Discrete Time SIR-model for COVID-19

Vladimir and Eunjin

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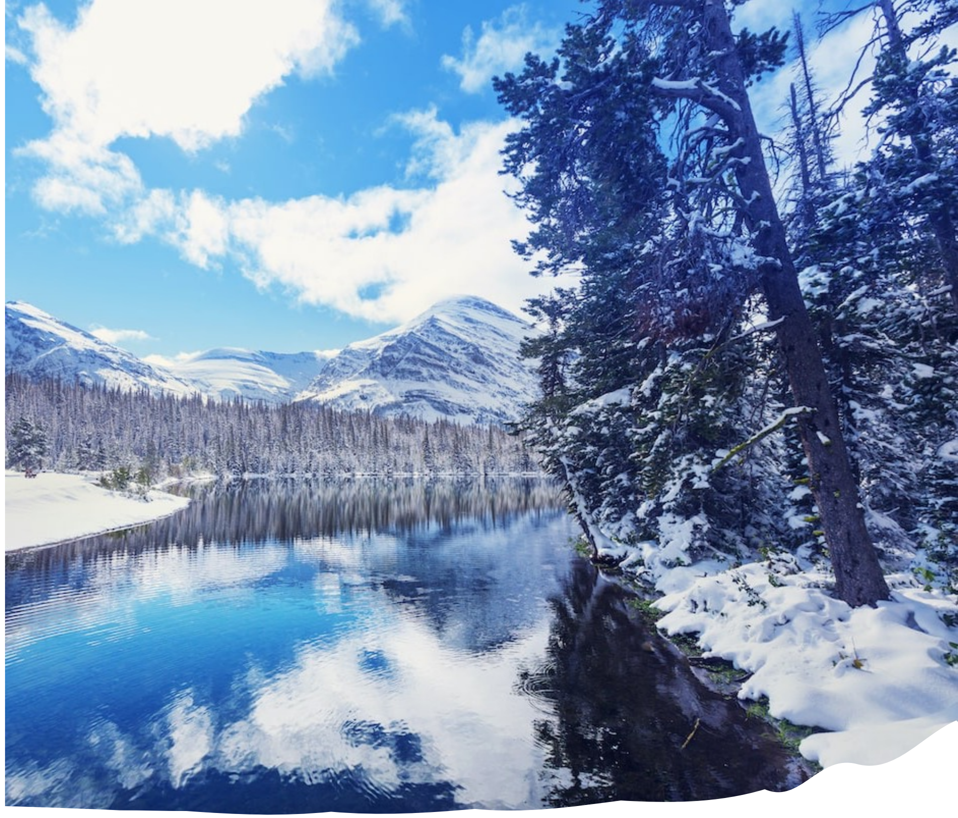

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Montana Winter

Montana Summer

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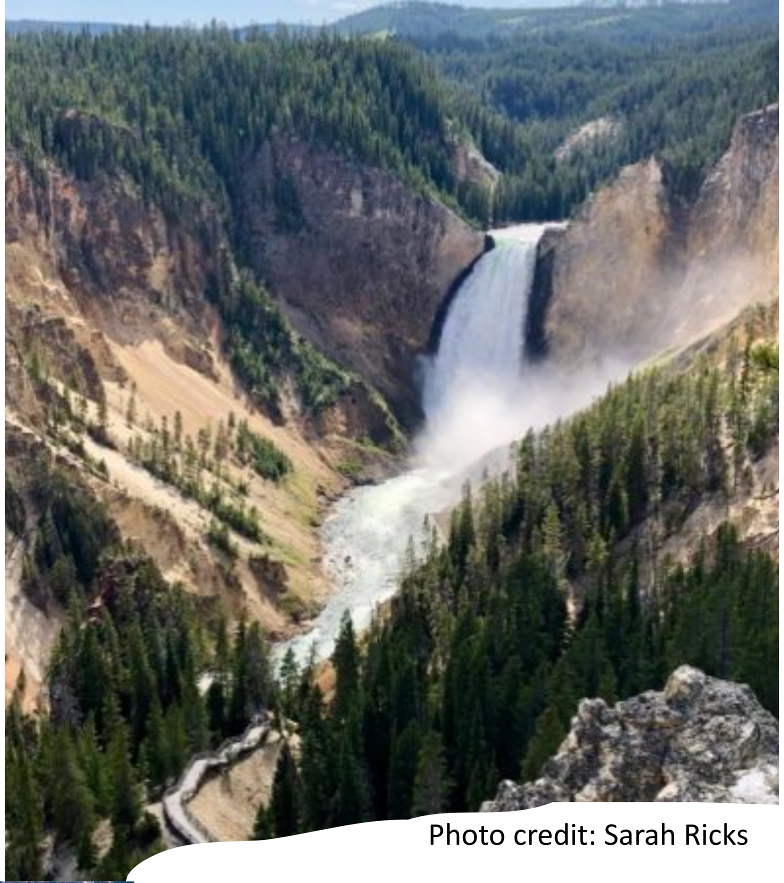

