1D Linear Advection - Sine Wave

Problem Specification

Use Case. Linear Advection
Spatial domain: $0 \leq x < 1$ meters, periodic boundary conditions
Governing equations: 1D Linear Advection Equation

$$\frac{\partial \phi}{\partial t} = -\mathbf{u} \cdot \nabla \phi$$

Initial conditions:

$$\phi(x) = \sin(2\pi x)$$

$$\mathbf{u} = 1 \text{ m/s}$$

The Saiph's code specification can be checked at:
[From local repository] **Click to Saiph code**
[From remote repository] **Click to Saiph code**

Simulation details

$\Delta x = 1$ mm
$\Delta t = 1$ ms
$nsteps = 1000$
Forward in-time integration using Euler method: $\mathcal{O}(t)$
Spatial differentiation accuracy (default): $\mathcal{O}(x^4)$

Results

Output results at three time-steps, $t = 0s$, $t = 0.5s$ and $t = 1s$ are presented in Figure 1. The L_2 norm has been computed over the ϕ variable taking the initial conditions as the analytic solution. The output simulation presents no truncation error $\mathbf{L_2 = 0}$

The Saiph's simulation animation can be checked at:
[From local repository] **Click to video**
[From remote repository] **Click to video**

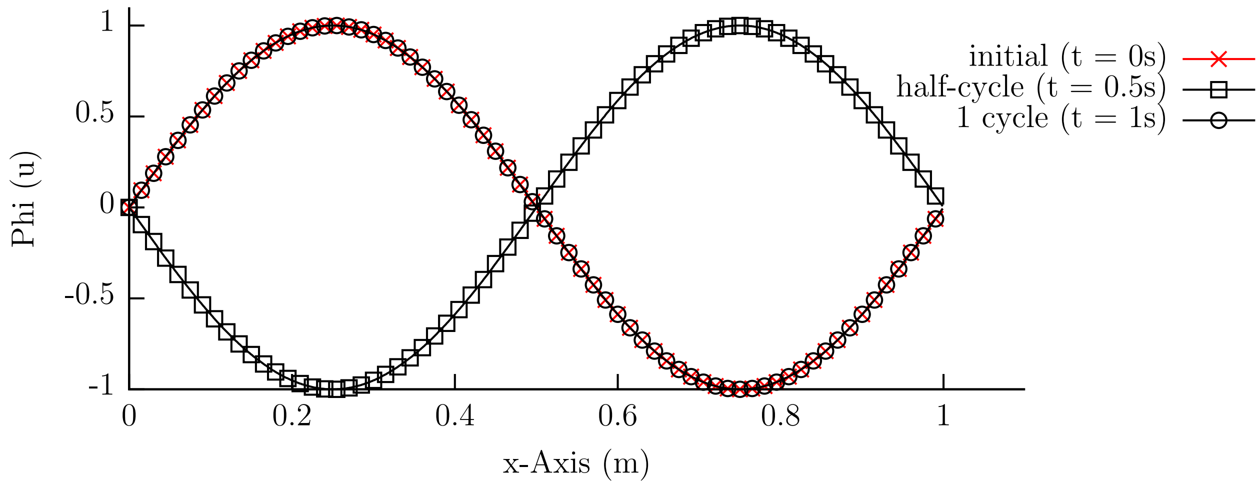

Figure 1: Phi profile at different time-steps.

1D Linear Advection - Discontinuous Waves

Problem Specification

Use Case. Linear Advection
Spatial domain: $-1 \leq x < 1$ meters, periodic boundary conditions
Governing equations: 1D Linear Advection Equation

$$\frac{\partial \phi}{\partial t} = -\mathbf{u} \cdot \nabla \phi$$

Initial conditions:

$$\phi(x) = \begin{cases} \exp\left(-\log(2)\frac{(x+7)^2}{0.0009}\right) & -0.8 \leq x \leq -0.6 \\ 1 & -0.4 \leq x \leq -0.2 \\ 1 - |10(x-0.1)| & 0 \leq x \leq 0.2 \\ \sqrt{1-100(x-0.5)^2} & 0.4 \leq x \leq 0.6 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{u} = 1 \text{ m/s}$$

The Saiph's code specification can be checked at:
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Simulation details

$\Delta x = 1$ mm
$\Delta t = 1$ ms
$nsteps = 2000$
Forward in-time integration using Euler method: $\mathcal{O}(t)$
Spatial differentiation accuracy (default): $\mathcal{O}(x^4)$

Results

Output results at two time-steps, $t = 0s$ and $t = 2s$ are presented in Figure 2.
The L_2 norm has been computed over the ϕ variable taking the initial conditions as the analytic solution. The output simulation presents no truncation error $\mathbf{L_2 = 0}$

The Saiph's simulation animation can be checked at:
[From local repository] **Click to video**
[From remote repository] **Click to video**

