ex5_sol

May 15, 2019

1 Exercise 5 - Convolutional Neural Networks and the MNIST dataset

This exercise is based on https://github.com/leriomaggio/deep-learning-keras-tensorflow

We want to solve the same multinomial classification problem as in last weeks exercise 4 using the MNIST dataset, but this time we want to use a convolutional neural network for it.

Before we start, we define a few useful functions, which we used in exercise 4:

```
In [1]: ##################################################################################

        import matplotlib.pyplot as plt
        %matplotlib inline

        def plot_history(network_history):
            plt.figure()
            plt.xlabel('Epochs')
            plt.ylabel('Loss')
            plt.plot(network_history.history['loss'])
            plt.plot(network_history.history['val_loss'])
            plt.legend(['Training', 'Validation'])

            plt.figure()
            plt.xlabel('Epochs')
            plt.ylabel('Accuracy')
            plt.plot(network_history.history['acc'])
            plt.plot(network_history.history['val_acc'])
            plt.legend(['Training', 'Validation'], loc='lower right')
            plt.show()

        ##################################################################################

        import itertools
        def plot_confusion_matrix(cm, classes,
                                  normalize=False,
                                  title='Confusion matrix',
                                  cmap=plt.cm.Blues):
            """
            This function prints and plots the confusion matrix.
```

```
    Normalization can be applied by setting `normalize=True`.
    """
    plt.imshow(cm, interpolation='nearest', cmap=cmap)
    plt.title(title)
    plt.colorbar()
    tick_marks = np.arange(len(classes))
    plt.xticks(tick_marks, classes, rotation=45)
    plt.yticks(tick_marks, classes)

    if normalize:
        cm = cm.astype('float') / cm.sum(axis=1)[:, np.newaxis]

    thresh = cm.max() / 2.
    for i, j in itertools.product(range(cm.shape[0]), range(cm.shape[1])):
        plt.text(j, i, cm[i, j],
                 horizontalalignment="center",
                 color="white" if cm[i, j] > thresh else "black")

    plt.tight_layout()
    plt.ylabel('True label')
    plt.xlabel('Predicted label')

########################################################################################
import matplotlib.cm as cm
def display_errors(errors_index,img_errors,pred_errors, obs_errors):
    """ This function shows 6 images with their predicted and real labels"""
    n = 0
    nrows = 2
    ncols = 3
    fig, ax = plt.subplots(nrows,ncols,sharex=True,sharey=True)
    for row in range(nrows):
        for col in range(ncols):
            error = errors_index[n]
            ax[row,col].imshow((img_errors[error]).reshape((28,28)), cmap=cm.Greys, in
            ax[row,col].set_title("Predicted label :{}\nTrue label :{}".format(pred_er
            n += 1

########################################################################################
```

1.1 Data Preparation

1.1.1 Very Important:

When dealing with images & convolutions, you need to handle the `image_data_format` properly, i.e. is the channel given first or last. The channel axis is an additional dimension of the input data used to access different views of the date, e.g. red/green/blue of a color image, left or right of a stereo sound file)

```
In [2]: from keras import backend as K
        K.image_data_format()
```

```
Using TensorFlow backend.
```

```
Out[2]: 'channels_last'
```

```
In [3]: img_rows, img_cols = 28, 28

        if K.image_data_format() == 'channels_first':
            shape_ord = (1, img_rows, img_cols)
        else:  # channel_last
            shape_ord = (img_rows, img_cols, 1)
```

```
In [4]: shape_ord
```

```
Out[4]: (28, 28, 1)
```

1.1.2 Task 1: Data preprocessing

- Load the mnist data of the keras datasets
- Scale the design matrix to values between 0 and 1
- Convert the design matrix to the expected (60000, 28, 28, 1) shape
- Convert the target vector to one-hot-vectors for the 10 classes
- Split the training data into 70% training and 30% validation data sets

1.1.3 Loading Data

```
In [5]: #Import the required libraries
        import numpy as np
        from keras.utils import np_utils

        np.random.seed(1338)  # for reproducibilty!!

        from keras.datasets import mnist

        (X_train, y_train), (X_test, y_test) = mnist.load_data()
```

1.1.4 Preprocess and Normalise Data

Convert the data to float, scale it with a MinMaxScaler and add the channel dimension

```
In [6]: X_train.shape
```

```
Out[6]: (60000, 28, 28)
```

Scaler need a 2D array, so we need reshape it first

```
In [7]: X_train = X_train.reshape(X_train.shape[0],img_rows*img_cols)
        X_test = X_test.reshape(X_test.shape[0],img_rows*img_cols)
        X_train.shape
```

```
Out[7]: (60000, 784)
```

```
In [8]: X_train = X_train.astype('float32')
        X_test = X_test.astype('float32')

        from sklearn.preprocessing import MinMaxScaler
        scaler = MinMaxScaler(feature_range=(0, 1))

        X_train = scaler.fit_transform(X_train)
        X_test = scaler.transform(X_test)

        X_train.shape
```

```
Out[8]: (60000, 784)
```

```
In [9]: X_train = X_train.reshape(X_train.shape[0],img_rows,img_cols)
        X_test = X_test.reshape(X_test.shape[0],img_rows,img_cols)
        X_train.shape
```

```
Out[9]: (60000, 28, 28)
```

And finally to the expected (60000, 28, 28, 1) shape for the CNN

```
In [10]: X_train = X_train.reshape((X_train.shape[0],) + shape_ord)
         X_test = X_test.reshape((X_test.shape[0],) + shape_ord)
```

```
In [11]: X_train.shape
```

```
Out[11]: (60000, 28, 28, 1)
```

1.1.5 Convert target vector

```
In [12]: from keras.utils import np_utils
         num_classes = 10
         Y_train = np_utils.to_categorical(y_train, num_classes)
         Y_test = np_utils.to_categorical(y_test, num_classes)
         Y_test[0]
```

```
Out[12]: array([0., 0., 0., 0., 0., 0., 0., 1., 0., 0.], dtype=float32)
```

1.1.6 Split Training and Validation Data

```
In [13]: from sklearn.model_selection import train_test_split

         X_train, X_val, Y_train, Y_val = train_test_split(X_train, Y_train, test_size=0.3, ra
```

1.2 A simple convolutional neural network

Convolution2D

```
from keras.layers.convolutional import Conv2D

Conv2D(filters, kernel_size, strides=(1, 1), padding='valid',
       data_format=None, dilation_rate=(1, 1), activation=None,
       use_bias=True, kernel_initializer='glorot_uniform',
       bias_initializer='zeros', kernel_regularizer=None,
       bias_regularizer=None, activity_regularizer=None,
       kernel_constraint=None, bias_constraint=None)
```

Arguments: filters: Integer, the dimensionality of the output space (i.e. the number output of filters in the convolution).

kernel_size: An integer or tuple/list of 2 integers, specifying the width and height of the 2D convolution window. Can be a single integer to specify the same value for all spatial dimensions.

strides: An integer or tuple/list of 2 integers, specifying the strides of the convolution along the width and height. Can be a single integer to specify the same value for all spatial dimensions. Specifying any stride value != 1 is incompatible with specifying any dilation_rate value != 1.

padding: one of "valid" or "same" (case-insensitive).

data_format: A string, one of channels_last (default) or channels_first. The ordering of the dimensions in the inputs. channels_last corresponds to inputs with shape (batch, height, width, channels) while channels_first corresponds to inputs with shape (batch, channels, height, width). It defaults to the image_data_format value found in your Keras config file at ~/.keras/keras.json. If you never set it, then it will be "channels_last".

dilation_rate: an integer or tuple/list of 2 integers, specifying the dilation rate to use for dilated convolution. Can be a single integer to specify the same value for all spatial dimensions. Currently, specifying any dilation_rate value != 1 is incompatible with specifying any stride value != 1.

activation: Activation function to use (see activations). If you don't specify anything, no activation is applied (ie. "linear" activation: a(x) = x).

use_bias: Boolean, whether the layer uses a bias vector.

kernel_initializer: Initializer for the kernel weights matrix (see initializers).

bias_initializer: Initializer for the bias vector (see initializers).

kernel_regularizer: Regularizer function applied to the kernel weights matrix (see regularizer).

bias_regularizer: Regularizer function applied to the bias vector (see regularizer).

activity_regularizer: Regularizer function applied to the output of the layer (its "activation"). (see regularizer).

kernel_constraint: Constraint function applied to the kernel matrix (see constraints).

bias_constraint: Constraint function applied to the bias vector (see constraints).

```
In [14]: from keras.models import Sequential
         from keras.layers.core import Dense, Dropout, Activation, Flatten

         from keras.layers.convolutional import Conv2D
         from keras.layers.pooling import MaxPooling2D
```