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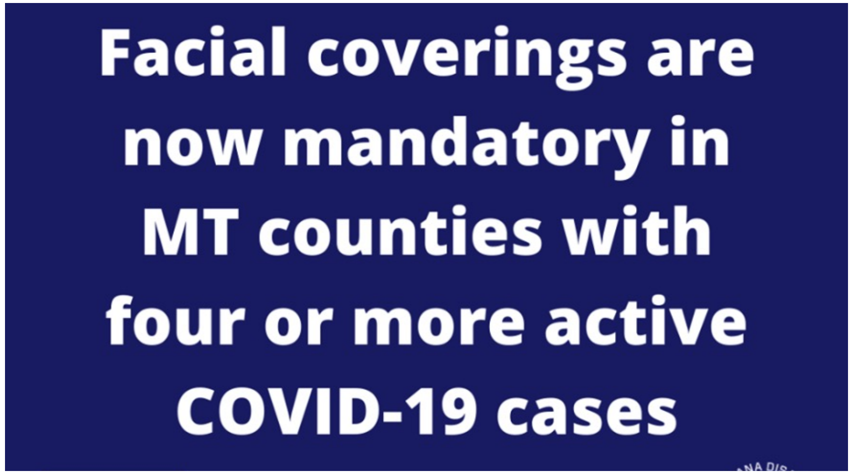

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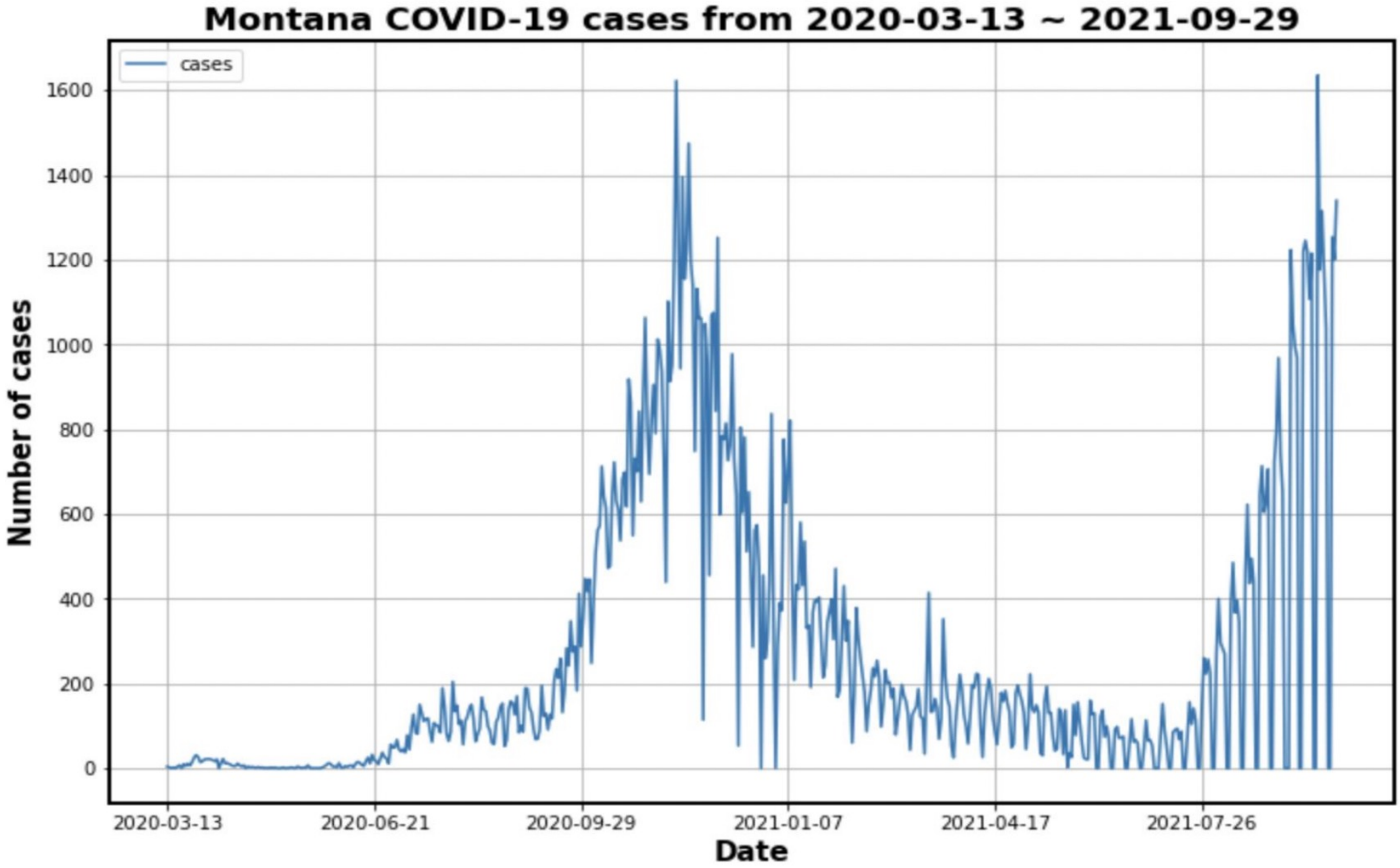
Anger over mask mandates, other covid rules, spurs states to curb power of public health officials

Republican lawmakers pass laws to restrict the power of health authorities to require masks, promote vaccinations and take other steps to protect the public health

