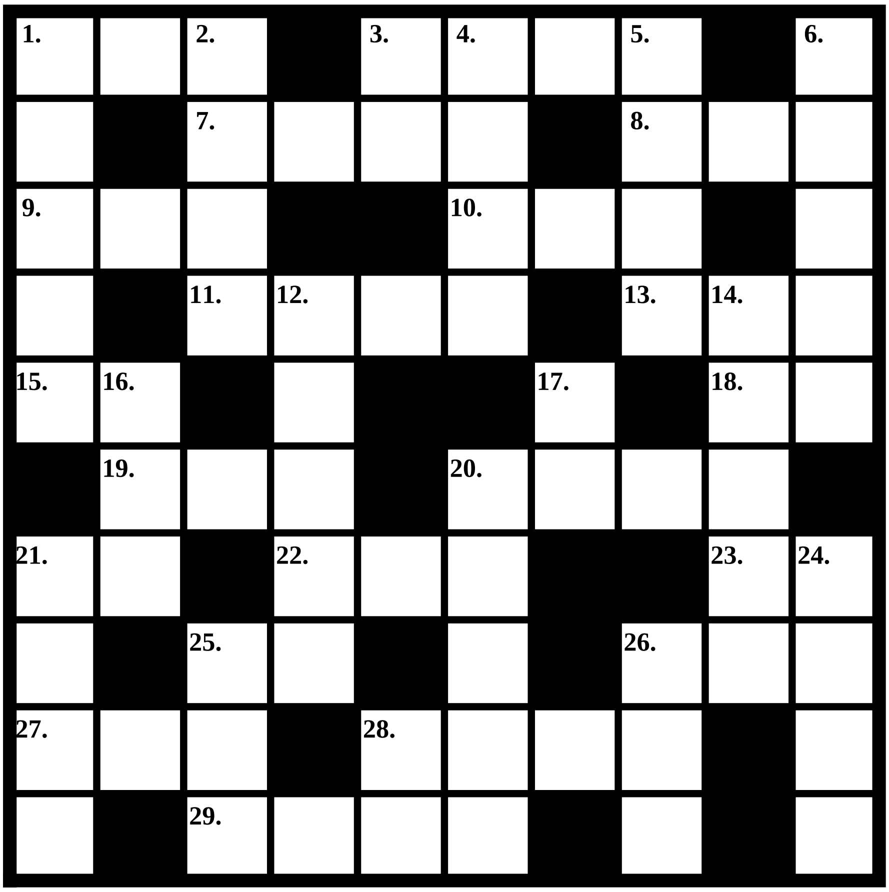

Figure 1:

Across Clues

1. The values of x and y such that $f(x,y) = -7750 - 65x^2 - 123y^2 + 1430x + 1968y - x^2y^2 + 16x^2y + 22xy^2 - 352xy$ is maximised.

3. The product of three distinct primes.

7. The standard Cauchy probability distribution has probability density function $f(x) = \frac{k}{1+x^2}$, $x \in (-\infty, \infty)$, where k is a constant. Find the value of k to 4 decimal places. Ignore the leading zero.

8. This number has the same digit sum as 3 Across and no repeated digits.

9. The number 184 is a prime number written to the base N. Find the decimal expression of this prime number.

10. This number has the same digit sum as 13 Across.

11. The largest possible number that can have the same digit sum as 2 Down.

13. A solution of the equation $\mu(\frac{N}{10}) = 0$, where $\mu(N)$ is the Mobius function (see 24 Down for a definition of the Mobius Function).

15. The general Cauchy probability distribution has probability density function $f(x) = \frac{\frac{b}{\pi}}{b^2+(x-m)^2}$, $x \in (-\infty, \infty)$, where b and m are constants. Given that $b = 2\pi$ and $m = 22$, find the median of this distribution.

18. A solution of the equation $\mu(N) = 0$, where $\mu(N)$ is the Mobius function (see 24 Down for a definition of the Mobius Function).

19. The value of n such that the cyclic group $\{1, x, x^2, x^3, \cdots, x^{n-1} | x^n \equiv 1\}$ has nine subgroups, two of which contain a prime number of elements.

20. The value of $\int_0^3 \int_0^y 70xy^2 dxdy$.

21. Merten's Function is defined for positive integers N as $M(N) = \sum_{r=1}^{N} \mu(r)$, where $\mu(N)$ is the Mobius function (see 24 Down for a definition of the Mobius Function). Find the smallest value of N such that $M(N) = -3$.

22. 1 Down plus 22 Across equals a multiple of 28 Across, with the added condition that the total number of sixes that appear in this crossnumber (upon completion) is ten.

