Humanoid Navigation using Control Barrier Functions

Eugenio Bugli, Damiano Imola, Salvatore Rago

November 29, 2024

Contents

1 Introduction **1**

2 3D LIP Model **2**

3 Model Predictive Control **2**

4 Conclusion **3**

1 Introduction

Humanoid robots are inherently underactuated thanks to to unilateral ground contacts thus a strong coupling exists between path planning and gait control. A path is considered safe if the robot does not collide with any obstacle and its dynamics and physical limitations are respected. Due to the high complexity of humanoids, we cannot decouple path planning from motion control without taking into account the dynamics. We should solve gait optimization problems based on the robot's full order model or the reduced one. Due to computational complexity, reduced order models such the Linear Inverted Pendulum (LIP) are often employed. In our case, since Control Barrier Functions are employed to ensure safety in path planning, we will pre-compute heading angles ans use approximated linead DCBFs. This is needed since we may have problems at computation level due to the non linearity of kinematics and path constraints.

2 3D LIP Model

This reduced model assumes that during the motion the Center of Mass (CoM) will have a constant height H.

$$\dot{v}_x = \frac{g}{H}(p_x - f_x) \qquad \dot{v}_y = \frac{g}{H}(p_y - f_y) \tag{1}$$

With (p_x, p_y) we denote the position of the CoM and with (v_x, v_y) its velocity with respect to the x-axis and y-axis. The stance foot position, which is the position in which both feets are in contact with the ground, is denoted with (f_x, f_y).
Given the position (p_{x_k}, p_{y_k}) and velocities (v_{x_k}, v_{y_k}) of the CoM at the k-th step, the closed-form solutions of the step-to-step discrete dynamics can be written as follows :

$$\begin{bmatrix} p_{x_{k+1}} \\ v_{x_{k+1}} \end{bmatrix} = A_d \begin{bmatrix} p_{x_k} \\ v_{x_k} \end{bmatrix} + B_d f_{x_k} \qquad \begin{bmatrix} p_{y_{k+1}} \\ v_{y_{k+1}} \end{bmatrix} = A_d \begin{bmatrix} p_{y_k} \\ v_{y_k} \end{bmatrix} + B_d f_{y_k} \tag{2}$$

Where $\beta = \sqrt{\frac{g}{H}}$ and the two matrices are:

$$A_d = \begin{bmatrix} cosh(\beta T) & \frac{sinh(\beta T)}{\beta} \\ \beta sinh(\beta T) & cosh(\beta T) \end{bmatrix} \qquad B_d = \begin{bmatrix} 1 - cosh(\beta T) & -\beta sinh(\beta T) \end{bmatrix} \tag{3}$$

By defining the state of our system as $x = [p_x, v_x, p_y, v_y, \theta]^T \in \mathbb{R}^5$ and the control input as $u = [f_x, f_y, \omega]^T \in \mathbb{R}^3$, where θ is the heading angle and ω is its turning rate, the step-to-step dynamics of the 3D-LIP model is written as follows:

$$x_{k+1} = A_L x_k + B_L u_k \tag{4}$$

Where the two matrices are defined as follows:

$$A_L = \begin{bmatrix} A_d & 0 & 0 \\ 0 & A_d & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B_d = \begin{bmatrix} B_d & 0 & 0 \\ 0 & B_d & 0 \\ 0 & 0 & T \end{bmatrix} \tag{5}$$

3 Model Predictive Control

The dynamic model is used to define a MPC problem to compute the optimal stepping positions for save nagivation and locomotion. The LIP-MPC is defined as:

$$J^* = \min_{u_{0:N-1}} \sum_{k=1}^{N} [(p_{x_k} - g_x)^2 + (p_{y_k} - g_y)^2]$$
$$\text{s. t} \quad x_{k+1} = A_L x_k + B_L u_k \qquad c_l \leq c(x_k, u_k) \leq c_u$$

Minimizing the cost function means that the robot is moving towards the goal position. This problem is subject to the satisfaction of the robot's dynamics and the kinematics and path constraints. The kinematics and path constraints are captured in $c(x_k, u_k)$ and they are often nonlinear. This nonlinearity is related to presence of the heading angle θ_k. These constraints are linearized by pre-computing the turning rates $\bar{\omega}_k$ for all the horizon length, in order to have them fixed in the MPC calculation.

4 Conclusion