1.2.1 Model Definition

```
In [15]: # -- Initializing the values for the convolution neural network

         nb_epoch = 10  # kept very low! Please increase if you can use a GPU

         batch_size = 256
         # number of convolutional filters to use
         nb_filters = 32
         # size of pooling area for max pooling
         nb_pool = 2
         # convolution kernel size
         nb_conv = 3
```

```
In [16]: model = Sequential()

         model.add(Conv2D(nb_filters, kernel_size=(nb_conv, nb_conv), padding='valid', activati
                          input_shape=shape_ord))  # note: the very first layer **must** alway
         model.add(Flatten())
         model.add(Dense(10, activation='softmax'))

         model.compile(loss='categorical_crossentropy', optimizer='adam', metrics=['accuracy']

         model.summary()
```

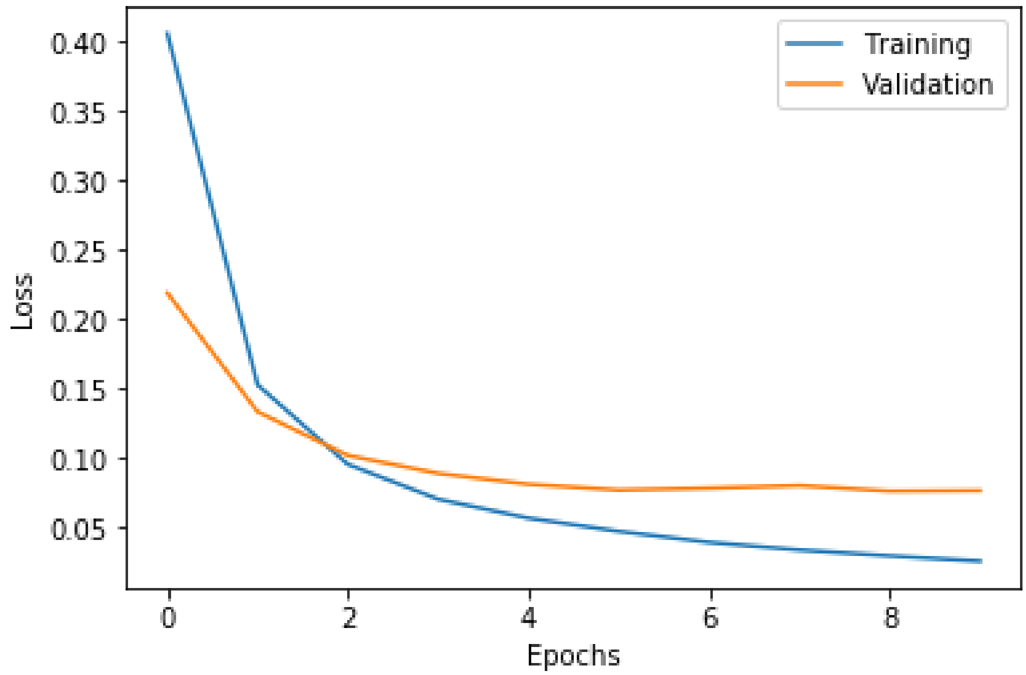
```
WARNING:tensorflow:From /home/nackenho/miniconda/envs/TUDortmundMLSeminar/lib/python2.7/site-pa
Instructions for updating:
Colocations handled automatically by placer.
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_1 (Conv2D)            (None, 26, 26, 32)        320
_________________________________________________________________
flatten_1 (Flatten)          (None, 21632)             0
_________________________________________________________________
dense_1 (Dense)              (None, 10)                216330
=================================================================
Total params: 216,650
Trainable params: 216,650
Non-trainable params: 0
_________________________________________________________________
```

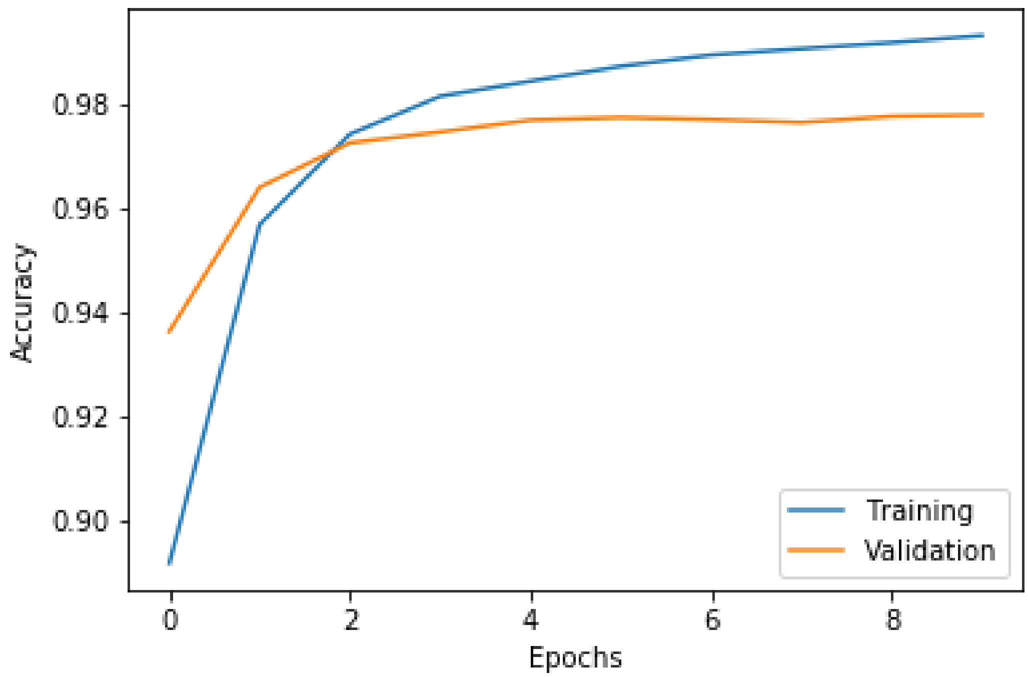
1.2.2 Training

```
In [17]: hist = model.fit(X_train, Y_train, batch_size=batch_size,
                          epochs=nb_epoch, verbose=1,
                          validation_data=(X_val, Y_val))
```

```
WARNING:tensorflow:From /home/nackenho/miniconda/envs/TUDortmundMLSeminar/lib/python2.7/site-pa
Instructions for updating:
Use tf.cast instead.
Train on 42000 samples, validate on 18000 samples
Epoch 1/10
42000/42000 [==============================] - 21s 506us/step - loss: 0.4060 - acc: 0.8918 - va
Epoch 2/10
42000/42000 [==============================] - 18s 430us/step - loss: 0.1530 - acc: 0.9569 - va
Epoch 3/10
42000/42000 [==============================] - 18s 433us/step - loss: 0.0958 - acc: 0.9743 - va
Epoch 4/10
42000/42000 [==============================] - 23s 541us/step - loss: 0.0706 - acc: 0.9816 - va
Epoch 5/10
42000/42000 [==============================] - 27s 640us/step - loss: 0.0570 - acc: 0.9845 - va
Epoch 6/10
42000/42000 [==============================] - 19s 445us/step - loss: 0.0475 - acc: 0.9874 - va
Epoch 7/10
42000/42000 [==============================] - 19s 446us/step - loss: 0.0395 - acc: 0.9895 - va
Epoch 8/10
42000/42000 [==============================] - 18s 433us/step - loss: 0.0340 - acc: 0.9908 - va
Epoch 9/10
42000/42000 [==============================] - 19s 448us/step - loss: 0.0298 - acc: 0.9919 - va
Epoch 10/10
42000/42000 [==============================] - 20s 472us/step - loss: 0.0260 - acc: 0.9933 - va
```