By Amy Goldstein

December 25, 2021 at 8:00 a.m. EST

Photo credit: Nora Mabie


Montana COVID-19 cases from 2020-03-13 ~ 2021-09-29
cases
Number of cases
0
200
400
600
800
1000
1200
1400
1600
2020-03-13
2020-06-21
2020-09-29
2021-01-07
2021-04-17
2021-07-26
Date

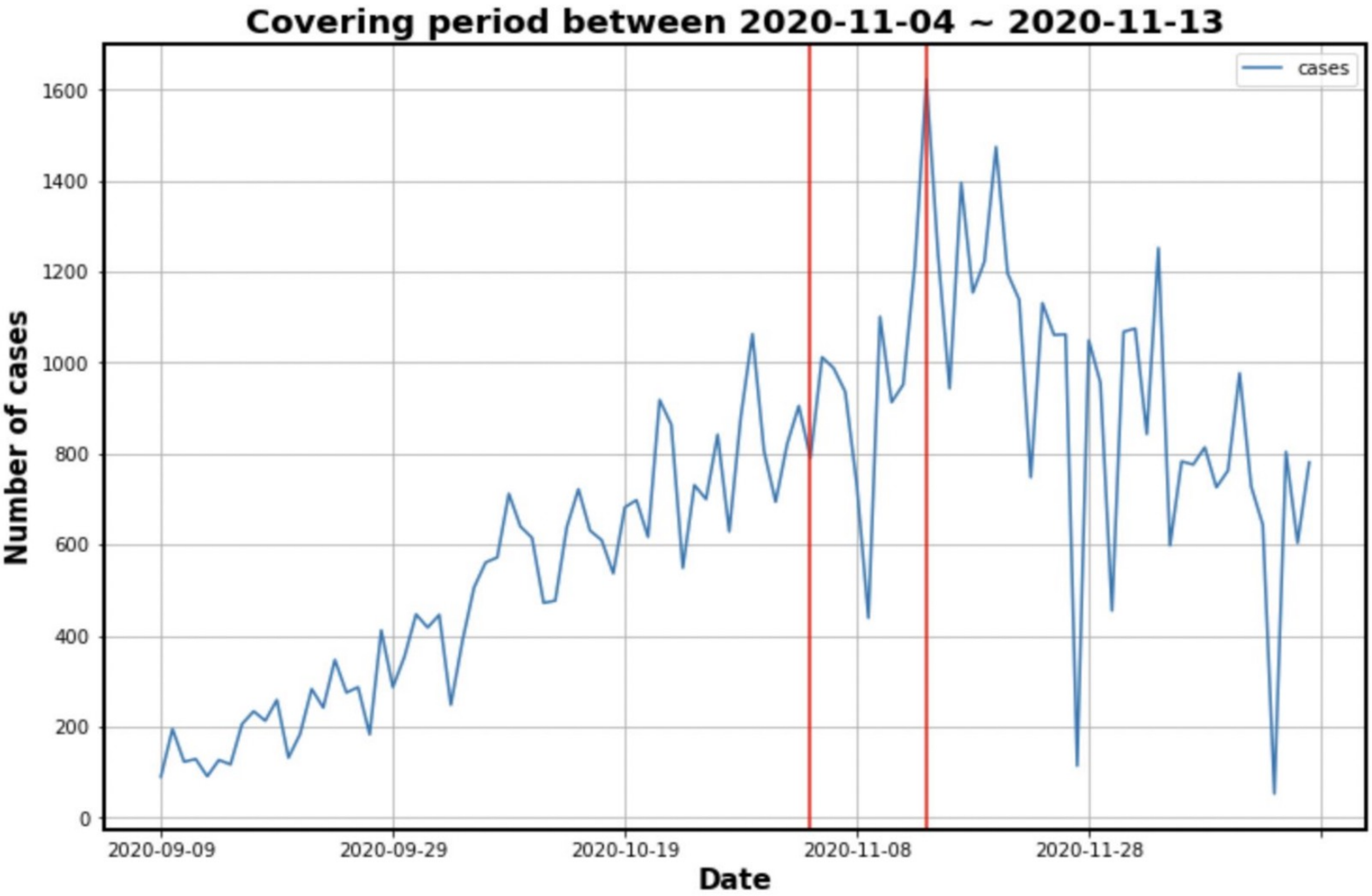
Using New Case Data to approximate infection rate, β

$$\beta I_n = \frac{D_{n+1}}{N};$$

where D_{n+1} … number of cases at *n+1* day

N … total population

$$I_n = \sum_{i=0}^{n} (1-\gamma)^{n-i} D_i \;;\; \gamma = \frac{1}{14}$$


Covering period between 2020-11-04 ~ 2020-11-13
cases
Number of cases
0
200
400
600
800
1000
1200
1400
1600
2020-09-09
2020-09-29
2020-10-19
2020-11-08
2020-11-28
Date

