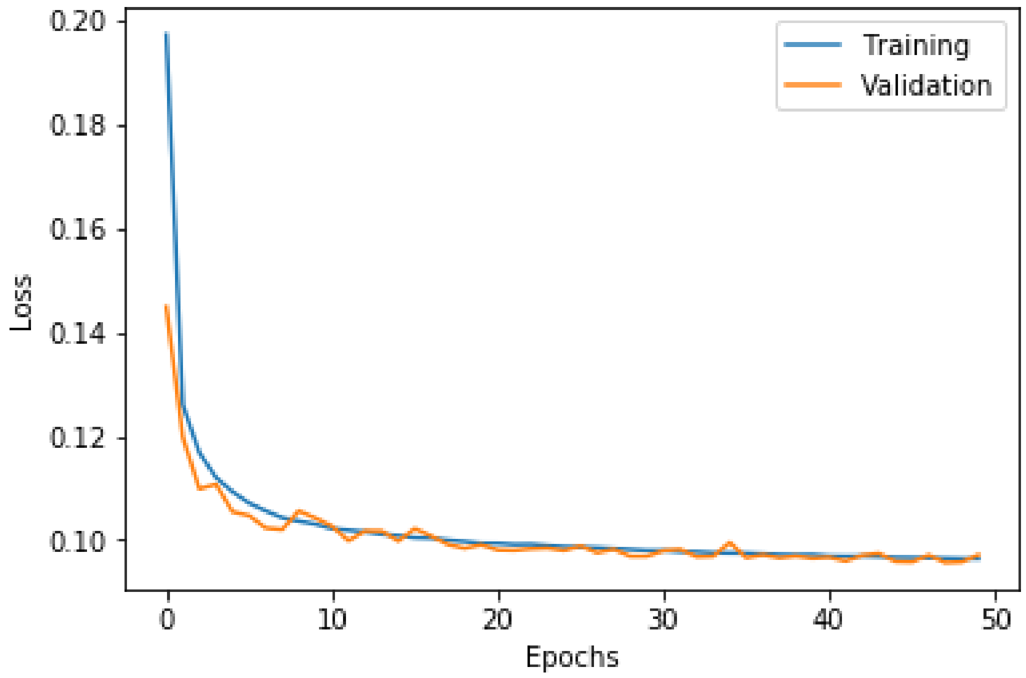
```
In [23]: history = autoencoder.fit(x_train_noisy, x_train,
                         epochs=50,
                         batch_size=128,
                         shuffle=True,
                         validation_data=(x_test_noisy, x_test),
                         callbacks=[TensorBoard(log_dir='/tmp/autoencoder_denoise',
                                                histogram_freq=0, write_graph=False)])
```

```
Train on 42000 samples, validate on 10000 samples
Epoch 1/50
42000/42000 [==============================] - 26s 619us/step - loss: 0.1973 - val_loss: 0.1449
Epoch 2/50
42000/42000 [==============================] - 26s 616us/step - loss: 0.1260 - val_loss: 0.1198
```

```
Epoch 3/50
42000/42000 [==============================] - 26s 625us/step - loss: 0.1168 - val_loss: 0.1099
Epoch 4/50
42000/42000 [==============================] - 26s 620us/step - loss: 0.1121 - val_loss: 0.1108
Epoch 5/50
42000/42000 [==============================] - 26s 629us/step - loss: 0.1093 - val_loss: 0.1054
Epoch 6/50
42000/42000 [==============================] - 26s 617us/step - loss: 0.1071 - val_loss: 0.1047
Epoch 7/50
42000/42000 [==============================] - 26s 611us/step - loss: 0.1056 - val_loss: 0.1023
Epoch 8/50
42000/42000 [==============================] - 26s 609us/step - loss: 0.1043 - val_loss: 0.1020
Epoch 9/50
42000/42000 [==============================] - 26s 608us/step - loss: 0.1036 - val_loss: 0.1056
Epoch 10/50
42000/42000 [==============================] - 26s 608us/step - loss: 0.1031 - val_loss: 0.1042
Epoch 11/50
42000/42000 [==============================] - 26s 611us/step - loss: 0.1022 - val_loss: 0.1027
Epoch 12/50
42000/42000 [==============================] - 26s 614us/step - loss: 0.1019 - val_loss: 0.0999
Epoch 13/50
42000/42000 [==============================] - 26s 608us/step - loss: 0.1016 - val_loss: 0.1017
Epoch 14/50
42000/42000 [==============================] - 27s 633us/step - loss: 0.1011 - val_loss: 0.1017
Epoch 15/50
42000/42000 [==============================] - 28s 659us/step - loss: 0.1008 - val_loss: 0.0998
Epoch 16/50
42000/42000 [==============================] - 28s 667us/step - loss: 0.1005 - val_loss: 0.1022
Epoch 17/50
42000/42000 [==============================] - 49s 1ms/step - loss: 0.1004 - val_loss: 0.1007
Epoch 18/50
42000/42000 [==============================] - 26s 615us/step - loss: 0.1000 - val_loss: 0.0991
Epoch 19/50
42000/42000 [==============================] - 25s 604us/step - loss: 0.0997 - val_loss: 0.0984
Epoch 20/50
42000/42000 [==============================] - 26s 611us/step - loss: 0.0994 - val_loss: 0.0991
Epoch 21/50
42000/42000 [==============================] - 26s 618us/step - loss: 0.0994 - val_loss: 0.0981
Epoch 22/50
42000/42000 [==============================] - 26s 608us/step - loss: 0.0991 - val_loss: 0.0980
Epoch 23/50
42000/42000 [==============================] - 26s 618us/step - loss: 0.0992 - val_loss: 0.0983