In [18]: `plot_history(hist)`





1.2.3 Task 2: Evaluating - Define an `evaluate` function, with the following properties:

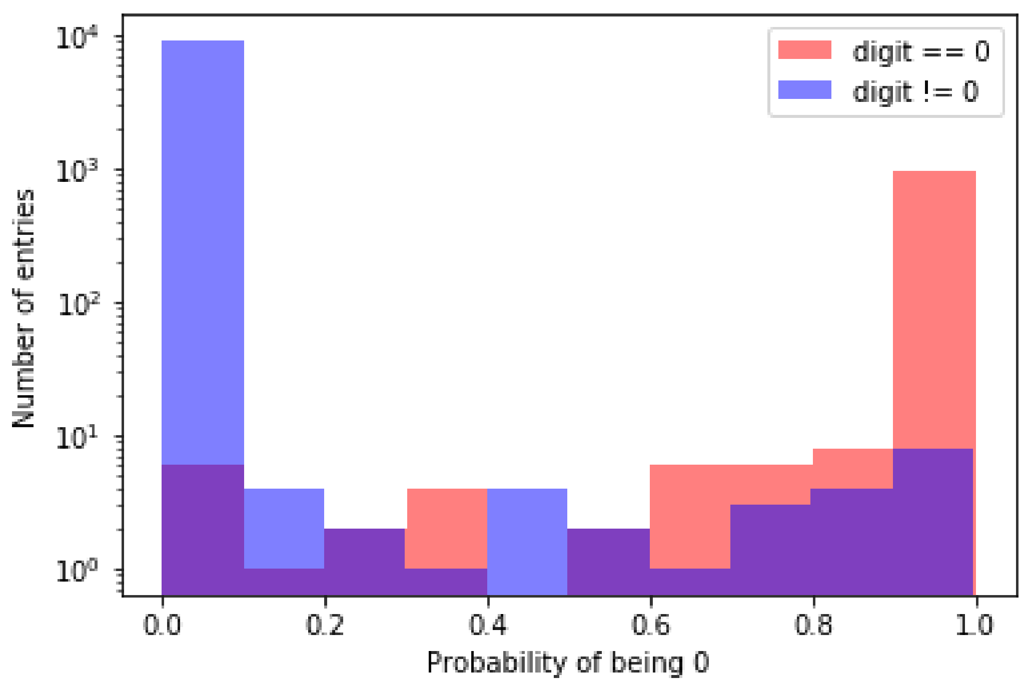
- It takes X_test and Y_test as arguments
- It calculates the loss, the accuracy and the classification report
- It plots the probability of being a zero for true zeros (red) and non-zeros (blue)
- It computes and plots the confusion matrix
- It plots the image, the prediction and the true value for the top 6 errors
- It plots image and predictions for the first 15 examples

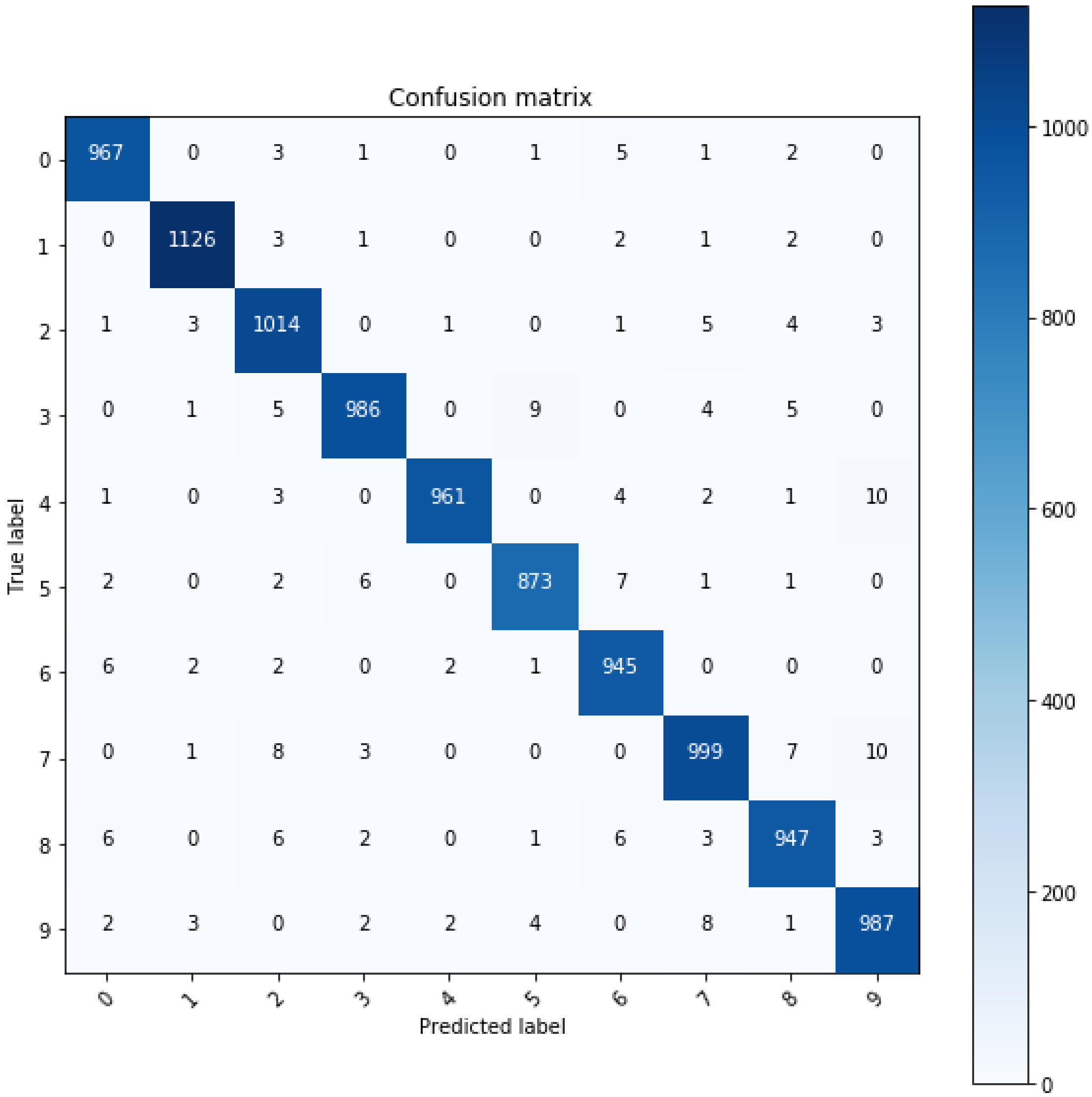
```
In [19]: from sklearn.metrics import confusion_matrix,classification_report

         def evaluate(X_test, Y_test):

             ##Evaluate loss and metrics
             loss, accuracy = model.evaluate(X_test, Y_test, verbose=0)
             print('Test Loss:', loss)
             print('Test Accuracy:', accuracy)
             # Predict the values from the test dataset
             Y_pred = model.predict(X_test)
             # Convert predictions classes to one hot vectors
             Y_cls = np.argmax(Y_pred, axis = 1)
             # Convert validation observations to one hot vectors
```

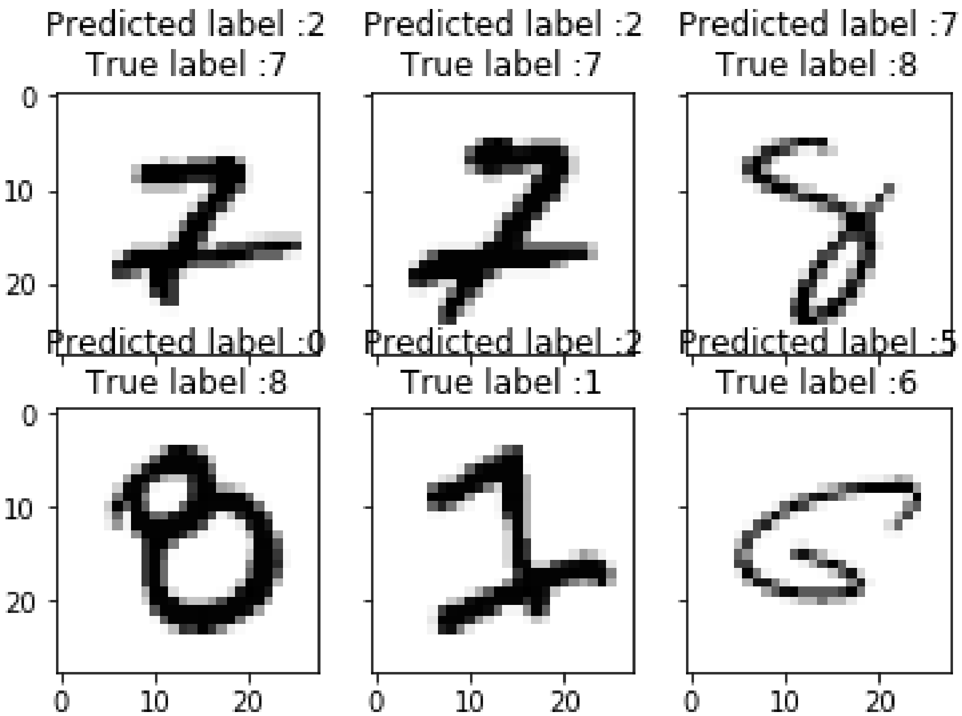
```
Y_true = np.argmax(Y_test, axis = 1)
print 'Classification Report:\n', classification_report(Y_true,Y_cls)

## Plot 0 probability
label=0
Y_pred_prob = Y_pred[:,label]
plt.hist(Y_pred_prob[Y_true == label], alpha=0.5, color='red', bins=10, log = True
plt.hist(Y_pred_prob[Y_true != label], alpha=0.5, color='blue', bins=10, log = Tru
plt.legend(['digit == 0', 'digit != 0'], loc='upper right')
plt.xlabel('Probability of being 0')
plt.ylabel('Number of entries')
plt.show()

# compute the confusion matrix
confusion_mtx = confusion_matrix(Y_true, Y_cls)
# plot the confusion matrix
plt.figure(figsize=(8,8))
plot_confusion_matrix(confusion_mtx, classes = range(10))

#Plot largest errors
errors = (Y_cls - Y_true != 0)
Y_cls_errors = Y_cls[errors]
Y_pred_errors = Y_pred[errors]
Y_true_errors = Y_true[errors]
X_test_errors = X_test[errors]
# Probabilities of the wrong predicted numbers
Y_pred_errors_prob = np.max(Y_pred_errors,axis = 1)
# Predicted probabilities of the true values in the error set
true_prob_errors = np.diagonal(np.take(Y_pred_errors, Y_true_errors, axis=1))
# Difference between the probability of the predicted label and the true label
delta_pred_true_errors = Y_pred_errors_prob - true_prob_errors
# Sorted list of the delta prob errors
sorted_dela_errors = np.argsort(delta_pred_true_errors)
# Top 6 errors
most_important_errors = sorted_dela_errors[-6:]
# Show the top 6 errors
display_errors(most_important_errors, X_test_errors, Y_cls_errors, Y_true_errors)

##Plot predictions
slice = 15
predicted = model.predict(X_test[:slice]).argmax(-1)
plt.figure(figsize=(16,8))
for i in range(slice):
    plt.subplot(1, slice, i+1)
    plt.imshow(X_test[i].reshape(28,28), interpolation='nearest')
    plt.text(0, 0, predicted[i], color='black',
             bbox=dict(facecolor='white', alpha=1))
    plt.axis('off')
```