2020-11-04 ~ 2020-11-12

Covid Cases

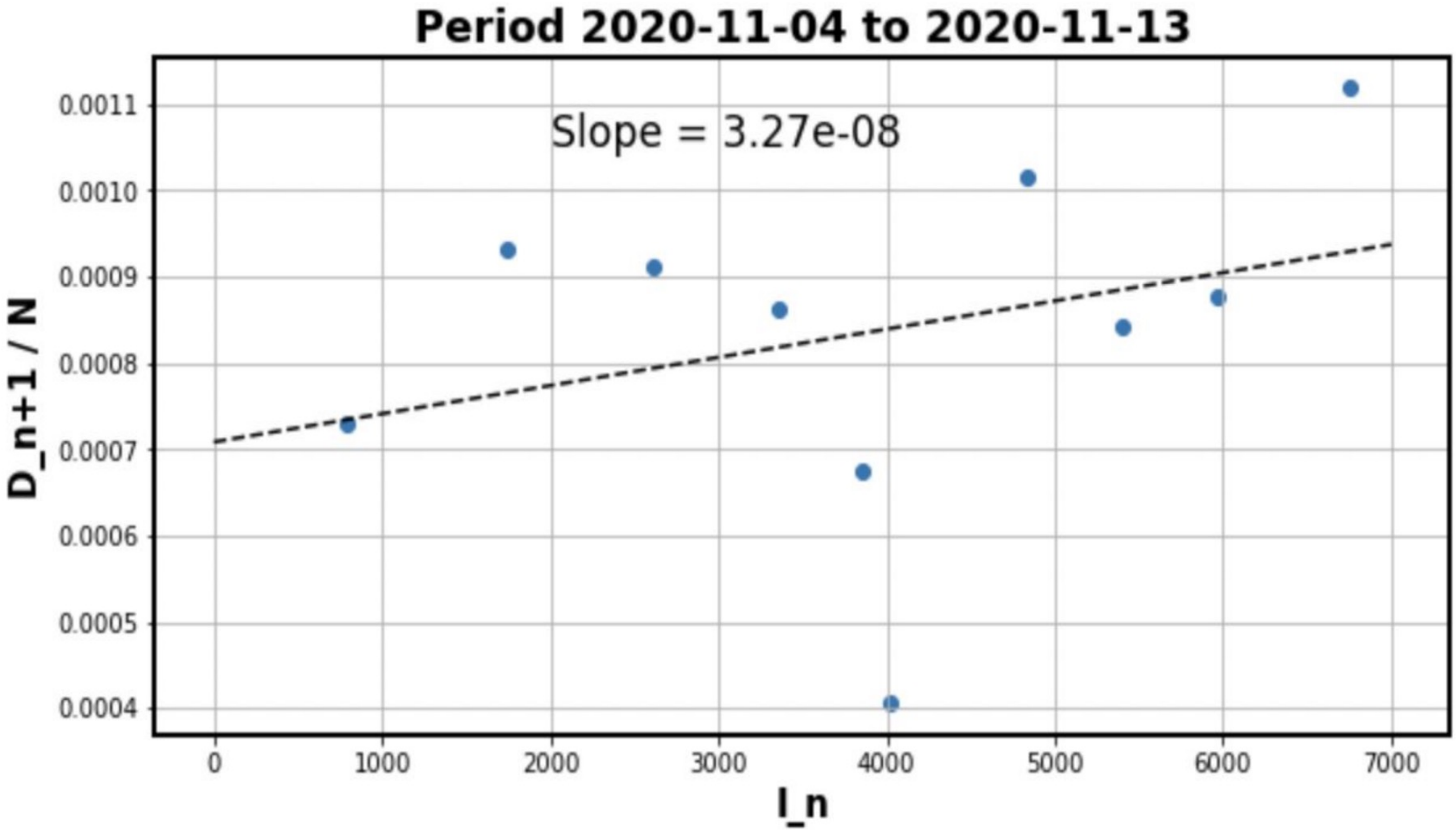
date	
2020-11-04	790
2020-11-05	1012
2020-11-06	988
2020-11-07	936
2020-11-08	731
2020-11-09	439
2020-11-10	1101
2020-11-11	913
2020-11-12	952
2020-11-13	1213