Epoch 24/50
42000/42000 [==============================] - 32s 764us/step - loss: 0.0989 - val_loss: 0.0985
Epoch 25/50
42000/42000 [==============================] - 31s 735us/step - loss: 0.0987 - val_loss: 0.0980
Epoch 26/50
42000/42000 [==============================] - 30s 703us/step - loss: 0.0986 - val_loss: 0.0988
```

```
Epoch 27/50
42000/42000 [==============================] - 31s 732us/step - loss: 0.0985 - val_loss: 0.0976
Epoch 28/50
42000/42000 [==============================] - 35s 834us/step - loss: 0.0984 - val_loss: 0.0982
Epoch 29/50
42000/42000 [==============================] - 31s 735us/step - loss: 0.0982 - val_loss: 0.0969
Epoch 30/50
42000/42000 [==============================] - 31s 738us/step - loss: 0.0980 - val_loss: 0.0969
Epoch 31/50
42000/42000 [==============================] - 29s 694us/step - loss: 0.0980 - val_loss: 0.0980
Epoch 32/50
42000/42000 [==============================] - 30s 703us/step - loss: 0.0977 - val_loss: 0.0982
Epoch 33/50
42000/42000 [==============================] - 29s 681us/step - loss: 0.0977 - val_loss: 0.0968
Epoch 34/50
42000/42000 [==============================] - 28s 673us/step - loss: 0.0975 - val_loss: 0.0969
Epoch 35/50
42000/42000 [==============================] - 34s 815us/step - loss: 0.0975 - val_loss: 0.0995
Epoch 36/50
42000/42000 [==============================] - 37s 885us/step - loss: 0.0975 - val_loss: 0.0966
Epoch 37/50
42000/42000 [==============================] - 33s 785us/step - loss: 0.0973 - val_loss: 0.0972
Epoch 38/50
42000/42000 [==============================] - 29s 700us/step - loss: 0.0972 - val_loss: 0.0966
Epoch 39/50
42000/42000 [==============================] - 29s 690us/step - loss: 0.0971 - val_loss: 0.0969
Epoch 40/50
42000/42000 [==============================] - 27s 632us/step - loss: 0.0972 - val_loss: 0.0965
Epoch 41/50
42000/42000 [==============================] - 27s 646us/step - loss: 0.0969 - val_loss: 0.0967
Epoch 42/50
42000/42000 [==============================] - 26s 629us/step - loss: 0.0969 - val_loss: 0.0960
Epoch 43/50
42000/42000 [==============================] - 31s 731us/step - loss: 0.0969 - val_loss: 0.0970
Epoch 44/50
42000/42000 [==============================] - 28s 664us/step - loss: 0.0968 - val_loss: 0.0974
Epoch 45/50
42000/42000 [==============================] - 25s 602us/step - loss: 0.0967 - val_loss: 0.0959
Epoch 46/50
42000/42000 [==============================] - 25s 605us/step - loss: 0.0966 - val_loss: 0.0958
Epoch 47/50
42000/42000 [==============================] - 25s 598us/step - loss: 0.0966 - val_loss: 0.0971
Epoch 48/50
42000/42000 [==============================] - 26s 612us/step - loss: 0.0965 - val_loss: 0.0957
Epoch 49/50
42000/42000 [==============================] - 25s 602us/step - loss: 0.0964 - val_loss: 0.0958
Epoch 50/50
42000/42000 [==============================] - 25s 592us/step - loss: 0.0964 - val_loss: 0.0971
```