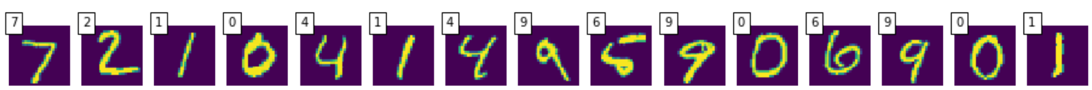
```
In [20]: evaluate(X_test, Y_test)
```

```
('Test Loss:', 0.06558480308502912)
('Test Accuracy:', 0.9805)
Classification Report:
              precision    recall  f1-score   support

           0       0.98      0.99      0.98       980
           1       0.99      0.99      0.99      1135
           2       0.97      0.98      0.98      1032
           3       0.99      0.98      0.98      1010
           4       0.99      0.98      0.99       982
           5       0.98      0.98      0.98       892
           6       0.97      0.99      0.98       958
           7       0.98      0.97      0.97      1028
           8       0.98      0.97      0.97       974
           9       0.97      0.98      0.98      1009

   micro avg       0.98      0.98      0.98     10000
   macro avg       0.98      0.98      0.98     10000
weighted avg       0.98      0.98      0.98     10000
```




Confusion matrix
True label
Predicted label
0
1
2
3
4
5
6
7
8
9
967
1126
1014
986
961
873
945
999
947
987
1000
800
600
400
200
0





1.3 Adding more Dense Layers and Dropout

1.3.1 Task 3: Adding additional classification layers

- Add a dense layer between the flatten layer and the output layer
- Add a 25% dropout layer before the flatten layer
- Add a 50% dropout layer between the two dense layers
- Build the model, train the NN, plot the loss and accuracy evolution
- Evaluate the new model

In [21]:
```
model = Sequential()

model.add(Conv2D(nb_filters, kernel_size=(nb_conv, nb_conv), padding='valid', activat
                 input_shape=shape_ord))
model.add(Dropout(0.25))
model.add(Flatten())
model.add(Dense(128, activation='relu'))
```

```
model.add(Dropout(0.5))
model.add(Dense(10, activation='softmax'))

model.compile(loss='categorical_crossentropy',
              optimizer='adam',
              metrics=['accuracy'])

model.summary()
```

```
WARNING:tensorflow:From /home/nackenho/miniconda/envs/TUDortmundMLSeminar/lib/python2.7/site-pa
Instructions for updating:
Please use `rate` instead of `keep_prob`. Rate should be set to `rate = 1 - keep_prob`.
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_2 (Conv2D)            (None, 26, 26, 32)        320
_________________________________________________________________
dropout_1 (Dropout)          (None, 26, 26, 32)        0
_________________________________________________________________
flatten_2 (Flatten)          (None, 21632)             0
_________________________________________________________________
dense_2 (Dense)              (None, 128)               2769024
_________________________________________________________________
dropout_2 (Dropout)          (None, 128)               0
_________________________________________________________________
dense_3 (Dense)              (None, 10)                1290
=================================================================
Total params: 2,770,634
Trainable params: 2,770,634
Non-trainable params: 0
_________________________________________________________________
```