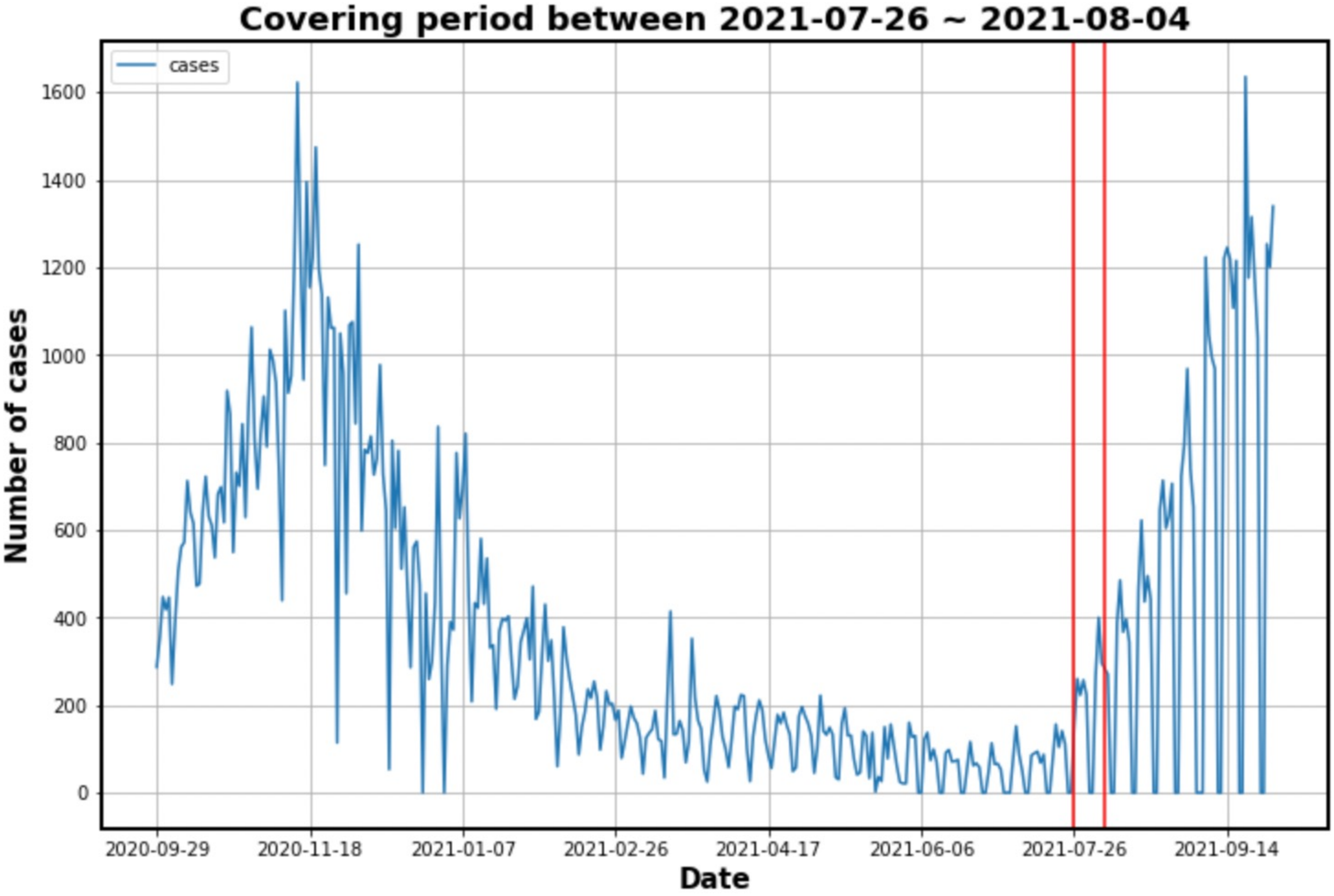
I_n Values

790.0
1745.5714285714284
2608.887755102041
3358.5386297376094
3849.6430133277804
4013.6685123757957
4827.977904348953
5396.122339752599
5962.685029770272
6749.778956215252

D_n/N Values (N=1,084,225)

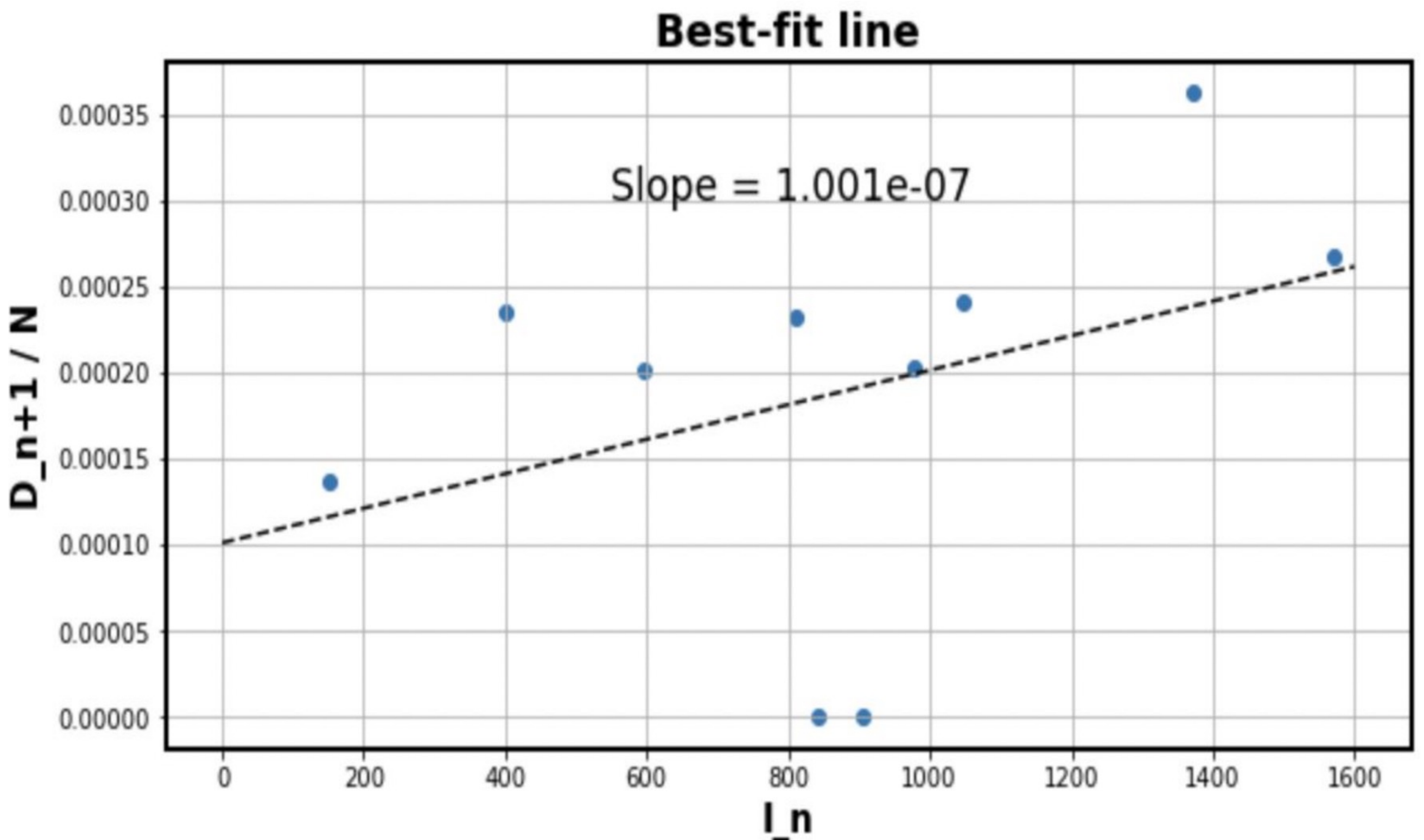
0.0007286310498282184
0.0009333855980077936
0.0009112499711775692
0.0008632894463787498
0.0006742143005372502
0.00040489750743618713
0.0010154718808365422
0.0008420761373331182
0.0008780465309322327
0.0011187714727109224