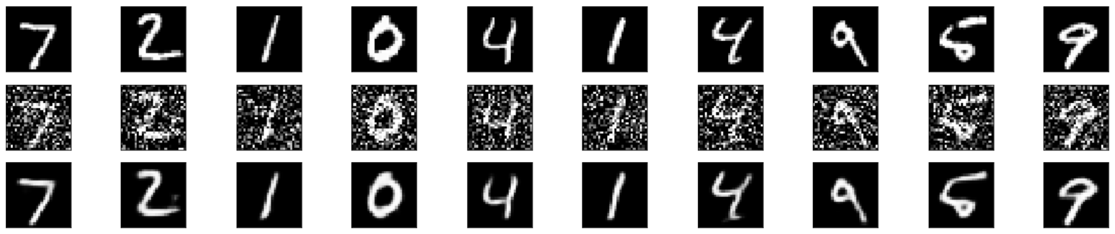
```
In [24]: plot_history(history)
```



Now let's take a look at the results. Top, the noisy digits fed to the network, and bottom, the digits are reconstructed by the network.

```
In [25]: decoded_imgs = autoencoder.predict(x_test_noisy)

         n = 10
         plt.figure(figsize=(20, 4))
         for i in range(n):
             # display original
             ax = plt.subplot(3, n, i+1)
             plt.imshow(x_test[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
             ax = plt.subplot(3, n, i + n + 1)
             plt.imshow(x_test_noisy[i].reshape(28, 28))
             plt.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
             # display reconstruction
             ax = plt.subplot(3, n, i + 2*n + 1)
             plt.imshow(decoded_imgs[i].reshape(28, 28))
```

```
        plt.gray()
        ax.get_xaxis().set_visible(False)
        ax.get_yaxis().set_visible(False)
    plt.show()
```



It seems to work pretty well. If you scale this process to a bigger convnet, you can start building document denoising or audio denoising models.

2 Bonus: Variational AutoEncoder

Reference *https://blog.keras.io/building-autoencoders-in-keras.html* and https://github.com/keras-team/keras/blob/master/examples/variational_autoencoder.py

Variational autoencoders are a slightly more modern and interesting take on autoencoding.

2.0.1 What is a variational autoencoder ?

It's a type of autoencoder with added constraints on the encoded representations being learned.

More precisely, it is an autoencoder that learns a latent variable model for its input data.

So instead of letting your neural network learn an arbitrary function, you are learning the parameters of a probability distribution modeling your data.

If you sample points from this distribution, you can generate new input data samples: a **VAE** is a **"generative model"**.

2.0.2 How does a variational autoencoder work?

First, an encoder network turns the input samples x into two parameters in a latent space, which we will note z_μ and z_{log_σ}.

Then, we randomly sample similar points z from the *latent normal distribution* that is assumed to generate the data, via $z = z_\mu + \exp(z_{log_\sigma}) * \epsilon$, where ϵ is a random normal tensor.

Finally, a decoder network maps these latent space points back to the original input data.

The parameters of the model are trained via two loss functions:

- a **reconstruction loss** forcing the decoded samples to match the initial inputs (just like in our previous autoencoders);
- and the **KL divergence** between the learned latent distribution and the prior distribution, acting as a regularization term.

You could actually get rid of this latter term entirely, although it does help in learning well-formed latent spaces and reducing overfitting to the training data.

2.1 Load MNIST

```
In [26]: from keras.layers import Lambda, Input, Dense
         from keras.models import Model
         from keras.datasets import mnist
         from keras.losses import binary_crossentropy
         from keras.utils import plot_model
         from keras import backend as K

         import numpy as np
         import matplotlib.pyplot as plt
         import argparse
         import os
```

```
In [27]: (x_train, y_train), (x_test, y_test) = mnist.load_data()

         image_size = x_train.shape[1]
         original_dim = image_size * image_size
         x_train = np.reshape(x_train, [-1, original_dim])
         x_test = np.reshape(x_test, [-1, original_dim])
         x_train = x_train.astype('float32') / 255
         x_test = x_test.astype('float32') / 255
```

2.2 Encoder Network

First, here's our encoder network, mapping inputs to our latent distribution parameters:

```
In [28]: # network parameters
         input_shape = (original_dim, )
         intermediate_dim = 512
         batch_size = 128
         latent_dim = 2
         epochs = 50
```

```
In [29]: # VAE model = encoder + decoder
         # build encoder model
         inputs = Input(shape=input_shape, name='encoder_input')
         x = Dense(intermediate_dim, activation='relu')(inputs)
         z_mean = Dense(latent_dim, name='z_mean')(x)
         z_log_var = Dense(latent_dim, name='z_log_var')(x)
```