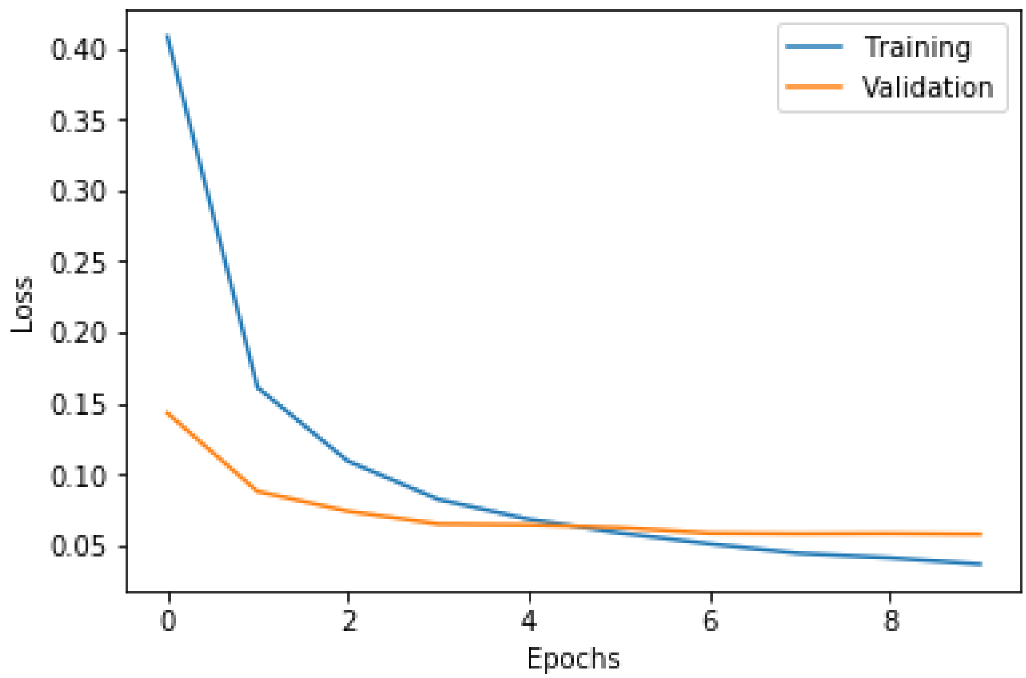
1.3.2 Training

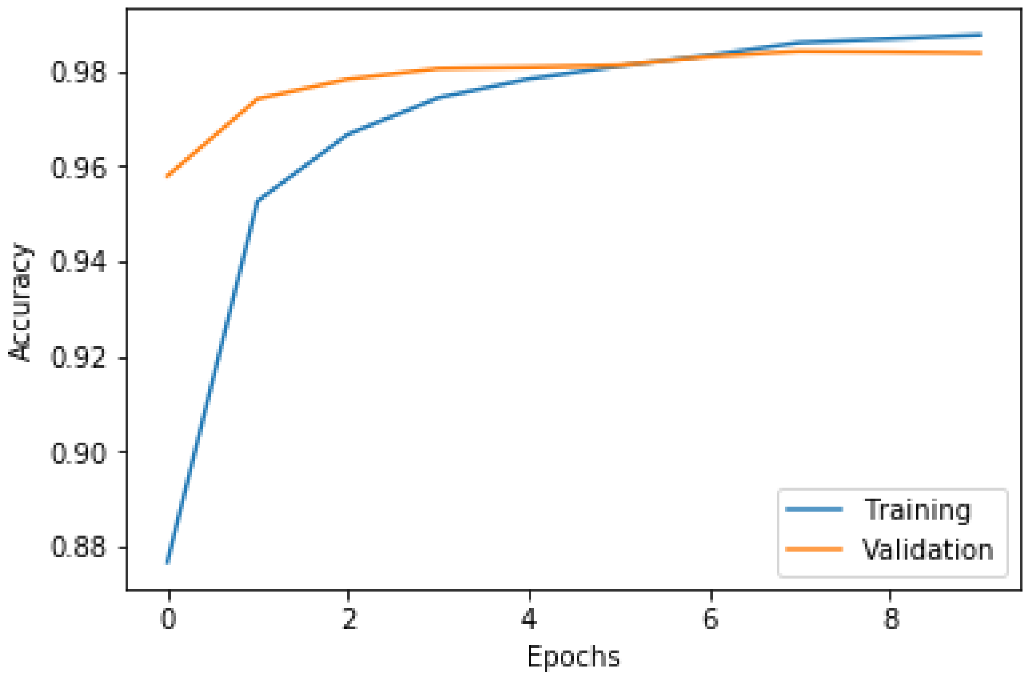
```
In [22]: hist = model.fit(X_train, Y_train, batch_size=batch_size,
                          epochs=nb_epoch, verbose=1,
                          validation_data=(X_val, Y_val))
```

```
Train on 42000 samples, validate on 18000 samples
Epoch 1/10
42000/42000 [==============================] - 70s 2ms/step - loss: 0.4081 - acc: 0.8765 - val_
Epoch 2/10
42000/42000 [==============================] - 69s 2ms/step - loss: 0.1611 - acc: 0.9527 - val_
Epoch 3/10
42000/42000 [==============================] - 63s 2ms/step - loss: 0.1095 - acc: 0.9667 - val_
Epoch 4/10
42000/42000 [==============================] - 66s 2ms/step - loss: 0.0824 - acc: 0.9745 - val_
```

```
Epoch 5/10
42000/42000 [==============================] - 68s 2ms/step - loss: 0.0684 - acc: 0.9784 - val_
Epoch 6/10
42000/42000 [==============================] - 62s 1ms/step - loss: 0.0589 - acc: 0.9811 - val_
Epoch 7/10
42000/42000 [==============================] - 73s 2ms/step - loss: 0.0511 - acc: 0.9834 - val_
Epoch 8/10
42000/42000 [==============================] - 80s 2ms/step - loss: 0.0444 - acc: 0.9860 - val_
Epoch 9/10
42000/42000 [==============================] - 70s 2ms/step - loss: 0.0413 - acc: 0.9868 - val_
Epoch 10/10
42000/42000 [==============================] - 73s 2ms/step - loss: 0.0368 - acc: 0.9877 - val_
```