Period 2020-11-04 to 2020-11-13
Slope = 3.27e-08
D_n+1 / N
I_n
0.0011
0.0010
0.0009
0.0008
0.0007
0.0006
0.0005
0.0004
0
1000
2000
3000
4000
5000
6000
7000


Covering period between 2021-07-26 ~ 2021-08-04
cases
Number of cases
0
200
400
600
800
1000
1200
1400
1600
2020-09-29
2020-11-18
2021-01-07
2021-02-26
2021-04-17
2021-06-06
2021-07-26
2021-09-14
Date

2021-7-26 ~ 2021-8-4

Covid Cases		I_n Values	D_n/N Values (N=1,104,271)
date			
2021-07-26	152	152	0.000137647370980493
2021-07-27	260	401.1428571428571	0.00023544945036136963
2021-07-28	223	595.4897959183673	0.00020194318242532856
2021-07-29	257	809.9548104956268	0.00023273272593412304
2021-07-30	224	976.1008954602248	0.00020284875723441075
2021-07-31	0	906.3794029273517	0.0
2021-08-01	0	841.6380170039695	0.0
2021-08-02	266	1047.5210157894003	0.00024088289921586278
2021-08-03	400	1372.6980860901574	0.00036222992363287634
2021-08-04	296	1570.6482227980034	0.0002680501434883285



Best-fit line
Slope = 1.001e-07
D_n+1 / N
I_n
0.00035
0.00030
0.00025
0.00020
0.00015
0.00010
0.00005
0.00000
0
200
400
600
800
1000
1200
1400
1600

Discrete Time SIR-model for infectious disease

Susceptible: $S_{n+1} = S_n - \beta I_n S_n$

Infected: $I_{n+1} = (1-\gamma)I_n + \beta I_n S_n$

Recovered: $R_{n+1} = R_n + \gamma I_n$

$$\vec{v} = A\vec{x}$$

$$\begin{bmatrix} S_{n+1} \\ I_{n+1} \\ R_{n+1} \end{bmatrix} = \begin{bmatrix} (1-\beta I_0) & -\beta S_0 & 0 \\ \beta I_0 & (1-\gamma+\beta S_0) & 0 \\ 0 & \gamma & 1 \end{bmatrix} \begin{bmatrix} S_n \\ I_n \\ R_n \end{bmatrix}$$

Matrices

- $\vec{A}_1 = \begin{bmatrix} 1 & -0.036 & 0 \\ 0 & 0.965 & 0 \\ 0 & 0.071 & 1 \end{bmatrix}$

- $\vec{A}_2 = \begin{bmatrix} 1 & -0.111 & 0 \\ 0 & 1.040 & 0 \\ 0 & 0.071 & 1 \end{bmatrix}$

Eigenvalues and Eigenvectors

$|A - \lambda I| = 0$ $\Rightarrow$ *characteristic polynomial*

First valley

$$\left|\vec{A}_1 - \lambda I\right| = \begin{bmatrix} (1-\lambda) & -0.036 & 0 \\ 0 & (0.965-\lambda) & 0 \\ 0 & 0.071 & (1-\lambda) \end{bmatrix} = 0$$

$$\lambda_{1,2} = 1\,; \qquad algebraic\ mult. = 2$$
$$\lambda_3 = 0.965\,; \qquad algebraic\ mult. = 1$$

for $\lambda_{1,2} = 1,$

$$\begin{bmatrix} (1-(1)) & -0.036 & 0 \\ 0 & (0.965-(1)) & 0 \\ 0 & 0.071 & (1-(1)) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad , \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \longrightarrow \textit{eigenvectors}$$

for $\lambda_3 = 0.965,$

$$\begin{bmatrix} (1-(0.965)) & -0.036 & 0 \\ 0 & (0.965-(0.965)) & 0 \\ 0 & 0.071 & (1-(0.965)) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -0.507 \\ -0.493 \\ 1 \end{bmatrix} \longrightarrow \textit{eigenvector}$$