We can use these parameters to sample new similar points from the latent space:

```
In [30]: # reparameterization trick
         # instead of sampling from Q(z|X), sample eps = N(0,I)
         # z = z_mean + sqrt(var)*eps
         def sampling(args):
             """Reparameterization trick by sampling fr an isotropic unit Gaussian.
             # Arguments:
```

```
        args (tensor): mean and log of variance of Q(z|X)
    # Returns:
        z (tensor): sampled latent vector
    """

    z_mean, z_log_var = args
    batch = K.shape(z_mean)[0]
    dim = K.int_shape(z_mean)[1]
    # by default, random_normal has mean=0 and std=1.0
    epsilon = K.random_normal(shape=(batch, dim))
    return z_mean + K.exp(0.5 * z_log_var) * epsilon

# use reparameterization trick to push the sampling out as input
# note that "output_shape" isn't necessary with the TensorFlow backend
z = Lambda(sampling, output_shape=(latent_dim,), name='z')([z_mean, z_log_var])

# instantiate encoder model
encoder = Model(inputs, [z_mean, z_log_var, z], name='encoder')
encoder.summary()
```

```
______________________________________________________________________________________________
Layer (type)                    Output Shape          Param #     Connected to
==============================================================================================
encoder_input (InputLayer)      (None, 784)           0
______________________________________________________________________________________________
dense_3 (Dense)                 (None, 512)           401920      encoder_input[0][0]
______________________________________________________________________________________________
z_mean (Dense)                  (None, 2)             1026        dense_3[0][0]
______________________________________________________________________________________________
z_log_var (Dense)               (None, 2)             1026        dense_3[0][0]
______________________________________________________________________________________________
z (Lambda)                      (None, 2)             0           z_mean[0][0]
                                                                  z_log_var[0][0]
==============================================================================================
Total params: 403,972
Trainable params: 403,972
Non-trainable params: 0
______________________________________________________________________________________________
```

2.3 Decoder Network

Finally, we can map these sampled latent points back to reconstructed inputs:

```
In [31]: # build decoder model
         latent_inputs = Input(shape=(latent_dim,), name='z_sampling')
         x = Dense(intermediate_dim, activation='relu')(latent_inputs)
         outputs = Dense(original_dim, activation='sigmoid')(x)
```

```
# instantiate decoder model
decoder = Model(latent_inputs, outputs, name='decoder')
decoder.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
z_sampling (InputLayer)      (None, 2)                 0
_________________________________________________________________
dense_4 (Dense)              (None, 512)               1536
_________________________________________________________________
dense_5 (Dense)              (None, 784)               402192
=================================================================
Total params: 403,728
Trainable params: 403,728
Non-trainable params: 0
_________________________________________________________________
```

2.4 Variational AutoEncoder

```
In [32]: # instantiate VAE model
         outputs = decoder(encoder(inputs)[2])
         vae = Model(inputs, outputs, name='vae_mlp')
```

We train the model using the end-to-end model, with a custom loss function: the sum of a reconstruction term, and the KL divergence regularization term.

```
In [33]: reconstruction_loss = binary_crossentropy(inputs,outputs)
         reconstruction_loss *= original_dim
         kl_loss = 1 + z_log_var - K.square(z_mean) - K.exp(z_log_var)
         kl_loss = K.sum(kl_loss, axis=-1)
         kl_loss *= -0.5
         vae_loss = K.mean(reconstruction_loss + kl_loss)
         vae.add_loss(vae_loss)
         vae.compile(optimizer='adam')
         vae.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
encoder_input (InputLayer)   (None, 784)               0
_________________________________________________________________
encoder (Model)              [(None, 2), (None, 2), (N 403972
_________________________________________________________________
decoder (Model)              (None, 784)               403728
=================================================================
Total params: 807,700
```

```
Trainable params: 807,700
Non-trainable params: 0
_________________________________________________________________
```