In [23]:
```
plot_history(hist)
```

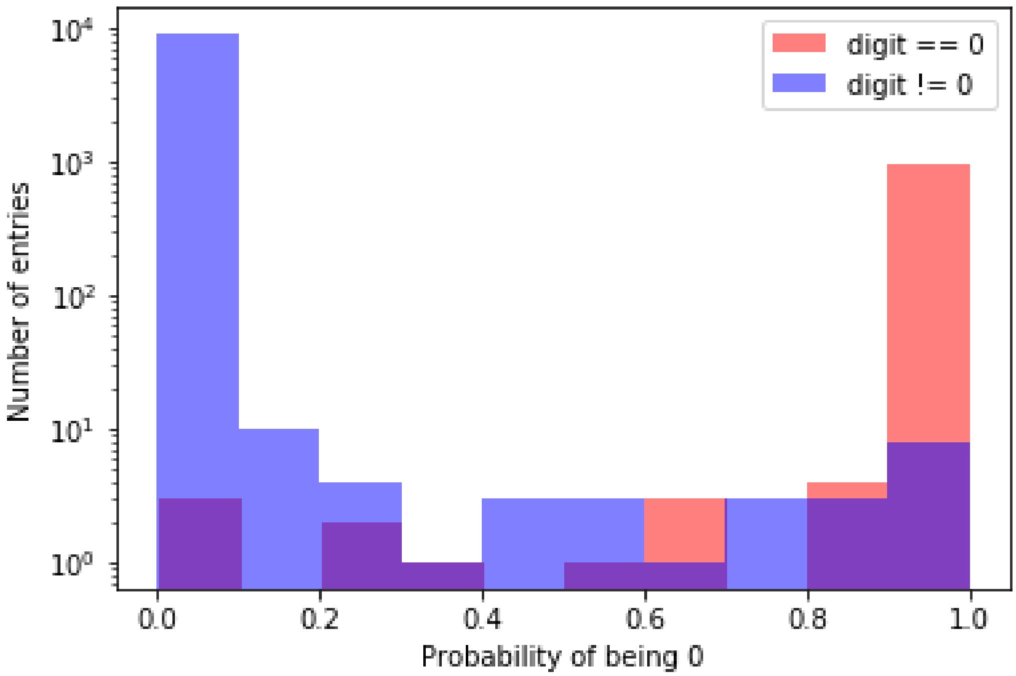




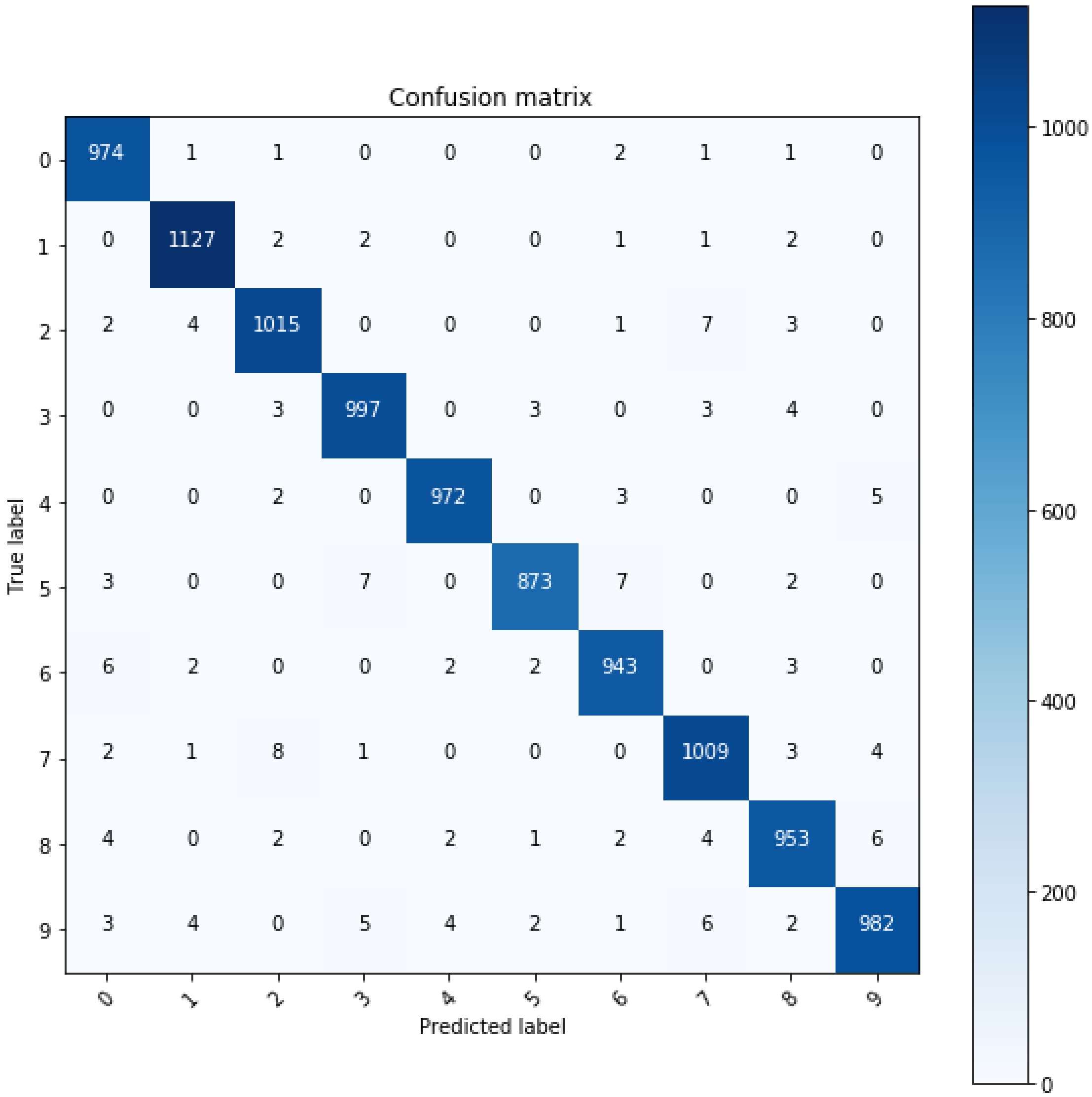
1.3.3 Evaluating

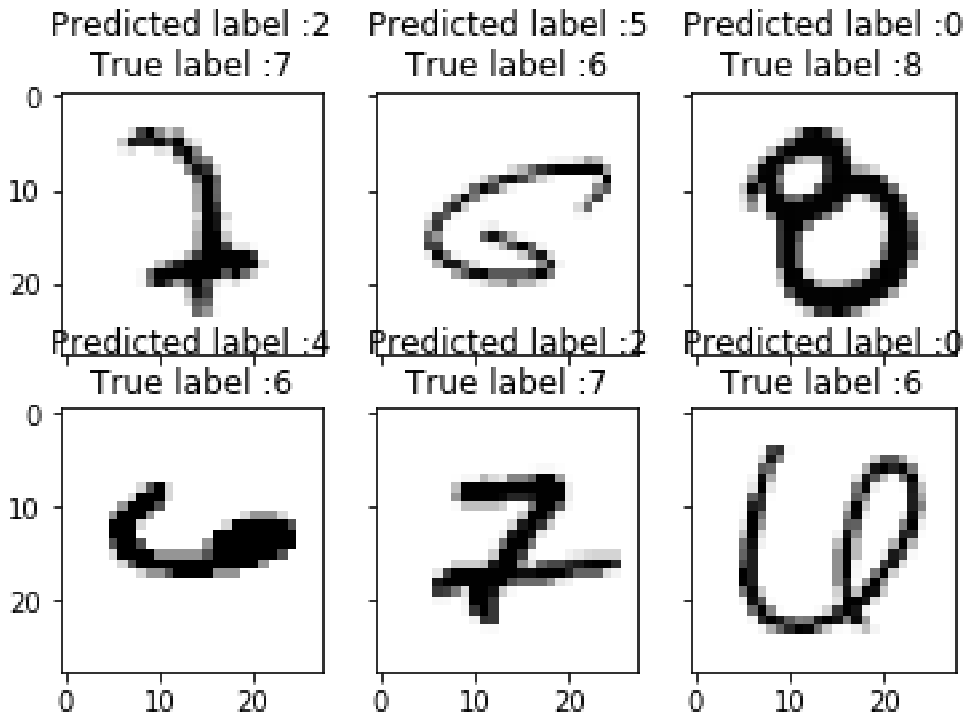
```
In [24]: # Evaluating the model on the test data
         evaluate(X_test, Y_test)
```

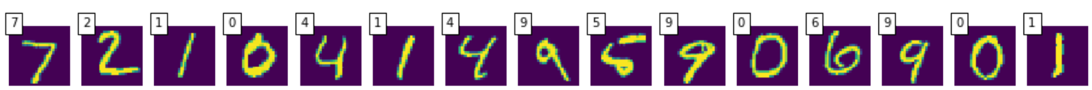
```
('Test Loss:', 0.04608184394863056)
('Test Accuracy:', 0.9845)
Classification Report:
              precision    recall  f1-score   support

           0       0.98      0.99      0.99       980
           1       0.99      0.99      0.99      1135
           2       0.98      0.98      0.98      1032
           3       0.99      0.99      0.99      1010
           4       0.99      0.99      0.99       982
           5       0.99      0.98      0.98       892
           6       0.98      0.98      0.98       958
           7       0.98      0.98      0.98      1028
           8       0.98      0.98      0.98       974
           9       0.98      0.97      0.98      1009

   micro avg       0.98      0.98      0.98     10000
   macro avg       0.98      0.98      0.98     10000
weighted avg       0.98      0.98      0.98     10000
```



digit == 0
digit != 0
Number of entries
Probability of being 0
10^4
10^3
10^2
10^1
10^0
0.0
0.2
0.4
0.6
0.8
1.0


Confusion matrix
True label
Predicted label
0
1
2
3
4
5
6
7
8
9
974
1127
1015
997
972
873
943
1009
953
982
1000
800
600
400
200





2 Adding an additional convolution layer and a pooling layer

2.0.1 Task 3: Adding additional classification layers

- Add another `Conv2D` layer after the first convolutional layer with 64 filter, 3x3 kernel and `valid` (no) padding
- Add another `MaxPooling2D` layer with a 2x2 pooling size
- Build the model, train the NN, plot the loss and accuracy evolution
- Evaluate the new model

```
In [25]: model = Sequential()

         model.add(Conv2D(nb_filters, kernel_size=(nb_conv, nb_conv), padding='valid', activati
                          input_shape=shape_ord))
         model.add(Conv2D(64, kernel_size=(nb_conv, nb_conv), padding='valid', activation='relu
         model.add(MaxPooling2D(pool_size=(2, 2)))
         model.add(Dropout(0.25))
```

```
model.add(Flatten())
model.add(Dense(128, activation='relu'))
model.add(Dropout(0.5))
model.add(Dense(10, activation='softmax'))

model.compile(loss='categorical_crossentropy',
              optimizer='adam',
              metrics=['accuracy'])

model.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_3 (Conv2D)            (None, 26, 26, 32)        320
_________________________________________________________________
conv2d_4 (Conv2D)            (None, 24, 24, 64)        18496
_________________________________________________________________
max_pooling2d_1 (MaxPooling2 (None, 12, 12, 64)        0
_________________________________________________________________
dropout_3 (Dropout)          (None, 12, 12, 64)        0
_________________________________________________________________
flatten_3 (Flatten)          (None, 9216)              0
_________________________________________________________________
dense_4 (Dense)              (None, 128)               1179776
_________________________________________________________________
dropout_4 (Dropout)          (None, 128)               0
_________________________________________________________________
dense_5 (Dense)              (None, 10)                1290
=================================================================
Total params: 1,199,882
Trainable params: 1,199,882
Non-trainable params: 0
_________________________________________________________________
```

