1D Linear Advection - Sine Wave

Problem Specification

Use Case. Linear Advection
Spatial domain: $0 \leq x < 1$ meters, periodic boundary conditions
Governing equations: 1D Linear Advection Equation

$$\frac{\partial \phi}{\partial t} = -\mathbf{u} \cdot \nabla \phi$$

Initial conditions:

$$\phi(x) = \sin(2\pi x)$$

$$\mathbf{u} = 1 \text{ m/s}$$

The Saiph's code specification can be checked at:
  [From local repository]   **Click to Saiph code**
  [From remote repository]   **Click to Saiph code**

Simulation details

$\Delta x = 1$ mm
$\Delta t = 1$ ms
$nsteps = 1000$
Forward in-time integration using Euler method: $\mathcal{O}(x)$
Spatial differentiation accuracy (default): $\mathcal{O}(x^4)$

Results

Output results at three time-steps, $t = 0s$, $t = 0.5s$ and $t = 1s$ are presented in Figure 1.
The L_2 norm has been computed over the ϕ variable taking the initial conditions as the
analytic solution. The output simulation presents no truncation error $L_2 = 0$

The Saiph's simulation animation can be checked at:
  [From local repository]   **Click to video**
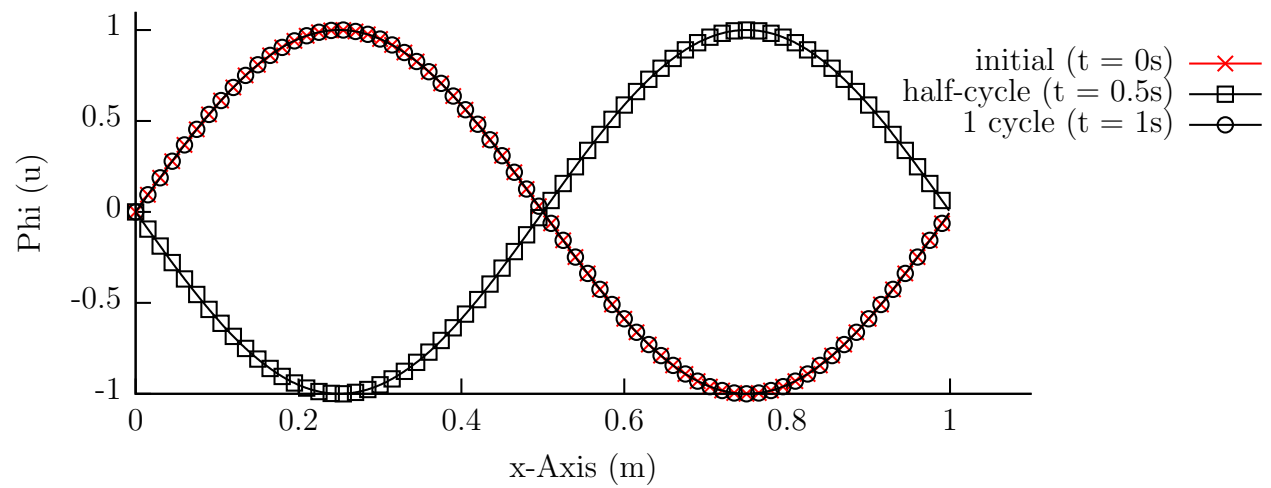 [From remote repository]   **Click to video**

Figure 1: Phi profile at different time-steps.

1D Linear Advection - Discontinuous Waves

Problem Specification

Use Case. Linear Advection
Spatial domain: $-1 \leq x < 1$ meters, periodic boundary conditions
Governing equations: 1D Linear Advection Equation

$$\frac{\partial \phi}{\partial t} = -\mathbf{u} \cdot \nabla \phi$$

Initial conditions:

$$\phi(x) = \begin{cases} \exp\left(-\log(2)\frac{(x+7)^2}{0.0009}\right) & -0.8 \leq x \leq -0.6 \\ 1 & -0.4 \leq x \leq -0.2 \\ 1 - |10(x-0.1)| & 0 \leq x \leq 0.2 \\ \sqrt{1 - 100(x-0.5)^2} & 0.4 \leq x \leq 0.6 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{u} = 1 \text{ m/s}$$

The Saiph's code specification can be checked at:
 [From local repository]   **Click to Saiph code**
 [From remote repository]   **Click to Saiph code**

Simulation details

$\Delta x = 1$ mm
$\Delta t = 1$ ms
$nsteps = 2000$
Forward in-time integration using Euler method: $\mathcal{O}(x)$
Spatial differentiation accuracy (default): $\mathcal{O}(x^4)$

Results

Output results at two time-steps, $t = 0s$ and $t = 2s$ are presented in Figure 2.
The L_2 norm has been computed over the ϕ variable taking the initial conditions as the analytic solution. The output simulation presents no truncation error $L_2 = 0$

The Saiph's simulation animation can be checked at:
 [From local repository]   **Click to video**
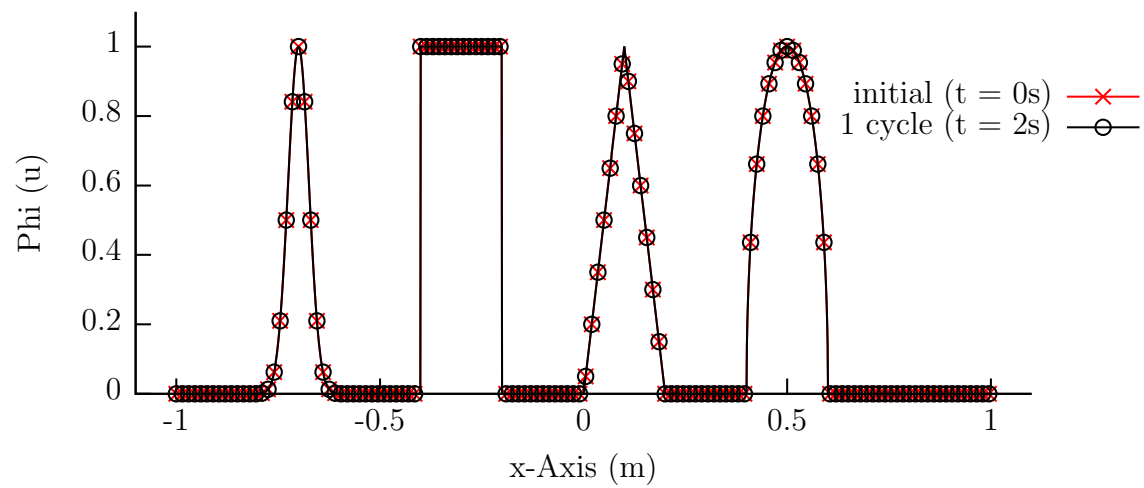 [From remote repository]   **Click to video**

Figure 2: Phi profile at different time-steps.