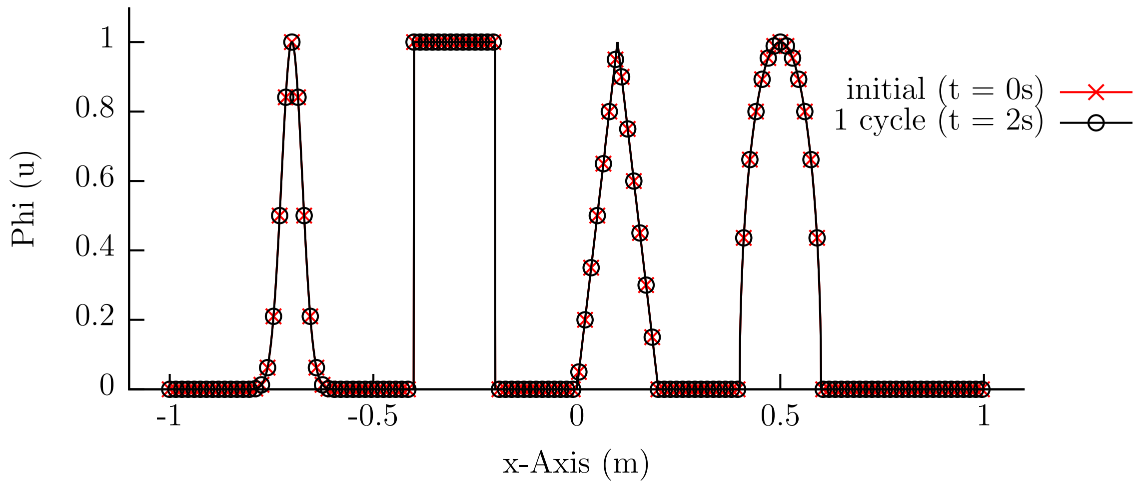

Figure 2: Phi profile at different time-steps.

1D Linear Diffusion - Sine Wave

Problem Specification

Use Case. Linear Diffusion
Spatial domain: $0 \leq x < 1$ meters, periodic boundary conditions
Governing equations: 1D Linear Diffusion Equation

$$\frac{\partial \phi}{\partial t} = \boldsymbol{\nabla} \cdot (\nu \nabla \phi)$$

Initial conditions:

$$\phi(x) = \sin(2\pi x)$$

$$\nu = 0.001 \ \mathrm{m}^2/\mathrm{s}$$

The Saiph's code specification can be checked at:
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[From remote repository] **Click to Saiph code**

Simulation details

$\Delta x = 12.5$ mm
$\Delta t = 50$ ms
$nsteps = 200$
Forward in-time integration using 3^{rd} order Runke-Kutta method: $\mathcal{O}(t^3)$
Spatial differentiation accuracy (default): $\mathcal{O}(x^4)$

Results

Output results at three time-steps, $t = 0s$, $t = 5s$ and $t = 10s$ are presented in Figure 3.
The L_2 norm has been computed over the ϕ variable taking the analytic solution as reference:

$$\phi(x,t) = e^{-\nu 4\pi^2 t} \sin(2\pi x)$$

The final output simulation ($t = 10s$), presents an error of $\mathbf{L_2 = 1.4 \cdot 10^{-7}}$

The Saiph's simulation animation can be checked at:
[From local repository] **Click to video**
[From remote repository] **Click to video**

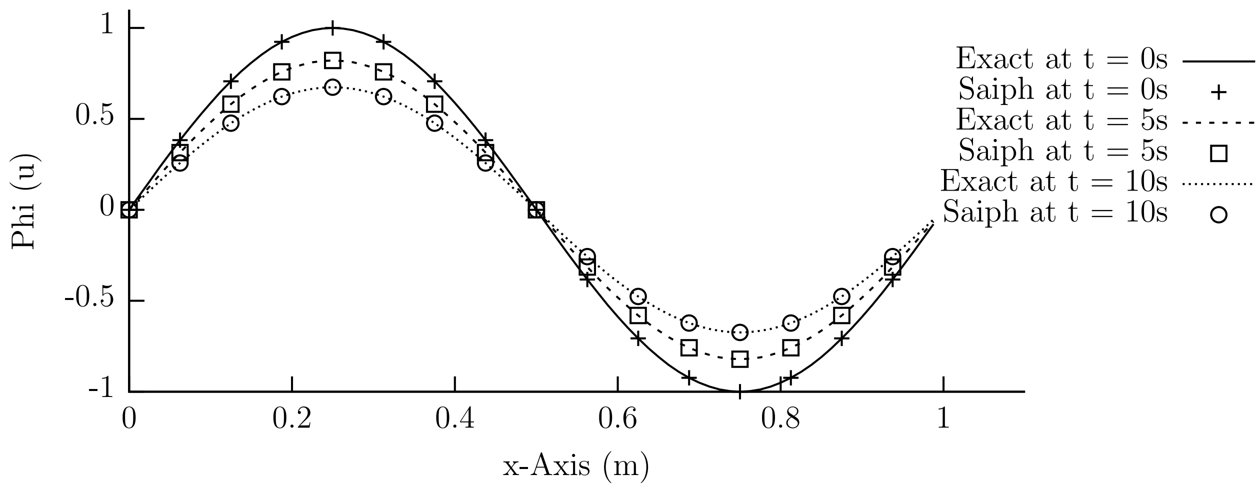

Figure 3: Phi profile at different time-steps.