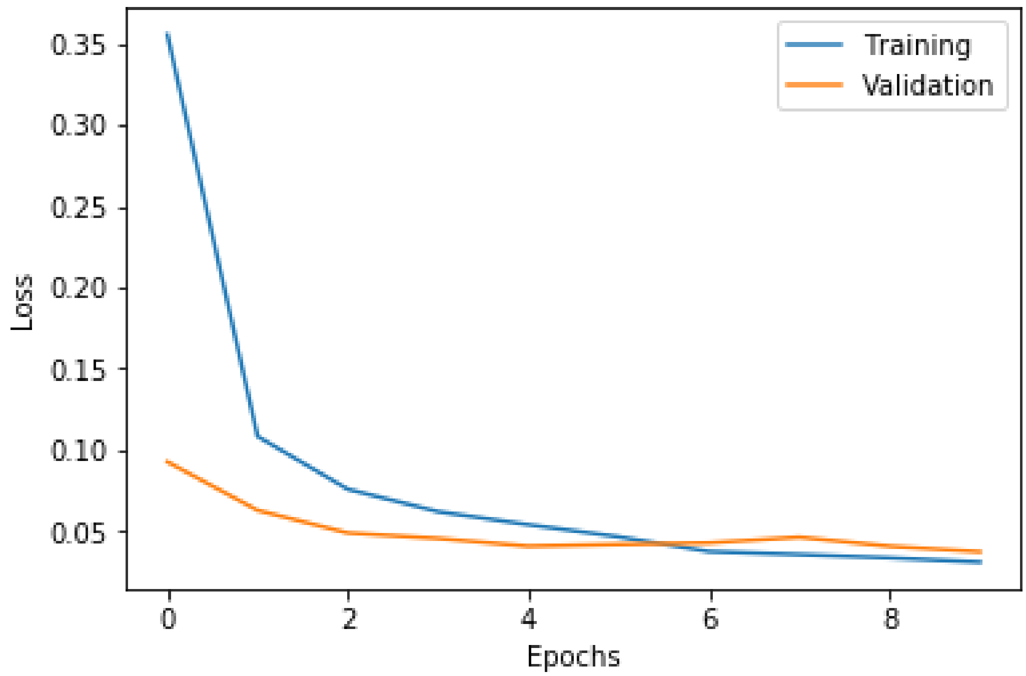
2.0.2 Training

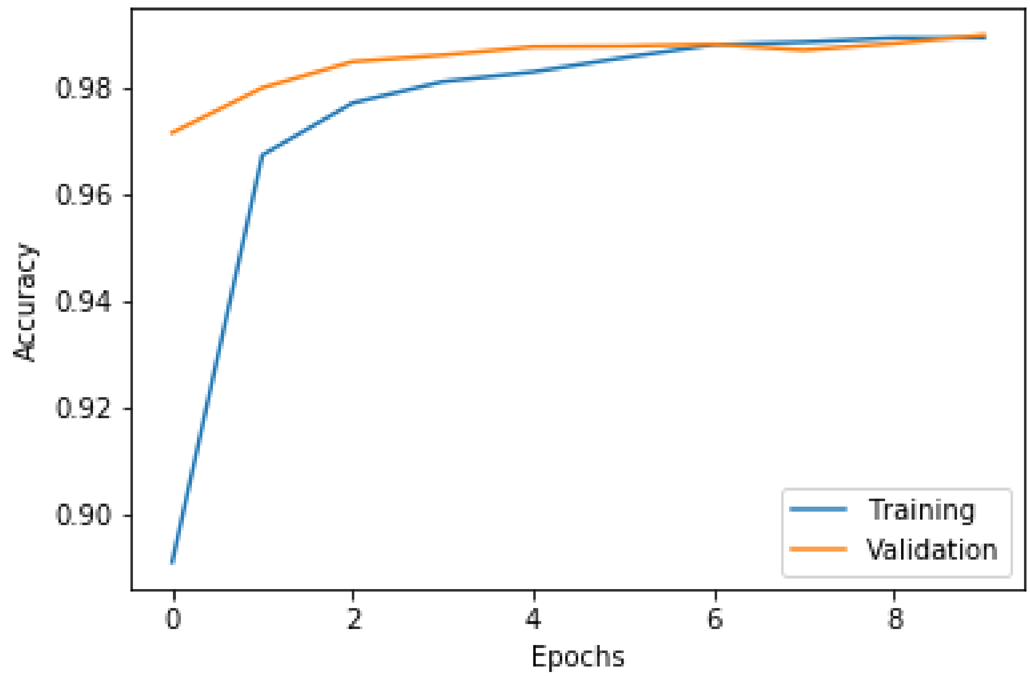
```
In [26]: hist = model.fit(X_train, Y_train, batch_size=batch_size,
                          epochs=nb_epoch, verbose=1,
                          validation_data=(X_val, Y_val))
```

```
Train on 42000 samples, validate on 18000 samples
Epoch 1/10
42000/42000 [==============================] - 103s 2ms/step - loss: 0.3555 - acc: 0.8910 - val
Epoch 2/10
42000/42000 [==============================] - 110s 3ms/step - loss: 0.1085 - acc: 0.9674 - val
Epoch 3/10
```

```
42000/42000 [==============================] - 135s 3ms/step - loss: 0.0759 - acc: 0.9772 - val
Epoch 4/10
42000/42000 [==============================] - 101s 2ms/step - loss: 0.0621 - acc: 0.9812 - val
Epoch 5/10
42000/42000 [==============================] - 94s 2ms/step - loss: 0.0541 - acc: 0.9830 - val_
Epoch 6/10
42000/42000 [==============================] - 98s 2ms/step - loss: 0.0466 - acc: 0.9857 - val_
Epoch 7/10
42000/42000 [==============================] - 89s 2ms/step - loss: 0.0376 - acc: 0.9881 - val_
Epoch 8/10
42000/42000 [==============================] - 95s 2ms/step - loss: 0.0357 - acc: 0.9886 - val_
Epoch 9/10
42000/42000 [==============================] - 90s 2ms/step - loss: 0.0337 - acc: 0.9894 - val_
Epoch 10/10
42000/42000 [==============================] - 95s 2ms/step - loss: 0.0311 - acc: 0.9894 - val_
```