Second valley

$$\left|\vec{A}_2 - \lambda I\right| = \begin{bmatrix} (1-\lambda) & -0.111 & 0 \\ 0 & (1.04-\lambda) & 0 \\ 0 & 0.071 & (1-\lambda) \end{bmatrix} = 0$$

eigenvalues: $\lambda_{1,2} = 1\,;$ $\quad algebraic\ mult. = 2$

$\lambda_3 = 1.04\,;$ $\quad algebraic\ mult. = 1$

for $\lambda_{1,2} = 1,$

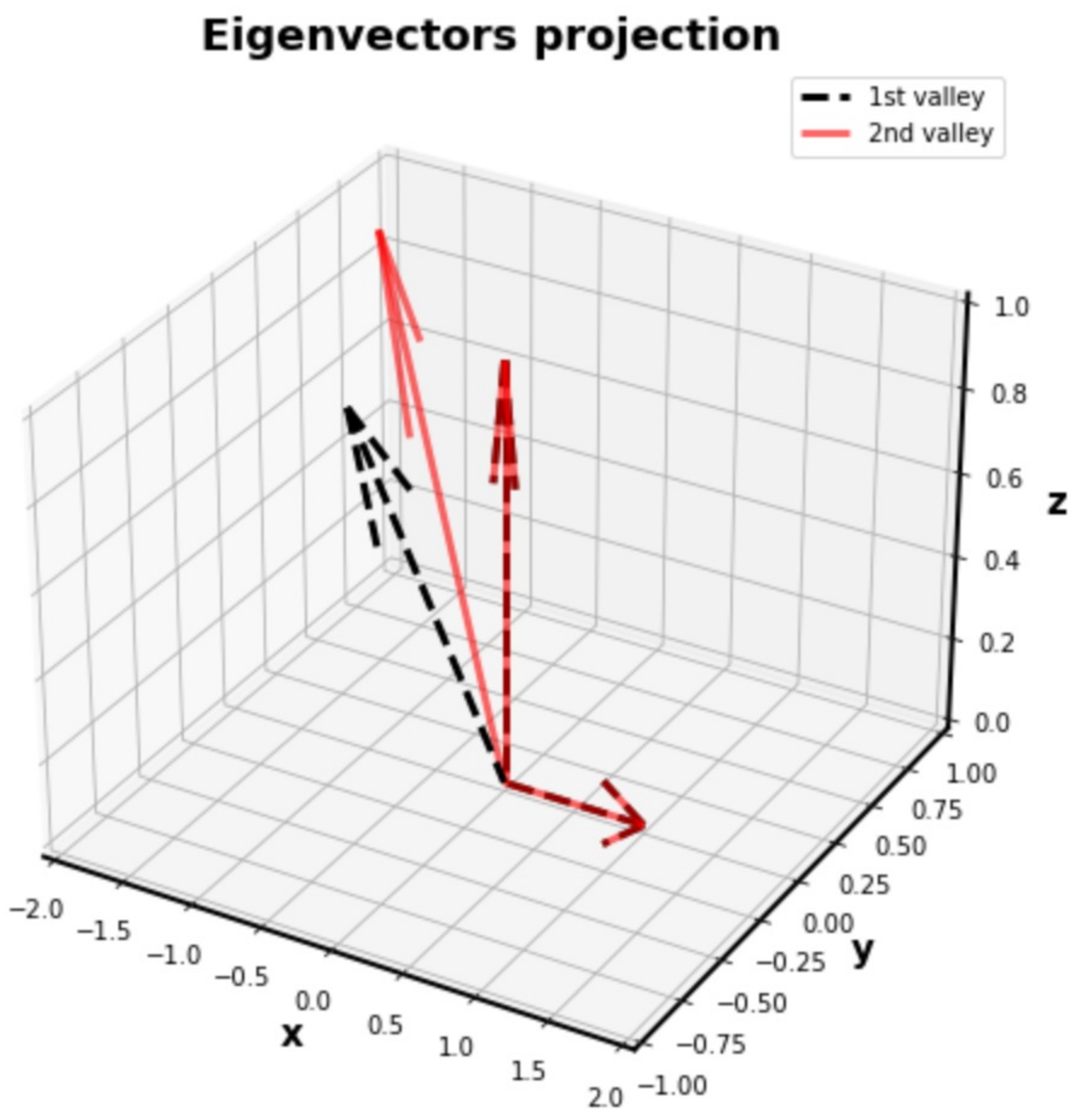
$$\begin{bmatrix} (1-(1)) & -0.111 & 0 \\ 0 & (1.04-(1)) & 0 \\ 0 & 0.071 & (1-(1)) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Longrightarrow \textit{eigenvectors}$$

for $\lambda_3 = 1.04$,

$$\begin{bmatrix} (1-(1.04)) & -0.111 & 0 \\ 0 & (1.04-(1.04)) & 0 \\ 0 & 0.071 & (1-(1.04)) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1.563 \\ 0.563 \\ 1 \end{bmatrix} \Rightarrow \textit{eigenvector}$$