23. The value of the base N in 9 Across.

25. The remainder when 22^{71} is divided by 24.

26. To calculate the Chirathivat number of a positive integer n you add together all the factors of n and then divide that sum by n. Positive integers which produce the same Chirathivat number are known as friends. Find a friend of 7440.

27. The decimal number 750 expressed in a different base.

28. 1 Down plus 22 Across equals a multiple of 28 Across, with the added condition that the total number of sixes that appear in this crossnumber (upon completion) is ten.

29. The value of $\sum_{r=0}^{n}(2^r \times {}^nC_r)$.

Down Clues

1. 1 Down plus 22 Across equals a multiple of 28 Across, with the added condition that the total number of sixes that appear in this crossnumber (upon completion) is ten.

2. 2 Down has the same digit sum as 11 Across.

3. Consider the set of natural numbers $\{1, 2\}$. The possible permutations of this set are $\{1, 2\}$ and $\{2, 1\}$. If we define a move as the act of moving one number in the set then no moves are required to reorder the first permutation into ascending order and one move is required to reorder the second permutation into ascending order; hence a minimum total of one move is needed to reorder all the permutations of $\{1, 2\}$ into ascending order. Find the minimum total number of moves required to reorder all the permutations of $\{1, 2, 3, 4\}$ into ascending order. [For clarity, it takes one move to reorder the set $\{3, 1, 2\}$ into ascending order: move the 3 to after the 2.]

4. Consider the set of natural numbers $\{1, 2\}$. The possible permutations of this set are $\{1, 2\}$ and $\{2, 1\}$. If we define a move as the act of moving one number in the set then no moves are required to reorder the first permutation into ascending order and one move is required to reorder the second permutation into ascending order; hence a minimum total of one move is needed to reorder all the permutations of $\{1, 2\}$ into ascending order. Now if we take this minimum total number of moves (1) and divide it by the number of permutations (2) and then by the size of the set (also 2) we get $\frac{1}{4}$ which is known as the second Wang Number, $W(2)$, where $W(N)$ denotes the Wang Number of the set $\{1, 2, \cdots, N\}$. Calculate $W(4)$, giving your answer to three decimal places and include the leading zero.

5. The value of $100(2n - 19) + \sum_{r=0}^{n} {}^{n}C_r$.

6. The value of $1 + \sum_{r=1}^{\infty} 10^{-n!}$, rounded to 4 decimal places.

12. Find the first five non-zero digits of $W(5)$. For a definition of Wang Numbers, $W(N)$, please refer to 4 Down.

14. A five-digit number that is divisible by nine.

16. The maximum value of $f(x,y)$ where $f(x,y) = -7750 - 65x^2 - 123y^2 + 1430x + 1968y - x^2y^2 + 16x^2y + 22xy^2 - 352xy$.

17. The value of $\sum_{r=0}^{\infty} \frac{r17^r e^{-17}}{r!}$.

20. To calculate the intermediate Hassan Number of a real number x, denoted $K(x)$, first concatenate, in ascending order, every prime number less than or equal to x. Then express this value in binary form. For example, to calculate $K(5)$ we concatenate the prime numbers less than or equal to 5 to get 235, and then convert this into the binary number 11101011. Hence $K(5) = 11101011$. Find $K(3)$.

21. The value of $\int_0^4 \int_1^3 24x^2y dxdy$.

24. The Mobius function of a positive integer N is defined as follows: $\mu(1) = 1$; if N is a composite number with no repeated prime factors, $\mu(N) = 1$; if N is a composite number with one or more repeated prime factors, $\mu(N) = 0$; if N is a prime number, $\mu(N) = -1$. The first digit of 24 Down is $\mu(6)$, the second digit is $\mu(4)$, the third digit is $\mu(9)$ and the fourth digit is $\mu(22)$.

25. A multiple of 9.

26. The hypothesised Hassan's Constant (not to be confused with the, related, intermediate Hassan Numbers, $K(x)$) is an under-studied mathematical constant, whose existence has not even been fully proven. To define Hassan's Constant, denoted H_0, we first take the digit sum of $K(p)$, where p is a prime number, then divide this by p. Hassan's Constant is then defined as the sum of all these values: defining the function $D(N)$ to be the digit sum of the positive integer N we can express Hassan's Constant as $H_0 = \sum_{p \in \{2,3,5,7,\cdots\}} \frac{D(K(p))}{p}$. Estimate H_0 by calculating $\sum_{p \in \{2,3,5,7,11\}} \frac{D(K(p))}{p}$, giving your answer to three significant figures.

28. An integer solution of the equation $K(N) = 1011001111010101010101001$, where $K(N)$ denotes the intermediate Hassan Number of N (see 20 Down for a full definition of intermediate Hassan Numbers).