Traing on MNIST Digits

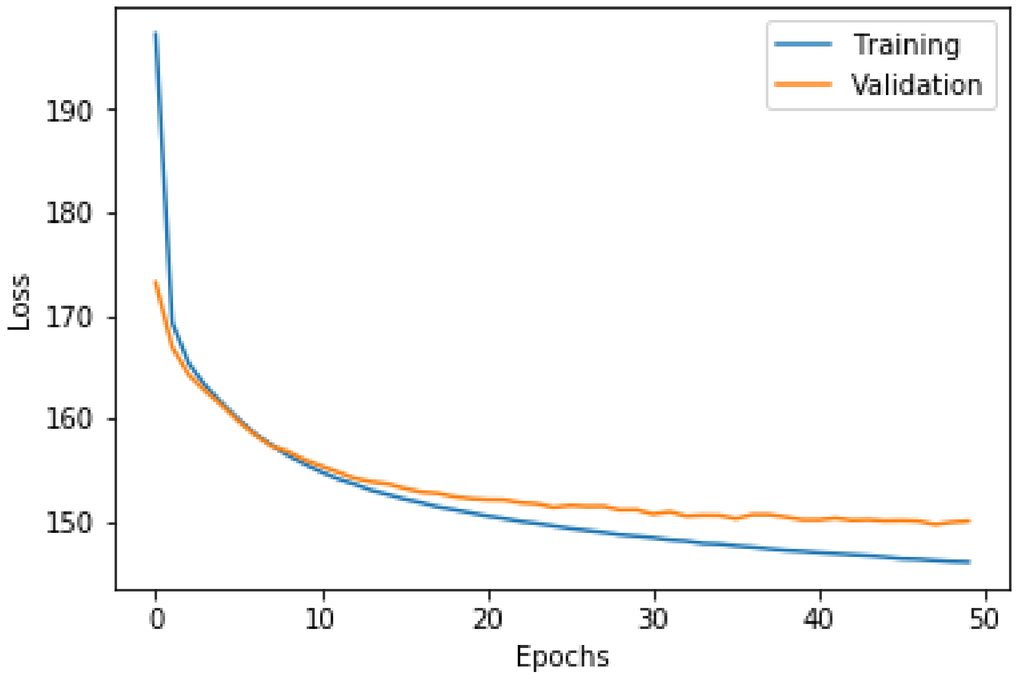
```
In [34]: history = vae.fit(x_train,
                          epochs=epochs,
                          batch_size=batch_size,
                          validation_data=(x_test, None))
```

```
Train on 60000 samples, validate on 10000 samples
Epoch 1/50
60000/60000 [==============================] - 10s 170us/step - loss: 197.3034 - val_loss: 173
Epoch 2/50
60000/60000 [==============================] - 9s 155us/step - loss: 169.2847 - val_loss: 166.
Epoch 3/50
60000/60000 [==============================] - 9s 153us/step - loss: 165.3224 - val_loss: 164.
Epoch 4/50
60000/60000 [==============================] - 9s 153us/step - loss: 163.1574 - val_loss: 162.
Epoch 5/50
60000/60000 [==============================] - 9s 154us/step - loss: 161.4838 - val_loss: 161.
Epoch 6/50
60000/60000 [==============================] - 9s 151us/step - loss: 159.9067 - val_loss: 159.
Epoch 7/50
60000/60000 [==============================] - 9s 150us/step - loss: 158.5133 - val_loss: 158.
Epoch 8/50
60000/60000 [==============================] - 9s 152us/step - loss: 157.4152 - val_loss: 157.
Epoch 9/50
60000/60000 [==============================] - 9s 155us/step - loss: 156.3856 - val_loss: 156.
Epoch 10/50
60000/60000 [==============================] - 9s 156us/step - loss: 155.5595 - val_loss: 155.
Epoch 11/50
60000/60000 [==============================] - 9s 151us/step - loss: 154.7978 - val_loss: 155.
Epoch 12/50
60000/60000 [==============================] - 9s 157us/step - loss: 154.1222 - val_loss: 154.
Epoch 13/50
60000/60000 [==============================] - 9s 150us/step - loss: 153.6171 - val_loss: 154.
Epoch 14/50
60000/60000 [==============================] - 9s 154us/step - loss: 153.0192 - val_loss: 153.
Epoch 15/50
60000/60000 [==============================] - 9s 153us/step - loss: 152.5997 - val_loss: 153.
Epoch 16/50
60000/60000 [==============================] - 9s 152us/step - loss: 152.1589 - val_loss: 153.
Epoch 17/50
60000/60000 [==============================] - 9s 152us/step - loss: 151.8143 - val_loss: 152.
Epoch 18/50
```

```
60000/60000 [==============================] - 9s 153us/step - loss: 151.4056 - val_loss: 152.7
Epoch 19/50
60000/60000 [==============================] - 9s 151us/step - loss: 151.1285 - val_loss: 152.3
Epoch 20/50
60000/60000 [==============================] - 9s 152us/step - loss: 150.8258 - val_loss: 152.2
Epoch 21/50
60000/60000 [==============================] - 9s 152us/step - loss: 150.5085 - val_loss: 152.1
Epoch 22/50
60000/60000 [==============================] - 11s 191us/step - loss: 150.2733 - val_loss: 152
Epoch 23/50
60000/60000 [==============================] - 11s 180us/step - loss: 149.9956 - val_loss: 151
Epoch 24/50
60000/60000 [==============================] - 11s 184us/step - loss: 149.7923 - val_loss: 151