Eigenvectors projection
1st valley
2nd valley
x
y
z
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0

Predicting the numbers of S_n, I_n, and R_n using A Matrix

$$\vec{x}_{n+1} = A\vec{x}_n$$

$$\begin{bmatrix} S_{n+1} \\ I_{n+1} \\ R_{n+1} \end{bmatrix} = A \begin{bmatrix} S_n \\ I_n \\ R_n \end{bmatrix}$$

Inverse Matrix

- $A_1{}^{-1} = \begin{bmatrix} 1 & 0.037 & 0 \\ 0 & 1.036 & 0 \\ 0 & -0.074 & 1 \end{bmatrix}$

- $A_2{}^{-1} = \begin{bmatrix} 1 & 0.107 & 0 \\ 0 & 0.962 & 0 \\ 0 & -0.068 & 1 \end{bmatrix}$

Predicting the numbers of S_n, I_n, and R_n using Inverse Matrix

$$\vec{x}_n = A^{-1}\vec{x}_{n+1}$$

$$\begin{bmatrix} S_n \\ I_n \\ R_n \end{bmatrix} = A^{-1} \begin{bmatrix} S_{n+1} \\ I_{n+1} \\ R_{n+1} \end{bmatrix}$$

Invertible Matrix, Q

- $Q_1 = \begin{bmatrix} 1 & 0.455 & 0 \\ 0 & 0.442 & 0 \\ 0 & -0.897 & -1.115 \end{bmatrix}$
- ${Q_1}^{-1} = \begin{bmatrix} 1 & -1.029 & 0 \\ 0 & 2.262 & 0 \\ 0 & -1.820 & -0.897 \end{bmatrix}$
- $Q_2 = \begin{bmatrix} 1 & 1 & 0.070 \\ 0 & -0.364 & 0 \\ 0 & -0.645 & 1.148 \end{bmatrix}$
- ${Q_2}^{-1} = \begin{bmatrix} 1 & 2.859 & -0.061 \\ 0 & -2.75 & 0 \\ 0 & -1.546 & 0.871 \end{bmatrix}$

Diagonal Matrix, *D*

- $D_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.965 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- $D_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.04 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Checks: $A = QDQ^{-1}$

<u>First valley</u>

$$\begin{bmatrix} 1 & -0.036 & 0 \\ 0 & 0.965 & 0 \\ 0 & 0.071 & 1 \end{bmatrix} \overset{\checkmark\checkmark}{=} \begin{bmatrix} 1 & 0.455 & 0 \\ 0 & 0.442 & 0 \\ 0 & -0.897 & -1.115 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.965 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1.029 & 0 \\ 0 & 2.262 & 0 \\ 0 & -1.820 & -0.897 \end{bmatrix}$$

<u>Second valley</u>

$$\begin{bmatrix} 1 & -0.111 & 0 \\ 0 & 1.040 & 0 \\ 0 & 0.071 & 1 \end{bmatrix} \overset{\checkmark\checkmark}{=} \begin{bmatrix} 1 & 1 & 0.070 \\ 0 & -0.364 & 0 \\ 0 & -0.645 & 1.148 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.04 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2.859 & -0.061 \\ 0 & -2.75 & 0 \\ 0 & -1.546 & 0.871 \end{bmatrix}$$

What is the fraction of the population of Montana needs to be vaccinated?

Revisiting our matrices

- $\vec{A}_1 = \begin{bmatrix} 1 & -0.036 & 0 \\ 0 & 0.965 & 0 \\ 0 & 0.071 & 1 \end{bmatrix}$ $\quad N = 1{,}084{,}225$

- $\vec{A}_2 = \begin{bmatrix} 1 & -0.111 & 0 \\ 0 & 1.040 & 0 \\ 0 & 0.071 & 1 \end{bmatrix}$ $\quad N = 1{,}104{,}271$

Initial population (2021) = 1,104,271
Initial λ = 1.04

5% decrease = 1049057	λ = 1.0335820793164285
10% decrease = 993843	λ = 1.0280552029614287
15% decrease = 938630	λ = 1.0225283266064287
20% decrease = 883416	λ = 1.0170014502514286
25% decrease = 828203	λ = 1.0114745738964286
30% decrease = 772989	λ = 1.0059476975414285
35% decrease = 717776	λ = 1.0004208211864285
40% decrease = 662562	λ = 0.9948939448314286
45% decrease = 607349	λ = 0.9893670684764286
50% decrease = 552135	λ = 0.9838401921214286
55% decrease = 496921	λ = 0.9783133157664285
60% decrease = 441708	λ = 0.9727864394114286
65% decrease = 386494	λ = 0.9672595630564286
70% decrease = 331281	λ = 0.9617326867014286
75% decrease = 276067	λ = 0.9562058103464286
80% decrease = 220854	λ = 0.9506789339914286
85% decrease = 165640	λ = 0.9451520576364286
90% decrease = 110427	λ = 0.9396251812814286
95% decrease = 55213	λ = 0.9340983049264285
100% decrease = 0	λ = 0.9285714285714286

40% of the total population of Montana needs to be vaccinated