In [27]:
```
plot_history(hist)
```

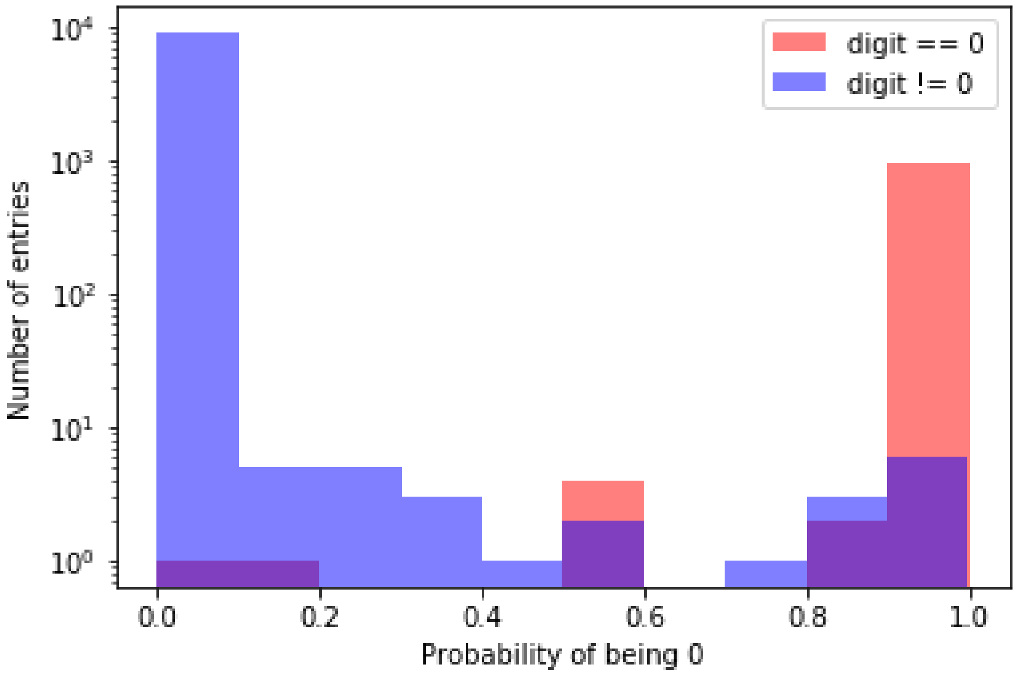




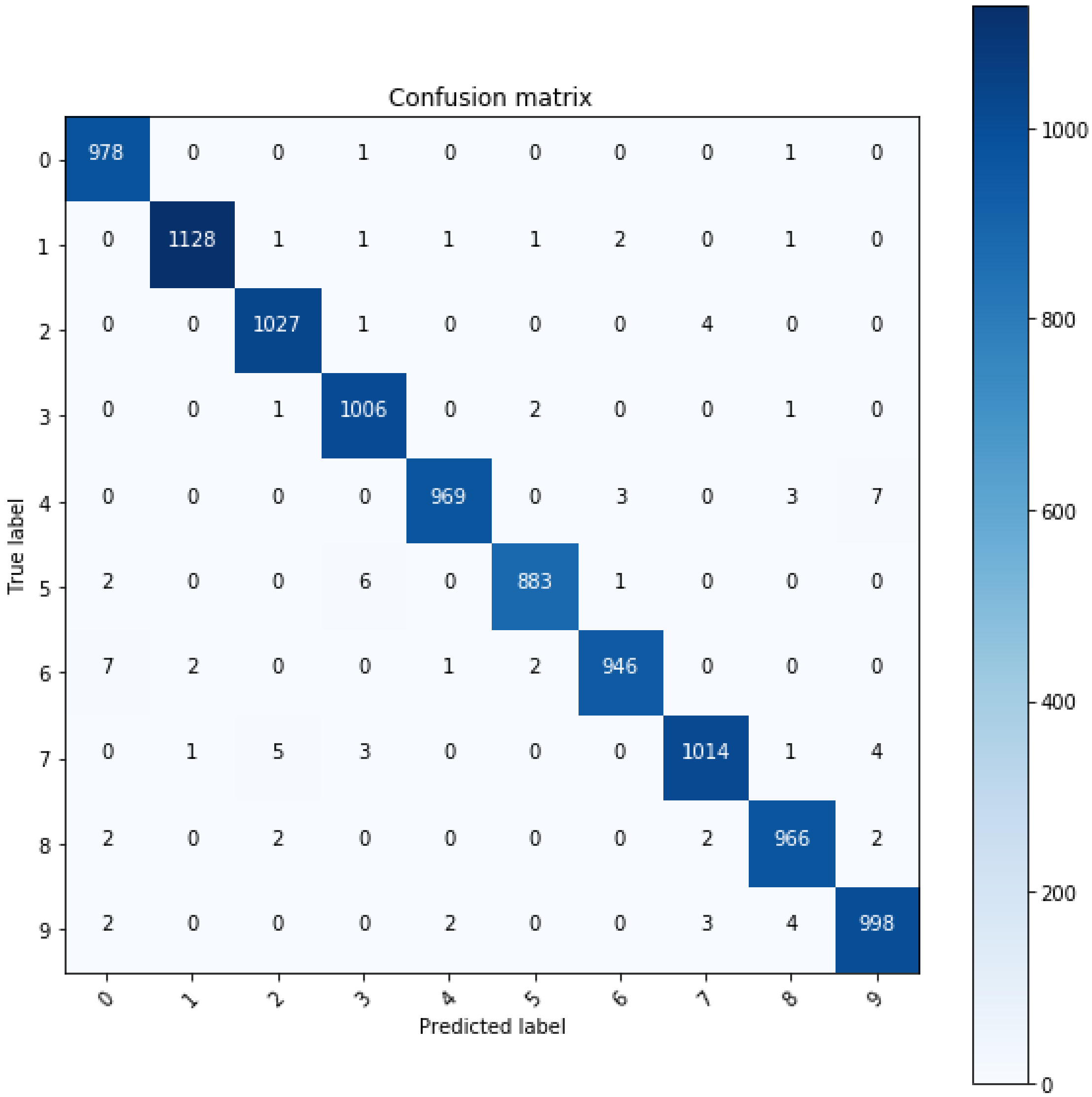
2.0.3 Evaluating

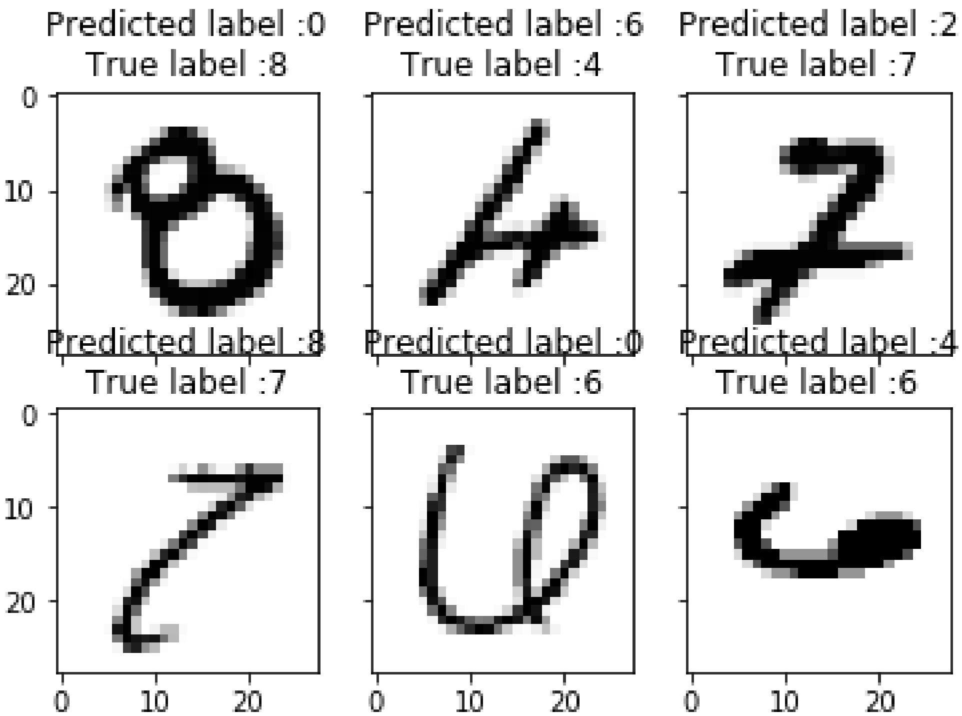
In [28]: evaluate(X_test, Y_test)

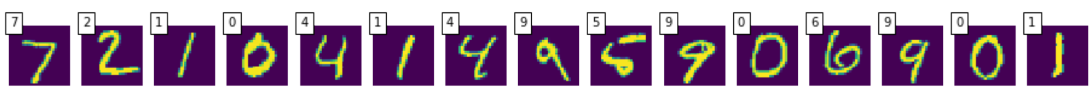
```
('Test Loss:', 0.030270598501911446)
('Test Accuracy:', 0.9915)
Classification Report:
              precision    recall  f1-score   support

           0       0.99      1.00      0.99       980
           1       1.00      0.99      1.00      1135
           2       0.99      1.00      0.99      1032
           3       0.99      1.00      0.99      1010
           4       1.00      0.99      0.99       982
           5       0.99      0.99      0.99       892
           6       0.99      0.99      0.99       958
           7       0.99      0.99      0.99      1028
           8       0.99      0.99      0.99       974
           9       0.99      0.99      0.99      1009

   micro avg       0.99      0.99      0.99     10000
   macro avg       0.99      0.99      0.99     10000
weighted avg       0.99      0.99      0.99     10000
```



digit == 0
digit != 0
Number of entries
10⁴
10³
10²
10¹
10⁰
0.0
0.2
0.4
0.6
0.8
1.0
Probability of being 0


Confusion matrix
True label
Predicted label
0 1 2 3 4 5 6 7 8 9
978 0 0 1 0 0 0 0 1 0
0 1128 1 1 1 1 2 0 1 0
0 0 1027 1 0 0 0 4 0 0
0 0 1 1006 0 2 0 0 1 0
0 0 0 0 969 0 3 0 3 7
2 0 0 6 0 883 1 0 0 0
7 2 0 0 1 2 946 0 0 0
0 1 5 3 0 0 0 1014 1 4
2 0 2 0 0 0 0 2 966 2
2 0 0 0 2 0 0 3 4 998
1000
800
600
400
200
0





---

3 Bonus: Understanding Convolutional Layers Structure

We will inspect and understand the convolutional layer of our previously defined quite shallow CNN, which contains two [Convolution, Convolution, MaxPooling] stages, and two Dense layers.

3.0.1 Understanding layer shapes

An important feature of Keras layers is that each of them has an `input_shape` attribute, which you can use to visualize the shape of the input tensor, and an `output_shape` attribute, for inspecting the shape of the output tensor.

As we can see, the input shape of the first convolutional layer corresponds to the `input_shape` attribute (which must be specified by the user).

In this case, it is a `28x28` image with three color channels.

Since this convolutional layer has the `padding` set to `same`, its output width and height will remain the same, and the number of output channel will be equal to the number of filters learned by the layer, 16.

The following convolutional layer, instead, have the default `padding`, and therefore reduce width and height by $(k-1)$, where k is the size of the kernel.

`MaxPooling` layers, instead, reduce width and height of the input tensor, but keep the same number of channels.

`Activation` layers, of course, don't change the shape.

```
In [29]: for i, layer in enumerate(model.layers):
             print ("Layer", i, "\t", layer.name, "\t\t", layer.input_shape, "\t", layer.outpu
```

```
('Layer', 0, '\t', 'conv2d_3', '\t\t', (None, 28, 28, 1), '\t', (None, 26, 26, 32))
('Layer', 1, '\t', 'conv2d_4', '\t\t', (None, 26, 26, 32), '\t', (None, 24, 24, 64))
('Layer', 2, '\t', 'max_pooling2d_1', '\t\t', (None, 24, 24, 64), '\t', (None, 12, 12, 64))
('Layer', 3, '\t', 'dropout_3', '\t\t', (None, 12, 12, 64), '\t', (None, 12, 12, 64))
('Layer', 4, '\t', 'flatten_3', '\t\t', (None, 12, 12, 64), '\t', (None, 9216))
('Layer', 5, '\t', 'dense_4', '\t\t', (None, 9216), '\t', (None, 128))
('Layer', 6, '\t', 'dropout_4', '\t\t', (None, 128), '\t', (None, 128))
('Layer', 7, '\t', 'dense_5', '\t\t', (None, 128), '\t', (None, 10))
```

3.0.2 Understanding weights shape

In the same way, we can visualize the shape of the weights learned by each layer.

In particular, Keras lets you inspect weights by using the `get_weights` method of a layer object.

This will return a list with two elements, the first one being the **weight tensor** and the second one being the **bias vector**.

In particular:

- **MaxPooling layer** don't have any weight tensor, since they don't have learnable parameters.
- **Convolutional layers**, instead, learn a (n_o, n_i, k, k) weight tensor, where k is the size of the kernel, n_i is the number of channels of the input tensor, and n_o is the number of filters to be learned.

For each of the n_o filters, a bias is also learned.

- **Dense layers** learn a (n_i, n_o) weight tensor, where n_o is the output size and n_i is the input size of the layer. Each of the n_o neurons also has a bias.

```
In [30]: for i, layer in enumerate(model.layers):
             if len(layer.get_weights()) > 0:
                 W, b = layer.get_weights()
                 print("Layer", i, "\t", layer.name, "\t\t", W.shape, "\t", b.shape)
```

```
('Layer', 0, '\t', 'conv2d_3', '\t\t', (3, 3, 1, 32), '\t', (32,))
('Layer', 1, '\t', 'conv2d_4', '\t\t', (3, 3, 32, 64), '\t', (64,))
('Layer', 5, '\t', 'dense_4', '\t\t', (9216, 128), '\t', (128,))
('Layer', 7, '\t', 'dense_5', '\t\t', (128, 10), '\t', (10,))
```