Epoch 25/50
60000/60000 [==============================] - 9s 154us/step - loss: 149.5510 - val_loss: 151.3
Epoch 26/50
60000/60000 [==============================] - 9s 152us/step - loss: 149.3002 - val_loss: 151.5
Epoch 27/50
60000/60000 [==============================] - 9s 153us/step - loss: 149.1172 - val_loss: 151.4
Epoch 28/50
60000/60000 [==============================] - 10s 158us/step - loss: 148.9169 - val_loss: 151
Epoch 29/50
60000/60000 [==============================] - 9s 154us/step - loss: 148.6982 - val_loss: 151.1
Epoch 30/50
60000/60000 [==============================] - 9s 152us/step - loss: 148.5547 - val_loss: 151.1
Epoch 31/50
60000/60000 [==============================] - 10s 160us/step - loss: 148.4056 - val_loss: 150
Epoch 32/50
60000/60000 [==============================] - 9s 153us/step - loss: 148.1931 - val_loss: 150.9
Epoch 33/50
60000/60000 [==============================] - 9s 150us/step - loss: 148.0830 - val_loss: 150.5
Epoch 34/50
60000/60000 [==============================] - 10s 169us/step - loss: 147.8503 - val_loss: 150
Epoch 35/50
60000/60000 [==============================] - 9s 154us/step - loss: 147.7861 - val_loss: 150.5
Epoch 36/50
60000/60000 [==============================] - 9s 154us/step - loss: 147.6073 - val_loss: 150.2
Epoch 37/50
60000/60000 [==============================] - 9s 155us/step - loss: 147.4830 - val_loss: 150.6
Epoch 38/50
60000/60000 [==============================] - 9s 155us/step - loss: 147.3167 - val_loss: 150.6
Epoch 39/50
60000/60000 [==============================] - 9s 153us/step - loss: 147.1767 - val_loss: 150.4
Epoch 40/50
60000/60000 [==============================] - 9s 151us/step - loss: 147.0808 - val_loss: 150.1
Epoch 41/50
60000/60000 [==============================] - 9s 153us/step - loss: 146.9549 - val_loss: 150.1
Epoch 42/50
```

```
60000/60000 [==============================] - 9s 153us/step - loss: 146.8595 - val_loss: 150.3
Epoch 43/50
60000/60000 [==============================] - 9s 153us/step - loss: 146.7603 - val_loss: 150.1
Epoch 44/50
60000/60000 [==============================] - 9s 152us/step - loss: 146.6619 - val_loss: 150.1
Epoch 45/50
60000/60000 [==============================] - 9s 152us/step - loss: 146.5239 - val_loss: 150.0
Epoch 46/50
60000/60000 [==============================] - 9s 150us/step - loss: 146.3688 - val_loss: 150.0
Epoch 47/50
60000/60000 [==============================] - 9s 152us/step - loss: 146.3166 - val_loss: 150.0
Epoch 48/50
60000/60000 [==============================] - 9s 152us/step - loss: 146.1968 - val_loss: 149.7
Epoch 49/50
60000/60000 [==============================] - 9s 156us/step - loss: 146.0999 - val_loss: 149.9
Epoch 50/50
60000/60000 [==============================] - 9s 158us/step - loss: 146.0464 - val_loss: 150.0
```

