《Computer Aided Geometric Design》
Assignment 6

Dec 4, 2025

1. Given the following cubic polynomial curve:

$$P(u) = -\begin{pmatrix}7/8\\5/8\end{pmatrix}u^3 + \begin{pmatrix}9\\15/4\end{pmatrix}u^2 - \begin{pmatrix}57/2\\9/2\end{pmatrix}u + \begin{pmatrix}30\\-1\end{pmatrix}$$

   1) Calculate its polar form and the vertices of its Bézier control polygon $P_0P_1P_2P_3$ within the interval$[2,4]$, and roughly sketch this control polygon;
   2) Use the de Casteljau algorithm to calculate the polynomial curve at sample $u = \{5/2\,, 3, 7/2\}$, and draw it in the figure in 1);
   3) Using the results from 2) to subdivide the curve at $u = 3$, then subdivide the right portion at its midpoint $u = 7/2$. Draw the control polygon in the figure in 1), and draw the curve $P(u)$.

2. Given the following cubic polynomial curve and parameter interval $[0,1]$

$$F(u) = \begin{pmatrix}15\\-6\end{pmatrix}u^3 + \begin{pmatrix}27\\10\end{pmatrix}u^2 - \begin{pmatrix}9\\9\end{pmatrix}u$$

   1) Calculate its first and second derivatives;
   2) Calculate its polar form $f(u_1, u_2, u_3)$ and the polar forms of the derivatives F' and F'', prove that they equal to $3f(u_1, u_2, \hat{1})$ and $6f(u_1, \hat{1}, \hat{1})$ respectively.
   Note that $f(u_1, u_2, \hat{1}) = f(u_1, u_2, 1) - f(u_1, u_2, 0)$.

3. Given a uniform B-spline defined by the following four points and knot vector $[0,0,1,2,3,4,5,5]$:

$$P_0 = \begin{pmatrix}-2\\-10\end{pmatrix}, \qquad P_1 = \begin{pmatrix}-4\\2\end{pmatrix}, \qquad P_2 = \begin{pmatrix}6\\5\end{pmatrix}, \qquad P_3 = \begin{pmatrix}4\\-7\end{pmatrix}$$

   1) Use the de Boor algorithm to calculate the curve position at $t = 2.5$. Sketch the control polygon and the relevant points constructed by this algorithm.
   2) For the B-spline in 1), calculate the corresponding Bézier control points that represent the same curve. Draw the control vertices and Bézier curve in the figure in 1).