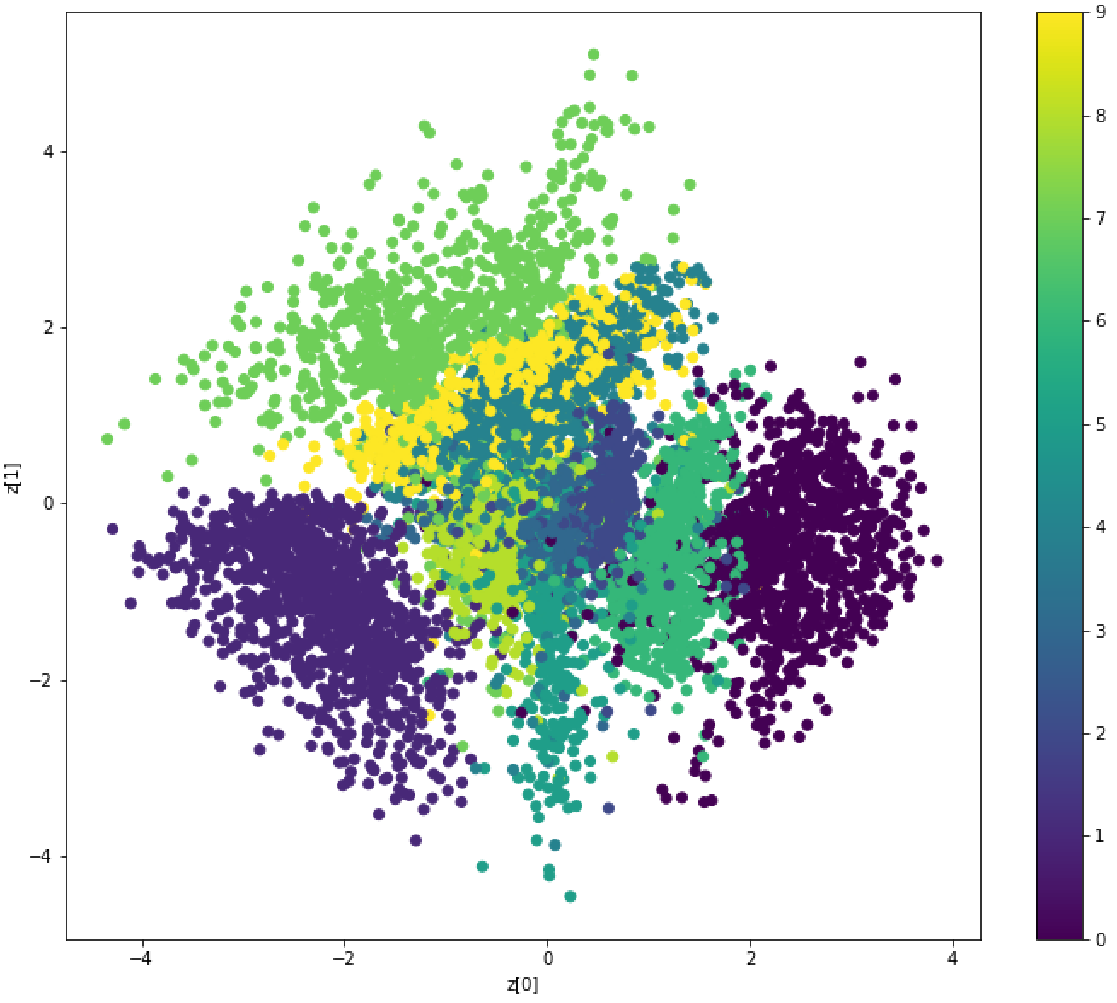
```
In [35]: plot_history(history)
```



Because our latent space is two-dimensional, there are a few cool visualizations that can be done at this point.

One is to look at the neighborhoods of different classes on the latent 2D plane:

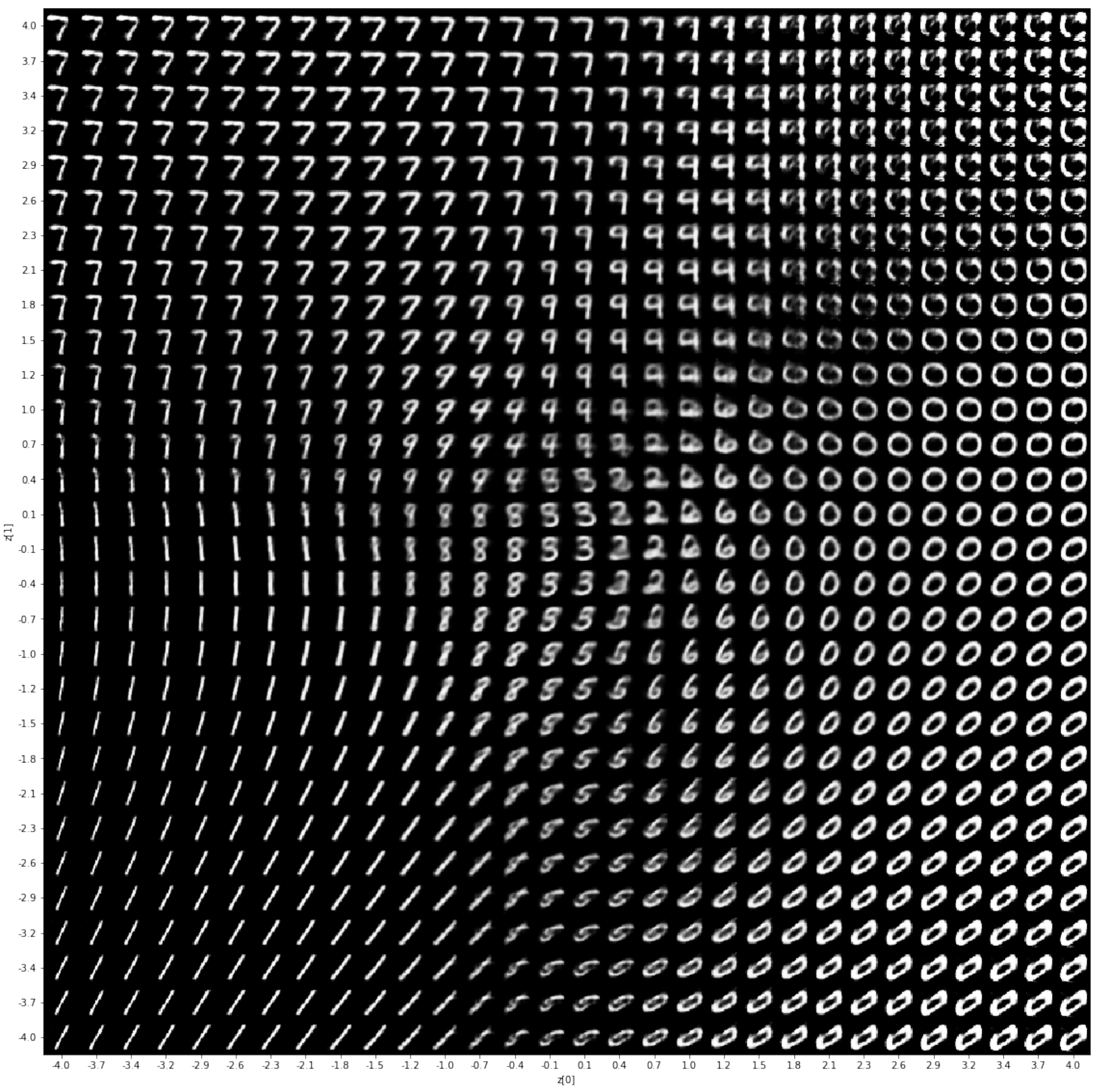
```
In [36]: x_test_encoded = encoder.predict(x_test, batch_size=batch_size)
In [37]: # display a 2D plot of the digit classes in the latent space
         z_mean, _, _ = encoder.predict(x_test,
                                        batch_size=batch_size)
         plt.figure(figsize=(12, 10))
         plt.scatter(z_mean[:, 0], z_mean[:, 1], c=y_test, cmap='viridis')
         plt.colorbar()
         plt.xlabel("z[0]")
         plt.ylabel("z[1]")
         plt.show()
```



Each of these colored clusters is a type of digit. Close clusters are digits that are structurally similar (i.e. digits that share information in the latent space).

Because the VAE is a generative model, we can also use it to generate new digits! Here we will scan the latent plane, sampling latent points at regular intervals, and generating the corresponding digit for each of these points. This gives us a visualization of the latent manifold that "generates" the MNIST digits.

```
In [38]: # display a 30x30 2D manifold of digits
         n = 30
         digit_size = 28
         figure = np.zeros((digit_size * n, digit_size * n))
         # linearly spaced coordinates corresponding to the 2D plot
         # of digit classes in the latent space
         grid_x = np.linspace(-4, 4, n)
         grid_y = np.linspace(-4, 4, n)[::-1]

         for i, yi in enumerate(grid_y):
             for j, xi in enumerate(grid_x):
                 z_sample = np.array([[xi, yi]])
                 x_decoded = decoder.predict(z_sample)
                 digit = x_decoded[0].reshape(digit_size, digit_size)
                 figure[i * digit_size: (i + 1) * digit_size,
                        j * digit_size: (j + 1) * digit_size] = digit

         plt.figure(figsize=(20, 20))
         start_range = digit_size // 2
         end_range = n * digit_size + start_range + 1
         pixel_range = np.arange(start_range, end_range, digit_size)
         sample_range_x = np.round(grid_x, 1)
         sample_range_y = np.round(grid_y, 1)
         plt.xticks(pixel_range, sample_range_x)
         plt.yticks(pixel_range, sample_range_y)
         plt.xlabel("z[0]")
         plt.ylabel("z[1]")
         plt.imshow(figure, cmap='Greys_r')
         plt.show()
```


4.0
3.7
3.4
3.2
2.9
2.6
2.3
2.1
1.8
1.5
1.2
1.0
0.7
0.4
0.1
-0.1
-0.4
-0.7
-1.0
-1.2
-1.5
-1.8
-2.1
-2.3
-2.6
-2.9
-3.2
-3.4
-3.7
-4.0
z[1]
-4.0
-3.7
-3.4
-3.2
-2.9
-2.6
-2.3
-2.1
-1.8
-1.5
-1.2
-1.0
-0.7
-0.4
-0.1
0.1
0.4
0.7
1.0
1.2
1.5
1.8
2.1
2.3
2.6
2.9
3.2
3.4
3.7
4.0
z[0]